Regularisation of the self-force: applications to Kerr spacetime and accelerated motion

Anna Heffernan
University College Dublin

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Collaborators: Adrian Ottewill, Barry Wardell

CAPRA16
UCD, Ireland

Motivation

Gravitational Wave Detection: EMRIs

- Verification of General Relativity in strong regime
- Intermediate mass black holes

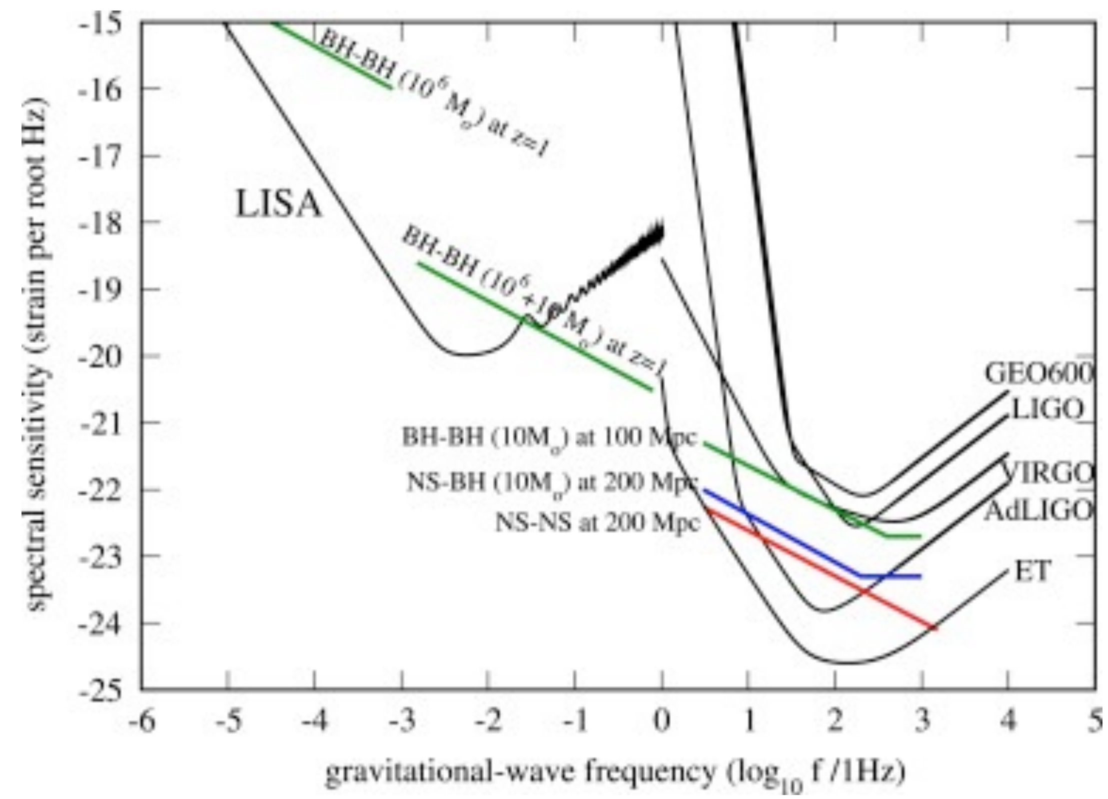


Diagram credit: N. Andersson, Elsevier 66 (2011)

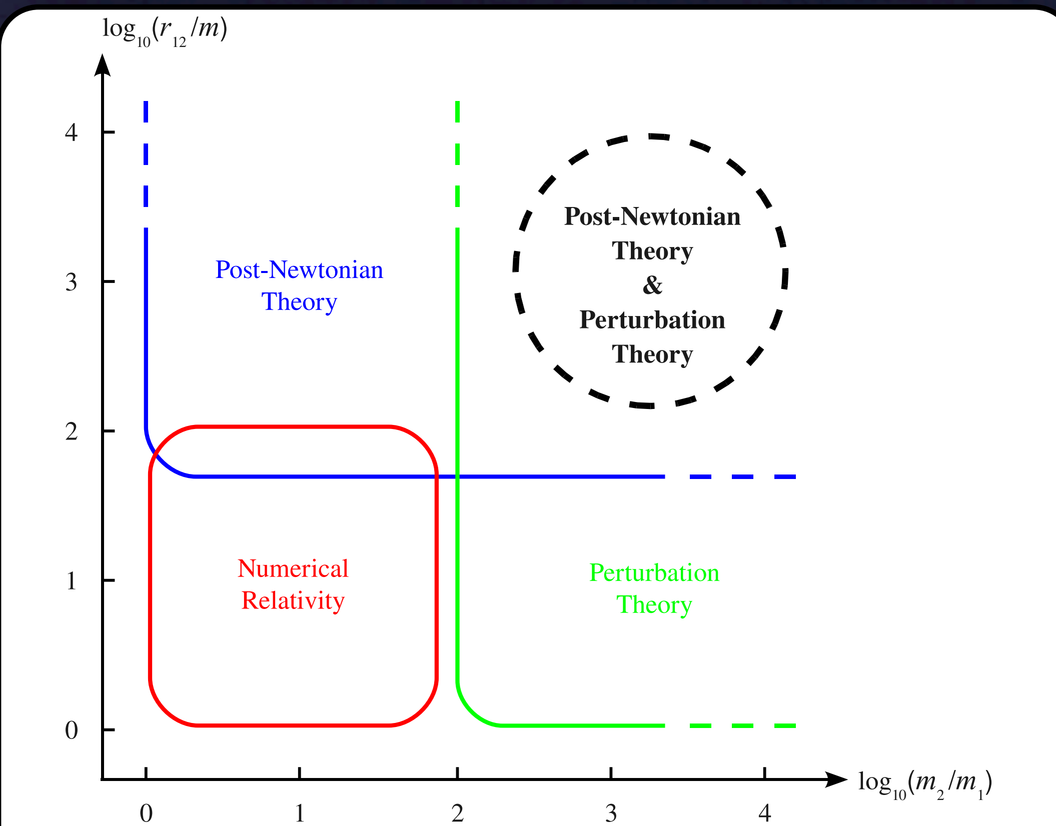


Diagram credit: L. Blanchet et al. Phys. Rev. D 81, 084033 (2010)

- Hubble Constant
- Galaxy formation
- Galaxy Census

Self-force

Retarded field satisfies

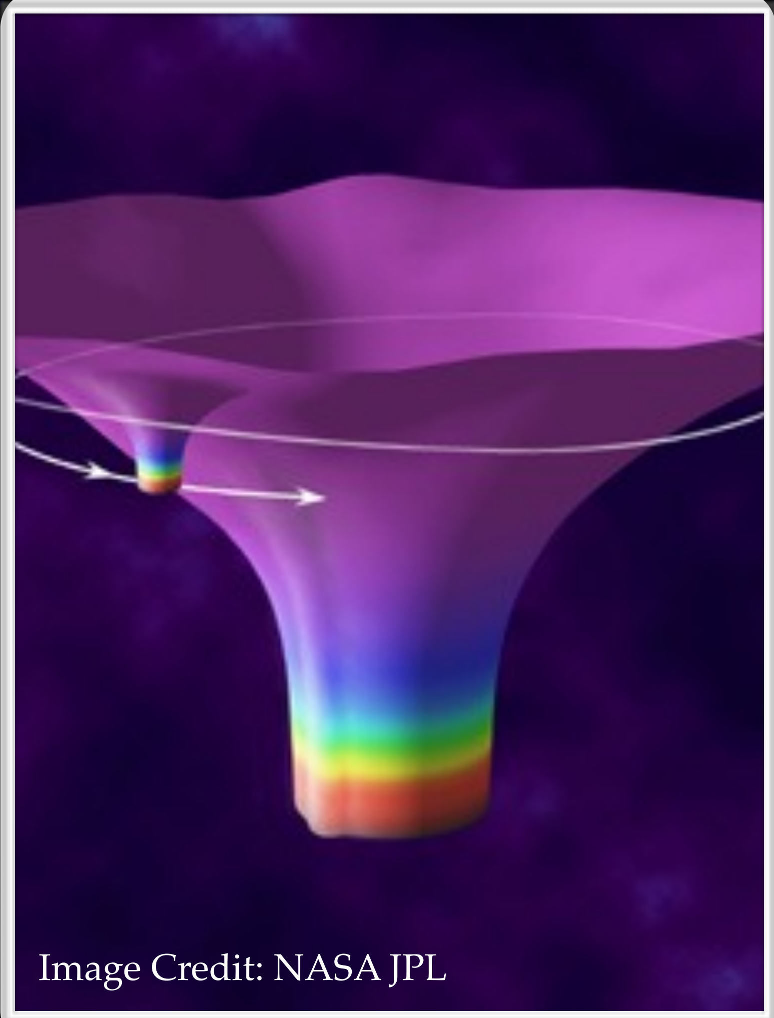
$$\mathcal{D}^A{}_B\varphi^B = -4\pi\mathcal{Q}\int u^A\delta_4\left(x, z(\tau')\right)d\tau' \qquad \text{where} \qquad \mathcal{D}^A{}_B = \delta^A{}_B(\Box - m^2) - P^A{}_B$$

Regular-singular split: $\psi^A_R = \psi^A_{ret} - \psi^A_S$

Detweiler-Whiting Singular field:

$$\varphi^A_{(\mathrm{S})} = \int_{\tau_{(\mathrm{adv})}}^{\tau_{(\mathrm{ret})}} G_{(\mathrm{S})}{}^A{}_{B'}\left(x, z(\tau')\right)u^{B'}d\tau'$$

The self-force: $f^a = p^a{}_A\varphi^A_{(R)}$



Detweiler-Whiting Green function:

$$G_{(\mathrm{S})}{}^A{}_{B'}\left(x, x'\right) = \frac{1}{2}\left\{U^A{}_{B'}\left(x, x'\right)\delta\left[\sigma\left(x, x'\right)\right] + V^A{}_{B'}\left(x, x'\right)\theta\left[\sigma\left(x, x'\right)\right]\right\}$$

Singular Field

The scalar singular field and self-force are

$$\Phi^{(\mathrm{S})}(x) = \left[\frac{U(x,x')}{2\sigma_{a'}u^{a'}}\right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V(x,z(\tau))d\tau$$

