Indirect integration in RW gauge and steps towards orbital evolution

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Plan of the talk

- Perturbations in RW gauge, Ψ properties and jump conditions
- Indirect method
- Perspectives

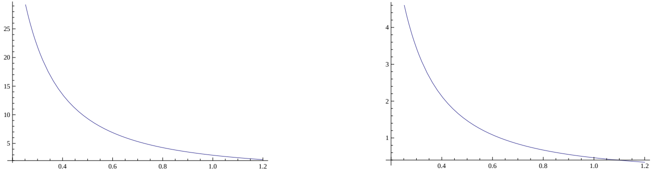
LISA-NGO and low frequencies

- The shift of LISA-NGO sensitivity towards higher frequency (shorter arm length) must concern the Capra community, the opportunity of EMRI detection being reduced.

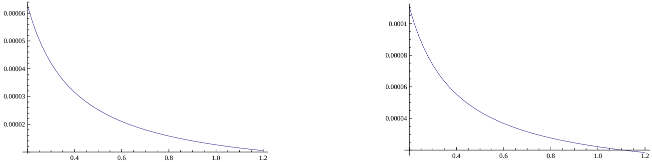
Are there new arguments, beyond EMRIs, to favour low-frequency sensitivity? Sequential detection of SMBHBs by pulsar timing array PTA (coalescence) and by LISA-NGO (merge and ringing).

- Individual detection of SMBHBs is at the limit of PTA actual state of the art (50 ns residuals).
- Sequential detection is also hindered by shift to higher frequencies.
- SMBHB of $n \cdot 10^{-8} M_{\odot}$ pass in few years from PTA to LISA-NGO band.
- Undergoing simulations (A. Sesana MPI) evaluate the probability of sequential detection.

LISA-NGO and low frequencies



Transfer time [y] versus SMBHB mass [$10^9 M_\odot$] for PTA-SKA band hight limit of respectively $4 \cdot 10^{-8}$ Hz and $8 \cdot 10^{-8}$ Hz.



Quasi-normal frequency [Hz] versus SMBHB mass [$10^9 M_\odot$], for Kerr spin parameter of respectively $a = 0.1$ and $a = 0.9$.

Perturbations in RW gauge

RW gauge $\Psi \in C^{-1}$, $h_{\mu\nu} \in C^0$ for radial infall, $h_{\mu\nu} \notin C^0$ for generic orbits.

- h even perturbations (radial case, only even) $\in C^0$ continuity class at z_u (1: integration over r of the Hamiltonian constraint, tt component of the Einstein equations; 2: structure of selected even perturbation equations (Lousto 2000, - and Nakano 2009).
- LN derive the jump conditions on Ψ and derivatives from RWZ equation.

For radial, there is an alternative: obtain jump conditions from formal solutions (inverse relations) of RWZ equation.

Determination of the jump conditions on *Psi* and derivatives, to get h even perturbations $\in C^0$.

Ψ properties

From the visual inspection of the Zerilli wave equation, $\Psi \in C^{-1}$ continuity class $\Longrightarrow$:

$$\frac{d^2\Psi_l(t,r)}{dr^{*2}} - \frac{d^2\Psi_l(t,r)}{dt^2} - V_l(r)\Psi_l(t,r) = S_l(t,r)$$

$$\Psi(t,r) = \Psi^+(t,r)\ \Theta_1 + \Psi^-(t,r)\ \Theta_2$$

$$\Psi_{,r} = \Psi^+_{,r}\Theta_1 + \Psi^-_{,r}\Theta_2 + \left(\Psi^+ - \Psi^-\right)\delta$$

$$\Psi_{,rr} = \Psi^+_{,rr}\Theta_1 + \Psi^-_{,rr}\Theta_2 + \left(\Psi^+_{,r} - \Psi^-_{,r}\right)\delta + \left(\Psi^+ - \Psi^-\right)_{z_u}\delta'$$

$$\Psi_{,t} = \Psi^+_{,t}\Theta_1 + \Psi^-_{,t}\Theta_2 - \left(\Psi^+ - \Psi^-\right)\dot{z}_u\delta$$

$$\Psi_{,tr} = \Psi^+_{,tr}\Theta_1 + \Psi^-_{,tr}\Theta_2 + \left(\Psi^+_{,t} - \Psi^-_{,t}\right)\delta - \left(\Psi^+ - \Psi^-\right)_{z_u}\dot{z}_u\delta'$$

