

REGULARISATION OF THE SELF-FORCE: APPLICATIONS TO KERR SPACETIME AND ACCELERATED MOTION

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UCD, Ireland

Motivation

Gravitational Wave Detection: EMRIs

- Verification of General Relativity in strong regime
- Intermediate mass black holes

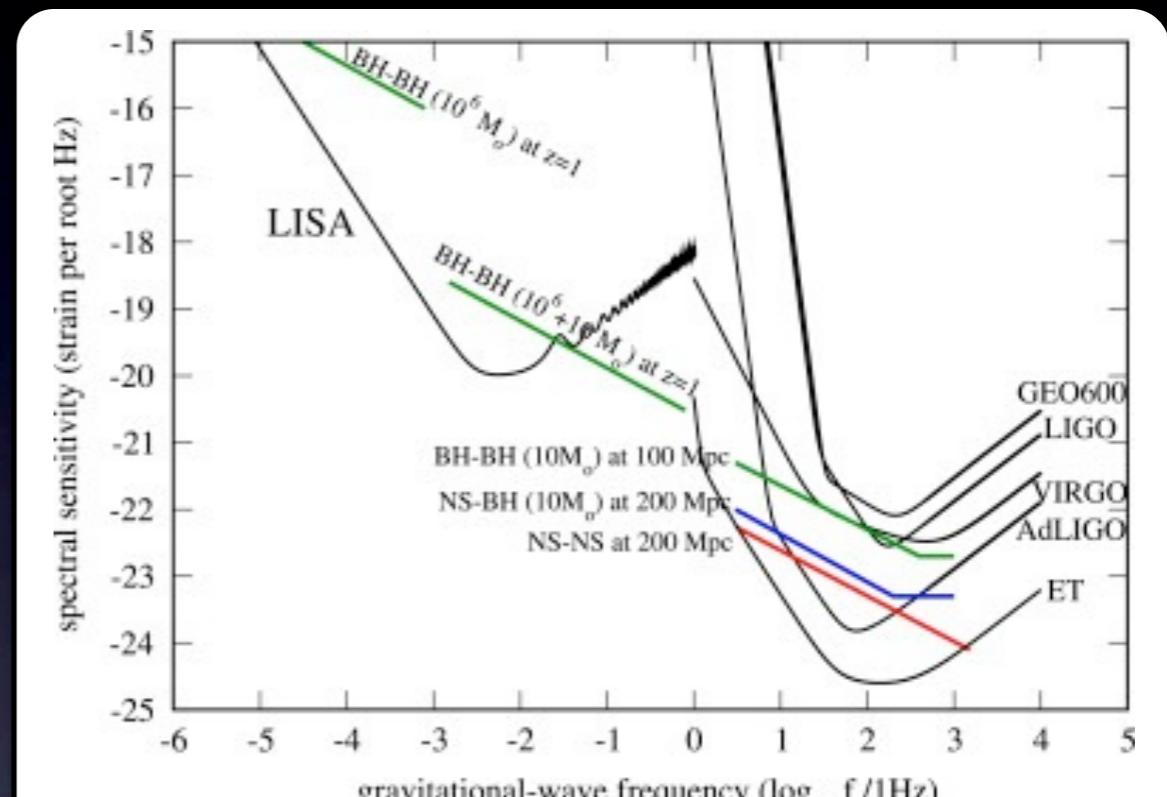
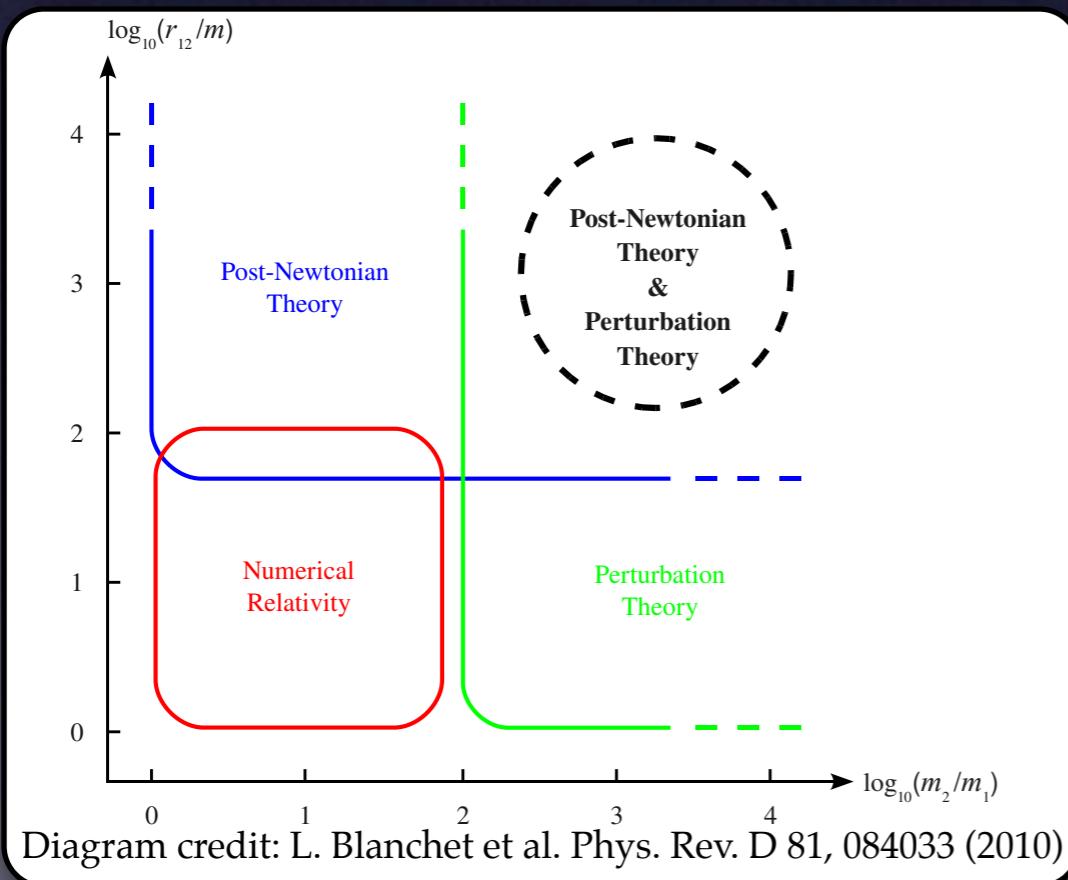


Diagram credit: N. Andersson, Elsevier 66 (2011)

- Hubble Constant
- Galaxy formation
- Galaxy Census

Self-force

Retarded field satisfies

$$\mathcal{D}^A{}_B \varphi^B = -4\pi Q \int u^A \delta_4(x, z(\tau')) d\tau' \quad \text{where} \quad \mathcal{D}^A{}_B = \delta^A{}_B (\square - m^2) - P^A{}_B$$

Regular-singular split: $\psi_R^A = \psi_{ret}^A - \psi_S^A$

Detweiler-Whiting Singular field:

$$\varphi_{(S)}^A = \int_{\tau_{(adv)}}^{\tau_{(ret)}} G_{(S)}{}^A{}_{B'}(x, z(\tau')) u^{B'} d\tau'$$

The self-force: $f^a = p^a{}_A \varphi_{(R)}^A$

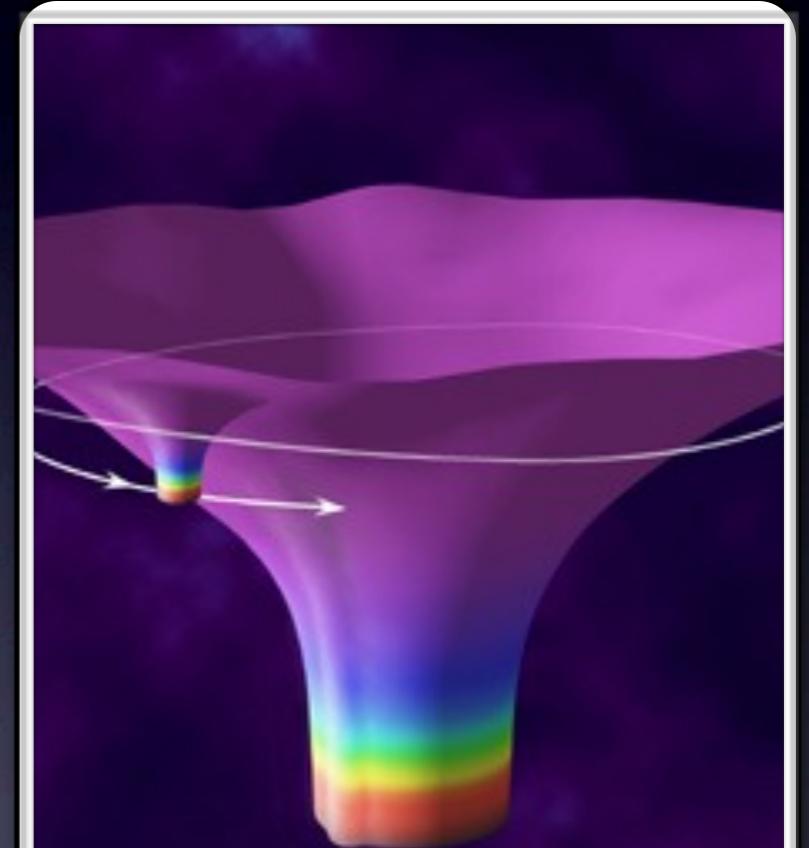


Image Credit: NASA JPL

Detweiler-Whiting Green function:

$$G_{(S)}{}^A{}_{B'}(x, x') = \frac{1}{2} \left\{ U^A{}_{B'}(x, x') \delta[\sigma(x, x')] + V^A{}_{B'}(x, x') \theta[\sigma(x, x')] \right\}$$