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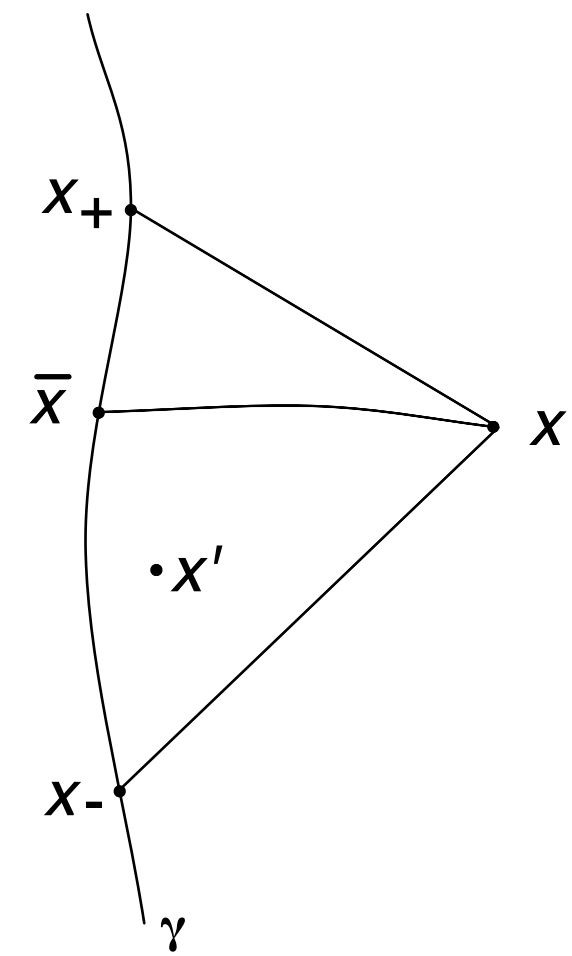

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$$\bar{h}^S_{ab} = \left[\frac{u^{a'}u^{b'}U^{ab}{}_{a'b'}(x,x')}{2\sigma_{a'}u^{a'}}\right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V^{ab}{}_{a'b'}(x,z(\tau))u^{a'}u^{b'}d\tau \qquad \text{and} \qquad f^a = k^{abcd}\bar{h}^{(R)}_{bc;d},$$

where

$$k^{abcd} \equiv \frac{1}{2}g^{ad}u^b u^c - g^{ab}u^c u^d - \frac{1}{2}u^a u^b u^c u^d + \frac{1}{4}u^a g^{bc}u^d + \frac{1}{4}g^{ad}g^{bc}.$$

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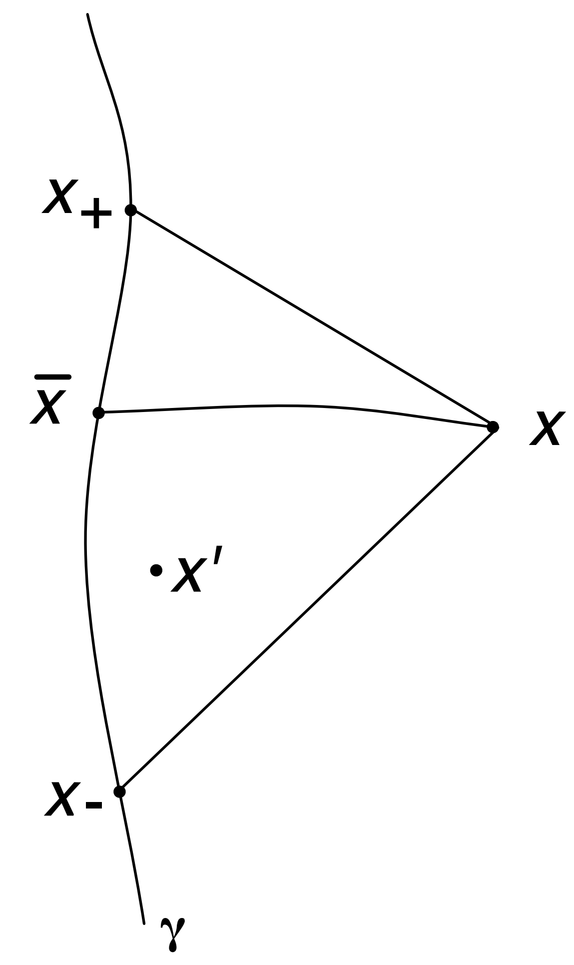

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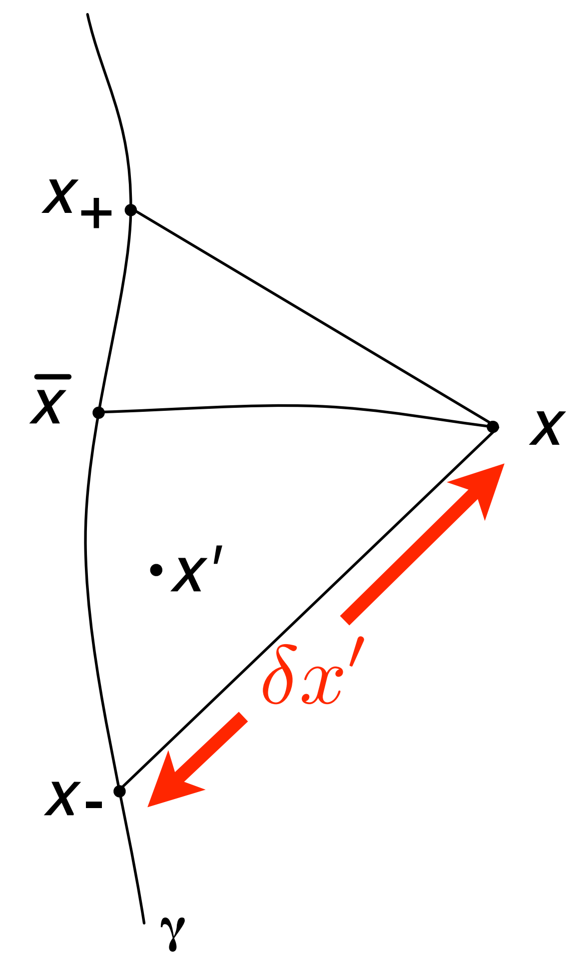

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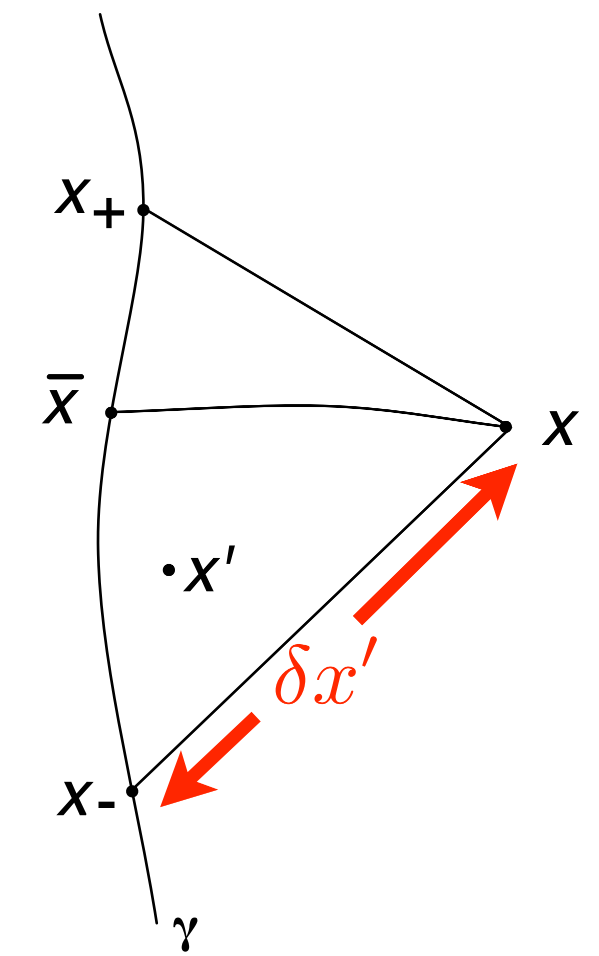

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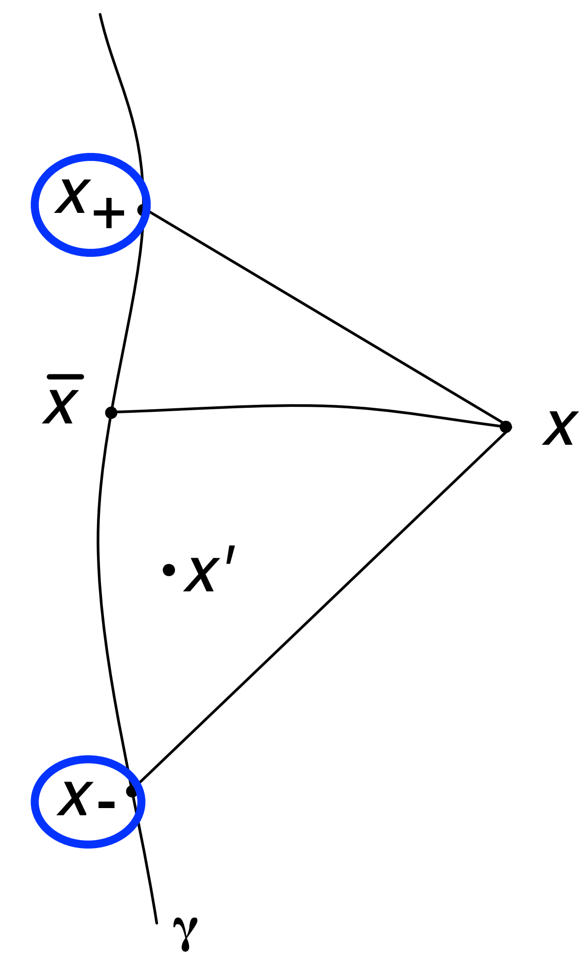

