Radiation Reaction Force of a Charged Particle in a Reissner-Nordström Space-time

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Outline

The idea

Consider a system:

- Composed of two massive, charged spherically symmetric bodies
- The body with bigger mass and charge is fixed in the coordinate origin
- The bodies are far enough from each other
- The other body can be considered as a point charge, and is going to radiate

Under these assumptions, the radiation reaction force is calculated by using the weak field limit method and the post-Newtonian potentials associated with the Reissner-Nordström metric.

The weak field limit

To know the radiation reaction force of a charged point particle q, that moves in a weakly curved space-time characterized by a Newtonian, time independent potential, it is necessary:

1. Find de retarded Green function for the electromagnetic field in a weakly curved space-time.
2. Evaluate the integral of the tail term over the past word line.

Some of the most important articles about the weak field limit are:

$$\text{Weak field} \rightarrow \left\{ \begin{array}{c} \text{De Witt-De Witt (1964) [2]} \\ \text{Wiseman (2000) [3]} \\ \text{Pfenning and Poisson (2002) [4]} \end{array} \right\}$$

where it is obtained a force composed by a conservative part and other that represents de radiation reaction force.

The Weak Field Limit

There are some assumptions considered to use this method:

- The force is going to be calculated to first order in the Newtonian potential $(\Phi \text{ or } \psi)$.
- The particle is gravitationally bounded to the matter distribution
- Also, by using the virial theorem, it is supposed that the squared velocity v^2 is of the same order of magnitude of the potential, which means that the motion is very slow.
- Terms that involves Φ^2, Φv^2, and v^4,... are going to be neglected.
- The superposition principle is valid under these assumptions.

THIS MEANS THAT IN THE CASE OF A CURVED SPACE-TIME DUE TO THE MASS AND THE CHARGE WE CAN USE THE POST-NEWTONIAN POTENTIALS ASSOCIATED TO SUCH SPACE-TIME

The Weak Field Limit

Also the space-time must be seen as a perturbation of the flat space-time, around some potentials that must be:

- Newtonian
- Time independent
- The matter distribution or whatever causes the curvature must be bounded
- At long distances from the mater, the potential $\longrightarrow -\frac{M}{r}$

The equations of motion

- It is known that the equation of motion for a charged particle is given by: (Quinn and Wald: [1])

$$
\begin{aligned}
m u^{\alpha}{}_{;\beta} u^{\beta} &= f^{\alpha}_{ext} + \frac{2}{3}\frac{e^2}{m}\left(\delta^{\alpha}{}_{\beta} + u^{\alpha} u_{\beta}\right) \dot{f}^{\beta}{}_{ext} \\
&\quad + \frac{1}{3} e^2 \left(R^{\alpha}{}_{\beta} u^{\beta} + u^{\alpha} R_{\beta\gamma} u^{\beta} u^{\gamma}\right) + f^{\alpha}_{em},
\end{aligned} \tag{1}
$$

where the electromagnetic self-force is:

$$
f^{\alpha}_{em} = -e^2 \int_{-\infty}^{\tau^-} \left(G^{\alpha}{}_{\gamma';\beta} - G^{\alpha}{}_{\beta;\gamma'}\right) u^{\beta} u^{\gamma'} d\tau'. \tag{2}
$$

- For this method it is assumed that the charged particle is in absence of any external force, and is in a region without matter, then the equations of motions are:

$$
m u^{\alpha}{}_{;\beta} u^{\beta} = f^{\alpha}_{em}. \tag{3}
$$

The Reissner-Nordström Space-time

The metric that describes such a system is given by:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} - d\Omega \tag{4}$$

On the other hand, the temporal component for the metric can be written as a function of a pair of potentials:

$$g_{00} = 1 - 2\left(\phi + \psi\right) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = 1 + 2\left(-\frac{M}{r} + \frac{Q^2}{2r^2}\right) \tag{5}$$

Therefore, the post-Newtonian potentials for the R-N metric are:

$$\phi = -\frac{M}{r}, \qquad \psi = \frac{Q^2}{2r^2} \tag{6}$$

These potentials fulfill Poisson´s equation and the equality

$$\nabla^2\psi = \frac{\partial^2\phi}{\partial t^2} + 4\pi\left(T^{00} + T^{ii}\right) = \frac{Q^2}{r^4}\ , \qquad \nabla^2\phi = 4\pi\rho \tag{7}$$