Singular Field

The scalar singular field and self-force are

$$\Phi^{(S)}(x) = \left[\frac{U(x, x')}{2\sigma_{a'} u^{a'}} \right]_{x' = x_-}^{x' = x_+} + \int_{\tau_-}^{\tau_+} V(x, z(\tau)) d\tau$$

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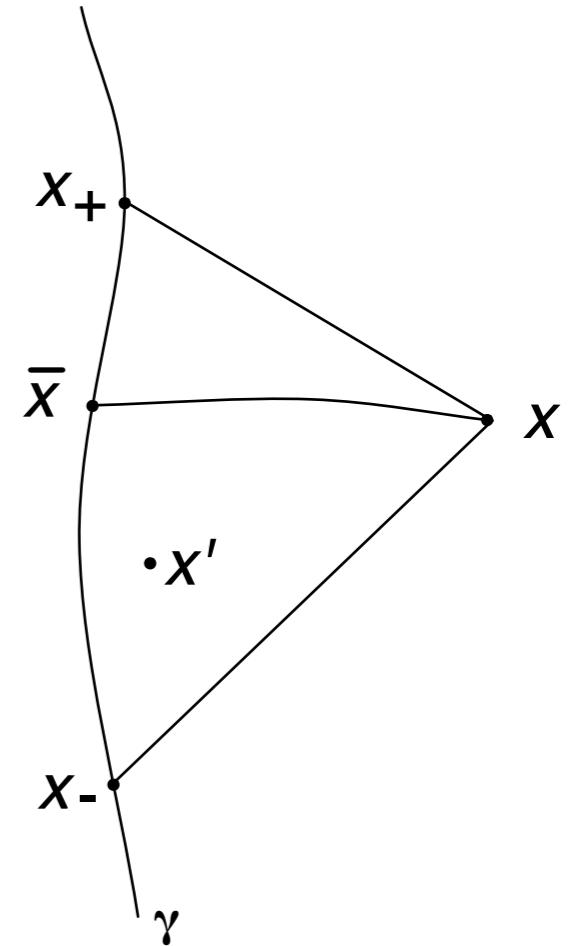
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where

$$k^{abcd} \equiv \frac{1}{2} g^{ad} u^b u^c - g^{ab} u^c u^d - \frac{1}{2} u^a u^b u^c u^d + \frac{1}{4} u^a g^{bc} u^d + \frac{1}{4} g^{ad} g^{bc}.$$



