Self-force loops

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Outline

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[arXiv:1307.3476]

The problem

We wish to determine the self-forced motion and field (e.g. energy and angular momentum fluxes) of a particle with scalar charge

$$\Box \psi^{\mathrm{ret}} = -4\pi q \int \delta^{(4)}(x - z(\tau))\, d\tau.$$

2 general approaches:

- Compute enough "geodesic"-based self-forces and then use this to drive the motion of the particle. (Post-processing, fast, accurate self-forces, relies on slow orbit evolution)
- Compute the "true" self-force while simultaneously driving the motion. (Slow and expensive, less accurate self-forces)

Effective source

... is a general approach to self-force and self-consistent orbital evolution that doesn't use any delta functions.

Key ideas

- Compute a regular field, ψ^{R}, such that the self-force is

$$F_\alpha = \nabla_\alpha \psi^{\mathsf{R}}|_{x=z},$$

where $\psi^{\mathsf{R}} = \psi^{\mathsf{ret}} - \psi^{\mathsf{S}}$, and ψ^{S} can be approximated via local expansions: $\psi^{\mathsf{S}} = \tilde{\psi}^{\mathsf{S}} + O(\epsilon^n)$.

- The effective source, S, for the field equation for ψ^{R} is regular at the particle location.

$$\Box \psi^{\mathsf{R}} = \Box \psi^{\mathsf{ret}} - \Box \psi^{\mathsf{S}} = S(x|z,u)$$

where $\Box \psi^{\mathsf{S}} = -4\pi q \int \delta^{(4)}(x - z(\tau))\, d\tau - S$.

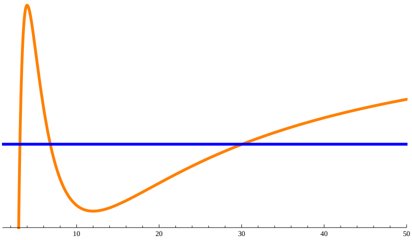
Evolution code

- A 3D multi-block scalar wave equation code.
- Schwarzschild background spacetime in Kerr-Schild coordinates.

$$\Box\psi^{\mathrm{R}} = S(x|z^\alpha(\tau), u^\alpha(\tau))$$
$$\frac{Du^\alpha}{d\tau} = 0\left(\frac{q}{m(\tau)}\left(g^{\alpha\beta} + u^\alpha u^\beta\right)\nabla_\beta\psi^{\mathrm{R}}\right)$$
$$\frac{dm}{d\tau} = 0(-qu^\beta\nabla_\beta\psi^{\mathrm{R}})$$

- Spherical inner boundary placed inside the black hole.
- We use 8th order summation by parts finite differencing and penalty boundary conditions at patch boundaries.
- We can evolve the orbit using the geodesic equations directly as well as using the osculating orbits framework.
- We use hyperboloidal slicings and place $\mathcal{J}^+$ at a finite coordinate radius.
- We extract the self-force by interpolation of $\nabla_\beta\psi^{\mathrm{R}}$ to the particle location and calculate energy and angular momentum fluxes through the horizon and $\mathcal{J}^+$.

e-p parametrization of the motion



- A bound orbit can be specified by its eccentricity (e) and semi-latus rectum (p):

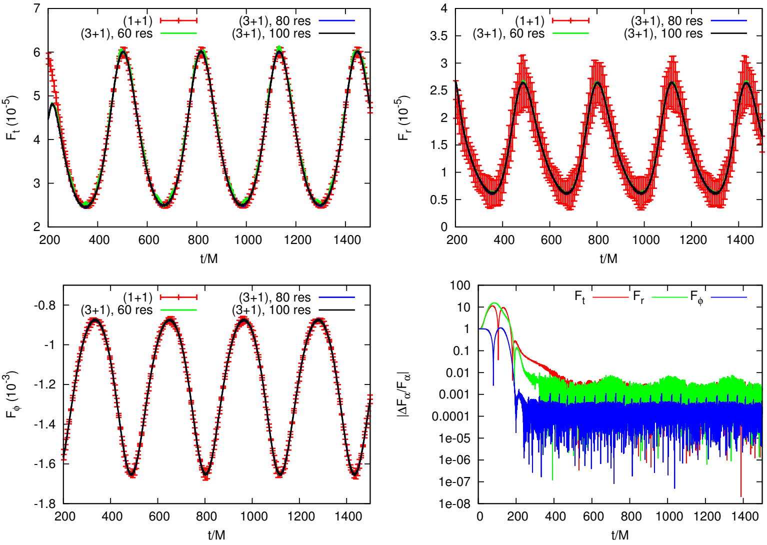
$$r_1 = \frac{pM}{1+e}, \;\; r_2 = \frac{pM}{1-e}$$

  where r_1 and r_2 are the turning points of the radial motion.

