Gravitational radiation at ultrahigh energies

Dmitry Gal'tsov

Moscow State University

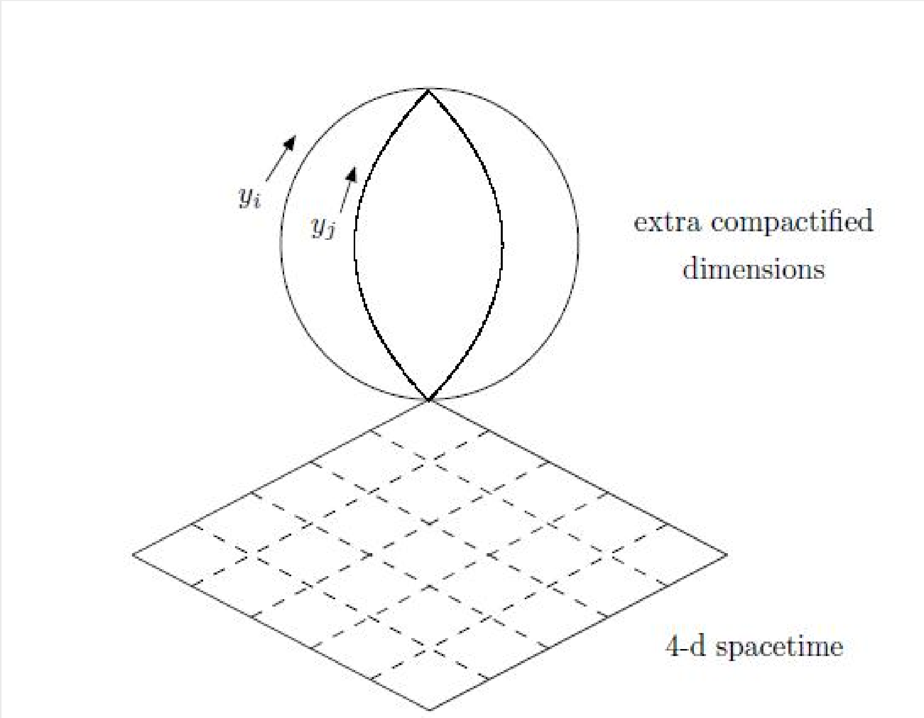
16th Capra Meeting on Radiation Reaction in General Relativity

School of Mathematical Sciences, University College Dublin

15-19 July 2013

Gravity with extra dimensions

- Space-time as a domain wall
  (Akama, Rubakov…)
- String theory motivation: supersymmetry breaking via mesoscopic compactifications
  (Antoniadis, Bachas, Lewellen and Tomaras)
- Solution to the hierarchy problem
- Search for "new physics" at TeV scale



ADD Tev-scale gravity

- Linearized D-dimensional gravity, $g_{MN} = \eta_{MN} + \kappa_D h_{MN}$ matter on the brane

$$\frac{1}{G_D}\int R_D \sqrt{|g_D|}\, d^D x = \frac{V_d}{G_D}\int R_4 \sqrt{-g_4}\, d^4 x$$

$D = 4 + d$

implying $G_D = V_d G_4$ $\quad M_{Pl}^2 = M_*^{d+2} V_d$ $\quad V = (2\pi R)^d$

For $M_* = 1\,\mathrm{TeV}$ size of extra dimensions $l_* = (V_d)^{1/d} \cong 10^{30/d-17}\,\mathrm{cm}$

n	R
1	1.5×10^{13} cm
2	$0.5\,\mathrm{mm} = 1/(10^{-4}\mathrm{eV})$
4	$3 \times 10^{-8}\,\mathrm{mm} = 3/(20\mathrm{KeV})$
6	$10^{-10}\,\mathrm{mm} = 1/(1\mathrm{MeV})$

Mode expansion

$$h_{MN} = \sum_{\vec{n}} \frac{1}{\sqrt{V_d}} h_{MN}^{(\vec{n})}(x) \exp\left(i \frac{\vec{n}\vec{y}}{R} \right)$$

$$(\partial_4^2 + m_{\vec{n}}^2) h_{MN}^{(n)} = 0 \qquad m_{\vec{n}}^2 = \left(\frac{\vec{n}}{R}\right)^2$$

Interaction

weak in each mode $\quad -\frac{\kappa_4}{2}\int \sum_{\vec{n}} h_{MN}^{(\vec{n})} T^{MN}$

Transplanckian scattering

- Planck energy is not the boundary for physics: energies above it are accessible in TeV-gravity, probably already at LHC level
- Since gravitational coupling increases with energy, gravity becomes the dominant force in TP
- It can be shown that TP gravity is well described by classical multidimensional Einstein equations
- For impact parameters less than gravitational radius of would be black hole, these should be created indeed (Penrose, Banks and Fichler, Giddings…)
- Formation of BH solves the problem of perturbative non-unitarity of quantum gravity by classicalization
- What about TP scattering with large impact parameteres when no black hole is formed?

