Electromagnetic Self Force

Circular Orbits in Schwarzschild Spacetime

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Motivation

- Working in Electromagnetism is a good warm-up for gravitational self-force
- Has its own physical motivation
- Circular orbits can be straightforwardly extended to eccentric ones

Field Equations

- The Self Force is given by the formula

$$F_\mu = -qF_{\mu\nu}u^\nu$$

with Field Tensor $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$

- Calculate this using the field equations for a spin one field, in Lorenz Gauge:

$$\Box A_\mu - R_\mu{}^\nu A_\nu = 0$$

Simplifying the problem: I

- To solve these PDEs, decompose fields into angular and radial components:

$$e^{i\omega t} A_\mu = \begin{pmatrix} 0 \\ 0 \\ R_4(r) X_\theta^{lm}(\theta,\phi) \\ -R_4(r) X_\phi^{lm}(\theta,\phi) \end{pmatrix} + \begin{pmatrix} R_1(r) Y^{lm}(\theta,\phi) \\ R_2(r) Y^{lm}(\theta,\phi) \\ R_3(r) Z_\theta^{lm}(\theta,\phi) \\ R_3(r) Z_\phi^{lm}(\theta,\phi) \end{pmatrix}$$

- Current J_μ decomposed similarly.

Simplifying the problem: 2

- Separating Even and Odd modes in equations leaves 3+1 ODEs to be solved
- We can use the gauge equation to eliminate one of our even sector fields:

$$R_3(r) \sim f(R_1(r) + R_2(r))$$

- System now has one decoupled, 2 coupled fields

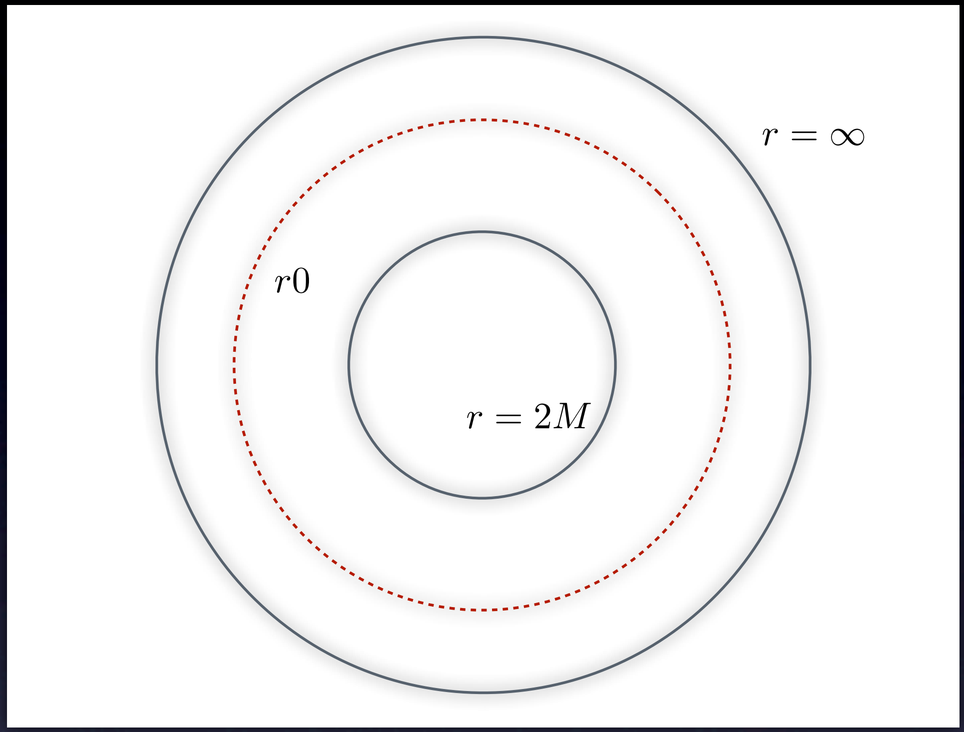
Solving for the Fields

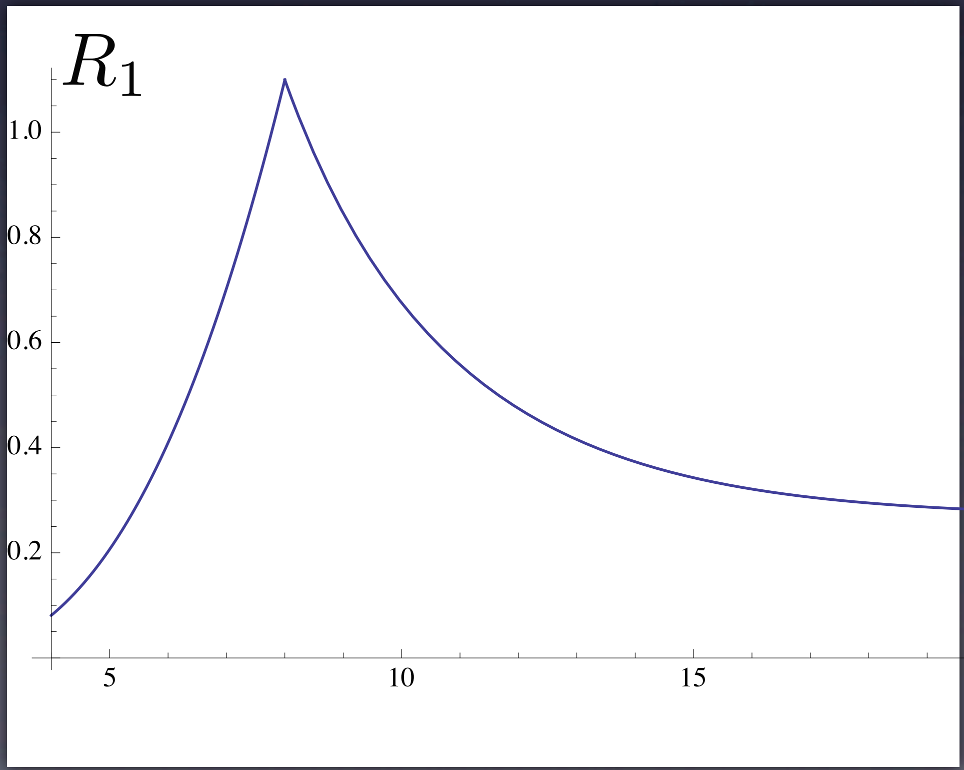
- We use a series expansion to approximate the fields at the boundaries:

INNER: $$e^{-i\omega r_*}\sum_{n=0}^{n_H} b_n^i (r-2M)^n$$

OUTER: $$e^{i\omega r_*}\sum_{n=0}^{n_\infty} \frac{a_n^i}{r^n}$$

- To match these solutions at the particle's orbit, we impose matching conditions