- $e = 0$, stable circular orbits
  $p = 6 + 2e$, (separatrix), unstable circular orbits
  $0 \leq e < 1, \;\; p \geq 6 + 2e$, bound orbit

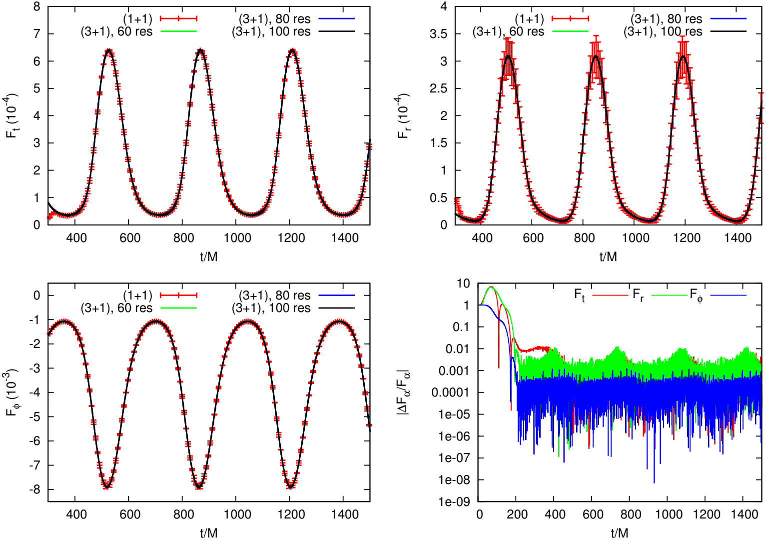
Comparison with (1+1) results

$e = 0.1, p = 9.9$



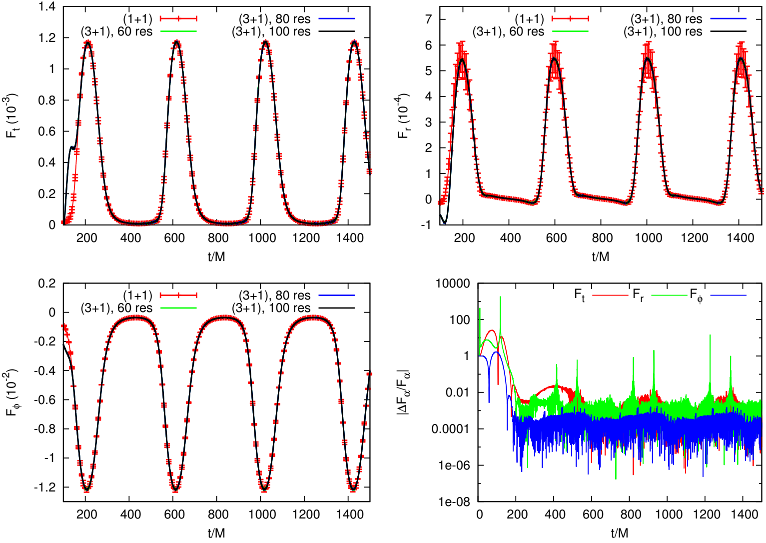
Comparison with (1+1) results

$e = 0.3, p = 7.0$



Comparison with (1+1) results

$e = 0.5, p = 7.2$



Energy and angular momentum losses

The dissipative pieces of the self force

$$F_t^{\rm diss}(r_o+\Delta r_p) = \frac{1}{2}\left[F_t^{\rm ret}(r_o+\Delta r_p) + F_t^{\rm ret}(r_o-\Delta r_p)\right]$$
$$F_\phi^{\rm diss}(r_o+\Delta r_p) = \frac{1}{2}\left[F_\phi^{\rm ret}(r_o+\Delta r_p) + F_\phi^{\rm ret}(r_o-\Delta r_p)\right]$$

In terms of which the energy and angular momentum losses are

$$-\Delta\mathcal{E} = \Delta u_t = 2\int_{r_{\rm min}}^{r_{\rm max}} \frac{F_t^{\rm diss}}{u^r} dr$$
$$\Delta\mathcal{L} = \Delta u_\phi = 2\int_{r_{\rm min}}^{r_{\rm max}} \frac{F_\phi^{\rm diss}}{u^r} dr.$$

Energy and angular momentum losses

Energy and angular momentum fluxes through the horizon and $\mathcal{J}^+$.