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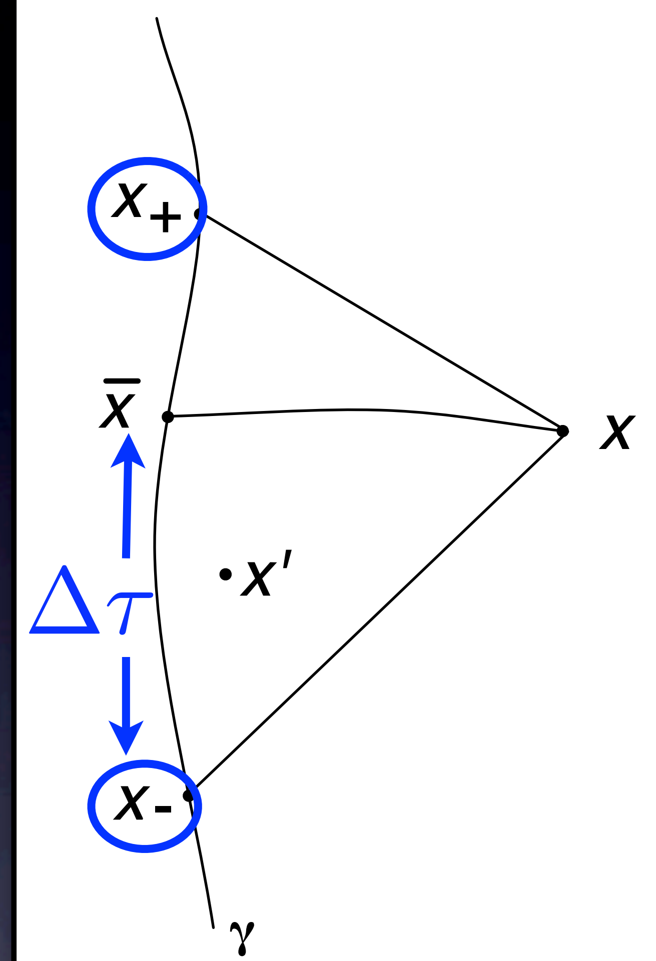

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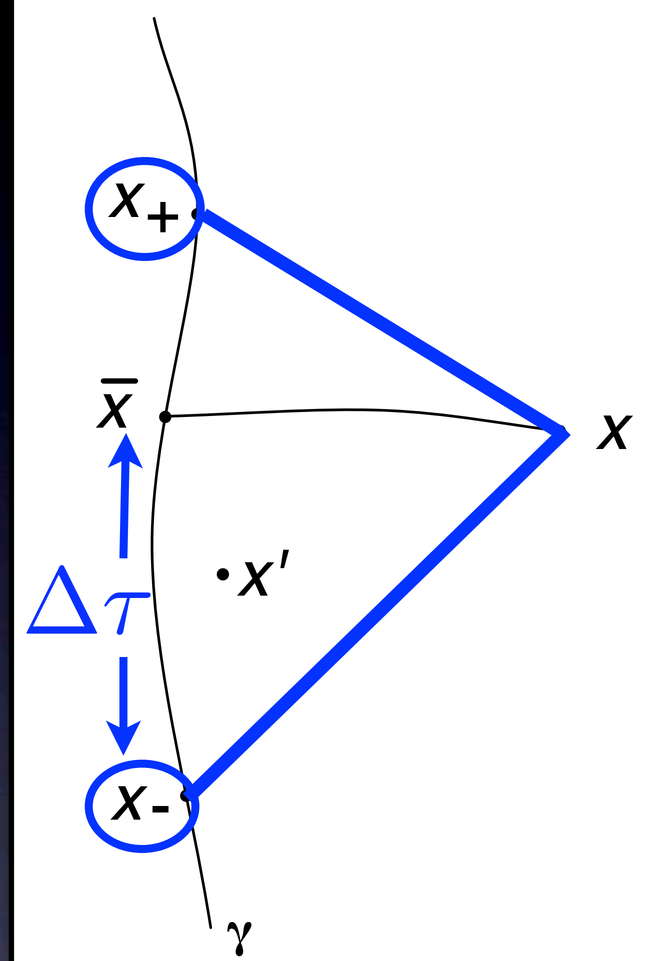

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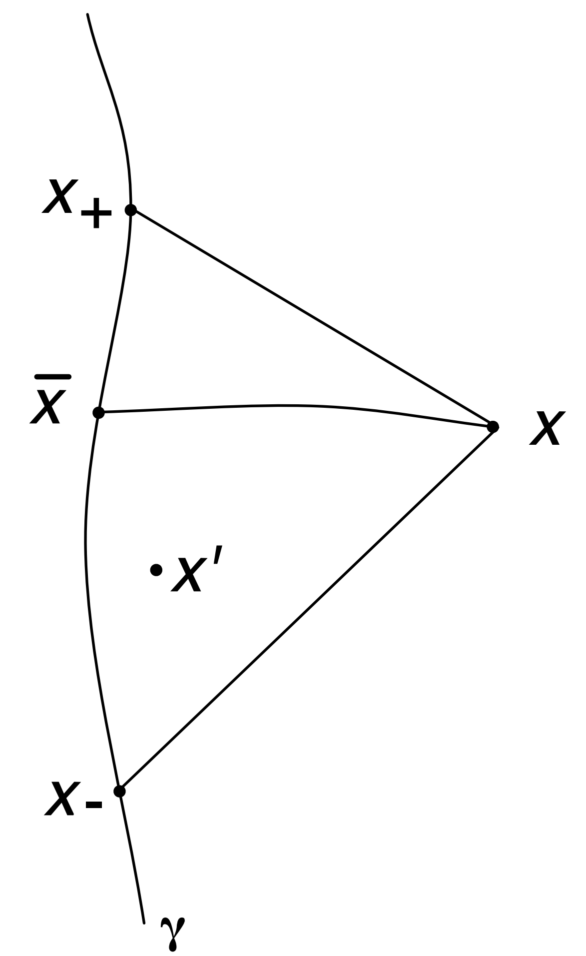

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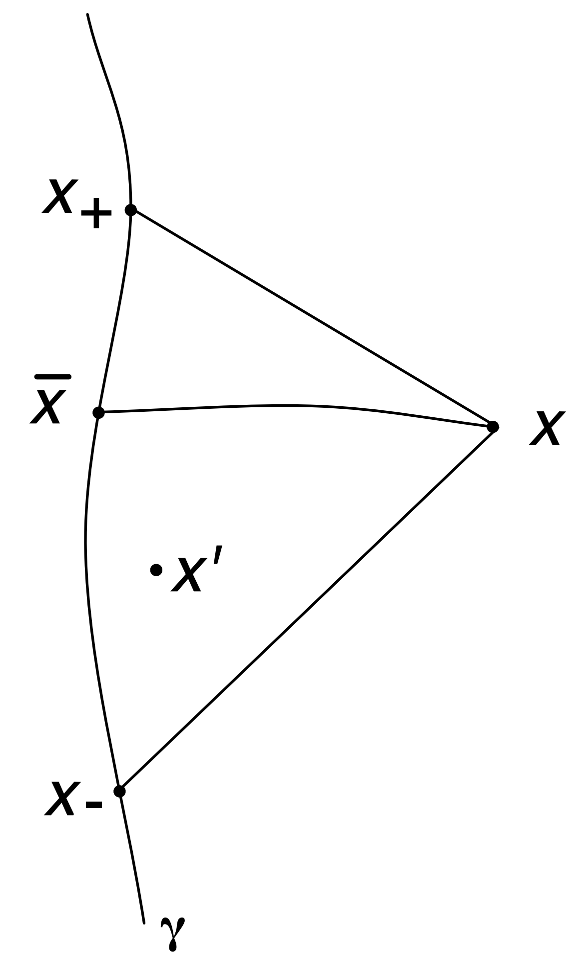

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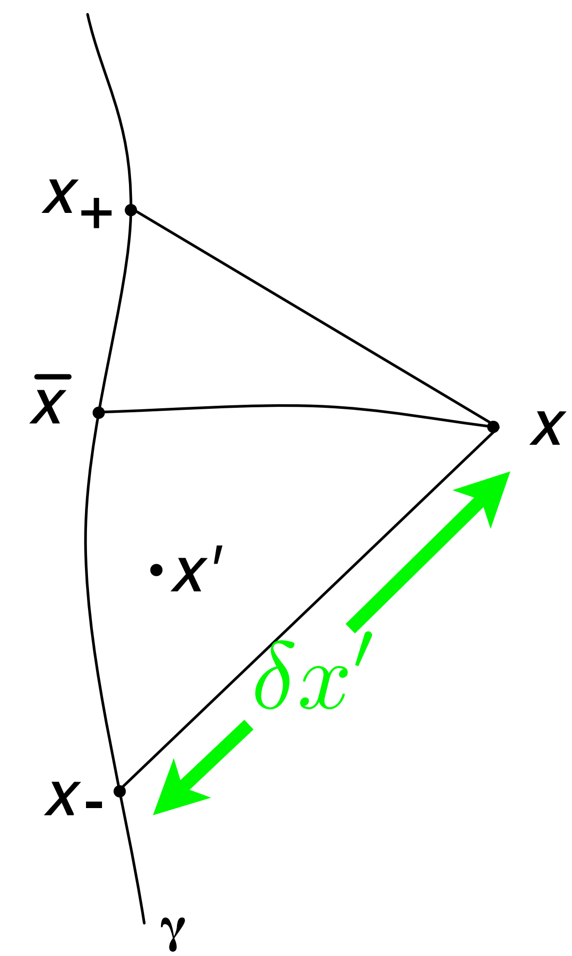

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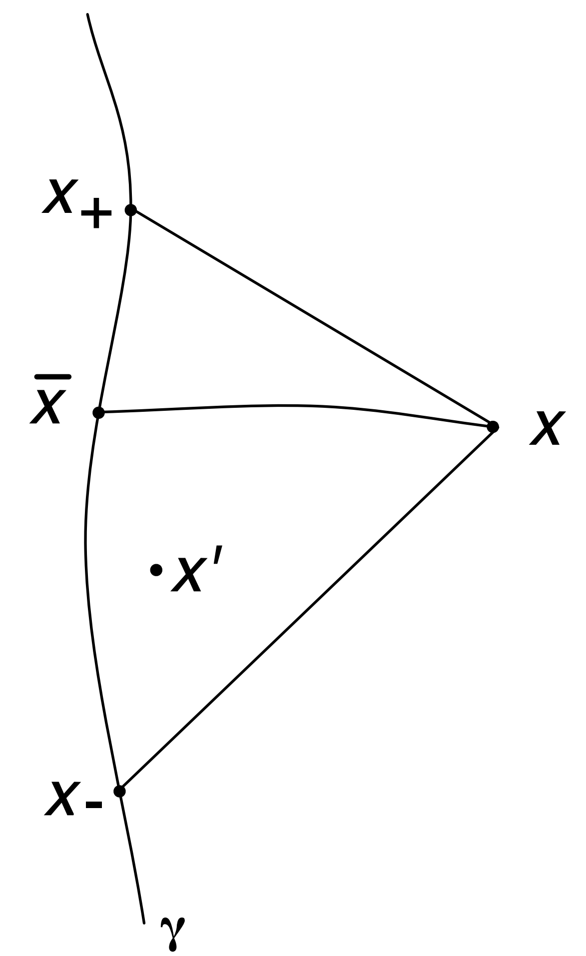