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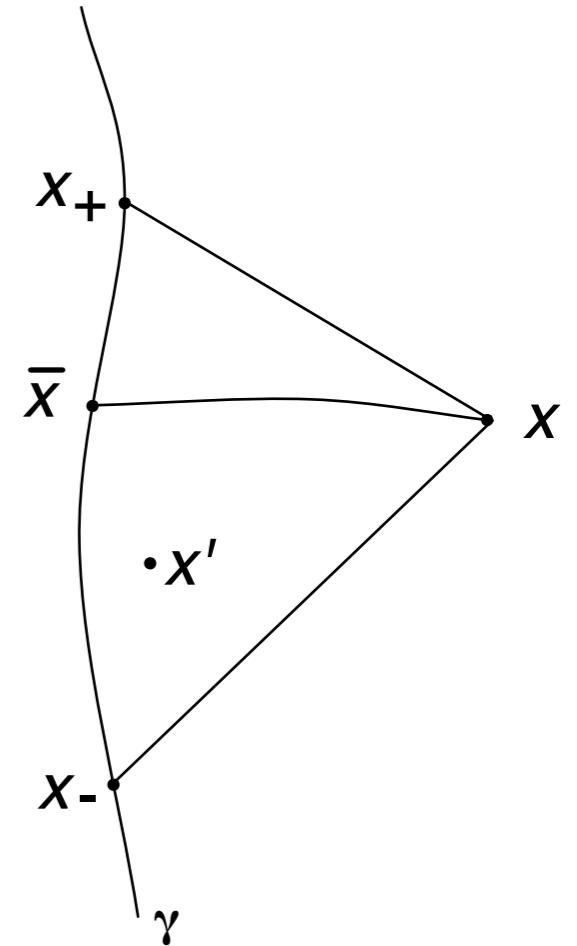
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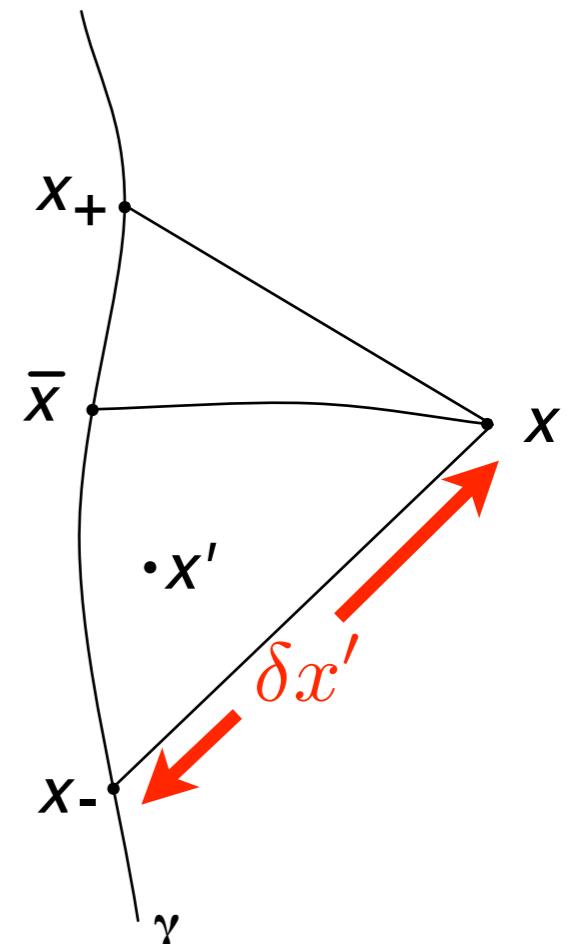
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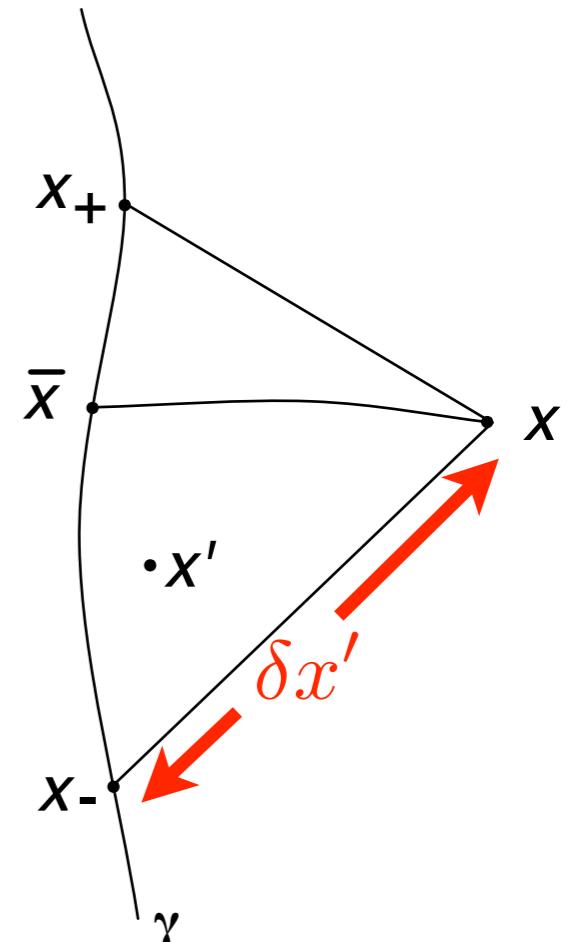
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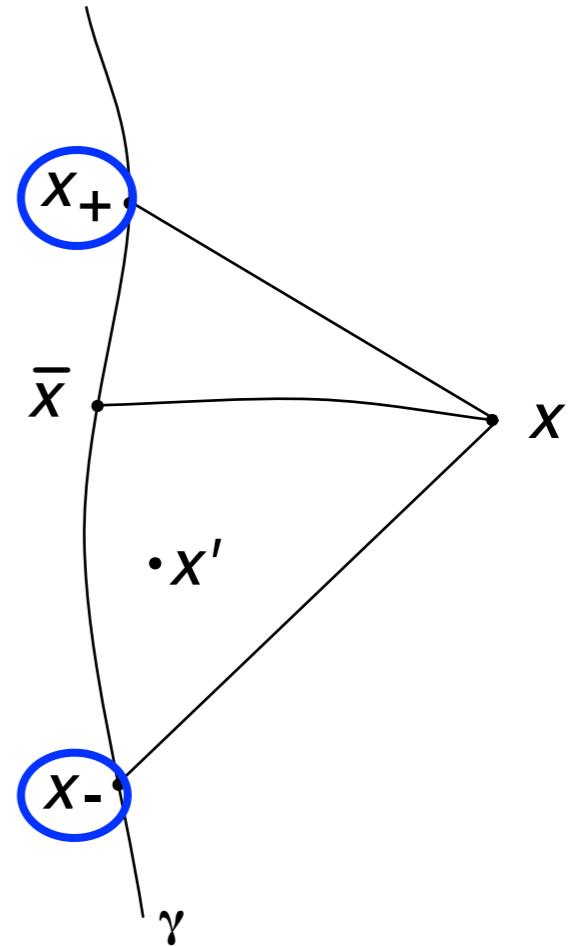
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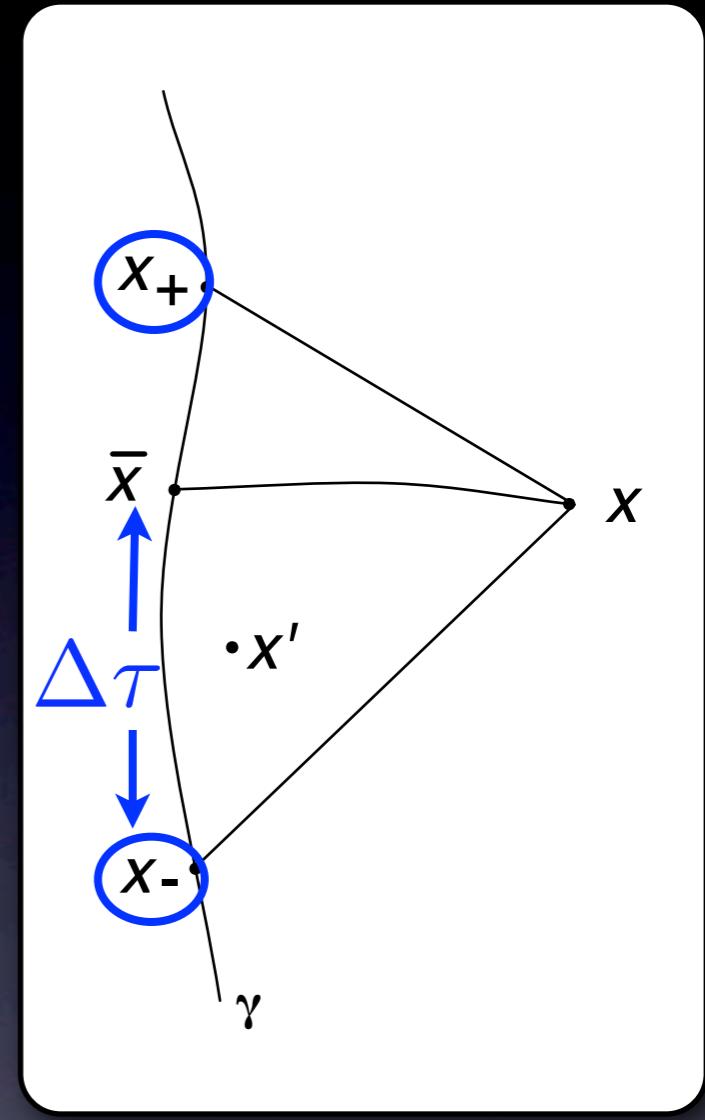
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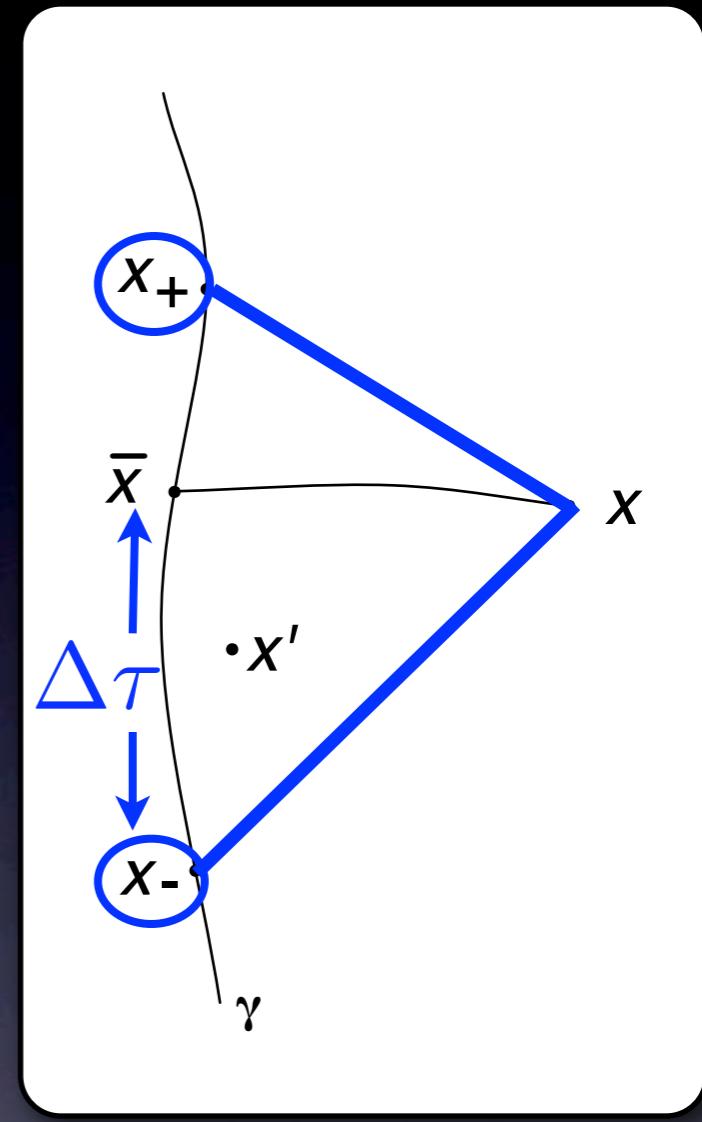
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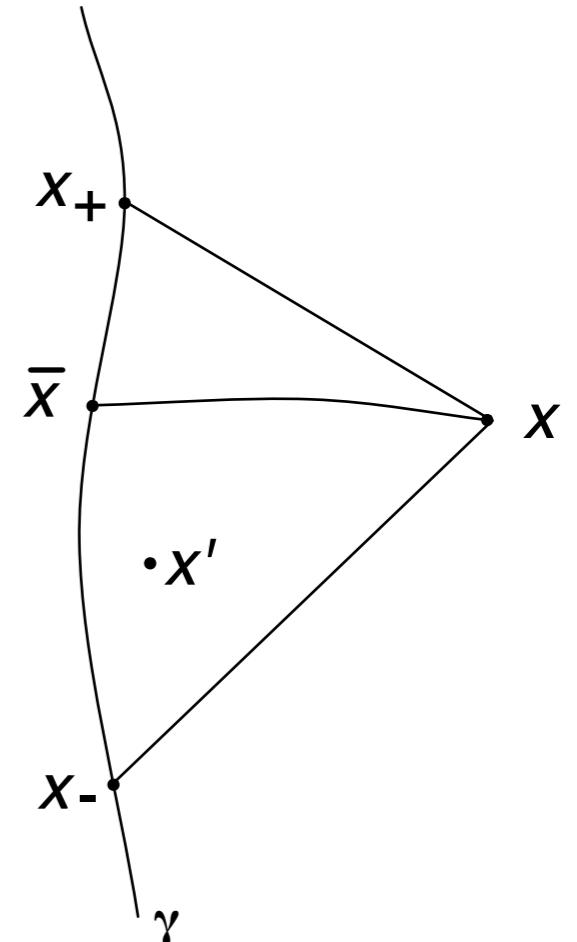
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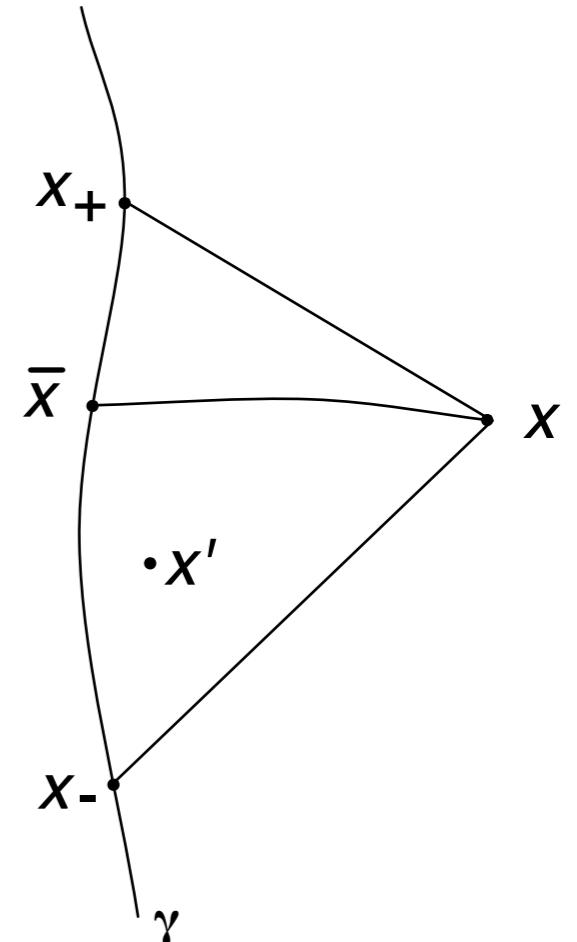
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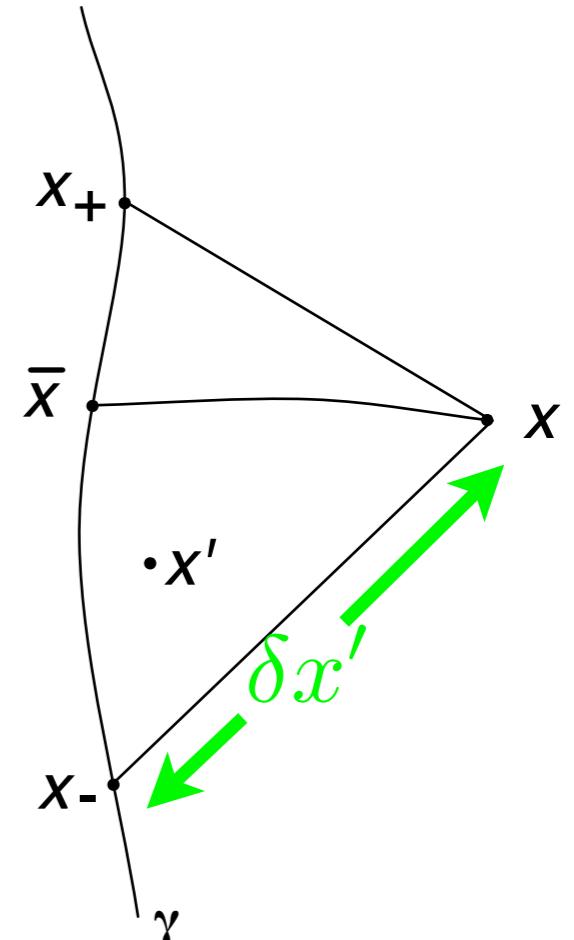
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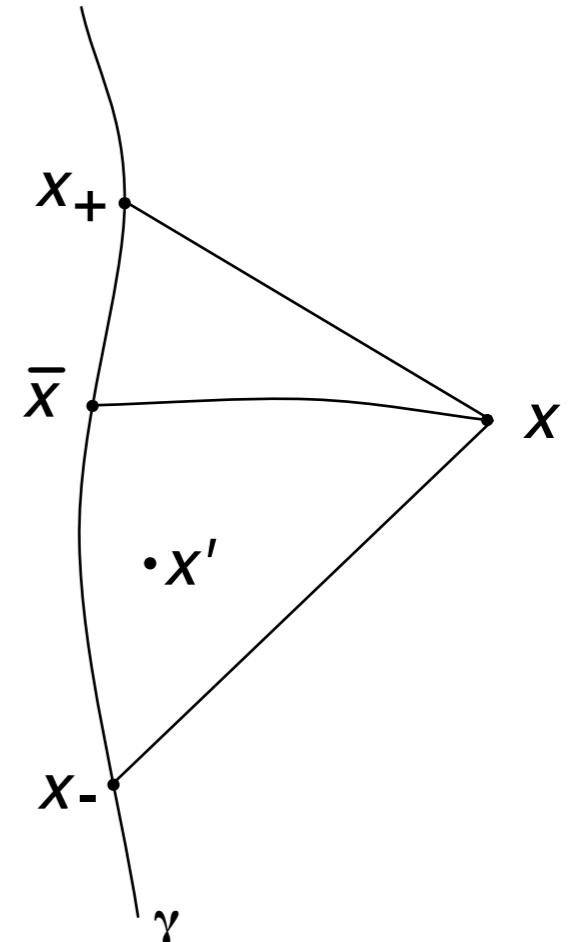
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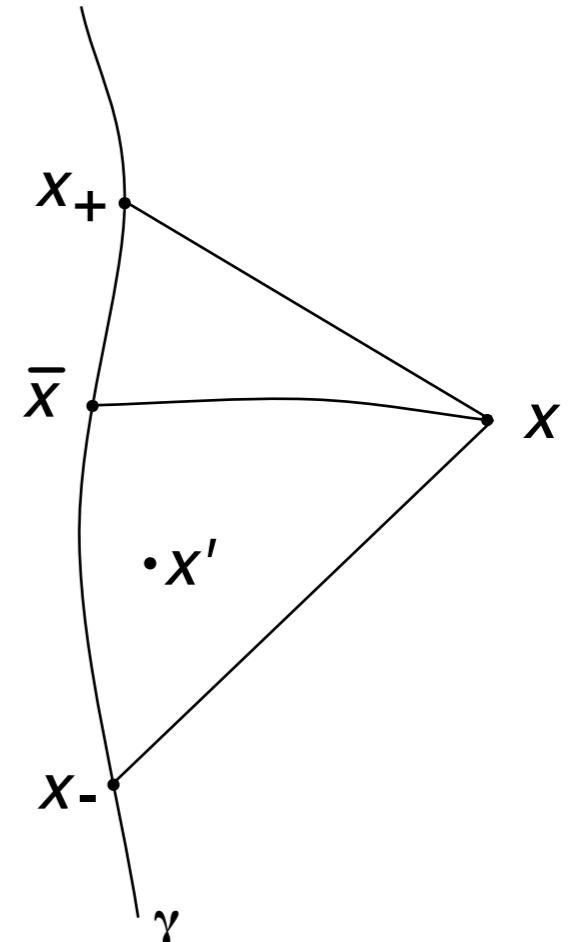
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$$f^a = g^{ab} u^c A^{(R)}_{[c,b]} \cdot V^{AB'}(x, x') = \sum_{n=0}^{\infty} V_n{}^{AB'}(x, x') \sigma^n(x, x')$$