Transplanckian scattering

- For $\sqrt{s} > M_*$ **CM energy exceeds the D-dimensional Planck mass**

Basic process for small impact parameters :

creation of black holes

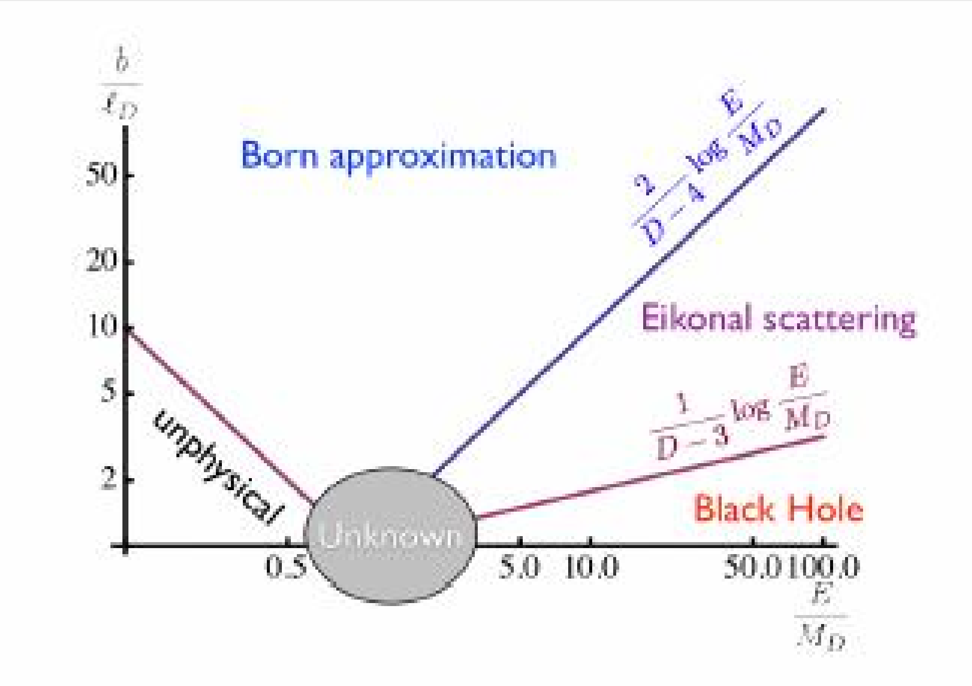
R. Penrose
P.C. Argyres, S. Dimopoulos, and J. March-Russell '98
Banks and Fischler '99
Aref'eva '99
Dimopoulos and Landsberg 2001
Giddings and Thomas

D-dimensional version of Thorne's hoop conjecture:
impact parameter *b* comparable to Schwarzschild radius of the CM energy of colliding particles

$$\boxed{r_S = k_S \left(\frac{G_D \sqrt{s}}{c^4} \right)^{\frac{1}{d+1}}}$$

$$k_S = \frac{1}{\sqrt{\pi}} \left(\frac{8\Gamma\left(\frac{d+3}{2}\right)}{d+2} \right)^{\frac{1}{d+1}}$$

TP scattering phases



- Different phases of gravitational scattering (from Giddings) depending on impact parameteres and energy
- Three TP phases exist: small b – black hole is formed; large b –eikonal phase, then the Born phase
- The black hole region is described classically solving non- unitarity of QG
- The eikonal phase is quasiclassical,

  and also may be well approximated by purely classical theory, but what about the unitarity issue?

At transplanckian energies gravity becomes not only dominant, but classical

De Broglie length, gravitational radius and planck length:

$$\lambda_B = \hbar c/\sqrt{s}$$

$$r_S = \frac{1}{\sqrt{\pi}} \left[\frac{8\Gamma\left(\frac{d+3}{2}\right)}{d+2} \right]^{\frac{1}{d+1}} \left(\frac{G_D\sqrt{s}}{c^4} \right)^{\frac{1}{d+1}}$$

$$l_* = \left(\hbar G_D/c^3\right)^{1/(d+2)} = \hbar/M_* c$$

satisfy classicality condition:

$$\lambda_B \ll l_* \ll r_S$$

if

$$s \gg G_D^{-2/(d+2)} = M_*^2$$

$$G_D = \text{fixed}$$

(Giudice, Rattazzi, Wells
Veneziano,…)

Shock wave as model of ultrarelativistic particle: Aichelburg-Sexl solution

Solution of the linearized gravity = exact solution (boosted Scwarzschild)