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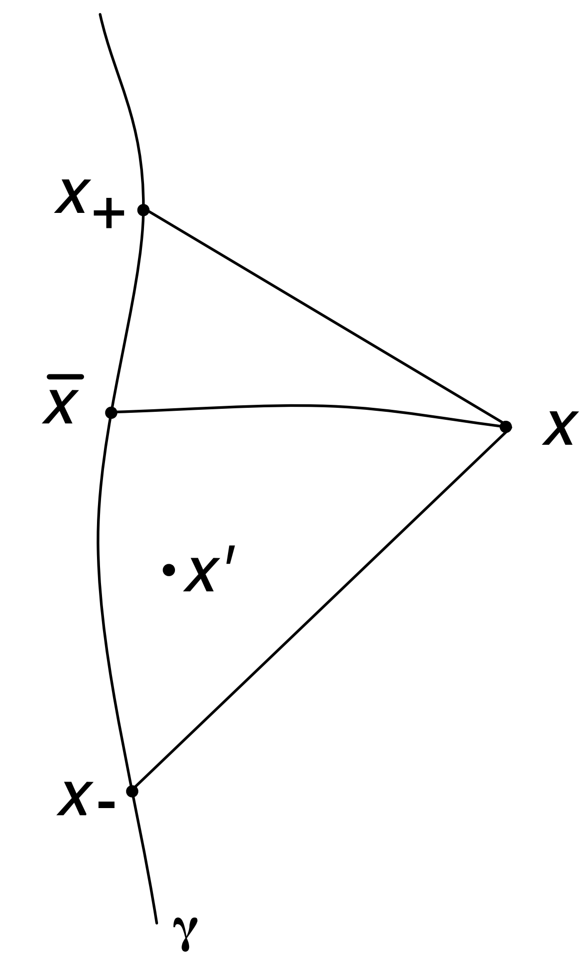
$$V^{AB'}(x,x') = \sum_{n=0}^{\infty} V_n{}^{AB'}(x,x')\,\sigma^n(x,x')$$



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$$\sigma^{;\alpha'}\left(\Delta^{-1/2}V_n^{AB'}\right)_{;\alpha'} + (n+1)\,\Delta^{-1/2}V_n^{AB'} + \frac{1}{2n}\Delta^{-1/2}\mathcal{D}^{B'}{}_{C'}V_{n-1}^{AC'} = 0$$

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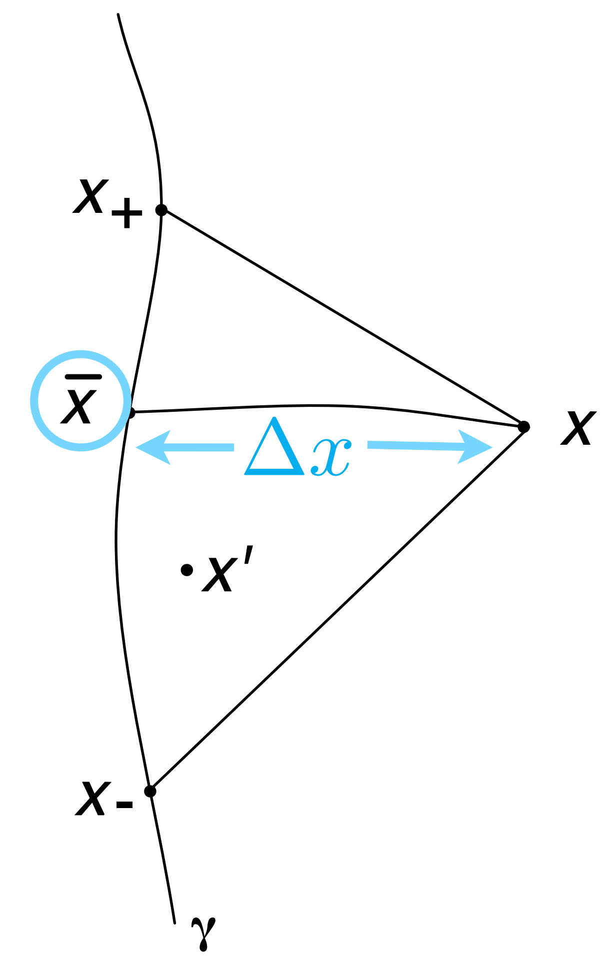
$$A_a^{\mathrm{S}} = \left[\frac{u^{a'}U^a{}_{a'}(x,x')}{2\sigma_{a'}u^{a'}}\right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V^a{}_{a'}(x,z(\tau))u^{a'}d\tau$$

$$f^a = g^{ab}u^c A^{(\mathrm{R})}{}_{[c,b]}.$$



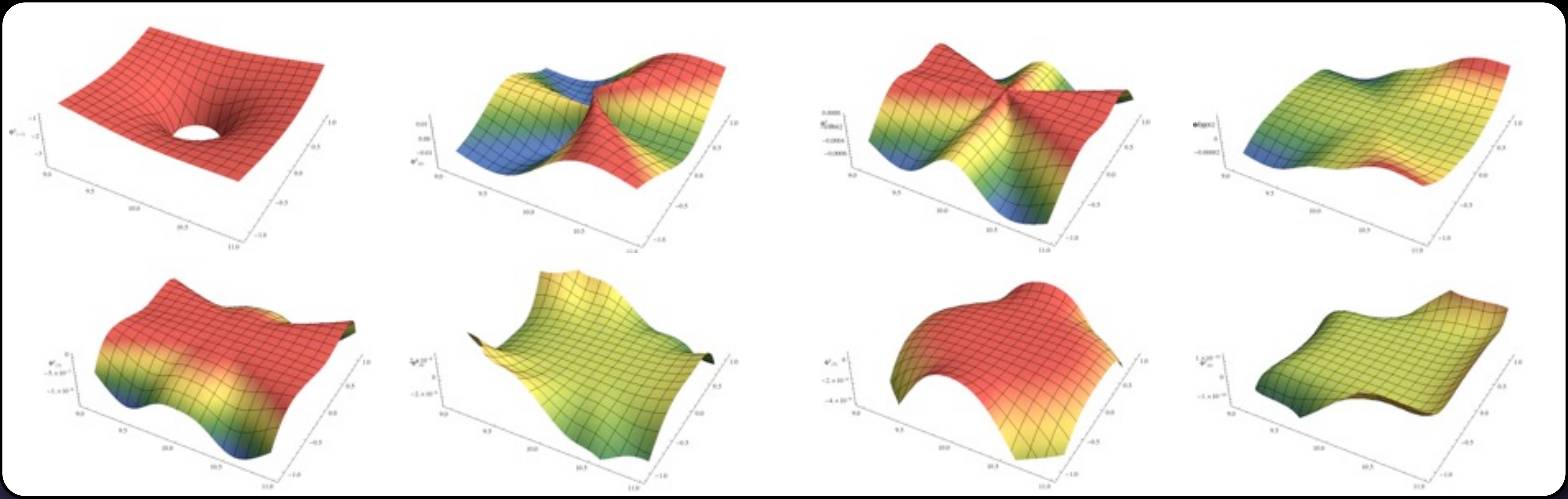
The gravitational singular field and self-force are

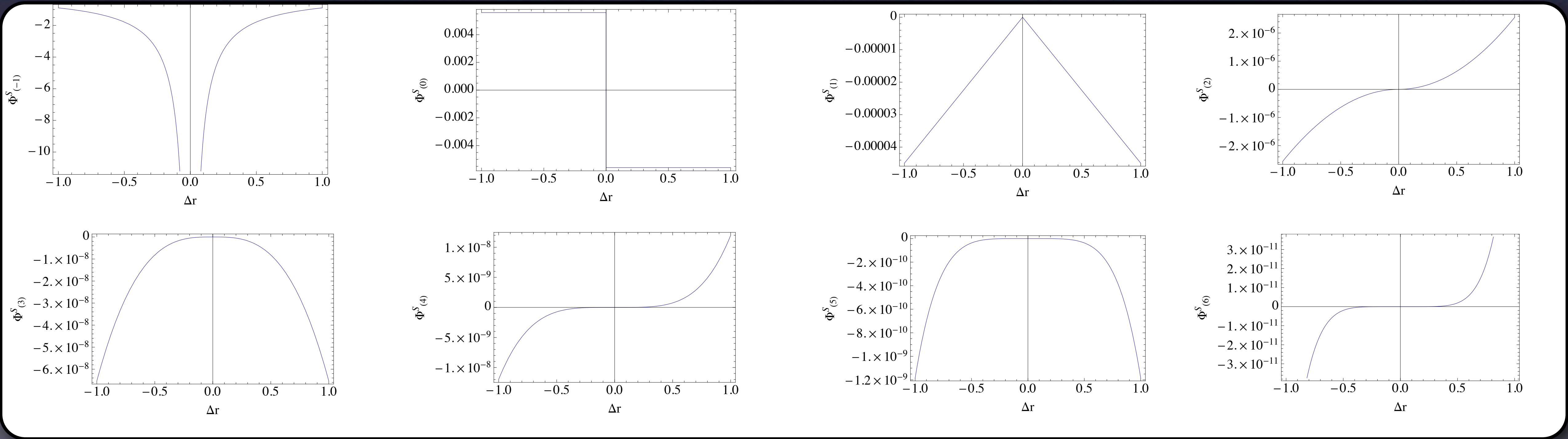
$$\bar{h}^S_{ab} = \left[\frac{u^{a'}u^{b'}U^{ab}{}_{a'b'}(x,x')}{2\sigma_{a'}u^{a'}}\right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V^{ab}{}_{a'b'}(x,z(\tau))u^{a'}u^{b'}d\tau \qquad \text{and} \qquad f^a = k^{abcd}\bar{h}^{(R)}_{bc;d},$$