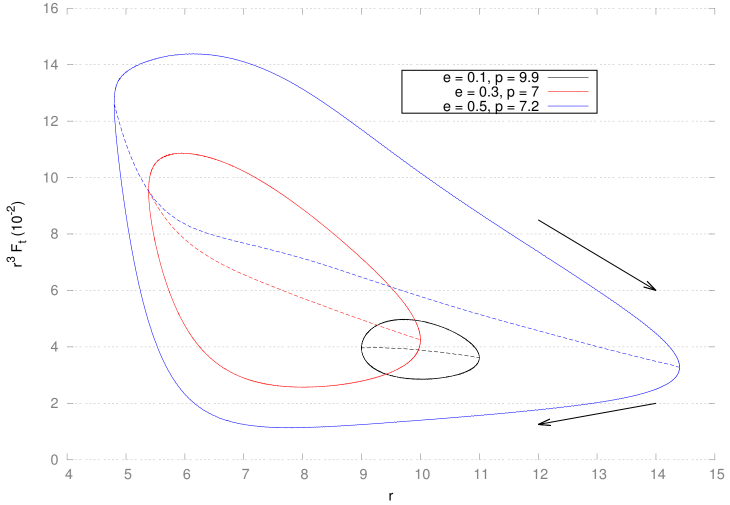
$$\left.\frac{dE}{dt}\right|_{\mathcal{H}} = -\frac{M^2}{\pi}\oint_{r=2M}\left(\frac{\partial\phi}{\partial t}\right)^2 d\Omega, \qquad \left.\frac{dL}{dt}\right|_{\mathcal{H}} = -\frac{M^2}{\pi}\oint_{r=2M}\frac{\partial\phi}{\partial t}\left(x\partial_y\phi - y\partial_x\phi\right) d\Omega.$$

$$\left.\frac{dE}{d\tau}\right|_{\mathcal{J}+} = -\frac{\rho^2_{\mathcal{J}+}}{4\pi}\oint_{\rho=\rho_{\mathcal{J}+}}\left(\frac{\partial\hat{\phi}}{\partial\tau}\right)^2 d\Omega, \qquad \left.\frac{dL}{d\tau}\right|_{\mathcal{J}+} = -\frac{\rho^2_{\mathcal{J}+}}{4\pi}\oint_{\rho=\rho_{\mathcal{J}+}}\frac{\partial\hat{\phi}}{\partial\tau}\left(\hat{x}\partial_{\hat{y}}\hat{\phi} - \hat{y}\partial_{\hat{x}}\hat{\phi}\right) d\Omega.$$

Results:

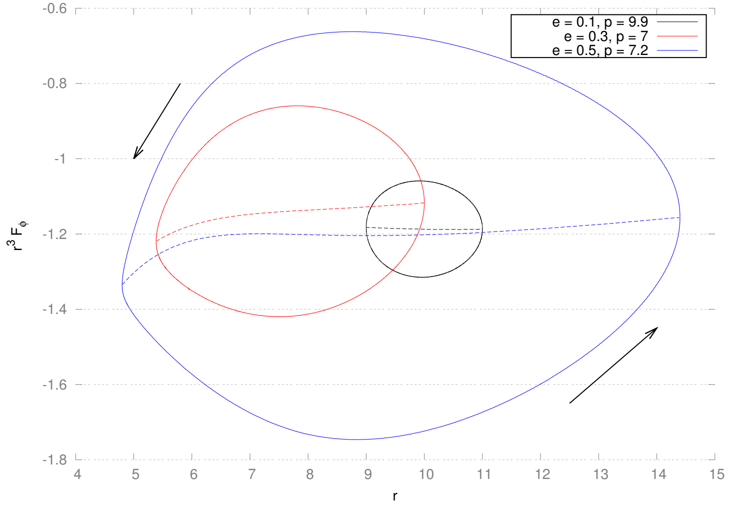
p	e	$10^4\langle\dot{\mathcal{E}}\rangle$ Self-force	$10^4\langle\dot{\mathcal{E}}\rangle$ Flux	$10^3\langle\dot{\mathcal{L}}\rangle$ Self-force	$10^3\langle\dot{\mathcal{L}}\rangle$ Flux
9.9	0.1	−0.32880	−0.32887	−1.01025	−1.01020
7.0	0.3	−1.6716	−1.6715	−2.6256	−2.6252
7.2	0.5	−1.9682	−1.9678	−2.5867	−2.5863