$\Theta_1 = \Theta\left[r - z_u(t)\right]$, $\Theta_2 = \Theta\left[z_u(t) - r\right]$ Heaviside step distributions

Property $f(r)\delta'[r - z_u(t)] = f_{z_u(t)}\delta'[r - z_u(t)] - f'_{z_u(t)}\delta[r - z_u(t)]$

Inverse relations

$$K = f_1(r)\Psi + f_2(r)\Psi_{,r} + f_3(r)\delta$$
$$H_2 = f_4(r)\Psi + f_5(r)\Psi_{,r} + f_6(r)\Psi_{,rr} + f_7(r)\delta + f_8(r)\delta'$$
$$H_1 = f_9(r)\Psi_{,t} + f_{10}(r)\Psi_{,tr} + f_{11}(r)\delta + f_{12}(r)\delta'$$

$$f_1 = \frac{6M^2 + 3M\lambda r + \lambda(\lambda+1)r^2}{r^2(\lambda r + 3M)} \qquad f_2 = \left(1 - \frac{2M}{r}\right) \qquad f_3 = -\frac{\kappa\ u^0 (r-2M)^2}{(\lambda+1)(\lambda r + 3M) r}$$

$$f_4 = -\frac{9M^3 + 9\lambda M^2 r + 3\lambda^2 M r^2 + \lambda^2(\lambda+1)r^3}{r^2(\lambda r + 3M)^2} \qquad f_5 = \frac{3M^2 - \lambda M r + \lambda r^2}{r(\lambda r + 3M)} \qquad f_6 = (r - 2M)$$

$$f_7 = \frac{\kappa u^0 (r-2M)(\lambda^2 r^2 + 2\lambda M r - 3Mr + 3M^2)}{r(\lambda+1)(\lambda r + 3M)^2} \qquad f_8 = -\frac{\kappa u^0 (r-2M)^2}{(\lambda+1)(\lambda r + 3M)}$$

$$f_9 = \frac{\lambda r^2 - 3M\lambda r - 3M^2}{(r-2M)(\lambda r + 3M)} \quad f_{10} = r \quad f_{11} = -\frac{\kappa\ u^0\ \dot{z}_u\ (\lambda r + M)}{(\lambda+1)(\lambda r + 3M)} \quad f_{12} = \frac{\kappa\ u^0\ \dot{z}_u\ r(r-2M)}{(\lambda+1)(\lambda r + 3M)}$$

$\delta = \delta\left[r - z_u(t)\right]$ and $\delta' = \delta'\left[r - z_u(t)\right]$

Jump conditions

Conditions on Ψ and its derivatives must cancel discontinuities in K, H_2 and H_1
Coefficients of Θ_1 must be = coefficients of Θ_2, Coefficients of δ and δ' must vanish

$$\left(\Psi^+ - \Psi^-\right)_{z_u} = \underbrace{-\frac{f_3}{f_2}}_{K} = \underbrace{-\frac{f_8}{f_6}}_{H_{0,2}} = \underbrace{\frac{f_{12}}{\dot{z}_u f_{10}}}_{H_1}$$

$$\left(\Psi^+_{,r} - \Psi^-_{,r}\right)_{z_u} = \underbrace{\frac{f_1 f_3}{f_2^2}}_{K} = \underbrace{\frac{1}{f_6}\left(\frac{f_5 f_8}{f_6} - f_7 + f_{8,r} - \frac{f_{6,r} f_8}{f_6}\right)}_{H_{0,2}}$$