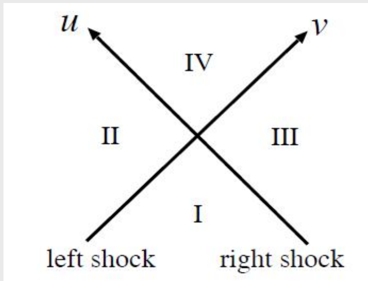
$$ds^2 = -du dv + d\rho^2 + \rho^2 d\Omega^2_{D-3} + \kappa\Phi(\rho)\delta(u)du^2$$

$$\Phi(\rho) = \begin{cases} -2\ln(\rho)\ , & D = 4 \\ \dfrac{2}{(D-4)\rho^{D-4}}\ , & D > 4 \end{cases}$$

$$\kappa \equiv 8\pi G_D \mu / \Omega_{D-3}$$

Non-vacuum: sourced by the particle energy-momentum tensor

Two waves can be superposed in the space-time region before collision



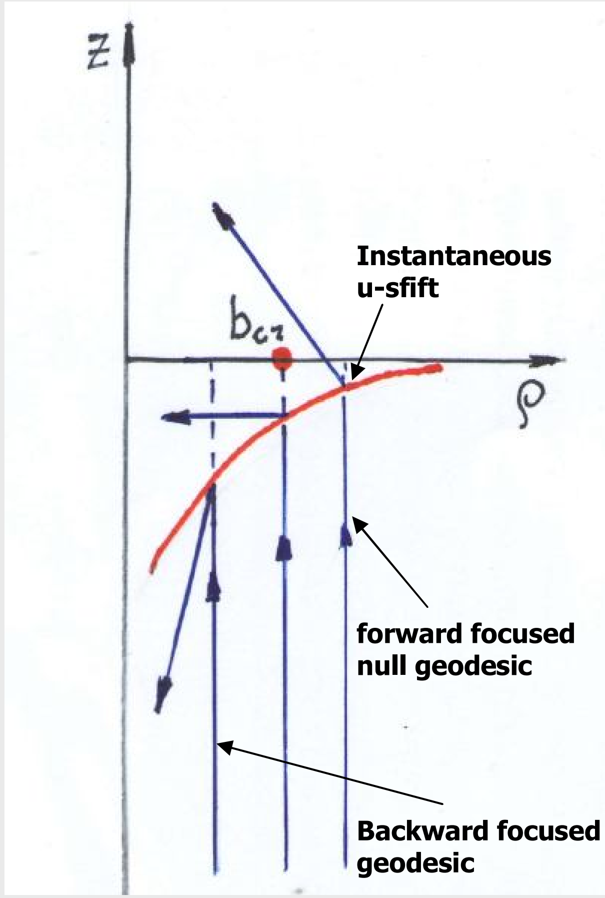
‘t Hooft picture of collision: particle scattered by shock wave

- Red line - instantaneous shift in u=t-z when crossing the wave front propagating in (–z) direction

- Geodesics impinging at impact parameters $b > b_{cr}$ are focused in the forward direction

- Geodesics falling at $b < b_{cr}$ are reflected

- Critical impact parameter b_{cr} marks position of the closed trapped surface in the forward collision of two shocks



Formation of apparent horizon

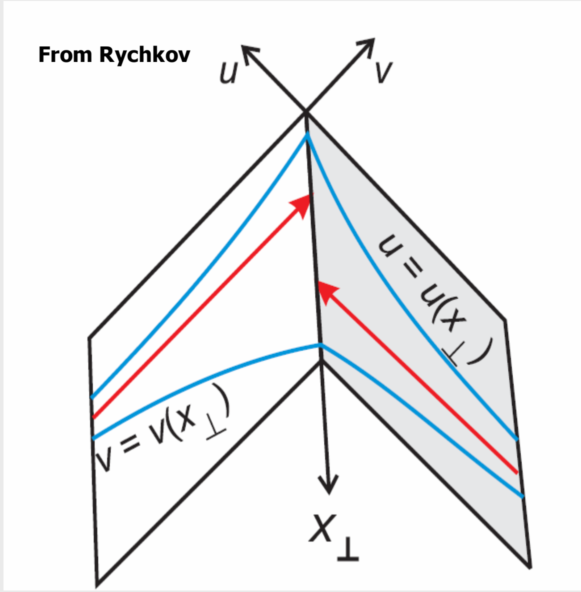
- **Conditions of formation of closed trapped surface**

$$\partial_\perp^2 u = \partial_\perp^2 v = 0$$

with matching on the boundary

$$u|_C = v|_C = 0$$
$$\nabla u \cdot \nabla v|_C = 4$$

Penrose '74,
Eardley and Giddings '02
Yoshino and Nambu '03
Nambu and Rychkov '05
....