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$$\sigma^{;\alpha'} (\Delta^{-1/2} V_n{}^{AB'})_{;\alpha'} + (n+1) \Delta^{-1/2} V_n{}^{AB'} + \frac{1}{2n} \Delta^{-1/2} \mathcal{D}^{B'}{}_{C'} V_{n-1}{}^{AC'} = 0$$

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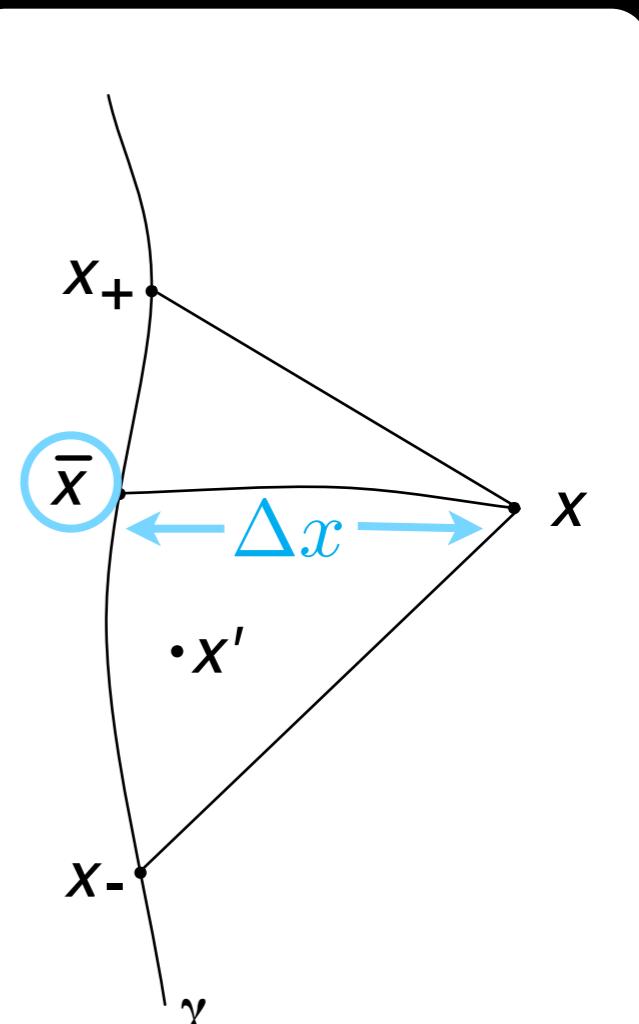
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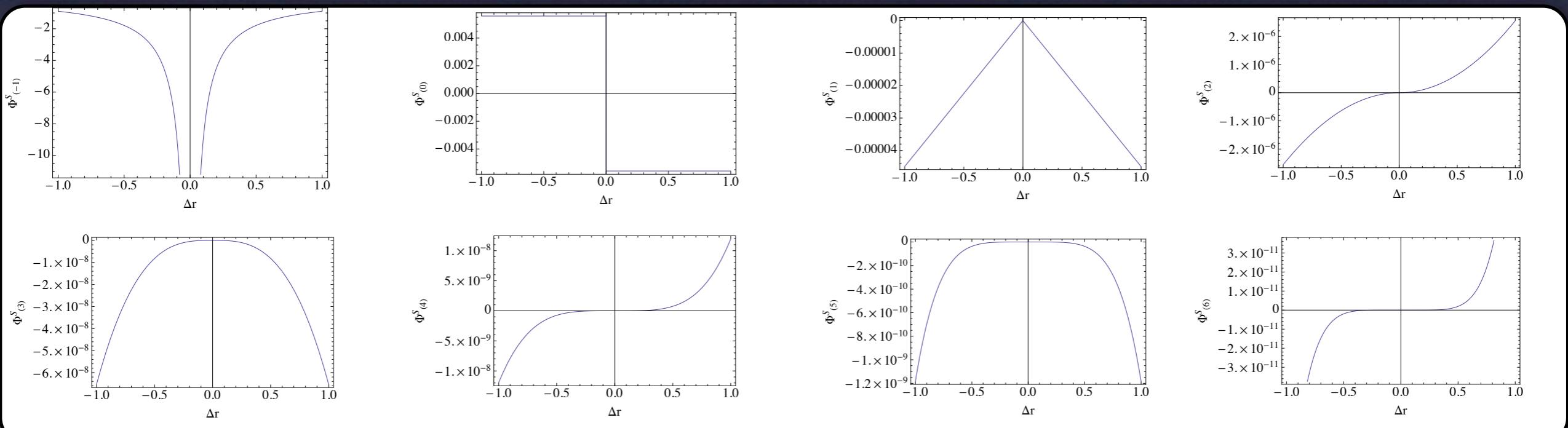
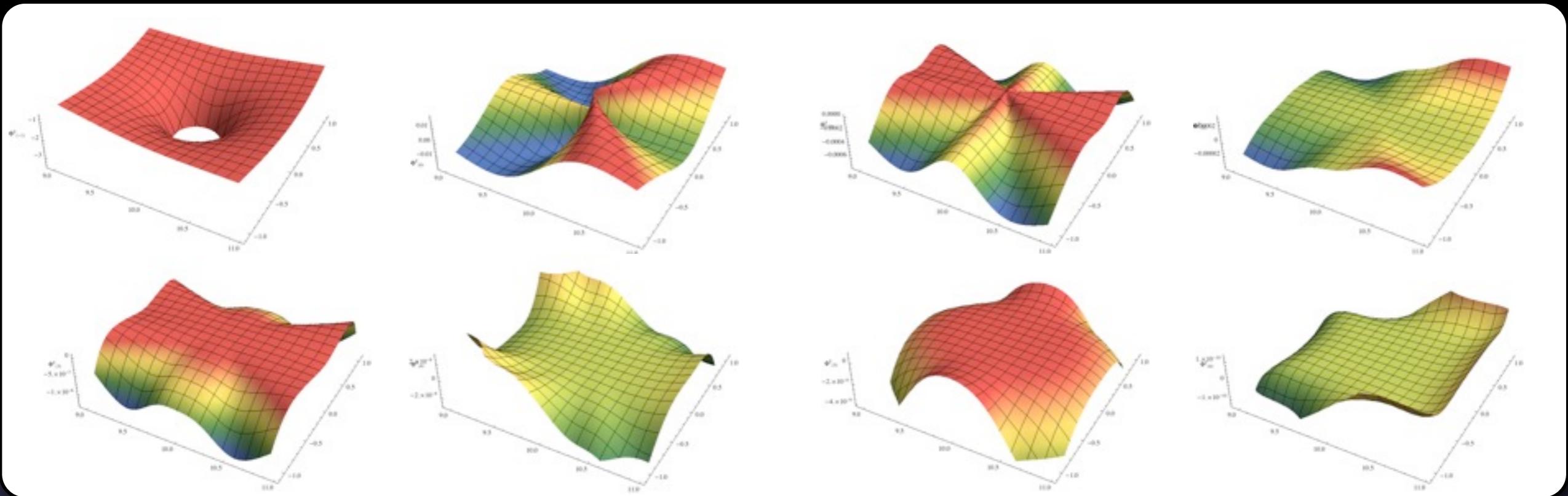
$$\bar{h}_{ab}^S = \left[\frac{u^{a'} u^{b'} U^{ab}{}_{a'b'}(x, x')}{2\sigma_{a'} u^{a'}} \right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V^{ab}{}_{a'b'}(x, z(\tau)) u^{a'} u^{b'} d\tau \quad \text{and} \quad f^a = k^{abcd} \bar{h}_{bc;d}^{(R)},$$