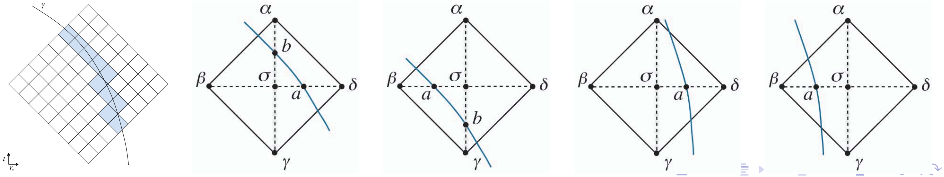
$$\left.\left(\Psi^+_{,rr} - \Psi^-_{,rr}\right)_{z_u} = -\frac{f_4\left(\Psi^+ - \Psi^-\right) + f_5\left(\Psi^+_{,r} - \Psi^-_{,r}\right)}{f_6}\right\} H_{0,2}$$

$$\left.\left(\Psi^+_{,t} - \Psi^-_{,t}\right)_{z_u} = \left.\frac{d\left(\Psi^+ - \Psi^-\right)}{dt}\right|_{z_u} - \left(\Psi^+_{,r} - \Psi^-_{,r}\right)\dot{z}_u = \frac{\left(f_9 - f_{10,r}\right)\dot{z}_u\left(\Psi^+ - \Psi^-\right) - f_{11} + f_{12,r}}{f_{10}}\right\} H_1$$

$$\left.\left(\Psi^+_{,tr} - \Psi^-_{,tr}\right)_{z_u} = -\frac{f_9\left(\Psi^+_{,t} - \Psi^-_{,t}\right)}{f_{10}}\right\} H_1$$

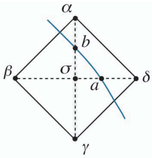
Indirect method I

- Integration domain h discretised by 2-dimensional uniform mesh $(t,\ r_*)$.
- Initial data and empty cells (Lousto and Price, Martel and Poisson, Martel, Lousto, Haas).
- The forward time value at the upper node of the (r^*, t) grid cell is obtained by
  i) the preceding node values of the same cell,
  ii) analytic expressions from the jump conditions on Ψ and its derivatives,
  iii) ONLY AT HIGH ORDERS: the values of the wave function at adjacent cells .
- The numerical integration does not deal with the source and potential terms directly, for cells crossed by the particle world line.
- The values of Ψ, $h_{\mu}\nu$ and derivatives at the particle positions are analytical.



Indirect method II

Case 1: particle crosses the β - δ line at a and γ - α line at b
$\epsilon_a = \min\{a\delta, a\beta\}$, $\epsilon_b = \min\{b\alpha, b\gamma\}$; $\Psi_b^\pm = \Psi^\pm(t_b, r_b^*)$, $\Psi_a^\pm = \Psi^\pm(t_a, r_a^*)$
6 analytic expressions and 6 numerical equations:



$$\left(\Psi^+ - \Psi^-\right)_a = [\Psi]_a \quad \left(\Psi^+_{,r^*} - \Psi^-_{,r^*}\right)_a = [\Psi_{,r^*}]_a \quad \left(\Psi^+_{,t} - \Psi^-_{,t}\right)_a = [\Psi_{,r^*}]_a$$

$$\left(\Psi^+ - \Psi^-\right)_b = [\Psi]_b \quad \left(\Psi^+_{,r^*} - \Psi^-_{,r^*}\right)_b = [\Psi_{,r^*}]_b \quad \left(\Psi^+_{,t} - \Psi^-_{,t}\right)_b = [\Psi_{,t}]_b$$

$$\Psi^+_\alpha = \Psi^+(t_b + \epsilon_b, r_b^*) = \Psi^+_b + \epsilon_b \, \Psi^+_{,t}\big|_b \quad (1)$$

$$\Psi^-_\sigma = \Psi^-(t_b - (h - \epsilon_b), r_b^*) = \Psi^-_b - (h - \epsilon_b) \, \Psi^-_{,t}\big|_b \quad (2)$$

$$\Psi^-_\gamma = \Psi^-(t_b - 2h + \epsilon_b, r_b^*) = \Psi^-(t_\sigma - h, r_b^*) = \Psi^-_\sigma - h \, \Psi^-_{,t}\big|_\sigma \quad (3)$$

$$\Psi^+_\delta = \Psi^+(t_a, r_a^* + \epsilon_a) = \Psi^+_a + \epsilon_a \, \Psi^+_{,r^*}\big|_a \quad (4)$$

$$\Psi^-_\sigma = \Psi^-(t_a, r_a^* - (h - \epsilon_a)) = \Psi^-_a - (h - \epsilon_a) \, \Psi^-_{,r^*}\big|_a \quad (5)$$

$$\Psi^-_\beta = \Psi^-(t_a, r_a^* - 2h + \epsilon_a) = \Psi^-(t_\sigma, r_\sigma^* - h) = \Psi^-_\sigma - h \, \Psi^-_{,r^*}\big|_\sigma \quad (6)$$