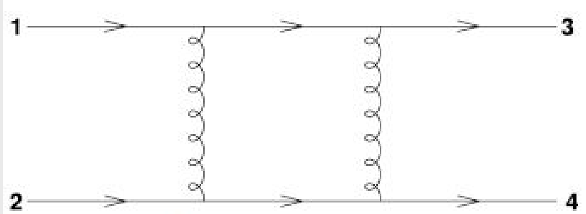


Calculations show that the apparent horizon radius differs from that of CTS by the factor of the order of unity

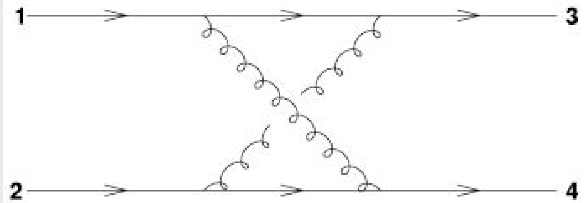
Elastic scattering: eikonalization

- One-graviton exchange amplitude diverges when summed up over KK massive states
- Two one-loop diagrams are finite in SUGRA-s (e.g. N=8)
- Summing up ladder and cross-ladder diagrams one obtains eikonal amplitude for s>>M* and –t/s<<1

$$M_{eik}(s,t) = 2is\int e^{i\mathbf{qb}}\left(1-e^{i\chi(s,t)}\right)d^2\mathbf{b}$$



Two graviton exchange.

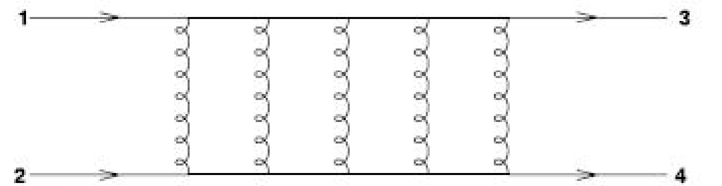


A two-graviton crossed ladder.

where

$$\chi(s,b) = \left(\frac{b_c}{b}\right)^d,$$

$$b_c \equiv \frac{1}{\sqrt{\pi}}\left(\frac{\varkappa_D^2\Gamma(d/2)\,s}{16\pi}\right)^{1/d}.$$



A ladder diagram with multiple graviton exchange.

For $b < r_S$ quantum description (Born), for $r_S < b < b_c$ – eikonal,
for $b > b_c$ plane waves

Remarkably, the eikonal phase is equal to shock wave amplitude up to factor !

$$\Phi(b) = \lambda_B \cdot \chi(s,b)$$

TP bremsstrahlung: methods of computing

Gravitational bremsstrahlung is the second important quasiclassical process inTP region. Various suggested methods include:

- **Estimates based on Hawking entropy** (Penrose, Eardley and Giddings…)

$$\epsilon_{\rm radiated} \leq 1 - \frac{1}{2}\left(\frac{D-2}{2}\frac{\Omega_{D-2}}{\Omega_{D-3}}\right)^{\frac{1}{D-2}}$$

- **Classical calculations using shock waves** (d'Eath '92,…, Herdeiro et al '12)
- **BH perturbations: infall and scattering of test bodies** (too many!)
- **Classical post-linear formalism** (Thorne and Kovacs '77, DG,Grats and Matiukhin '78, DG, Kofinas, PS, Tomaras, 2010,…)
- **Imaginary part of eikonal in string theory** (Amati, Ciafaloni,Veneziano)
- **Furry's picture in shock wave, (quantum)** (Lodone and Rytchkov)
- **Numerical simulations** (Pretorius, Berti et al,…)

Bremsstrahlung via eikonal

In models with extra dimensions eikonal approximation is bound both sides: $r_s < b < b_c$

The real eikonal phase is found form Born amplitude: $$\chi(s,b) = \frac{1}{2s}\int \mathrm{e}^{-i\mathbf{q}\cdot\mathbf{b}} \mathcal{M}_{\mathrm{Born}}(s,t)\,\frac{d^2q}{(2\pi)^2}$$

Classical result (DG Kofinas Spirin Tomaras '09) corresponds to stationary phase point: $$b_s = \left(\frac{db_c^d}{q}\right)^{1/(d+1)}$$

Imaginary part due to bremsstrahlung (ACV) is $$\mathrm{Im}\chi \sim \left(\frac{b_r}{b}\right)^{3d+2}$$

where $\frac{b_r}{r_s} = \left(\frac{b_c}{r_s}\right)^{\frac{d}{3d+2}}$ so that $r_s \ll b \ll b_r$

If interpreted as number of emitted gravitons radiation would be large for b>>r_s

Only if frequencies are bound by $\omega_b = 1/b$ radiation is not catastrophic: $$\epsilon = \frac{\Delta E}{E} \sim \left(\frac{r_s}{b}\right)^{\frac{d}{3d+3}}$$

(Giudice,Ratazzi and Wells)

But classical calculations show that bremsstrahlung spectrum at small angle scattering is dominated by $\omega \gg \omega_b$

Particles falling into black holes

- D=4: Zerilli, Chranowski, Misner,
- Higher D: Cardoso,Lemos….