where

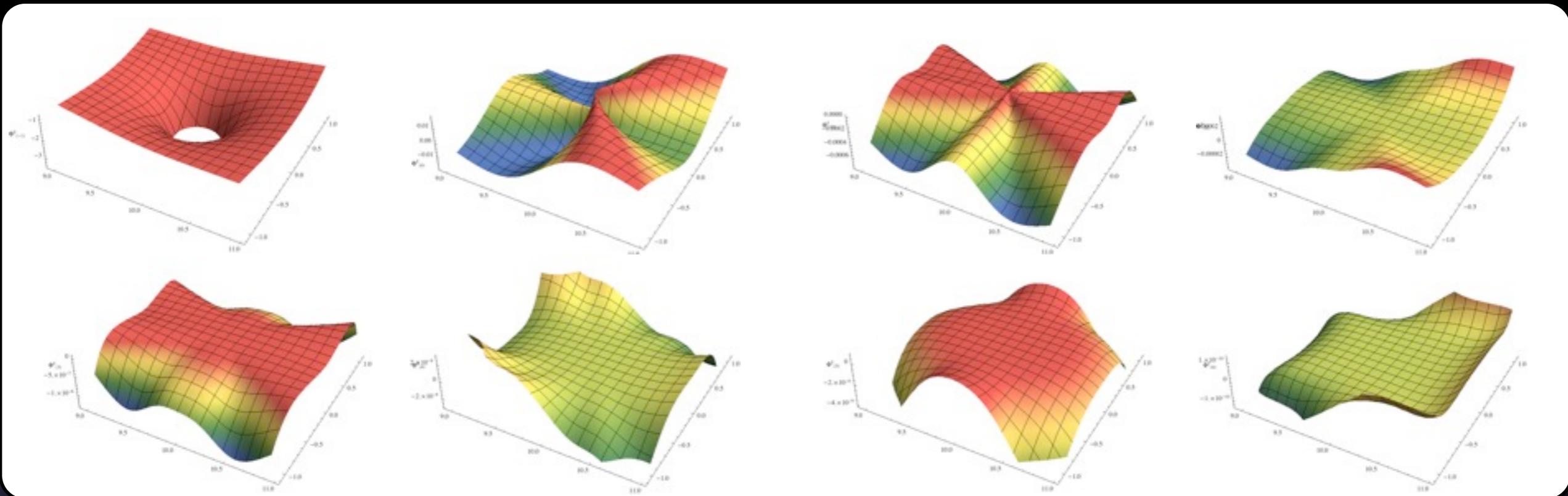
$$k^{abcd} \equiv \frac{1}{2} g^{ad} u^b u^c - g^{ab} u^c u^d - \frac{1}{2} u^a u^b u^c u^d + \frac{1}{4} u^a g^{bc} u^d + \frac{1}{4} g^{ad} g^{bc}.$$