where

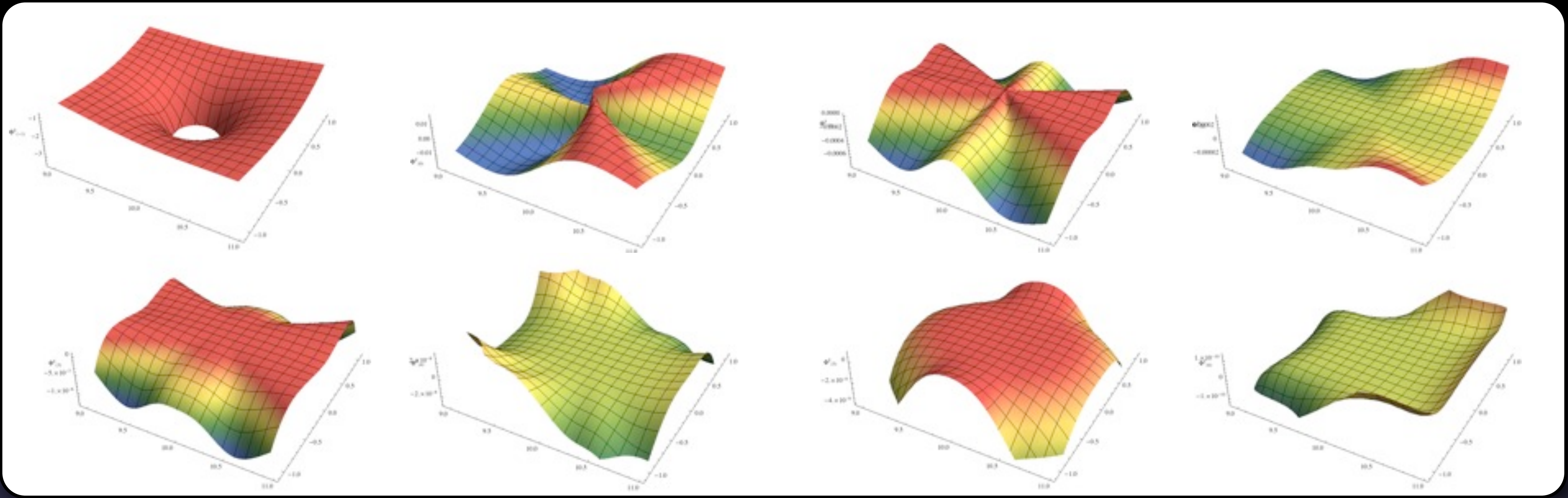
$$k^{abcd} \equiv \frac{1}{2}g^{ad}u^b u^c - g^{ab}u^c u^d - \frac{1}{2}u^a u^b u^c u^d + \frac{1}{4}u^a g^{bc}u^d + \frac{1}{4}g^{ad}g^{bc}.$$

Singular Field



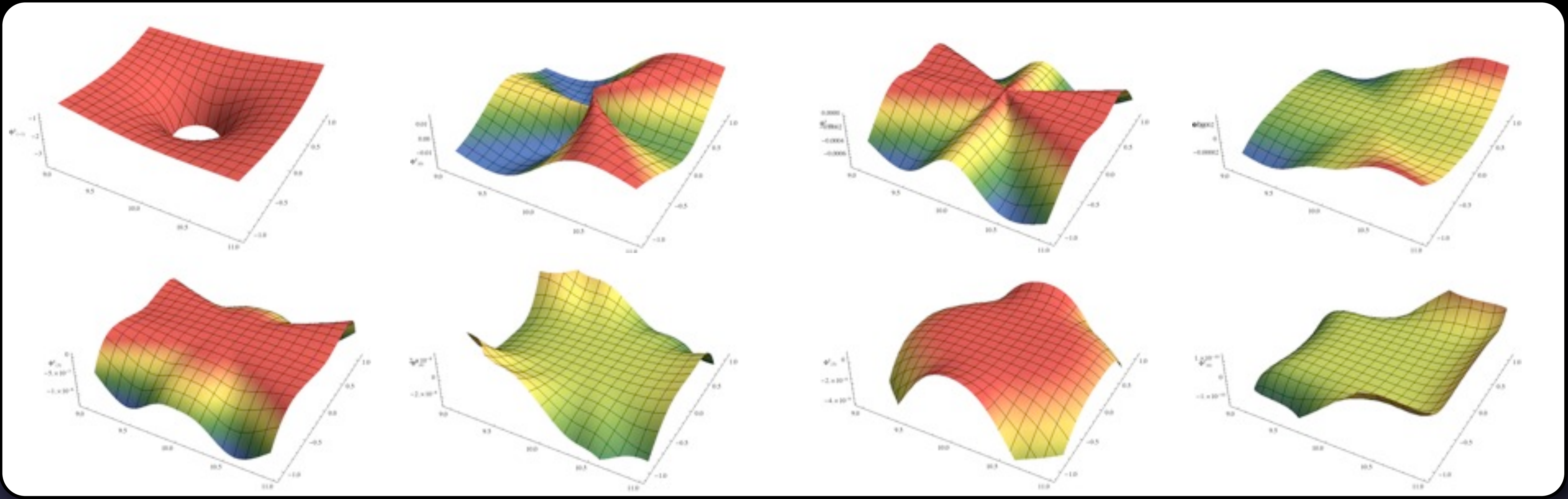


Singular Field



Regular-singular split: $\psi^A_R = \psi^A_{ret} - \psi^A_S$

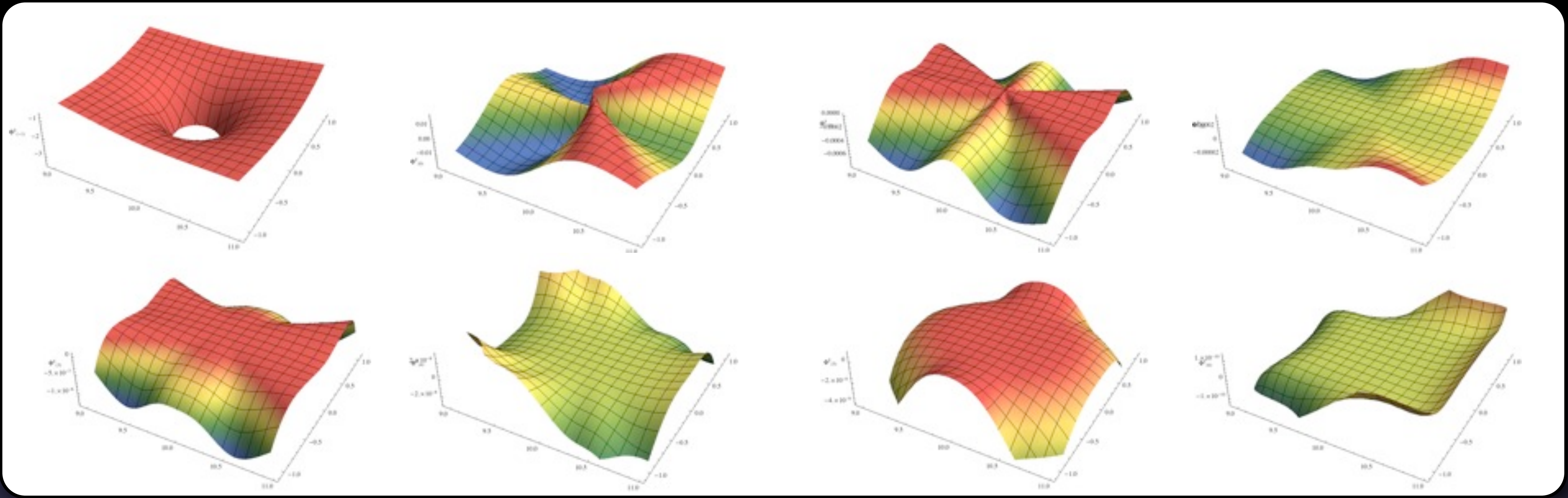
Singular Field



Regular-singular split: $\psi_R^A = \psi_{ret}^A - \psi_S^A$

Matched Expansions: Poisson & Wiseman, Casals et al.

Singular Field

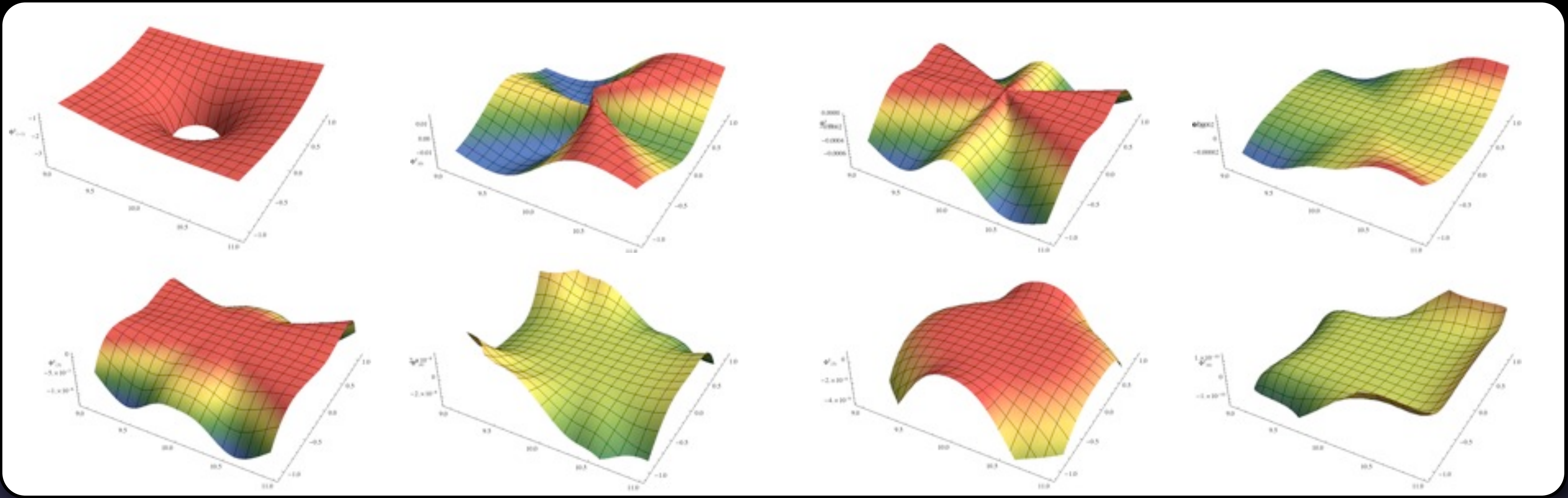


Regular-singular split: $\psi^A_R = \psi^A_{ret} - \psi^A_S$

Matched Expansions: Poisson & Wiseman, Casals et al.

Mode-Sum: Barack & Ori, Sago, Warburton, Shah, Friedman et al.

Singular Field



Regular-singular split: $\psi_R^A = \psi_{ret}^A - \psi_S^A$

Matched Expansions: Poisson & Wiseman, Casals et al.