- Using these potentials the metric take the form:

$$ds^2 = -\left(1 + 2\left(\Phi + \psi\right)\right) dt^2 + \left(1 - 2\left(\Phi + \psi\right)\right)\left(dx^2 + dy^2 + dz^2\right). \tag{8}$$

- The components of the metric can be written as

$$g_{\alpha\beta} = \eta_{\alpha\beta} - 2\left(\Phi + \psi\right)\chi_{\alpha\beta}, \tag{9}$$

where

$$\chi_{\alpha\beta} \equiv \eta_{\alpha\beta} + 2t_\alpha t_\beta, \tag{10}$$

with $\eta_{\alpha\beta}$ Minkowski's metric and t^α the timelike killing vector

- These are the tensors and scalars related with the metric

Determinant	$\sqrt{1-4(\Phi+\psi)} \approx 1-2(\Phi+\psi)$
Metric tensor	$g^{\alpha\beta} = \eta^{\alpha\beta} + 2(\Phi+\psi)\chi^{\alpha\beta}$
SDC	$\Gamma^{\mu}_{\alpha\beta} = -\chi^{\mu}_{\alpha}(\Phi+\psi)_{,\beta} - \chi^{\mu}_{\beta}(\Phi+\psi)_{,\alpha} + \chi_{\alpha\beta}(\Phi+\psi)^{,\mu} + O(\Phi^2, \psi^2, \Phi\psi)$
Riemann Tensor	$R^{\mu}{}_{\alpha\beta\gamma} = -\chi^{\mu}_{\gamma}(\Phi+\psi)_{,\alpha\beta} + \chi^{\mu}_{\beta}(\Phi+\psi)_{,\alpha\gamma} - \chi_{\alpha\beta}(\Phi+\psi)^{,\mu}_{\gamma} + \chi_{\alpha\gamma}(\Phi+\psi)^{,\mu}_{\beta}$
Ricci Tensor	$R_{\alpha\gamma} = \chi_{\alpha\gamma}\Box(\Phi+\psi)$
Ricci Scalar	$R = 2\Box(\Phi+\psi)$ $(\Box(\Phi+\psi) = \eta^{\alpha\beta}(\Phi+\psi)_{,\alpha\beta})$

Table 1.Tensors that describe the geometry of the space-time

- By using the superposition principle, it is observed that the total self-force can be written as

$$f^{\alpha} = f^{\alpha}_{\Phi} + f^{\alpha}_{\psi}. \tag{11}$$

- The part associated with Φ, is the self force found by Pfenning and Poisson [4],

$$\overrightarrow{f}_{\Phi} = q^2 \frac{M}{r^3}\widehat{r} + \frac{2}{3} q^2 \frac{d\overrightarrow{g}}{dt} \tag{12}$$

where $\widehat{r} = \frac{\overrightarrow{x}}{r}$ and $\overrightarrow{g} = -\nabla\Phi$.

- Given the potential A^α, that represents the electromagnetic field and fulfill the Lorenz Gauge $A^\alpha_{;\alpha} = 0$, its field equations are:

$$g^{\mu\nu} A^{\alpha}{}_{;\mu\nu} - R^{\alpha}_{\mu} A^{\mu} = -4\pi j^{\alpha}. \tag{13}$$

- In its densitized form, is written as:

$$E^{\alpha}\left[A\right] = -4\pi\sqrt{-g} j^{\alpha} \tag{14}$$

where,

$$\begin{aligned} E^{\alpha}\left[A\right] &= \Box A^{\alpha} + 4\psi t^{\mu} t^{\nu} A^{\alpha}{}_{,\mu\upsilon} \\ &\quad -2\left(\chi^{\alpha}_{\rho}\psi_{,\upsilon}A^{\rho,\upsilon} + \chi^{\alpha}_{\upsilon}\psi_{,\rho}A^{\rho,\nu} - \chi_{\rho\upsilon}\psi^{,\alpha}A^{\rho,\upsilon}\right) \\ &\quad -\left(\chi^{\alpha\upsilon}\psi_{,\rho\nu} + 2\chi^{\alpha}_{\rho}\Box\psi - \chi^{\upsilon}_{\rho}\psi^{,\alpha}{}_{\upsilon}\right) A^{\rho}. \end{aligned} \tag{15}$$

- To solve this equation the Green function is used.