- Radiation is about 14% in radial infall D=4 increasing up to 40% in higher D

- Radiation grows with non-zero impact parameter being maximal in grazing collisions when particle make revolutions around an unstable photon orbit

- Constant radiation power of GSR implies possibility of large radiation (not fully explored yet)

Continuation of colliding shock wave metrics (D'Eath)

- Metric in future sector of two superposed SW computed perturbatively in the frame where the energy of one wave is much less that another.

- In D=4 extensively studied by D'Eath and Payne '92 for b=0, recently generalized to higher D and b=0 (Herdeiro, Sampiaio, Rebelo)

First order approximation gives bremsstrahlung loss varying from 25% in D=4 to 41,2% in D=10, consistent with entropy bounds. Second order gives about 2/3 of this

SW metric is continued as vacuum solution, no account for the matter source

D-dimensional PLF setting (ADD and Minkowskian) (DG,Kofinas.Spirin.Tomaras)

$$S = -\frac{1}{\kappa_D^2}\int \sqrt{-g}\, R\, d^D x - \sum_a \frac{1}{2}\int \left(e_a\, g_{MN}(z_a)\, \dot{z}_a^M \dot{z}_a^N + \frac{m_a^2}{e_a} \right) d\tau$$

$$g_{MN} = \eta_{MN} + \kappa_D h_{MN}$$

Metric deviation (considered as Minkowski tensor) is further expanded in terms of gravitational coupling

$$h_{MN} = \overset{(1)}{h}_{MN} + \overset{(2)}{h}_{MN} + \ldots$$

Particles world lines are presented similarly

$$z^M(\tau) = \overset{(0)}{z^M} + \overset{(1)}{z^M} + \ldots$$

Post-linear formalism in D=4

- Based on expansion of the metric up to the second order and constructing metric and trajectories by iterations

- Valid for large b, applicability in D=4 restricted by small angle scattering $\theta_s << 1/\gamma$
  (Thorne and Kovacs ‘77, DG, Grats,Matiukhin ’78 also agree with Peters ’70)

Energy loss in the rest frame of one mass $$\Delta E \sim \frac{G^3 M^2 m^2 \gamma^3}{b^3 c^4}$$

PLF valid for arbitrary masses, for $\gamma >> 1$ gives zero efficiency at the limit of applicability! But precise limit on allowed b is not quite clear: no higher order available.

Massless limit puzzling: in the CM frame $$\Delta E \sim \frac{G^3 m^4 \gamma_{cm}^5}{b^3 c^4}$$

In the limit m=0, $\gamma_{CM} >> 1$ and finite $m\gamma_{CM}$ diverges for finite b, though goes to zero at the limit of applicability

Perturbation expansions and iterations

$$G_{MN} = -\frac{\kappa_D}{2}\partial^2 \overset{(1)}{\psi}_{MN} - \frac{\kappa_D^2}{2} S_{MN} + \text{cubic terms}$$

harmonic gauge $\quad \partial^N \overset{(k)}{\psi}_{MN} = 0 \qquad \overset{(k)}{\psi}_{MN} \equiv \overset{(k)}{h}_{MN} - \frac{1}{2}\eta_{MN}\overset{(k)}{h}$

EOMs

$$\partial_D^2\left(\overset{(k)}{h}_{MN} - \frac{1}{2}\eta_{MN}\overset{(k)}{h}\right) = -\kappa_D \overset{(k-1)}{\tau}_{MN}$$

$$\overset{(k)}{\ddot{z}}_M(\tau) = -\kappa_D\left(\overset{(k_1)}{h}_{MN,P} - \frac{1}{2}\overset{k_1}{h}_{NP,M}\right)\overset{(k_2)}{\dot{z}^N}\,\overset{(k_3)}{\dot{z}^P} \qquad k_1 + k_2 + k_3 = k$$

0-th order

$$\overset{(0)}{z^M} = u^M\tau + b^M \qquad \overset{(0)}{z'^M} = u'^M\tau \qquad \overset{(0)}{\tau}_{MN} = \overset{(0)}{T}_{MN} = \sum_a \int e_a \overset{(0)}{\dot{z}^a_M}\,\overset{(0)}{\dot{z}^a_N}\,\delta^D(x - \overset{(0)}{z}_a)\,d\tau$$

lab frame

1-st order

$$\overset{(1)}{h}_{MN} = \sum_a {}_a\overset{(1)}{h}_{MN} \qquad {}_a\overset{(1)}{\ddot{z}}_M(\tau) = -\kappa_D \left({}_{a'}\overset{(1)}{h}_{MN,P} - \frac{1}{2}\, {}_{a'}\overset{(1)}{h}_{NP,M} \right) {}_a\overset{(0)}{\dot{z}}{}^N \; {}_a\overset{(0)}{\dot{z}}{}^P$$

$$\overset{(1)}{\tau}_{MN} = \sum_{a=1,2} {}_a\overset{(1)}{T}_{MN} + S_{MN}$$

2-nd order (radiation)

$$\partial_D^2 \overset{(2)}{\psi}_{MN} = -\kappa_D \left(\overset{(1)}{T}_{MN} + S_{MN} \right)$$