Singular Field

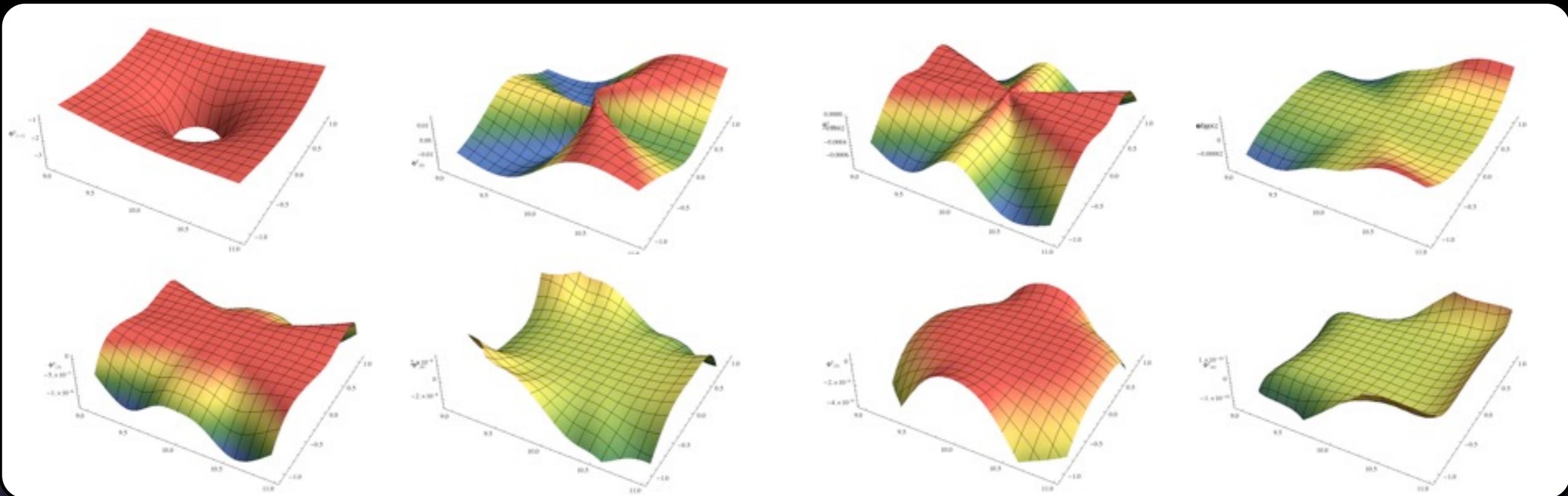


Singular Field



Regular-singular split: $\psi_R^A = \psi_{ret}^A - \psi_S^A$

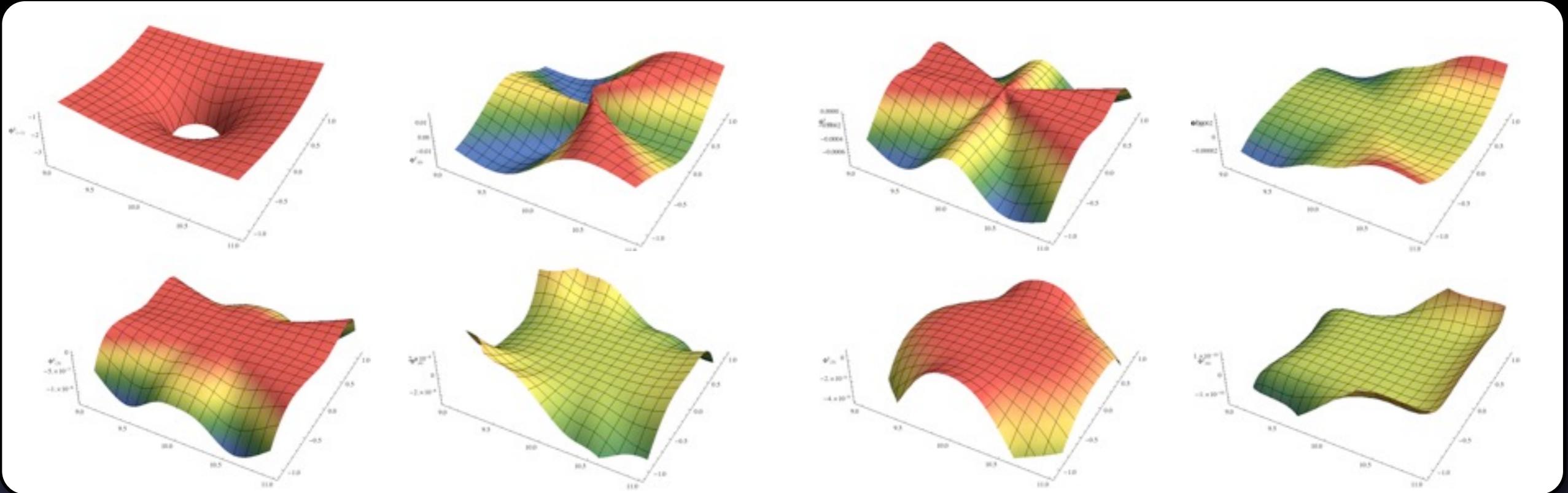
Singular Field



Regular-singular split: $\psi_R^A = \psi_{ret}^A - \psi_S^A$

Matched Expansions: Poisson & Wiseman, Casals et al.

Singular Field

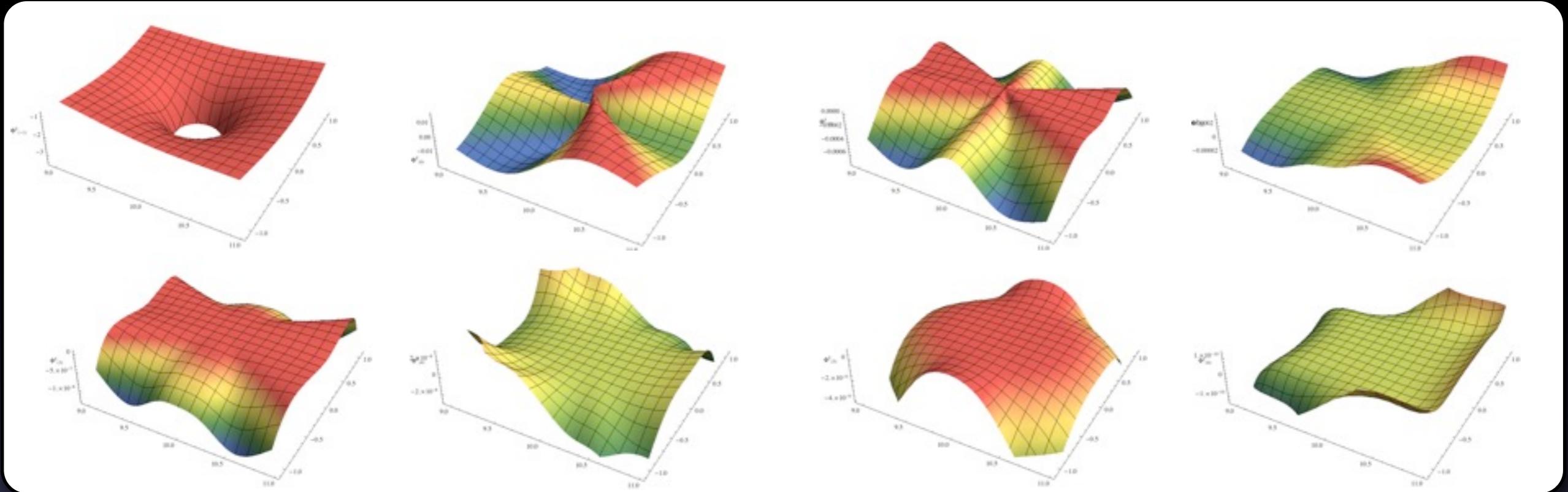


Regular-singular split: $\psi_R^A = \psi_{ret}^A - \psi_S^A$

Matched Expansions: Poisson & Wiseman, Casals et al.

Mode-Sum: Barack & Ori, Sago, Warburton, Shah, Friedman et al.

Singular Field



Regular-singular split: $\psi_R^A = \psi_{ret}^A - \psi_S^A$

Matched Expansions: Poisson & Wiseman, Casals et al.

Mode-Sum: Barack & Ori, Sago, Warburton, Shah, Friedman et al.

Effective Source: Barack & Golbourn, Detweiler & Vega, Barack & Dolan

Mode Sum

$$\begin{aligned}
 f_a^l(r_0, t_0) &= \lim_{\Delta r \rightarrow 0} \sum_m f_a^{lm}(r_0 + \Delta r, t_0) Y^{lm}(\alpha_0, \beta_0) \\
 &= \frac{2l+1}{4\pi} \lim_{\Delta r \rightarrow 0} \int f_a(r_0 + \Delta r, t_0, \alpha, \beta) P_l(\cos \alpha) d\Omega
 \end{aligned}$$

$$\begin{aligned}
 \sin \theta \cos \phi &= \cos \alpha \\
 \sin \theta \sin \phi &= \sin \alpha \sin \beta \\
 \cos \theta &= \sin \alpha \cos \beta
 \end{aligned}$$

Singular field contribution:

where $\mathcal{B}_a^{(k)} = b_{a_1 a_2 \dots a_k}(\bar{x}) \Delta x^{a_1} \Delta x^{a_2} \dots \Delta x^{a_k}$

$$f_a(r, t, \alpha, \beta) = \sum_{n=1} \frac{\mathcal{B}_a^{(3n-2)}}{\rho^{2n+1}} \epsilon^{n-3}$$

$$\begin{aligned}
 \rho(r, t_0, \alpha, \beta)^2 &= (g_{\bar{a}\bar{b}} u^{\bar{b}} \Delta x^b)^2 + g_{\bar{a}\bar{b}} \Delta x^a \Delta x^b \\
 &= \frac{r_0 [E r_0 (a^2 + r_0^2) + 2aM(aE - L)]^2}{[r_0 (L^2 + a^2) + 2a^2 M + r_0^3] (r_0^2 - 2Mr_0 + a^2)^2} \Delta r^2 + r_0^2 \Delta w_2^2 \\
 &\quad + \left(L^2 + r_0^2 + a^2 + \frac{2a^2 M}{r_0} \right) \left[\Delta w_1 + \frac{L r_0^3 \dot{r}_0}{(r_0^2 - 2Mr_0 + a^2) (2a^2 M + a^2 r_0 + L^2 r_0 + r_0^3)} \Delta r \right]^2
 \end{aligned}$$

$$\Delta w_1 \rightarrow \Delta w_1 + \mu \Delta r \Rightarrow \rho(r, t_0, \alpha, \beta) = \nu^2 \Delta r + \zeta^2 \Delta w_1^2 + r_0^2 \Delta w_2^2$$

$$F_{a[-1]}^l(r_0, t_0) = (l + 1/2) \frac{b_{a_r} sgn(\Delta r)}{\zeta \nu r_0}$$

Mode Sum

$$\begin{aligned}
f_a^l(r_0, t_0) &= \lim_{\Delta r \rightarrow 0} \sum_m f_a^{lm}(r_0 + \Delta r, t_0) Y^{lm}(\alpha_0, \beta_0) \\
&= \frac{2l+1}{4\pi} \lim_{\Delta r \rightarrow 0} \int f_a(r_0 + \Delta r, t_0, \alpha, \beta) P_l(\cos \alpha) d\Omega
\end{aligned}$$

$$w_1 = 2 \sin\left(\frac{\alpha}{2}\right) \cos \beta$$

$$w_2 = 2 \sin\left(\frac{\alpha}{2}\right) \sin \beta$$

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&\quad + \left(L^2 + r_0^2 + a^2 + \frac{2a^2 M}{r_0} \right) \left[\Delta w_1 + \frac{L r_0^3 \dot{r}_0}{(r_0^2 - 2Mr_0 + a^2) (2a^2 M + a^2 r_0 + L^2 r_0 + r_0^3)} \Delta r \right]^2
\end{aligned}$$

$$\Delta w_1 \rightarrow \Delta w_1 + \mu \Delta r \Rightarrow \rho(r, t_0, \alpha, \beta) = \nu^2 \Delta r + \zeta^2 \Delta w_1^2 + r_0^2 \Delta w_2^2$$

$$F_{a[-1]}^l(r_0, t_0) = (l + 1/2) \frac{b_{a_r} sgn(\Delta r)}{\zeta \nu r_0}$$