Mode-Sum: Barack & Ori, Sago, Warburton, Shah, Friedman et al.

Effective Source: Barack & Golbourn, Detweiler & Vega, Barack & Dolan

Mode Sum

$$f_a^l(r_0,t_0) = \lim_{\Delta r\to 0}\sum_m f_a^{lm}(r_0+\Delta r,t_0)\,Y^{lm}(\alpha_0,\beta_0)$$
$$= \frac{2l+1}{4\pi}\lim_{\Delta r\to 0}\int f_a(r_0+\Delta r,t_0,\alpha,\beta)\,P_l(\cos\alpha)\,d\Omega$$

$$\sin\theta\cos\phi = \cos\alpha$$
$$\sin\theta\sin\phi = \sin\alpha\sin\beta$$
$$\cos\theta = \sin\alpha\cos\beta$$

Singular field contribution:

$$f_a(r,t,\alpha,\beta) = \sum_{n=1}\frac{\mathcal{B}_a^{(3n-2)}}{\rho^{2n+1}}\epsilon^{n-3}$$

where $\mathcal{B}_a^{(k)} = b_{a_1 a_2 \dots a_k}(\bar{x})\Delta x^{a_1}\Delta x^{a_2}\dots\Delta x^{a_k}$

$$\rho(r,t_0,\alpha,\beta)^2 = (g_{\bar{a}\bar{b}}u^{\bar{b}}\Delta x^b)^2 + g_{\bar{a}\bar{b}}\Delta x^a\Delta x^b$$
$$= \frac{r_0[Er_0(a^2+r_0^2)+2aM(aE-L)]^2}{[r_0(L^2+a^2)+2a^2M+r_0^3](r_0^2-2Mr_0+a^2)^2}\Delta r^2 + r_0^2\Delta w_2^2$$
$$+\left(L^2+r_0^2+a^2+\frac{2a^2M}{r_0}\right)\left[\Delta w_1 + \frac{Lr_0^3\dot{r}_0}{(r_0^2-2Mr_0+a^2)(2a^2M+a^2r_0+L^2r_0+r_0^3)}\Delta r\right]^2$$

$$\Delta w_1 \to \Delta w_1 + \mu\Delta r \Rightarrow \rho(r,t_0,\alpha,\beta) = \nu^2\Delta r + \zeta^2\Delta w_1^2 + r_0^2\Delta w_2^2$$

$$F^l_{a[-1]}(r_0,t_0) = (l+1/2)\,\frac{b_{a_r}sgn(\Delta r)}{\zeta\nu r_0}$$

Mode Sum

$$
\begin{aligned}
f_a^l(r_0,t_0) &= \lim_{\Delta r\to 0}\sum_m f_a^{lm}(r_0+\Delta r,t_0)\,Y^{lm}(\alpha_0,\beta_0)\\
&= \frac{2l+1}{4\pi}\lim_{\Delta r\to 0}\int f_a(r_0+\Delta r,t_0,\alpha,\beta)\,P_l(\cos\alpha)\,d\Omega
\end{aligned}
$$

$$w_1 = 2\sin\left(\frac{\alpha}{2}\right)\cos\beta$$

$$w_2 = 2\sin\left(\frac{\alpha}{2}\right)\sin\beta$$

Singular field contribution:

$$f_a(r,t,\alpha,\beta) = \sum_{n=1}\frac{\mathcal{B}_a^{(3n-2)}}{\rho^{2n+1}}\epsilon^{n-3}$$

where $\mathcal{B}_a^{(k)} = b_{a_1 a_2\ldots a_k}(\bar{x})\Delta x^{a_1}\Delta x^{a_2}\ldots\Delta x^{a_k}$

$$
\begin{aligned}
\rho(r,t_0,\alpha,\beta)^2 &= (g_{\bar a\bar b}u^{\bar b}\Delta x^b)^2 + g_{\bar a\bar b}\Delta x^a\Delta x^b\\
&= \frac{r_0[Er_0(a^2+r_0^2)+2aM(aE-L)]^2}{[r_0(L^2+a^2)+2a^2M+r_0^3](r_0^2-2Mr_0+a^2)^2}\Delta r^2 + r_0^2\Delta w_2^2\\
&+\left(L^2+r_0^2+a^2+\frac{2a^2M}{r_0}\right)\left[\Delta w_1+\frac{Lr_0^3\dot{r}_0}{(r_0^2-2Mr_0+a^2)(2a^2M+a^2r_0+L^2r_0+r_0^3)}\Delta r\right]^2
\end{aligned}
$$

$$\Delta w_1 \to \Delta w_1 + \mu\Delta r \Rightarrow \rho(r,t_0,\alpha,\beta) = \nu^2\Delta r + \zeta^2\Delta w_1^2 + r_0^2\Delta w_2^2$$

$$F^l_{a[-1]}(r_0,t_0) = (l+1/2)\,\frac{b_{a_r}sgn(\Delta r)}{\zeta\nu r_0}$$

Mode Sum

$$f_a^l(r_0,t_0) = \frac{2l+1}{4\pi}\left[\epsilon^{-2}\lim_{\Delta r\to 0}\int \frac{B_a^{(1)}}{\rho^3}P_l(\cos\alpha)\,d\Omega + \epsilon^{n-3}\sum_{n=2}\int \rho_0^{n-3}c_{a(n)}(r_0,\beta)\,P_l(\cos\alpha)\,d\Omega\right]$$

$$w_1 = 2\sin\left(\frac{\alpha}{2}\right)\cos\beta$$

$$w_2 = 2\sin\left(\frac{\alpha}{2}\right)\sin\beta$$

Singular field contribution:

$$f_a(r,t,\alpha,\beta) = \sum_{n=1}\frac{\mathcal{B}_a^{(3n-2)}}{\rho^{2n+1}}\epsilon^{n-3}$$

where $\mathcal{B}_a^{(k)} = b_{a_1a_2\ldots a_k}(\bar{x})\Delta x^{a_1}\Delta x^{a_2}\ldots\Delta x^{a_k}$

$$\begin{aligned}\rho(r,t_0,\alpha,\beta)^2 &= (g_{\bar{a}\bar{b}}u^{\bar{b}}\Delta x^b)^2 + g_{\bar{a}\bar{b}}\Delta x^a\Delta x^b \\ &= \frac{r_0[Er_0(a^2+r_0^2)+2aM(aE-L)]^2}{[r_0(L^2+a^2)+2a^2M+r_0^3](r_0^2-2Mr_0+a^2)^2}\Delta r^2 + r_0^2\Delta w_2^2 \\ &+ \left(L^2+r_0^2+a^2+\frac{2a^2M}{r_0}\right)\left[\Delta w_1 + \frac{Lr_0^3\dot{r}_0}{(r_0^2-2Mr_0+a^2)(2a^2M+a^2r_0+L^2r_0+r_0^3)}\Delta r\right]^2\end{aligned}$$

$$\Delta w_1 \to \Delta w_1 + \mu\Delta r \Rightarrow \rho(r,t_0,\alpha,\beta) = \nu^2\Delta r + \zeta^2\Delta w_1^2 + r_0^2\Delta w_2^2$$

$$F^l_{a[-1]}(r_0,t_0) = (l+1/2)\,\frac{b_{a_r}sgn(\Delta r)}{\zeta\nu r_0}$$

Mode Sum

$$f_a^l(r_0, t_0) = \frac{2l+1}{4\pi}\left[\epsilon^{-2}\lim_{\Delta r \to 0}\int \frac{B_a^{(1)}}{\rho^3} P_l(\cos\alpha)\, d\Omega \right.$$
$$\left. + \epsilon^{n-3}\sum_{n=2}\int \rho_0^{n-3} c_{a(n)}(r_0, \beta) P_l(\cos\alpha)\, d\Omega\right]$$

$$w_1 = 2\sin\left(\frac{\alpha}{2}\right)\cos\beta$$
$$w_2 = 2\sin\left(\frac{\alpha}{2}\right)\sin\beta$$

Singular field contribution:

$$f_a(r, t, \alpha, \beta) = \sum_{n=1}\frac{\mathcal{B}_a^{(3n-2)}}{\rho^{2n+1}}\epsilon^{n-3}$$

where $\mathcal{B}_a^{(k)} = b_{a_1 a_2 \ldots a_k}(\bar{x})\Delta x^{a_1}\Delta x^{a_2}\ldots\Delta x^{a_k}$

$$\begin{aligned}\rho(r, t_0, \alpha, \beta)^2 &= (g_{\bar{a}\bar{b}}u^{\bar{b}}\Delta x^b)^2 + g_{\bar{a}\bar{b}}\Delta x^a \Delta x^b \\ &= \frac{r_0[E r_0(a^2 + r_0^2) + 2aM(aE - L)]^2}{[r_0(L^2 + a^2) + 2a^2 M + r_0^3](r_0^2 - 2M r_0 + a^2)^2}\Delta r^2 + r_0^2 \Delta w_2^2 \\ &+ \left(L^2 + r_0^2 + a^2 + \frac{2a^2 M}{r_0}\right)\left[\Delta w_1 + \frac{L r_0^3 \dot{r}_0}{(r_0^2 - 2M r_0 + a^2)(2a^2 M + a^2 r_0 + L^2 r_0 + r_0^3)}\Delta r\right]^2\end{aligned}$$

$-\mu$

$$\Delta w_1 \to \Delta w_1 + \mu\Delta r \Rightarrow \rho(r, t_0, \alpha, \beta) = \nu^2 \Delta r + \zeta^2 \Delta w_1^2 + r_0^2 \Delta w_2^2$$

$$F^l_{a[-1]}(r_0, t_0) = (l + 1/2)\frac{b_{a_r} sgn(\Delta r)}{\zeta \nu r_0}$$

Mode Sum

$$f_a^l(r_0,t_0) = \frac{2l+1}{4\pi}\left[\epsilon^{-2}\lim_{\Delta r\to 0}\int \frac{B_a^{(1)}}{\rho^3}P_l(\cos\alpha)\,d\Omega + \epsilon^{n-3}\sum_{n=2}\int \rho_0^{n-3}c_{a(n)}(r_0,\beta)\,P_l(\cos\alpha)\,d\Omega\right]$$

$$w_1 = 2\sin\left(\frac{\alpha}{2}\right)\cos\beta$$

$$w_2 = 2\sin\left(\frac{\alpha}{2}\right)\sin\beta$$

Singular field contribution:

$$f_a(r,t,\alpha,\beta) = \sum_{n=1}\frac{\mathcal{B}_a^{(3n-2)}}{\rho^{2n+1}}\epsilon^{n-3}$$

where $\mathcal{B}_a^{(k)} = b_{a_1a_2\ldots a_k}(\bar{x})\Delta x^{a_1}\Delta x^{a_2}\ldots\Delta x^{a_k}$