Indirect method III

Our aim: determination of the value of Ψ^+_α, knowing those of Ψ^-_β, Ψ^-_γ, Ψ^+_δ, ϵ_a, ϵ_b, $[\Psi]_{a,b}$, $[\Psi_{,r}]_{a,b}$ and $[\Psi_{,t}]_{a,b}$
Algebraic manipulation. Subtracting (1) and (2):

$$\Psi^+_\alpha = \Psi^-_\sigma + [\Psi]_b + [\Psi_{,t}]_b + h\ \Psi^-_{,t}\Big|_b \tag{7}$$

Subtracting (4) and (5):

$$\Psi^+_\delta = \Psi^-_\sigma + [\Psi]_a + [\Psi_{,r^*}]_a + h\ \Psi^-_{,r^*}\Big|_a \tag{8}$$

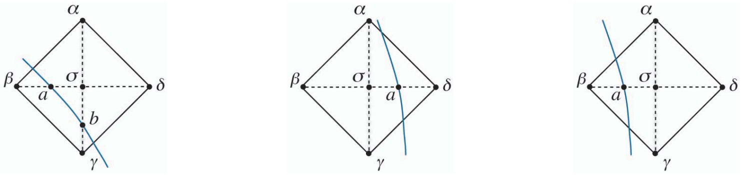
Summing (3) and (7), (6) and (8), and combining the results, it provides:

$$\Psi^+_\alpha = \Psi^-_\beta - \Psi^-_\gamma + \Psi^+_\delta - [\Psi]_a + [\Psi]_b - \epsilon_a [\Psi_{,r^*}]_a + \epsilon_b [\Psi_{,t}]_b + \mathcal{O}(h^2)$$

- No need of direct integration of the singular source
- Top cell value depending upon analytic expressions (and other cell's corners)

Indirect method IV

Similar relations for the other three cases

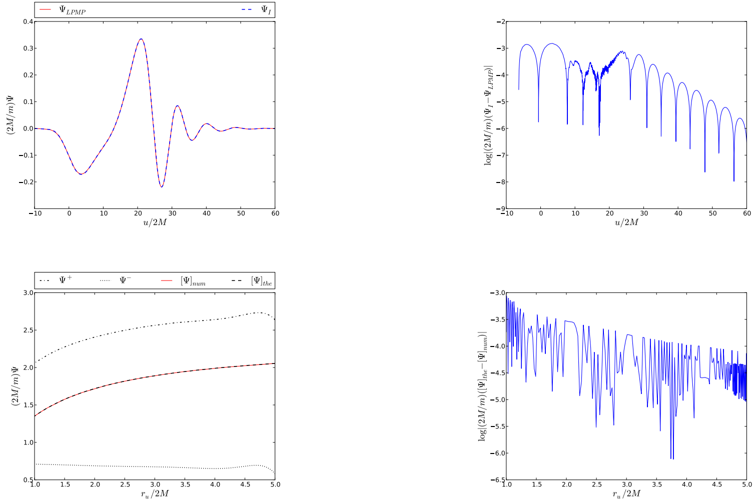


$$\Psi_\alpha^+ = \Psi_\beta^- - \Psi_\gamma^- + \Psi_\delta^+ - [\Psi]_a + [\Psi]_b - \epsilon_a [\Psi_{,r*}]_a + \epsilon_b [\Psi_{,t}]_b + \mathcal{O}(h^2)$$

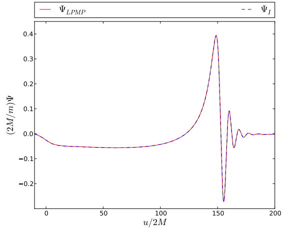
$$\Psi_\alpha^+ = \Psi_\beta^- - \Psi_\gamma^- + \Psi_\delta^+ - [\Psi]_a - \epsilon_a [\Psi_{,r*}]_a + \mathcal{O}(h^2)$$