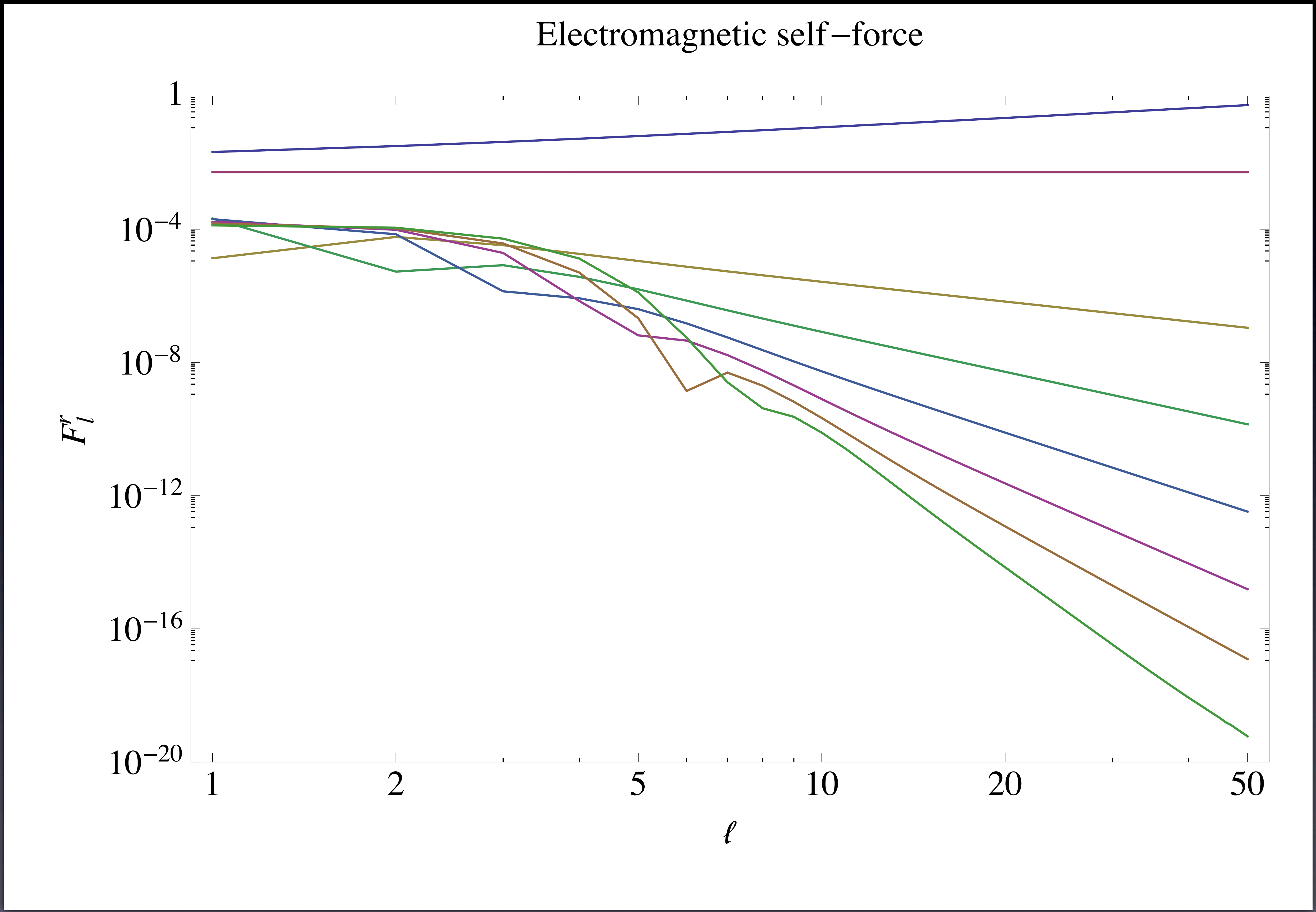


Construct the Self Force

- Having solved for the l-mode fields, it is now straightforward to construct the l-mode Faraday Tensor, and hence the self force:

$$F_\mu = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} F_{\mu\nu}^{lm}$$

Regularised Self Force:



Self Force EM

- $F_r =$ 0.0012098217906065(1)

  (circular orbit with r0=10, M=1)

- Much more accurate than current EM data
- Successfully applies the new regularisation parameters

Comparing to Gravity

- Method mostly extends directly to gravity
- coupling in both sectors
- static mode complications