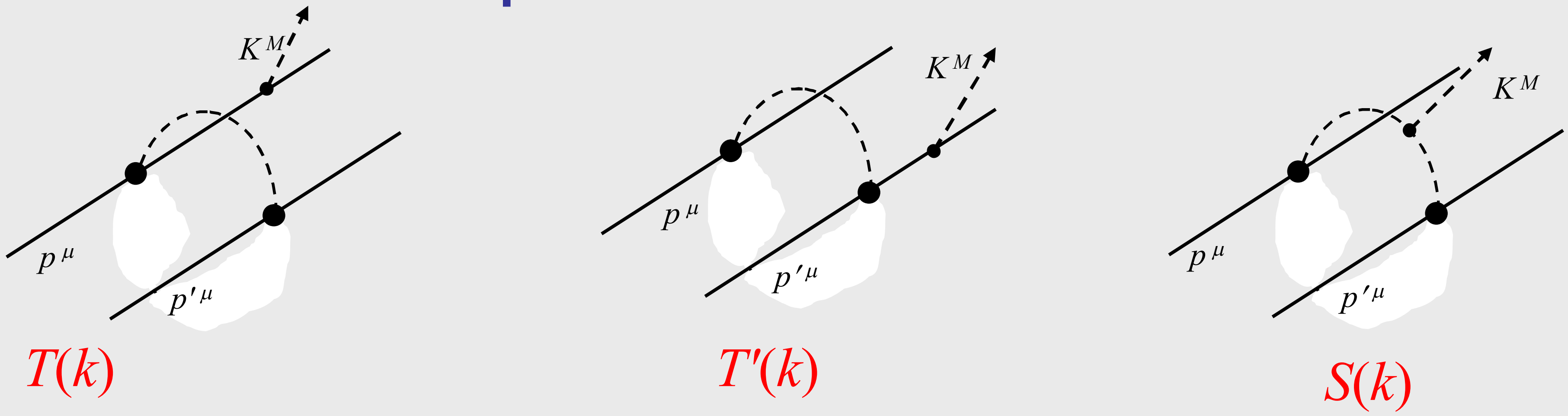
In coordinate space

$$\Delta P_M = -\frac{1}{2} \int \overset{(2)}{h}_{PQ,M} \partial^2 \overset{(2)}{\psi}{}^{PQ} d^D x$$

In momentum space

$$\Delta P^M = \frac{\kappa_D^2}{4(2\pi)^{D-1}} \sum_{\text{pol}} \int |T_D^{(\text{pol})}|^2 k^M \frac{d^{D-1}\mathbf{k}}{k^0}$$