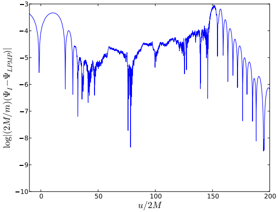
$$\Psi_\alpha^+ = \Psi_\beta^- - \Psi_\gamma^- + \Psi_\delta^+ + [\Psi]_a - \epsilon_a [\Psi_{,r*}]_a + \mathcal{O}(h^2)$$

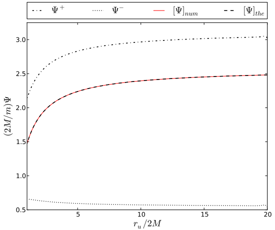
Indirect method V: Radial fall from 5M

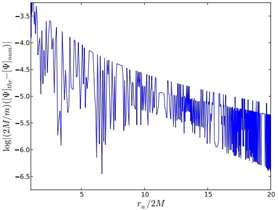


Indirect method VI: Radial fall from 20M

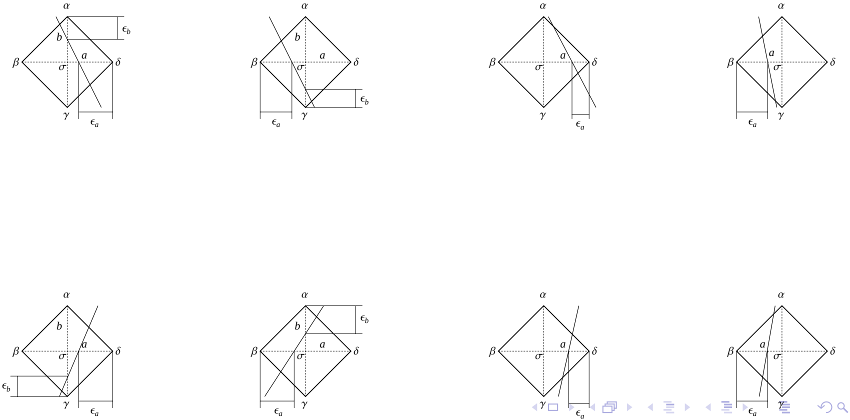








Indirect method VII: Generic orbits



Indirect method VIII: Generic orbits

$r^* = r + 2M\ln(r/2M - 1)$; $T^{(}t), R(t), \Theta(t), \Phi(t)$; $\lambda = (\ell - 1)(\ell + 2)/2,\ f = 1 - 2M/r$.

$$\frac{\partial^2 \Psi^\ell(t,r)}{\partial r^{*2}} - \frac{\partial^2 \Psi^\ell(t,r)}{\partial t^2} - V^\ell(r)\Psi^\ell(t,r) = S^\ell(t,r) ,$$

$$V_e^\ell = 2f\,\frac{\left[\lambda^2(\lambda+1)r^3 + 3\lambda^2 M r^2 + 9\lambda M^2 r + 9M^3\right]}{r^3(\lambda r + 3M)^2} \qquad V_o^\ell = 2f\left(\frac{\lambda+1}{r^2} - 3\frac{M}{r^3}\right)$$

$$S^\ell(t,r) = G^\ell(t,r)\delta[r - R(t)] + F^\ell(t,r)\delta'[r - R(t)] .$$

$$F_e^l = -8K\,\frac{(R - 2\,M)\left[(R\dot{R})^2 - (R - 2\,M)^2\right]}{R^2\,(\lambda\,R + 3\,M)}\,\bar{Y}_{\ell m} \qquad F_o^l = 8K\,\frac{\left[(R\dot{R})^2 - (R - 2\,M)^2\right]}{\lambda\,R}\,\mathrm{ang}(t)$$

$$G_e^l = K\left\{4(R - 2M)\left[4\,\frac{\dot{R}}{(\lambda R + 3\,M)}\,\frac{d}{dt}\bar{Y}_{\ell m} - 2\frac{\dot{\Theta}\dot{\Phi}}{\lambda}\bar{X}_{\ell m} - \frac{\left(\dot{\Theta}^2 - \sin^2\theta\,\dot{\Phi}^2\right)}{\lambda}\bar{X}_{\ell m}\right] + \right.$$