Self-force loops (F_t)



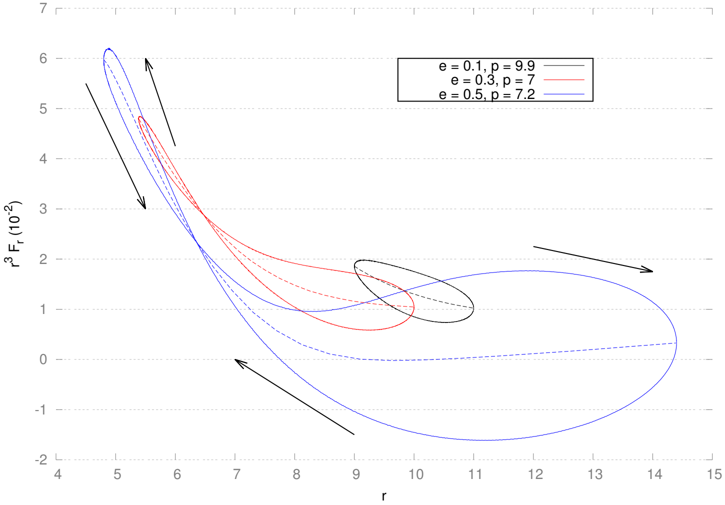
$$F_t^{\text{diss}}(r_o + \Delta r_p) = \frac{1}{2}\left[F_t^{\text{ret}}(r_o + \Delta r_p) + F_t^{\text{ret}}(r_o - \Delta r_p)\right]$$

Self-force loops (F_ϕ)



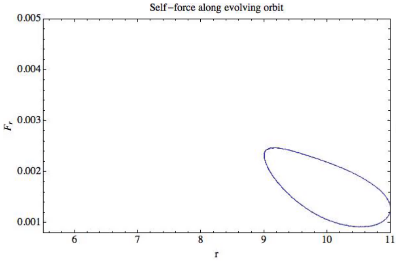
$$F_\phi^{\mathrm{diss}}(r_o + \Delta r_p) = \frac{1}{2}\left[F_\phi^{\mathrm{ret}}(r_o + \Delta r_p) + F_\phi^{\mathrm{ret}}(r_o - \Delta r_p)\right]$$

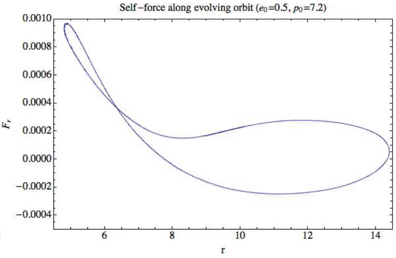
Self-force loops (F_r)



$$F_r^{\text{cons}}(r_o + \Delta r_p) = \frac{1}{2}\left[F_r^{\text{ret}}(r_o + \Delta r_p) + F_r^{\text{ret}}(r_o - \Delta r_p)\right].$$

Self-force loops (movies)





Conclusions and future work

Conclusions

- We get agreement to better than 1% (F_t and F_r) and 0.1% (F_ϕ) for the extracted self-force for eccentric orbits.
- The internal consistency checks for energy and angular momentum losses are good (0.02% for E and 0.015% for L).
- The self-force loops is a new way of plotting self-force data for eccentric orbits, that may help provide physical insights.

Future work.

- In order to compare with the "geodesic evolutions" we need to increase the accuracy by using a smoother effective source (this seems feasible now after recent optimizations to the effective source routine).
- We would probably also have to add the acceleration dependence to the effective source (see Heffernan's talk).
- Generalization to a scalar charge around a Kerr black hole.
- Generalization to the gravitational case.