ν^2

$$\begin{aligned}\rho(r,t_0,\alpha,\beta)^2 &= (g_{\bar{a}\bar{b}}u^{\bar{b}}\Delta x^b)^2 + g_{\bar{a}\bar{b}}\Delta x^a\Delta x^b \\ &= \frac{r_0[Er_0(a^2+r_0^2)+2aM(aE-L)]^2}{[r_0(L^2+a^2)+2a^2M+r_0^3](r_0^2-2Mr_0+a^2)^2}\Delta r^2 + r_0^2\Delta w_2^2 \\ &+ \left(L^2+r_0^2+a^2+\frac{2a^2M}{r_0}\right)\left[\Delta w_1 + \frac{Lr_0^3\dot{r}_0}{(r_0^2-2Mr_0+a^2)(2a^2M+a^2r_0+L^2r_0+r_0^3)}\Delta r\right]^2\end{aligned}$$

ζ^2

$$\Delta w_1 \to \Delta w_1 + \mu\Delta r \Rightarrow \rho(r,t_0,\alpha,\beta) = \nu^2\Delta r + \zeta^2\Delta w_1^2 + r_0^2\Delta w_2^2$$

$$F_{a[-1]}^l(r_0,t_0) = (l+1/2)\,\frac{b_{a_r}sgn(\Delta r)}{\zeta\nu r_0}$$

Mode Sum

$$f_a^l(r_0,t_0) = \frac{2l+1}{4\pi}\left[\epsilon^{-2}\lim_{\Delta r\to 0}\int \frac{B_a^{(1)}}{\rho^3}P_l(\cos\alpha)\,d\Omega\right. \quad F^l_{a[-1]}(r_0,t_0)$$

$$\left. + \epsilon^{n-3}\sum_{n=2}\int \rho_0^{n-3}c_{a(n)}(r_0,\beta)\,P_l(\cos\alpha)\,d\Omega\right]$$

$$w_1 = 2\sin\left(\frac{\alpha}{2}\right)\cos\beta$$

$$w_2 = 2\sin\left(\frac{\alpha}{2}\right)\sin\beta$$

Singular field contribution:

$$f_a(r,t,\alpha,\beta) = \sum_{n=1}\frac{\mathcal{B}_a^{(3n-2)}}{\rho^{2n+1}}\epsilon^{n-3}$$

where $\mathcal{B}_a^{(k)} = b_{a_1 a_2 \ldots a_k}(\bar{x})\Delta x^{a_1}\Delta x^{a_2}\ldots\Delta x^{a_k}$

$$\rho(r,t_0,\alpha,\beta)^2 = (g_{\bar{a}\bar{b}}u^{\bar{b}}\Delta x^b)^2 + g_{\bar{a}\bar{b}}\Delta x^a\Delta x^b$$

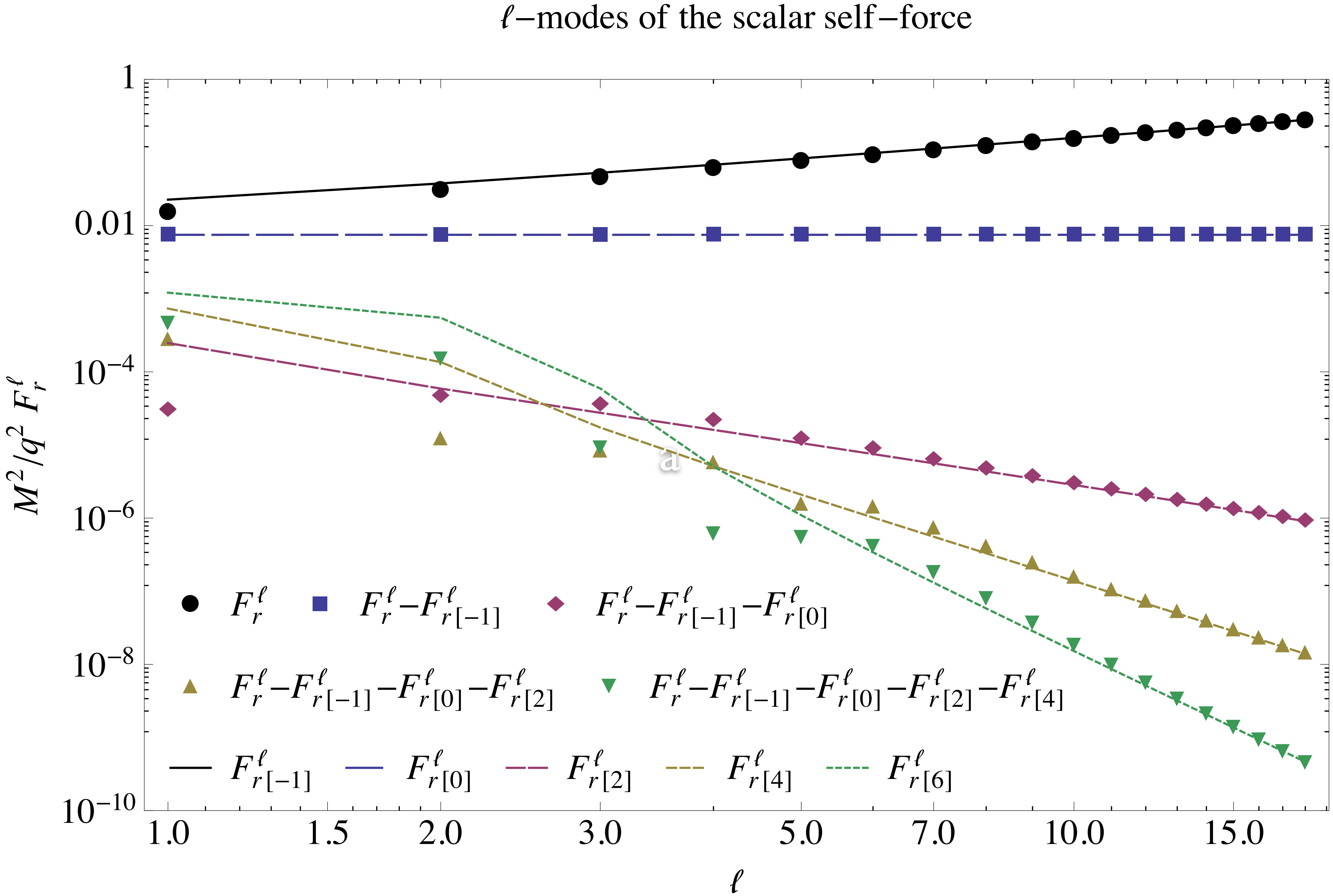
$$= \underbrace{\frac{r_0[Er_0(a^2+r_0^2)+2aM(aE-L)]^2}{[r_0(L^2+a^2)+2a^2M+r_0^3](r_0^2-2Mr_0+a^2)^2}}_{\nu^2}\Delta r^2 + r_0^2\Delta w_2^2$$

$$+ \underbrace{\left(L^2+r_0^2+a^2+\frac{2a^2M}{r_0}\right)}_{\zeta^2}\left[\Delta w_1 + \frac{Lr_0^3\dot{r}_0}{(r_0^2-2Mr_0+a^2)(2a^2M+a^2r_0+L^2r_0+r_0^3)}\Delta r\right]^2$$

$$\Delta w_1 \to \Delta w_1 + \mu\Delta r \Rightarrow \rho(r,t_0,\alpha,\beta) = \nu^2\Delta r + \zeta^2\Delta w_1^2 + r_0^2\Delta w_2^2$$

$$F^l_{a[-1]}(r_0,t_0) = (l+1/2)\,\frac{b_{a_r}sgn(\Delta r)}{\zeta\nu r_0}$$

Kerr Scalar



Data supplied by Niels Warburton

Accelerated Motion

Schwarzschild (geodesic):

$$\rho\left(r,t_0,\alpha,\beta\right)^2=\frac{E^2r_0^4}{\left(L^2+r_0^2\right)\left(r_0-2M\right)^2}\Delta r^2+\left(L^2+r_0^2\right)\left(\Delta w_1+\frac{Lr_0\dot{r}_0}{\left(r_0-2M\right)\left(L^2+r_0^2\right)}\Delta r\right)^2+r_0^2\Delta w_2^2$$

f(r) metric (non-geodesic):

$$\rho\left(r,t_0,\alpha,\beta\right)^2=\frac{\dot{t}_0^{\,2}}{1+r_0^2\dot{\phi}_0^{\,2}}\Delta r^2+\left(r_0^2+r_0^4\dot{\phi}_0^{\,2}\right)\left[\Delta w_1+\frac{Lr_0\dot{\phi}_0}{f(r_0)\left(1+r_0^2\dot{\phi}_0\right)}\Delta r\right]^2+r_0^2\Delta w_2^2$$

Reissner-Nordstrom:

$$\rho\left(r,t_0,\alpha,\beta\right)^2=\frac{r_0^4(qQ-Er_0)^2}{\left(L^2+r_0^2\right)\left(r_0^2-2Mr_0+Q^2\right)^2}\Delta r^2+r_0^2\Delta w_2^2$$

$$+\left(L^2+r_0^2\right)\left[\Delta w_1+\frac{Lr_0^2\dot{r}_0}{\left(Q^2-2Mr_0+a^2\right)\left(L^2+r_0^2\right)}\Delta r\right]^2$$

Accelerated Motion

Schwarzschild (geodesic):

$$\rho\left(r,t_0,\alpha,\beta\right)^2 = \underbrace{\frac{E^2 r_0^4}{\left(L^2+r_0^2\right)\left(r_0-2M\right)^2}}_{\nu^2}\Delta r^2 + \underbrace{\left(L^2+r_0^2\right)}_{\zeta^2}\left(\Delta w_1 + \underbrace{\frac{L r_0 \dot{r}_0}{\left(r_0-2M\right)\left(L^2+r_0^2\right)}}_{-\mu}\Delta r\right)^2 + r_0^2\Delta w_2^2$$

f(r) metric (non-geodesic):

$$\rho\left(r,t_0,\alpha,\beta\right)^2 = \underbrace{\frac{\dot{t}_0^{\,2}}{1+r_0^2\dot{\phi}_0^{\,2}}}_{\nu^2}\Delta r^2 + \underbrace{\left(r_0^2 + r_0^4\dot{\phi}_0^{\,2}\right)}_{\zeta^2}\left[\Delta w_1 + \underbrace{\frac{L r_0 \dot{\phi}_0}{f(r_0)\left(1+r_0^2\dot{\phi}_0\right)}}_{-\mu}\Delta r\right]^2 + r_0^2\Delta w_2^2$$