$$\left.\frac{8\,(R - 2\,M)^2}{(\lambda R + 3\,M)}\,\bar{Y}_{\ell m}\left[\dot{\Theta}^2 + \sin^2\theta\,\dot{\Phi}^2 + \frac{\left(R^2\lambda + 6\,RM + 6\,\lambda R\,M + 3\,M^2 + R^2\lambda^2\right)\dot{R}^2}{R\,(\lambda R + 3\,M)\,(R - 2\,M)^2} - \frac{\left(R^2\lambda + R^2\lambda^2 + 15\,M^2 + 6\,\lambda R\,M\right)}{R^3\,(\lambda R + 3\,M)}\right]\right\}$$

$$G_o^l = -K\left\{\frac{8R\dot{R}}{\lambda}\frac{d}{dt}\mathrm{ang}(t) - \frac{8}{\lambda R}\mathrm{ang}(t)\left[R^2\dot{R}\frac{\dot{U}^0}{U^0} + R^2\ddot{R} + 2M - R + R\dot{R}^2\right]\right\}$$

Indirect method IX: Generic orbits

$$K = (\pi\, m\, U^0)/(\lambda + 1)$$

$$X^{\ell m} = 2\partial_\varphi \Big(\partial_\theta - \cot\theta\Big) Y^{\ell m}$$

$$W^{\ell m} = \left(\partial_\theta^2 - \cot\theta\,\partial_\theta - \frac{1}{\sin^2\theta}\partial_\varphi^2\right) Y^{\ell m}$$

$\bar{Y}^{\ell m}$ represents the complex conjugation of spherical harmonics and it is defined by

$$\bar{Y}^{\ell m} = \quad .$$

$$\mathrm{ang}(t) = \frac{1}{\sin\Theta}\,\dot{\Theta}\,\partial_\varphi\,\bar{Y}^{\ell m} - \sin\Theta\,\dot{\Phi}\,\partial_\theta\,\bar{Y}^{\ell m}$$

$\dot{\Theta} = 0$ and $\Theta = \pi/2$ without loss of generality.

Indirect method X: Generic orbits

$$\Psi = \Psi^+\mathcal{H}_1 + \Psi^-\mathcal{H}_2$$

The wave equation is cast in the following form

$$ff'\partial_r\Psi + f^2\partial_r^2\Psi - \frac{\partial^2\Psi}{\partial t^2} - V\Psi = S(t,r) = G(t,r)\delta + F(t,r)\delta' = \tilde{G}_R\delta + F_R\delta'$$

where $\tilde{G}_R = G_R - F'_R$ (since $\phi(r)\delta'\left[r - R(t)\right] = \phi_{R(t)}\delta'\left[r - R(t)\right] - \phi'_{R(t)}\delta\left[r - R(t)\right]$)

$$ff'\partial_r\Psi + f^2\partial_r^2\Psi = \left[ff'\Psi^+_{,r} + f^2\Psi^+_{,rr}\right]\mathcal{H}_1 + \left[ff'\Psi^-_{,r} + f^2\Psi^-_{,rr}\right]\mathcal{H}_2 + ff'\left(\Psi^+ - \Psi^-\right)\delta + 2f^2\left(\Psi^+_{,r} - \Psi^-_{,r}\right)\delta + f^2\left(\Psi^+ - \Psi^-\right)$$

$$\Psi^+_{,tt}\mathcal{H}_1 + \Psi^+_{,tt}\Theta_2 - 2\dot{r}_u\partial_t\left(\Psi^+ - \Psi^-\right)\delta - \ddot{R}\left(\Psi^+ - \Psi^-\right)\delta + \dot{R}^2\left(\Psi^+ - \Psi^-\right)\delta'$$

$$V\Psi = V\Psi^+\mathcal{H}_1 + V\Psi^-\mathcal{H}_2$$

Indirect method XI: Generic orbits

Equating the coefficients of δ', and owing to the above mentioned property of the delta derivative for which $\left(\Psi^+ - \Psi^-\right)\delta' = [\Psi]\,\delta' - [\Psi_{,r}]\,\delta$, we get the jump condition for Ψ

$$[\Psi] = \frac{1}{f_R^2 - \dot{R}^2} F_R \quad . \tag{9}$$

Equating the coefficients of δ, we get the jump condition on the space derivative

$$[\Psi_{,r}] = \frac{1}{f_R^2 - \dot{R}^2}\left[\tilde{G}_R + \left(f_R f_R' - \ddot{R}\right)[\Psi] - 2\dot{R}\frac{d}{dR}[\Psi]\right] \quad , \tag{10}$$

and therefore the jump condition on the first time derivative

$$[\Psi_{,t}] = \dot{R}\frac{d}{dR}[\Psi] - \dot{R}\,[\Psi_{,r}] \quad . \tag{11}$$

