Electromagnetic self-force in the vicinity of a charged black hole

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4 July 2011

Motivation

- The cosmic censorship conjecture states that spacetime singularities are hidden behind event horizons, making them invisible to all observers in the external universe.
- Hubeny (1999) proposed a black hole destruction mechanism in which a test charge was dropped onto a near extremal BH to create conditions where $M < Q$.
- Hubeny's analysis was inconclusive due to an inadequate treatment of the electromagnetic self-force.
- Does the self-force act as a cosmic censor?
- Recent work (Barausse et al, 2010) establishes that the dissipative part of the self-force does not play this role.

We aim to address the question more completely by computing the *full* electromagnetic self-force and checking how this influences the outcome of the Hubeny scenario.

Infalling test-charges in Reissner-Nördstrom

- A particle of mass m and charge $q \gg m$ follows a radial path towards a RN black hole having mass M and charge $Q = M - 2\epsilon^2$ for small positive ϵ.

- The RN spacetime admits a timelike Killing vector $t^\alpha \equiv \frac{\partial}{\partial t}$ that gives rise to a conserved quantity $E_0 = -t^\alpha(mu_\alpha + qA_\alpha)$. Using this quantity, we express the equations of motion $ma^\alpha = qF^\alpha{}_\beta u^\beta$ in first order form

$$
\begin{aligned}
m\dot{t} &= \frac{1}{f}(E_0 - qQ/r)\,, \\
m\dot{r} &= -\sqrt{(E_0 - qQ/r)^2 - m^2 f}\,,
\end{aligned}
$$

where $$f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}\,.$$

Overcharging Conditions

- The overcharging conditions are
  1. $\dot{r}^2 > 0\,, \quad \forall r \geq r_+$
  2. $Q + q > M + E_0$

- By setting $M \equiv 1$, $Q \equiv 1 - 2\epsilon^2$, Hubeny showed that the parameter space for overcharging is the three-parameter family characterized by

$$\begin{aligned} q &= a\epsilon & a &> 1\,, \\ E_0 &= a\epsilon - 2b\epsilon^2 & 1 < b &< a\,, \\ m &= c\epsilon & c &< \sqrt{a^2 - b^2}\,. \end{aligned}$$

Is the self-force relevant to the motion of a falling charge?

- For most cases, it is not.

$$f_{\text{self}} \sim \frac{q^2 M}{r^3} \sim \frac{q^2 M}{r_+^3} \sim \left(\frac{q}{M}\right)^2 ,$$
$$f_{\text{BH}} \sim \frac{qQ}{r^2} \sim \frac{qQ}{r_+^2} \sim \frac{q}{M} ,$$

 which imply $$\frac{f_{\text{self}}}{f_{\text{BH}}} \sim \frac{q}{M} \sim \epsilon ,$$

 and that self-force effects are subdominant.

- But for Hubeny's scenario it is!
 Examination of the equation of motion close to the event horizon, with Hubeny's parameters, reveals that

$$m\ddot{r} \sim \epsilon^2 .$$

 Hence the self-force corrections, which are also of order ϵ^2, cannot be neglected in a self-consistent analysis.

The Retarded Field

- For radial infall, F_{tr} is the relevant part of the electromagnetic field tensor.
- The multipole decomposition of the electromagnetic field tensor takes the form

$$F_{tr} = \sum_{\ell\, m} \left(-\frac{\Phi_1^{\ell m}}{r^2} \right) Y^{\ell m}$$

- The scalar function, Φ_1, evolves according to

$$-\partial_t^2 \Phi_1 + \partial_{r^*}^2 \Phi_1 - \frac{l(l+1)}{r^2} f \Phi_1 = 4\pi r^2 f (\partial_t j_r - \partial_r j_t - 2 j_t / r)$$

where

$$j_t = G(R)\delta(r - R(t))$$
$$j_r = H(R, \dot{R})\delta(r - R(t))$$

- We solve the scalar wave equation for Φ_1 using the second-order numerical code developed by R. Haas (Haas, 2007).

The Self-Field

- The regular self-field is calculated using mode-sum regularization:

$$F_{tr}^{\mathsf{R}} = \sum_{\ell=0}^{\infty} \left((F_{tr})_\ell - (\ell + \tfrac{1}{2})A - B - \frac{D}{\left(\ell - \frac{1}{2}\right)\left(\ell + \frac{3}{2}\right)} \right),$$

- The self-field contains contributions from the BH and the particle

$$F_{tr} = F_{tr}^{\mathrm{BH}} + F_{tr}^{\mathsf{R}} = -\frac{Q}{r^2} - \frac{q^2 M}{r^3} K(r)\,,$$

  where K is a worldline-dependent, dimensionless quantity that must be computed numerically.