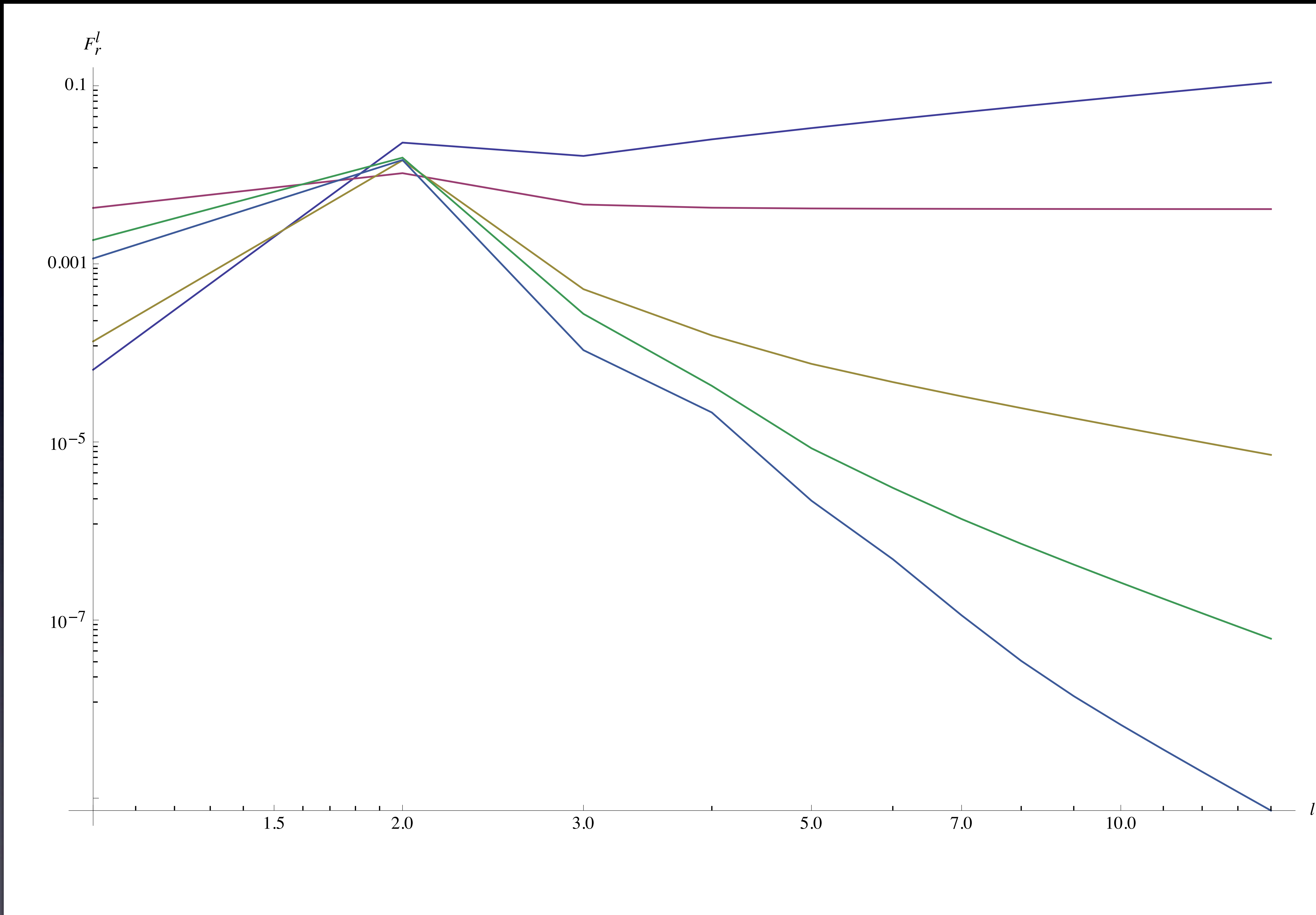
EM	Gravity
3+1 fields	7+3 fields
1+0 gauge	3+1 gauge
2+1 to solve	4+2 to solve

Static Terms

- EM static (m=0) modes are known analytically
- Only known for gravity odd sector
- even sector requires asymptotic expansion
- Outgoing ansatz must be changed:

$$R^i_{inf} = \sum_{n=0}^{n_\infty} \frac{a^i_n + b^i_n log(r)}{r^n}$$

Gravity Regularised:



Gravitational Self Force

$$F_r = 0.013389(5)$$
$$F_t = -0.000091907(6)$$

(circular orbit with r0=10, M=1)

- Very close to making use of new regularisation parameters
- Expect to have more accurate data than is currently available

To-Do List

- Need to extend gravity data to include higher l-modes
- Want to check data against independent calculation

$\rightarrow$ Regge-Wheeler Comparison