Self-force from equivalent periodic sources

Barak Kol
Hebrew University, Jerusalem
Capra 16 Dublin, July 2013

Based on arXiv:1307.xxxx

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Outline

- Orbits and frequencies
- Equivalent sources
- Regularization
- Discussion

Issues

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- Regularization

MiSaTaQuWa
Mode sum Regularization (Barack & Ori)
Detweiler-Whiting decomposition

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- Computational cost

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- Computational cost

- Characterization of conservative sector

Hinderer-Flanagan
Brinholtz-Hadar-BK

Goal

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Trajectory Parameters $(E,\, l,\, t_0,\, \phi_0)$

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- Obtain the adiabatic flow in the space of trajectories - first order EMR

Osculating orbits

Orbits and frequencies

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Quasi periodic frequency spectrum

$$\omega_{mn} = m\,\Omega_\phi + n\,\Omega_r$$

$$\Omega_r = \frac{2\pi}{P_t}$$

$$\Omega_\phi = \frac{P_\phi}{P_t}$$

Orbits and frequencies

Quasi periodic frequency spectrum

$$\omega_{mn} = m\,\Omega_\phi + n\,\Omega_r$$

$$\Omega_r = \frac{2\pi}{P_t}$$

$$\Omega_\phi = \frac{P_\phi}{P_t}$$

Frequency requirement - constructive interference in azimuthal direction

A stroboscope

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- Imagine a stroboscope flashing every P_{mn}

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- The body traces a curve

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- The body traces a curve
- It is equivalent to folding the trajectory over a periodic time coordinate

(m,n) equivalent source

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ergodic: time average to ensemble average

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equation $$0 = \psi(r,\phi) = \omega_{mn}\, t(r) + m\,(\phi - \phi(r))$$

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equation $$0 = \psi(r, \phi) = \omega_{mn}\, t(r) + m\,(\phi - \phi(r))$$

parametric form

$$\sigma = -m\,\alpha$$

$$\phi = n\,\alpha + \left(\phi(\sigma) - \frac{P_\phi}{2\pi}\,\sigma\right)$$

(m,n) equivalent source

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equation $$0 = \psi(r,\phi) = \omega_{mn}\, t(r) + m\,(\phi - \phi(r))$$

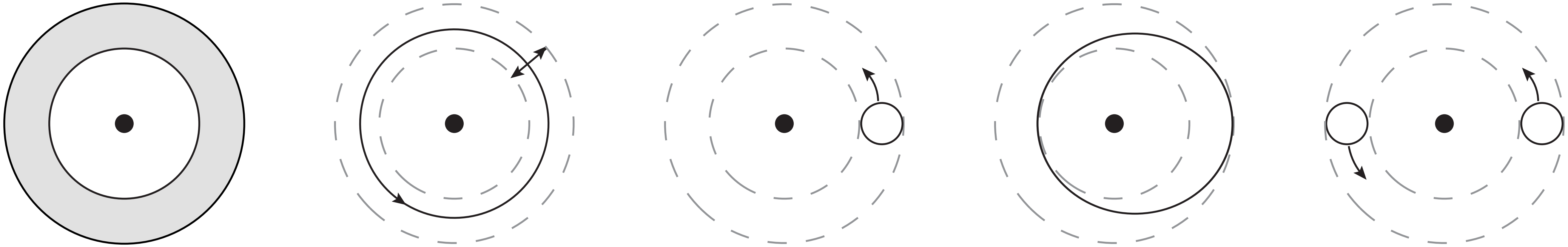
parametric form

$$\sigma = -m\,\alpha$$

$$\phi = n\,\alpha + \left(\phi(\sigma) - \frac{P_\phi}{2\pi}\,\sigma\right)$$

winds (-m) times around phi and n times around r

Examples



=	$(0\,0)$	$(0\,1)$	$(1\,0)$	$(1\,1)$	$(2\,0)$
=	0	Ω_r	Ω_Φ	$\Omega_\Phi - \Omega_r$	$2\Omega_\Phi$

Zero frequency

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$$\bar{\rho} := \langle \rho \rangle_t = \frac{q}{2\pi\, P_t\, r\, (dr/d\tau)} \delta(z)$$

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- The dissipative part $\rho' := \rho - \bar{\rho}$

Solving the field equations

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$$\left(\triangle - f^{-1}\,\partial_t^2\right)\Phi = 4\pi\,\rho$$

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- **Frequency domain** - natural here, elliptic equations

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- Frequency domain - natural here, elliptic equations
- Time domain

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- Less singular (1d density) for aperiodic motion and equivalent source

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- Zero freq. sector: surface charge density

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- Zero freq. sector: surface charge density
- In frequency space - similar to electrostatics with singular source

Electrostatics

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$$\Phi = \Phi_S + \Phi_R$$

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Electrostatics

$$\Phi = \Phi_S + \Phi_R$$

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$$[\triangle + f^{-1}\, \omega^2]\Phi_R = 4\pi\, \rho_R$$

Time domain in Progress - generalizing Hadamard's local construction

$$\Phi_S \sim \lambda\, \log \Gamma$$

Edge

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Charge density near r_{min} (or r_{max})

$$\rho(x, y, z) = \frac{\sigma_{-1/2}}{\sqrt{x}}\delta(z) \qquad x \geq 0$$

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$$\rho(x, y, z) = \frac{\sigma_{-1/2}}{\sqrt{x}} \delta(z) \qquad x \geq 0$$

Solution

$$\Phi = 2\pi \, \sigma_{-1/2} \, \Re\sqrt{-w}$$

$$w = x + i \, z$$

Outgoing radiation and self-force

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- Outgoing radiation

$$\Phi \sim \frac{\Phi_\infty(\theta,\phi)}{r}\, e^{-i\omega\, r} + 0 \cdot e^{i\omega\, r} \text{ for } r \to \infty$$

Outgoing radiation and self-force

- Outgoing radiation

$$\Phi \sim \frac{\Phi_\infty(\theta,\phi)}{r}\, e^{-i\omega\, r} + 0 \cdot e^{i\omega\, r} \text{ for } r \to \infty$$

- Self-force throughout trajectory: drift in

$(E,\, l,\, t_0,\, \phi_0)$

Method summary

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Method summary

- Goal: drift in trajectory parameters
- Equivalent periodic source
- Conservative is zero frequency
- Regularization
- Self-force computed throughout at once

Generalizations

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- Electromagnetism and gravity: source, waves

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- Rotating BH (Kerr)

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- Electromagnetism and gravity: source, waves
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- Higher EMR orders

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- Relativistic - or else hierarchy of scales

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- For freq. space: few freq. (low eccen.)

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- Relativistic - or else hierarchy of scales
- For freq. space: few freq. (low eccen.)
- Nearly incommensurate - the rational w. smallest denominator within the freq. ratio range

Paths for continuation

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- Gauge choice

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- Invitation for collaboration: Numerical evaluation

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- Time domain regularization - local expansion generalizing Hadamard

Paths for continuation

- Gauge choice
- Invitation for collaboration: Numerical evaluation
- Time domain regularization - local expansion generalizing Hadamard
- Formulate conservative sector of EMR


Thank you for your attention!

Givat Brenner

My home Kibbutz (village)

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