Mode Sum

$$f_a^l(r_0, t_0) = \frac{2l+1}{4\pi} \left[\epsilon^{-2} \lim_{\Delta r \rightarrow 0} \int \frac{B_a^{(1)}}{\rho^3} P_l(\cos \alpha) d\Omega + \epsilon^{n-3} \sum_{n=2} \int \rho_0^{n-3} c_{a(n)}(r_0, \beta) P_l(\cos \alpha) d\Omega \right]$$

$$w_1 = 2 \sin\left(\frac{\alpha}{2}\right) \cos \beta$$

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$$\begin{aligned} \rho(r, t_0, \alpha, \beta)^2 &= (g_{\bar{a}\bar{b}} u^{\bar{b}} \Delta x^b)^2 + g_{\bar{a}\bar{b}} \Delta x^a \Delta x^b \\ &= \frac{r_0 [E r_0 (a^2 + r_0^2) + 2aM(aE - L)]^2}{[r_0 (L^2 + a^2) + 2a^2 M + r_0^3] (r_0^2 - 2Mr_0 + a^2)^2} \Delta r^2 + r_0^2 \Delta w_2^2 \\ &\quad + \left(L^2 + r_0^2 + a^2 + \frac{2a^2 M}{r_0} \right) \left[\Delta w_1 + \frac{L r_0^3 \dot{r}_0}{(r_0^2 - 2Mr_0 + a^2) (2a^2 M + a^2 r_0 + L^2 r_0 + r_0^3)} \Delta r \right]^2 \end{aligned}$$

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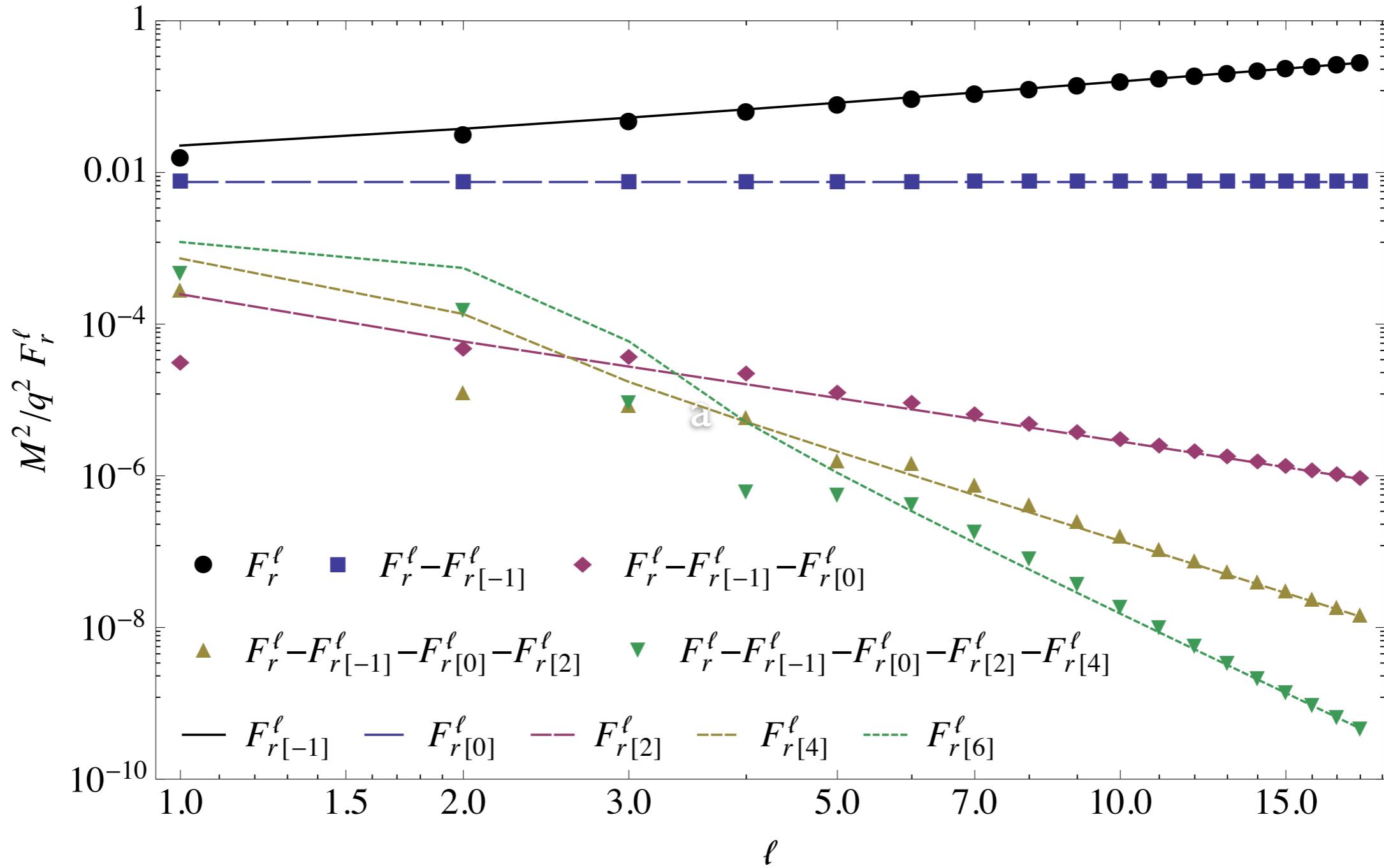
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$$F_{a[-1]}^l(r_0, t_0) = (l + 1/2) \frac{b_{a_r} sgn(\Delta r)}{\zeta \nu r_0}$$

Kerr Scalar

ℓ -modes of the scalar self-force



Data supplied by Niels Warburton

Accelerated Motion

Schwarzschild (geodesic):

$$\rho(r, t_0, \alpha, \beta)^2 = \frac{E^2 r_0^4}{(L^2 + r_0^2)(r_0 - 2M)^2} \Delta r^2 + (L^2 + r_0^2) \left(\Delta w_1 + \frac{L r_0 \dot{r}_0}{(r_0 - 2M)(L^2 + r_0^2)} \Delta r \right)^2 + r_0^2 \Delta w_2^2$$

f(r) metric (non-geodesic):

$$\rho(r, t_0, \alpha, \beta)^2 = \frac{\dot{t}_0^2}{1 + r_0^2 \dot{\phi}_0^2} \Delta r^2 + \left(r_0^2 + r_0^4 \dot{\phi}_0^2 \right) \left[\Delta w_1 + \frac{L r_0 \dot{\phi}_0}{f(r_0) (1 + r_0^2 \dot{\phi}_0)} \Delta r \right]^2 + r_0^2 \Delta w_2^2$$

Reissner-Nordstrom:

$$\begin{aligned} \rho(r, t_0, \alpha, \beta)^2 &= \frac{r_0^4 (qQ - Er_0)^2}{(L^2 + r_0^2)(r_0^2 - 2Mr_0 + Q^2)^2} \Delta r^2 + r_0^2 \Delta w_2^2 \\ &\quad + (L^2 + r_0^2) \left[\Delta w_1 + \frac{L r_0^2 \dot{r}_0}{(Q^2 - 2Mr_0 + a^2)(L^2 + r_0^2)} \Delta r \right]^2 \end{aligned}$$

Accelerated Motion

Schwarzschild (geodesic):

$$\rho(r, t_0, \alpha, \beta)^2 = \frac{E^2 r_0^4}{(L^2 + r_0^2)(r_0 - 2M)^2} \zeta^2 \nu^2 + (L^2 + r_0^2) \left(\Delta w_1 + \frac{-\mu}{(r_0 - 2M)(L^2 + r_0^2)} \Delta r \right)^2 + r_0^2 \Delta w_2^2$$

f(r) metric (non-geodesic):

$$\rho(r, t_0, \alpha, \beta)^2 = \frac{\dot{t}_0^2}{1 + r_0^2 \dot{\phi}_0^2} \zeta^2 \nu^2 + (r_0^2 + r_0^4 \dot{\phi}_0^2) \left[\Delta w_1 + \frac{-\mu}{f(r_0)(1 + r_0^2 \dot{\phi}_0)} \Delta r \right]^2 + r_0^2 \Delta w_2^2$$

Reissner-Nordstrom:

$$\begin{aligned} \rho(r, t_0, \alpha, \beta)^2 = & \frac{r_0^4 (qQ - Er_0)^2}{(L^2 + r_0^2)(r_0^2 - 2Mr_0 + Q^2)^2} \zeta^2 \nu^2 + r_0^2 \Delta w_2^2 \\ & + (L^2 + r_0^2) \left[\Delta w_1 + \frac{-\mu}{(Q^2 - 2Mr_0 + a^2)(L^2 + r_0^2)} \Delta r \right]^2 \end{aligned}$$

Accelerated Motion

Accelerated motion is a type of motion where an object's velocity changes over time. This change in velocity can be either positive (increasing speed) or negative (decreasing speed). Acceleration is defined as the rate of change of velocity with respect to time. In other words, it measures how quickly an object's speed is increasing or decreasing.

The formula for calculating acceleration is:

$$a = \frac{\Delta v}{\Delta t}$$

where a is acceleration, Δv is the change in velocity, and Δt is the change in time. Acceleration is a vector quantity, meaning it has both magnitude and direction. The direction of acceleration is always in the same direction as the change in velocity.