Indirect method XII: Generic orbits

For case 5, we have

$$\Psi_\alpha^- = \Psi_\beta^- - \Psi_\gamma^+ + \Psi_\delta^+ + [\Psi]_a - [\Psi]_b - \epsilon_a [\Psi_{,r^*}]_a - \epsilon_b [\Psi_{,t}]_b$$

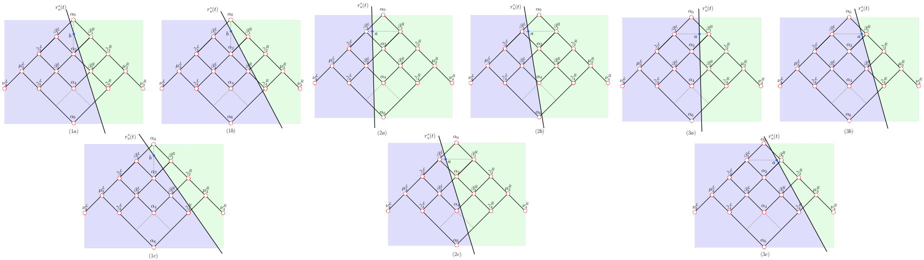
For case 6, we have

$$\Psi_\alpha^- = \Psi_\beta^- - \Psi_\gamma^+ + \Psi_\delta^+ - [\Psi]_a + [\Psi]_b - \epsilon_a [\Psi_{,r^*}]_a - \epsilon_b [\Psi_{,t}]_b$$

- Waveforms for circular orbits at first order have been obtained. Work slowly in progress (SA - ET, LISA).

Indirect method XIII: 4^{th} order

- The indirect method may be extended to higher orders for any orbit.
- Coding state: radial 2^{nd}, 3^{rd} are working (4^{th} unstable on July 8^{th}).
- Warning: slightly different approach (1 point-based, ϵ definition).



Indirect method XIV: 4^{th} order

$$\Psi^{+}_{\alpha_0} = \sum_{n=0}^{4} c_n \partial_t^n \Psi^{+}_{b} + \mathcal{O}\left(h^5\right) = \sum_{n=0}^{4} c_n \left(\partial_t^n \Psi^{-}_{b} + [\partial_t^n \Psi]_b\right) + \mathcal{O}\left(h^5\right) =$$

$$= c_0 \Psi^{-}_{b} + c_1 \partial_t \Psi^{-}_{b} + c_2 \partial_t^2 \Psi^{-}_{b} + c_3 \partial_t^3 \Psi^{-}_{b} + c_4 \partial_t^4 \Psi^{-}_{b} + \sum_{n=0}^{4} c_n [\partial_t^n \Psi]_b + \mathcal{O}\left(h^5\right)$$

After considerable manipulation, we get for the first of the nine sub-cases

$$\Psi^{+}_{\alpha_0} = \quad x_1 \Psi^{-}_{\alpha_2} + x_2 \Psi^{-}_{\alpha_4} + x_3 \Psi^{-}_{\alpha_6} + x_4 \left(\Psi^{-}_{\beta_1^L} + \Psi^{+}_{\beta_1^R}\right) + x_5 \left(\Psi^{-}_{\beta_3^L} + \Psi^{+}_{\beta_3^R}\right)$$
$$x_6 \left(\Psi^{-}_{\gamma_2^L} + \Psi^{+}_{\gamma_2^R}\right) + x_7 \left(\Psi^{-}_{\gamma_4^L} + \Psi^{+}_{\gamma_4^R}\right) + x_8 \left(\Psi^{-}_{\mu_3^L} + \Psi^{+}_{\mu_3^R}\right) + \Phi^{(1)}_{r_u^*(t_b)}$$