Reissner-Nordstrom:

$$\rho\left(r,t_0,\alpha,\beta\right)^2 = \underbrace{\frac{r_0^4(qQ-Er_0)^2}{\left(L^2+r_0^2\right)\left(r_0^2-2Mr_0+Q^2\right)^2}}_{\nu^2}\Delta r^2 + r_0^2\Delta w_2^2$$

$$+\underbrace{\left(L^2+r_0^2\right)}_{\zeta^2}\left[\Delta w_1 + \underbrace{\frac{L r_0^2 \dot{r}_0}{\left(Q^2-2Mr_0+a^2\right)\left(L^2+r_0^2\right)}}_{-\mu}\Delta r\right]^2$$

Accelerated Motion

Accelerated Motion

- F(r) metric accelerated motion: $F^{l}_{a[0]}(r_0, t_0)$

Accelerated Motion

- F(r) metric accelerated motion: $F^{l}_{a[0]}(r_0, t_0)$
- F(r) metric radial infall: $F^{l}_{a[4]}(r_0, t_0)$

Accelerated Motion

- F(r) metric accelerated motion: $F^{l}_{a[0]}(r_0, t_0)$
- F(r) metric radial infall: $F^{l}_{a[4]}(r_0, t_0)$
- Schwarzschild accelerated motion: $F^{l}_{a[2]}(r_0, t_0)$

Accelerated Motion

- F(r) metric accelerated motion: $F^l_{a[0]}(r_0, t_0)$
- F(r) metric radial infall: $F^l_{a[4]}(r_0, t_0)$
- Schwarzschild accelerated motion: $F^l_{a[2]}(r_0, t_0)$
- Reissner-Nordstrom charged particle: $F^l_{a[4]}(r_0, t_0)$

Effective Source

Splitting the retarded field into approximate singular and regularized parts

$$\varphi^A_{(\mathrm{ret})} = \tilde{\varphi}^A_{(\mathrm{S})} + \tilde{\varphi}^A_{(\mathrm{R})}$$

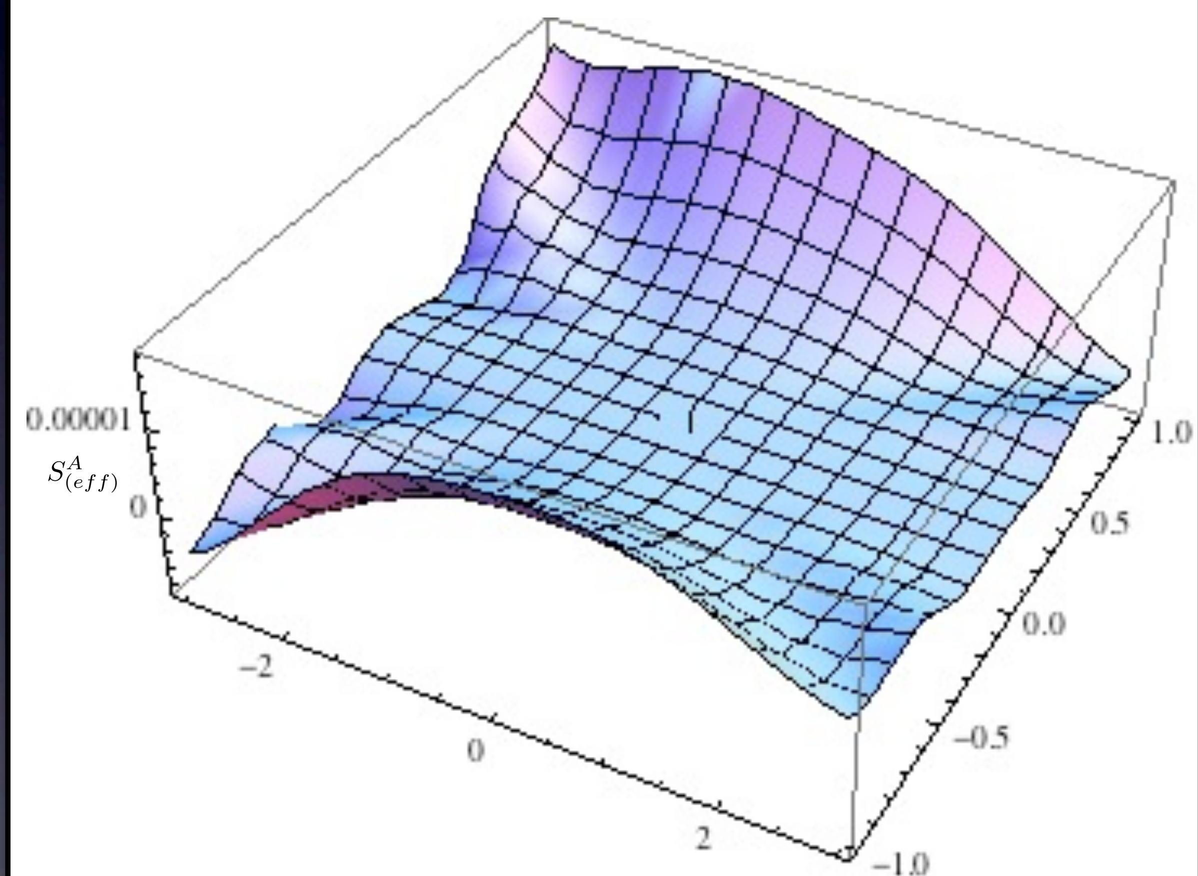
Substituting into the wave equation gives

$$\mathcal{D}^A{}_B \tilde{\varphi}^B_{(\mathrm{R})} = S^A_{(\mathrm{eff})}$$

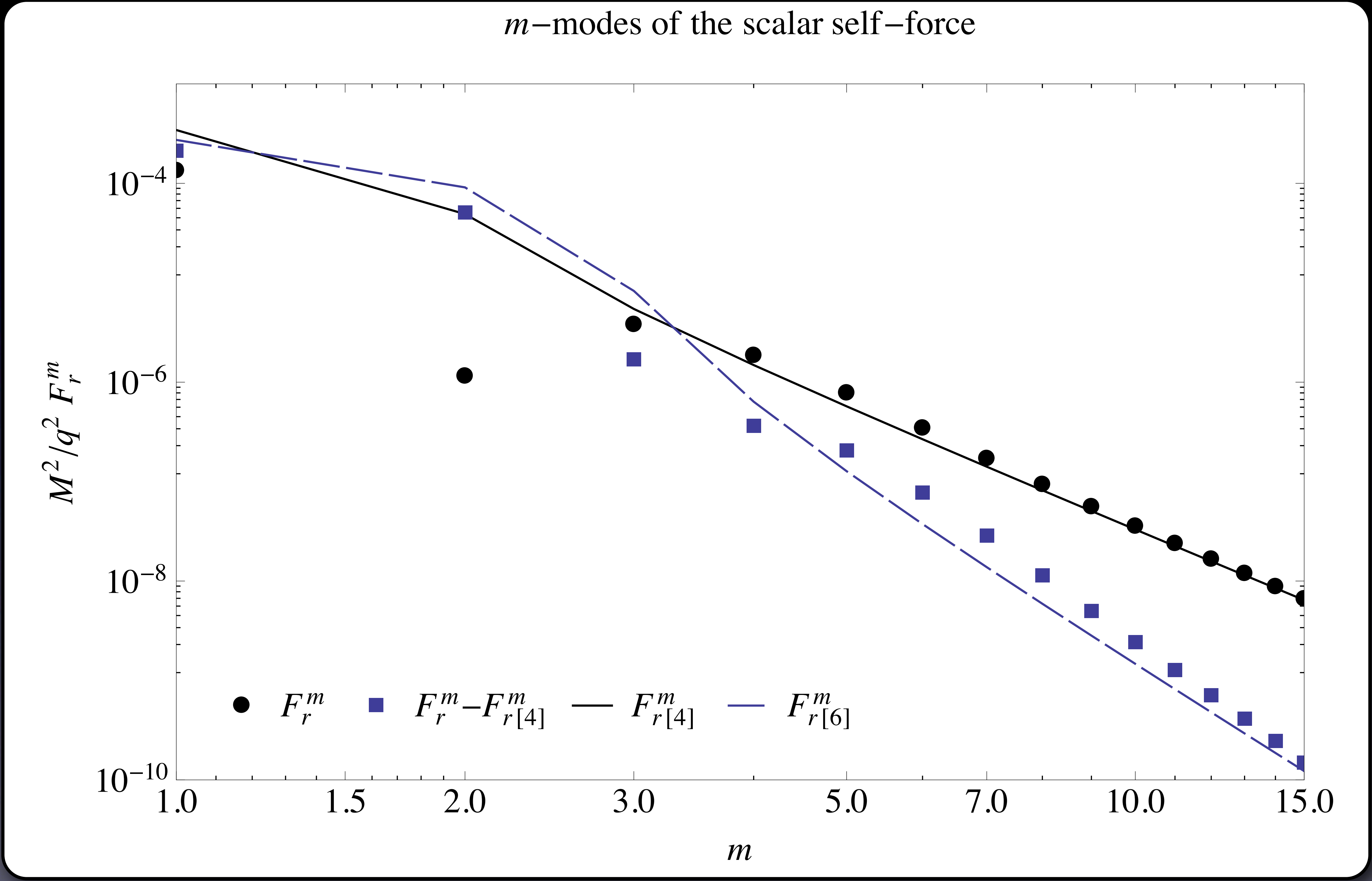
with effective source,

$$S^A_{(\mathrm{eff})} = \mathcal{D}^A{}_B \tilde{\varphi}^B - 4\pi \mathcal{Q} \int u^A \delta_4 \left(x, z(\tau')\right) d\tau'.$$

Effective Source Kerr Scalar



Kerr Scalar



Data supplied by Jonathan Thornburg

More on the way!

More on the way!

- More accurate calculations for Kerr gravitational case (Lorentz gauge): mode-sum (Patrick Nolan), effective source (Barack & Dolan)
- Regularisation in radiation gauge: application to matched expansions (Casals et al.), mode-sum (Shah, Friedman et al.)
- Cosmic Censorship