There are three types of accelerated motion:

- Uniformly Accelerated Motion:** This occurs when an object's velocity changes at a constant rate over time. The acceleration is constant and acts in the same direction as the change in velocity.
- Non-uniformly Accelerated Motion:** This occurs when an object's velocity changes at a non-constant rate over time. The acceleration is not constant and may even change direction.
- Free Fall:** This is a specific type of uniformly accelerated motion where an object falls under the influence of gravity. The acceleration is constant and acts vertically downwards.

Accelerated Motion

- $F(r)$ metric accelerated motion: $F_{a[0]}^l(r_0, t_0)$

Accelerated Motion

- $F(r)$ metric accelerated motion: $F_{a[0]}^l(r_0, t_0)$
- $F(r)$ metric radial infall: $F_{a[4]}^l(r_0, t_0)$

Accelerated Motion

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- Schwarzschild accelerated motion: $F_{a[2]}^l(r_0, t_0)$

Accelerated Motion

- $F(r)$ metric accelerated motion: $F_{a[0]}^l(r_0, t_0)$
- $F(r)$ metric radial infall: $F_{a[4]}^l(r_0, t_0)$
- Schwarzschild accelerated motion: $F_{a[2]}^l(r_0, t_0)$
- Reissner-Nordstrom charged particle: $F_{a[4]}^l(r_0, t_0)$

Effective Source

Splitting the retarded field into approximate singular and regularized parts

$$\varphi_{(\text{ret})}^A = \tilde{\varphi}_{(\text{S})}^A + \tilde{\varphi}_{(\text{R})}^A$$

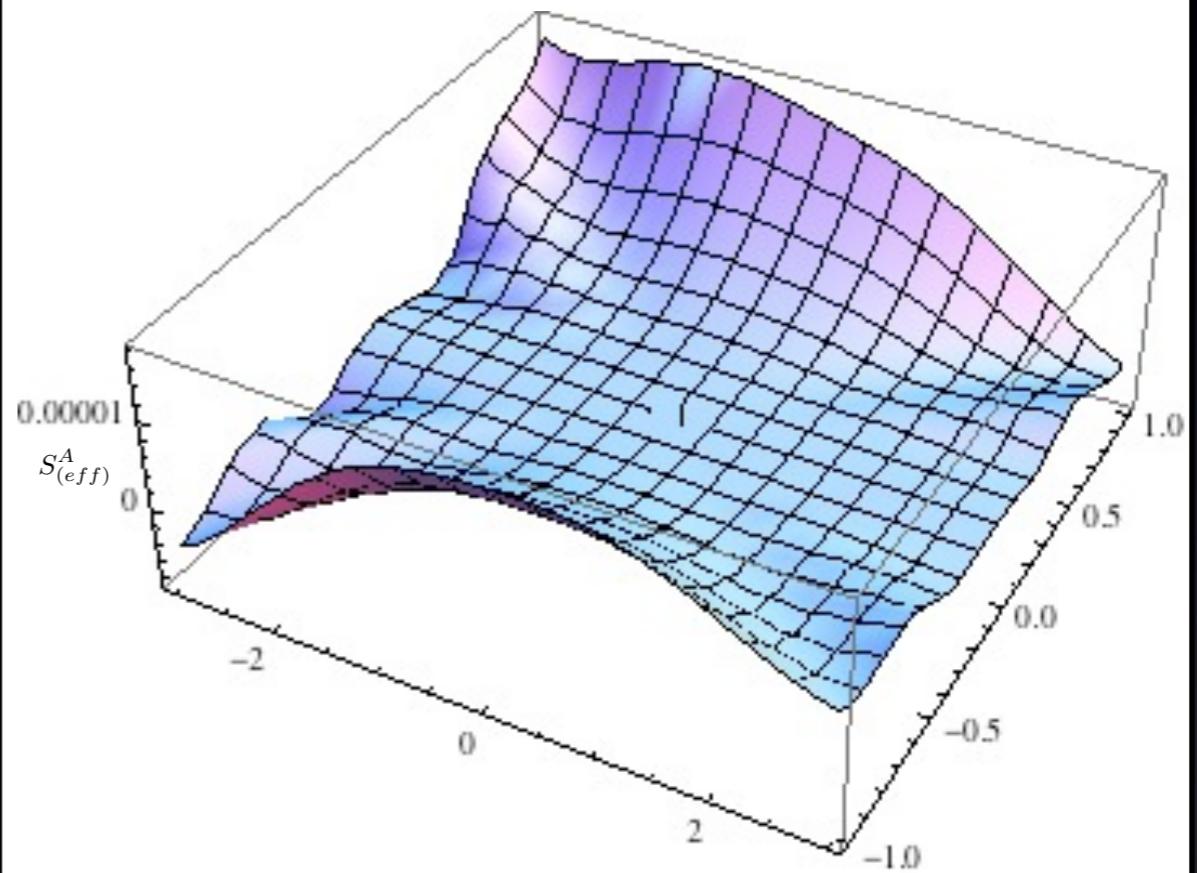
Substituting into the wave equation gives

$$\mathcal{D}^A_B \tilde{\varphi}_{(\text{R})}^B = S_{(\text{eff})}^A$$

with effective source,

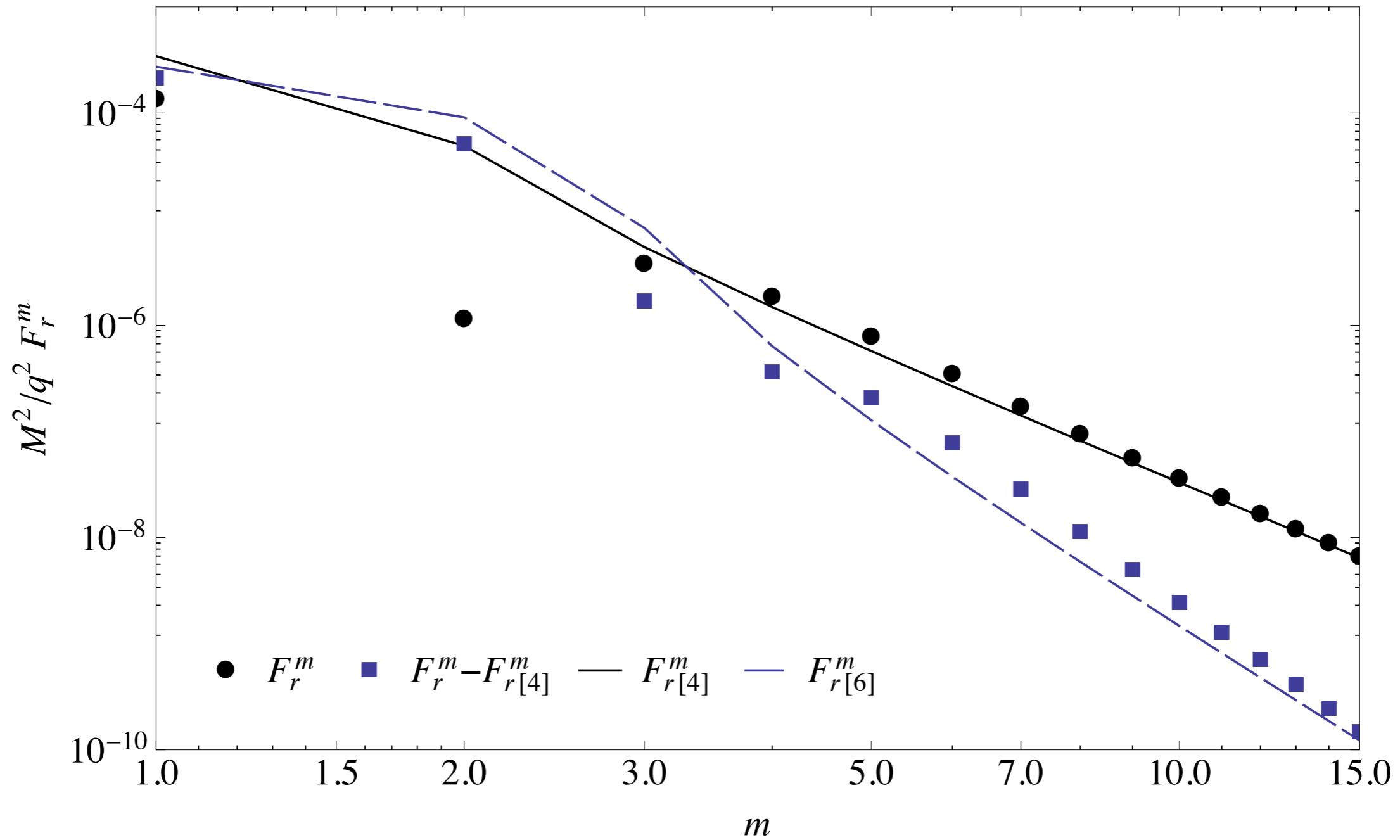
$$S_{(\text{eff})}^A = \mathcal{D}^A_B \tilde{\varphi}^B - 4\pi Q \int u^A \delta_4(x, z(\tau')) d\tau'.$$

Effective Source Kerr Scalar



Kerr Scalar

m-modes of the scalar self-force



Data supplied by Jonathan Thornburg

More on the way!



More on the way!

- More accurate calculations for Kerr gravitational case (Lorentz gauge): mode-sum (Patrick Nolan), effective source (Barack & Dolan)
- Regularisation in radiation gauge: application to matched expansions (Casals et al.), mode-sum (Shah, Friedman et al.)
- Cosmic Censorship