Radiation Amplitudes



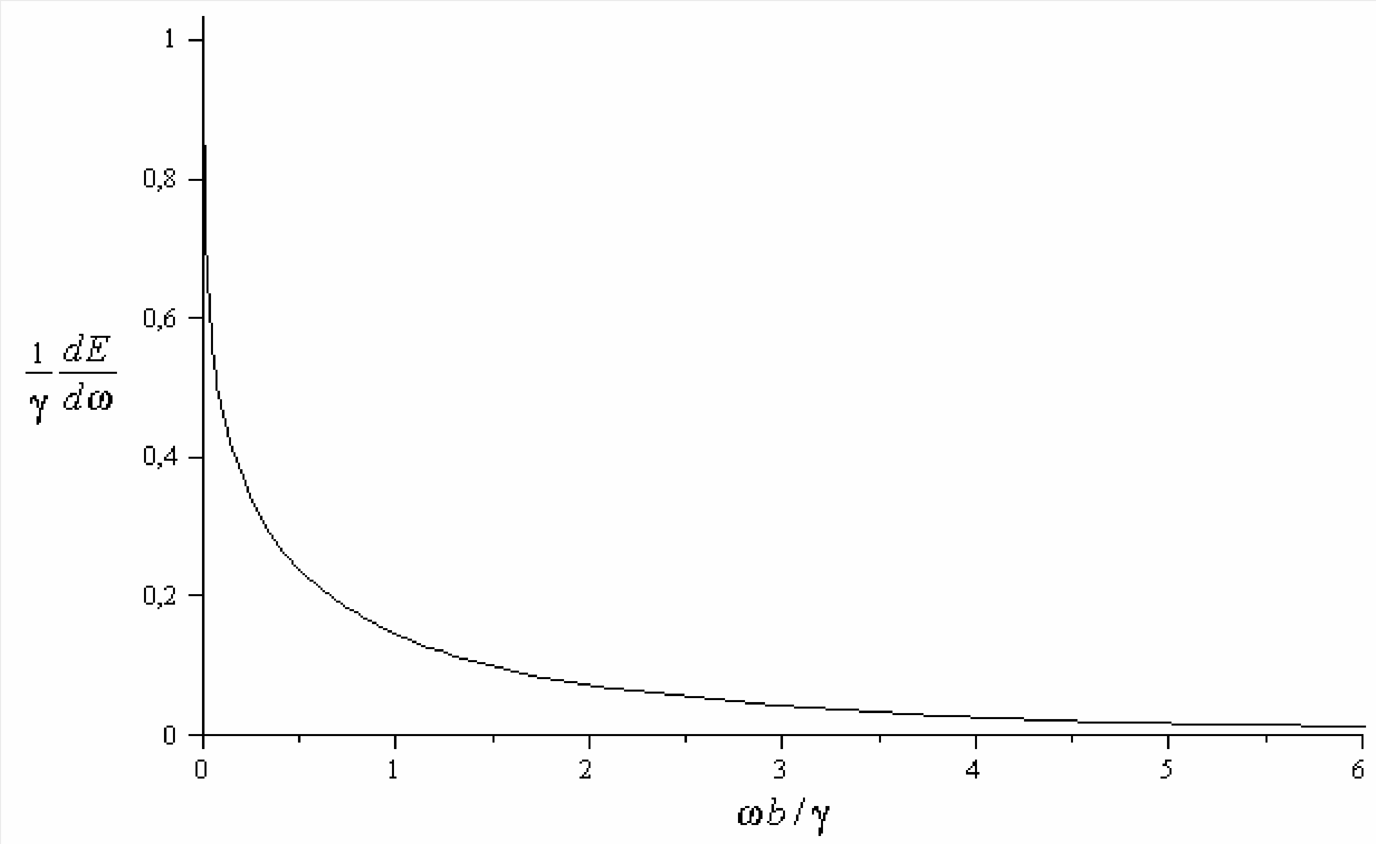
Destructive Interference

$$\omega = \gamma / b \ldots \gamma^2 / b \qquad \vartheta < 1/\gamma$$

$$S = S^z + S^{z'} \qquad S^z \approx -T \qquad S^{z'} \approx -T'$$

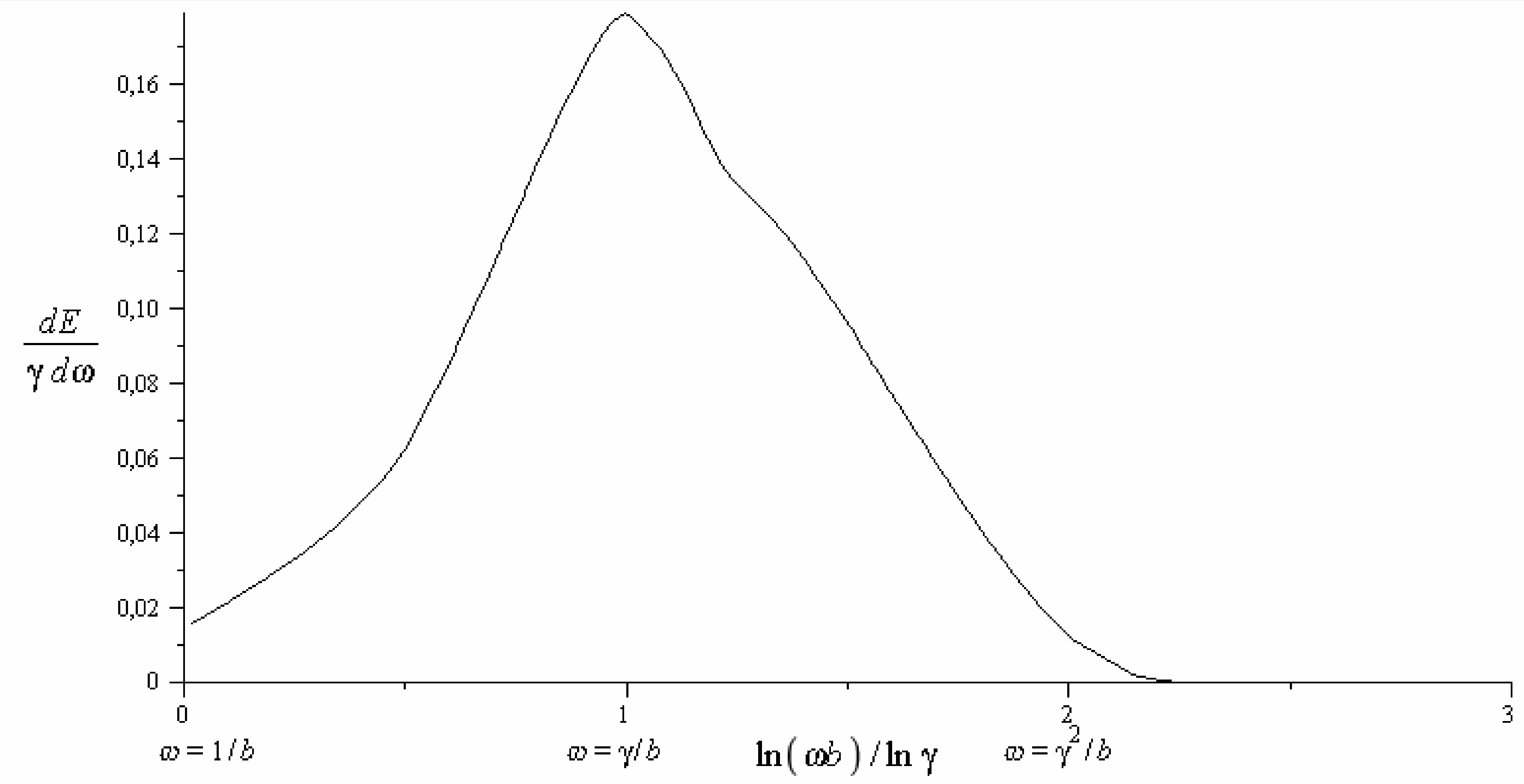
$\vartheta \backslash \omega$	$\omega \sim 1/b$	$\omega \sim \gamma/b$	$\omega \sim \gamma^2/b$
γ^{-1}	no destructive interference $\tau \sim T \gg S$	no destructive interference $S^{[z]} \sim T \sim S^{[z']} \sim \gamma$	destructive interference: $T \approx -S^{[z]}$ $S^{[z']} \sim \exp(-\gamma), \quad \tau = \mathcal{O}(T/\gamma^2) \sim 1/\gamma$
1	no destructive interference $\tau \sim T \sim S$	destructive interference: $S^{[z]} \approx T \sim \exp(-\gamma)$ $\tau = S = S^{[z']} \sim \gamma^{-1}$	destructive interference $T \sim S \sim \tau \sim \exp(-\gamma)$

Frequency distribution in 4D



Angular distribution: beaming at angle $\theta < 1/\gamma$ (along fast-particle's motion direction) for all dimensions

Frequency distribution in 6D in logarithmic scale



Total PLF bremsstrahlung loss

$$E_{\text{rad}} = C_D \frac{(\kappa_D^3 m m')^2}{b^{3d+3}} \begin{cases} \gamma^3, & D=4 \\ \gamma^3 \ln\gamma, & D=5\,, \\ \gamma^{D-2}, & D>5 \end{cases} \qquad C_D \cong 10^{-4}$$