$$\Phi^{(1a)}_{r_u^*(t_b)} = y_1 [\Psi]_b + y_2(h, \epsilon_b) [\partial_t \Psi]_b + y_3(h^2, \epsilon_b^2, h\epsilon_b) \left[\partial_t^2 \Psi\right]_b + y_4(h^3, h^2, \epsilon_b^3, \epsilon_b^2) \left[\partial_t^3 \Psi\right]_b$$
$$+ y_5(h^4, h^3, h^2, \epsilon_b^4, \epsilon_b^3, \epsilon_b^2) \left[\partial_t^4 \Psi\right]_b + y_6 [\partial_{r^*} \Psi]_b + y_8(h^3) \left[\partial_{r^*}^3 \Psi\right]_b + y_9(h^2, h\epsilon_b) [\partial_{r^*} \partial_t \Psi]_b$$
$$+ y_{10}(h^3, h^2\epsilon_b, h\epsilon_b^2) \left[\partial_{r^*}^3 \partial_t \Psi\right]_b + y_{11}(h^4, h^3\epsilon_b) \left[\partial_{r^*}^2 \partial_t^2 \Psi\right]_b + y_{12}(h^4, h^3\epsilon_b, h^2\epsilon_b^2, h\epsilon_b^3) \left[\partial_{r^*} \partial_t^3 \Psi\right]_b$$

Perspectives

- The indirect method appears to be an (close to) analytical development scheme and applicable to any orbit.
- Our aim is the orbital evolution. Although, a self-consistent evolution *à la* Gralla-Wald of the self-force is conceivable in the harmonic gauge, an **iterative** evolution of the back-action may be conceived in other gauges.

References

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Orbital evolution I

The geodesic deviation expression has been rigorously derived by Gralla and Wald. Here a non-rigorous (kludgy ? quick and dirty) derivation of the same expression.

$$\frac{D^2 z^\alpha}{d\tau^2} = \frac{d^2 z^\alpha}{d\tau^2} + {}^b\Gamma^\alpha_{\mu\nu} u^\mu u^\nu = \frac{d^2 z^\alpha}{d\tau^2} + {}^b\Gamma^\alpha_{\mu\nu} \frac{dz^\mu}{d\tau}\frac{dz^\nu}{d\tau} = 0$$

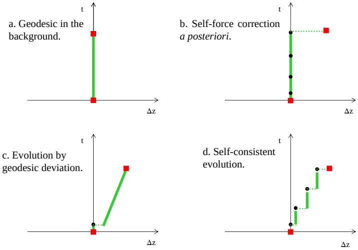
$$\frac{D^2 \hat{z}^\alpha}{d\lambda^2} = \frac{d^2 \hat{z}^\alpha}{d\lambda^2} + {}^f\Gamma^\alpha_{\mu\nu} \hat{u}^\mu \hat{u}^\nu = \frac{d^2 \hat{z}^\alpha}{d\lambda^2} + {}^f\Gamma^\alpha_{\mu\nu} \frac{d\hat{z}^\mu}{d\lambda}\frac{d\hat{z}^\nu}{d\lambda} = 0$$

being τ an d λ proper time in the background (b) and full (f) metric, respectively. $\hat{z}^\alpha = z^\alpha + \Delta z^\alpha$ is the coordinates of the particle in the full metric $g_{\mu\nu} + h^R_{\mu\nu}$, where $h^R_{\mu\nu}$ is the DeWh radiative effective or the MiSaTuQuWa tail perturbation.

Subtracting the two geodesics, after some manipulation:

$$\frac{D^2 \Delta z^\alpha}{d\tau^2} = \underbrace{-R_{\mu\beta\nu}{}^\alpha u^\mu \Delta z^\beta u^\nu}_{Background\ \ geodesic\ \ deviation} \underbrace{-\frac{1}{2}(g^{\alpha\beta} + u^\alpha u^\beta)(2h^R_{\mu\beta;\nu} - h^R_{\mu\nu;\beta})u^\mu u^\nu}_{Self-acceleration} .$$

Orbital evolution II



- Self-consistent evolution corresponds to the bottom-right figure: the geodesic is constantly updated at each instant (iteration) by the self-force term, having neglected the background geodesic deviation.
- It has been shown in specific cases the correspondence between the pragmatic RW gauge and the self-consistent approaches, term by term. Meaningfulness under scrutiny (proper $\rightarrow$ coordinate time, harmonic $\rightarrow$ RW gauge).