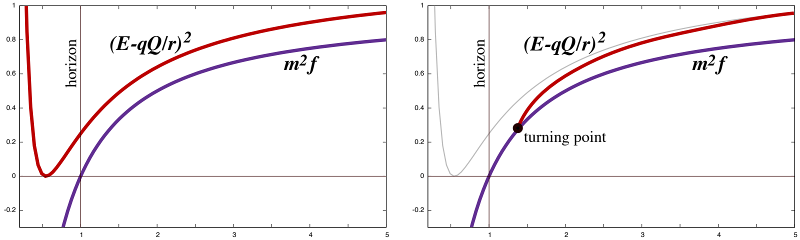
- A generalized energy, $E(r)$, is computed from $K(r)$ via

$$E(r) = E_0 - q^2 M \int_r^{\infty} \frac{K(r')}{r'^3}\, dr'$$

Turning point due to the self-force

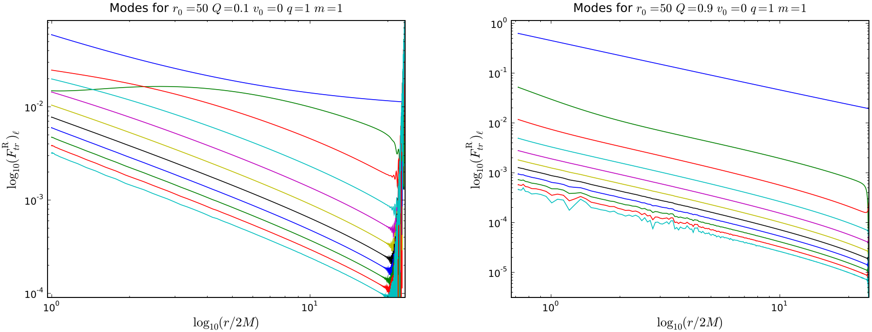
- Under the influence of the self-field, the particle evolves according to the generalized radial equation

$$m\dot{r} = -\sqrt{(E(r) - qQ/r)^2 - m^2 f}$$



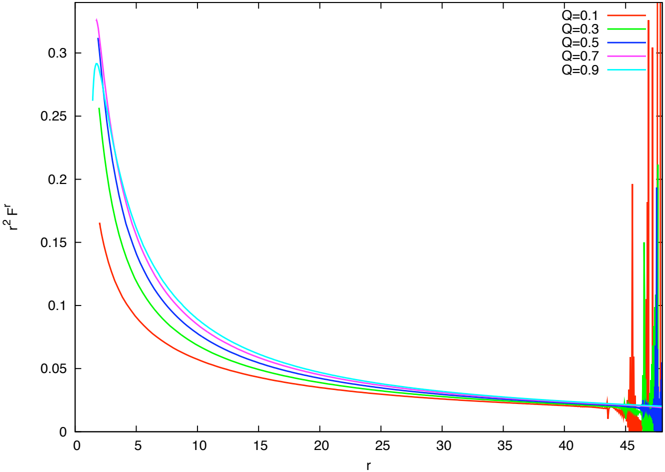
The existence of a turning point at $r > r_+$ would prevent the particle from falling in to the BH. Such an occurrence would provide evidence that the self-force plays a role enforcing cosmic censorship.

Results: regularized modes for the infalling charge



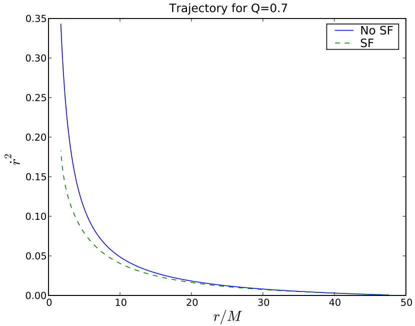
The regularized modes for low Q look qualitatively similar to the scalar case (Barack & Burko, 2000). The behavior of the dipole mode is what's most sensitive to Q.

Results: Radial self-force

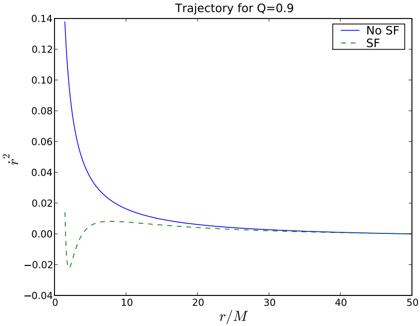


The self-force on a charge that falls from rest is always positive. Moreover, it tends to increase closer to the event horizon.

Results: $\dot{r}^2$ versus r



$M = 1, Q = 0.7, q = m = 1$



$M = 1, Q = 0.9, q = m = 1$

The self-force appears to make a significant impact on the trajectory for high values of Q.

Conclusion and Future work

- Self-force is a potential enforcer of cosmic censorship.
- Whether or not the self-force *really* is the cosmic censor remains to be seen. (We promise to keep you posted!).
- We have all the tools in place, but presently have problems with high initial starting velocities.