Notice non-universal dependence of Lorentz factor in D<6

For D>5 radiation efficiency is (d=D-4):

$$\epsilon = \frac{E_{rad}}{m\gamma} \sim (r_S/b)^{3(d+1)}\,\gamma^{d-1/2}$$

At minimal allowed impact parameter $b = r_S \gamma^{\frac{1}{2(d+1)}}$

one has $\epsilon \sim \gamma^{d-2}$

becoming catastrophic in dimensions higher than d>2 !!

APPLICABILITY WINDOW (including quantum bounds)

$$\omega \sim \gamma/b \, : \omega \ll m\gamma \to b \gg 1/m$$

$$r_S\,\gamma^\nu \qquad 1/m \qquad \overbrace{\qquad b \qquad}^{} \qquad b_c \qquad R \quad \longrightarrow$$

$$r_S\,\gamma^\nu \ll 1/m \ll b_c$$

SATISFIED in a window depending on s, d, m, M*

e.g. d=2 $M_* \sim 1\,TeV,\ m \sim 100\,GeV,\ \sqrt{s} \sim 10\,TeV$

Conclusions

- TP scattering in ADD with $d>2$ has two main phases: the black hole phase and radiation damping phase, the second being another manifestation of classicalization.
- In both cases unitarity is resolved
- Observationally: either black hole, or nothing
- Cases $d=1$ excluded, $d=2$ problematic

Heun function radial solutions

- D.V. Gal'tsov, A.A. Ershov,
- {\em Exact solutions of Teukolsky and Klein—Gordon radial equations in the class of type--D vacuum metrics},
- Izvestiya VUZ Fizika, {\bf 32} (1989) 13--18, (Sov. Phys. Journ.,
- {\bf 32} N 10 (1990) 764--769).