- The Green functions are obtained by using differential operators over a pair of functions $A(x,x')$ and $B(x,x')$ defined as:

$$A_\psi(x,x') = \frac{1}{2\pi}\int G^{flat}(x,x'')\,\psi(x'')\,G^{flat}(x'',x')\,d^4x'', \tag{16}$$

$$B_\psi(x,x') = \frac{1}{2\pi}\int G^{flat}(x,x'')\,\nabla^2\psi(x'')\,G^{flat}(x'',x')\,d^4x'', \tag{17}$$

where $G^{flat}(x,x')$ is the retarded Green function for the wave operator in a flat space-time

$$G^{flat}(x,x') = \frac{\delta\left(t-t'-|\mathbf{x}-\mathbf{x}'|\right)}{|\mathbf{x}-\mathbf{x}'|}, \tag{18}$$

$$\Box G^{flat}(x,x') = -4\pi\delta_4(x-x'). \tag{19}$$

- The solution to the field equations (14), in terms of the Green function are of the form:

$$A^{\alpha}\left(x\right)=\int G^{\alpha}{}_{\beta'}\left(x,x'\right)j^{\beta'}\left(x'\right)\sqrt{-g'}d^{4}x' \tag{20}$$

Where $G^{\alpha}{}_{\beta'}\left(x,x'\right)$ is the solution to the equation

$$E^{\alpha}{}_{\beta'}\left[G\right]=-4\pi\delta^{\alpha}{}_{\beta'}\delta_{4}\left(x-x'\right) \tag{21}$$

- To find the Green function to firs order in ψ it is written

$$G^{\alpha}{}_{\beta'}(x,x')=G^{flat}(x,x')\delta^{\alpha}{}_{\beta'}+\dot{G}^{\alpha}_{\psi\,\beta'}(x,x')+O\left(\psi^{2}\right) \tag{22}$$

- By using the Green function perturbation in (14), it is obtained

$$\begin{aligned} \Box \dot{G}^{\alpha}_{\psi\ \beta'}(x,x') &= -4\psi\partial_{tt}G^{flat}(x,x')\delta^{\alpha}{}_{\beta} \\ &+2\nabla^2\psi\chi^{\alpha}_{\beta}G^{flat}(x,x') \\ &+2\left(\chi^{\alpha}_{\upsilon}\psi_{,\beta}+\chi^{\alpha}_{\beta}\psi_{,\upsilon}-\chi_{\beta\upsilon}\psi^{,\alpha}\right)G^{flat,\upsilon}(x,x'). \end{aligned} \tag{23}$$

- The solution for this equation is:

$$\dot{G}^{\alpha}{}_{\beta'}\left(x,x'\right) = -\frac{1}{4\pi}\int G^{flat}\left(x,x''\right)\Box\dot{G}^{\alpha}_{\psi\ \beta'}(x,x')d^4x''. \tag{24}$$

- By using relations between the derivatives of the functions A_ψ and B_ψ, it is written

$$\begin{aligned} \dot{G}^{\alpha}_{\psi\ \beta'}(x,x') &= -2\partial_{tt'}A_\psi(x,x')\delta^{\alpha}{}_{\beta} + \left(\partial^{\alpha'}{}_{\beta} - \partial^{\alpha}{}_{\beta'}\right)A_\psi\left(x,x'\right) \\ &\quad +2t^{\alpha}\left(\partial_{t'\beta} - \partial_{t\beta'}\right)A_\psi\left(x,x'\right) \\ &\quad +2t_{\beta}\left(\partial^{\alpha'}{}_{t} - \partial^{\alpha}{}_{t'}\right)A_\psi\left(x,x'\right) \\ &\quad +\chi^{\alpha}_{\beta}\left(\triangle\psi G^{flat}\left(x,x'\right) - \frac{1}{2}B_\psi\left(x,x'\right)\right), \end{aligned} \tag{25}$$

where

$$\triangle\psi = \psi\left(x\right) - \psi\left(x'\right). \tag{26}$$

- Or by components:

$$\begin{aligned}
\dot{G}^{t}_{\psi\, t'}(x,x') &= -2\partial_{tt}A_{\psi}(x,x') + \triangle\psi G^{flat}\left(x,x'\right) - \frac{1}{2}B_{\psi}\left(x,x'\right), \quad (27)\\
\dot{G}^{t}_{\psi\, a'}(x,x') &= \left(\partial^{t'}{}_{a} - \partial^{t}{}_{a'}\right)A_{\psi}\left(x,x'\right),\\
\dot{G}^{a}_{\psi\, t'}(x,x') &= \left(\partial^{a'}{}_{t} - \partial^{a}{}_{t'}\right)A_{\psi}\left(x,x'\right),\\
\dot{G}^{a}_{\psi\, b'}(x,x') &= \delta^{a}{}_{b}\left(\triangle\psi G^{flat}\left(x,x'\right) - 2\partial_{tt'}A_{\psi}(x,x') - \frac{1}{2}B\left(x,x'\right)\right)\\
&\quad + \left(\partial^{a'}{}_{b} - \partial^{a}{}_{b'}\right)A_{\psi}\left(x,x'\right).
\end{aligned}$$

- The self force associated with a charged particle q is:

$$f_\psi^\alpha = -q^2 \int_{-\infty}^{\tau-} \left(G_\psi^\alpha{}_{\gamma';\beta} - G_{\psi\beta\gamma'}{}^{;\alpha} \right) u^\beta u^{\gamma'} \sqrt{1 - v'^2} dt'. \qquad (28)$$

- By using eq. (22),

$$f_\psi^\alpha = -q^2 \int_{-\infty}^{t} \left(\dot{G}_\psi{}^\alpha{}_{\gamma',\beta} - \dot{G}_{\psi}{}_{\beta\gamma'}{}^{;\alpha} \right) u^\beta u^{\gamma'} dt'. \qquad (29)$$

- Or by using the components of the perturbation for the Green function

$$\begin{aligned} f_{\psi}^{\alpha} &= -q^2 \int_{-\infty}^{t} \left(\dot{G}_{\psi\, at',t} - \dot{G}_{\psi\, tt',a} \right) dt' \\ & -q^2 \int_{-\infty}^{t} \left(\dot{G}_{\psi\, a\ t',b} - \dot{G}_{\psi\, bt',a} \right) v^b dt' \\ & -q^2 \int_{-\infty}^{t} \left(\dot{G}_{\psi ab',t} - \dot{G}_{\psi\, tb',a} \right) v^{b'} dt'. \end{aligned} \tag{30}$$

- It is observed that the self-force can be divided in two parts

$$f_{\psi}^{\alpha} = f_{\psi A}^{\alpha} + f_{\psi B}^{\alpha}. \tag{31}$$

- The first part is given by:

$$
\begin{aligned}
f_A^a \quad = \quad & -q^2 \int_{-\infty}^{t} \Big[A_{\psi,a'tt} - A_{\psi,at't} - 2A_{\psi,tt'a} + \left(\triangle\psi G^{flat} \right)_{,a} \\
& + \left(A_{\psi,a'tb} - A_{\psi,at'b} - A_{\psi,b'ta} + A_{\psi,bt'a} \right) v^b \\
& + \delta^a{}_b \left(\left(\triangle\psi G^{flat} \right)_{,t} - 2A_{\psi,tt't} \right) v^{b'} \\
& + \left(A_{\psi,a'bt} - A_{\psi,ab't} - A_{\psi,t'ba} + A_{\psi,tb'a} \right) v^{b'} \Big] dt'.
\end{aligned}
\tag{32}
$$

- By integrating considering ellipsoidal coordinates, a Taylor series expansion of A_ψ, and the coincidence limit it is obtained

$$f^a_{\psi A} = q^2 \left(-\frac{2}{3} \psi_{,ab} \left(\mathbf{x} \right) \right) v^b, \tag{33}$$

or

$$\overrightarrow{f}_{\psi A} = -q^2 \frac{2}{3} \frac{d \nabla \psi}{dt} = -q^2 Q^2 \frac{1}{3} \frac{d}{dt} \nabla \frac{1}{r^2} \tag{34}$$

- On the other hand, the second part is given by

$$f^a_{\psi B} = -\frac{1}{2} q^2 \int_{-\infty}^{t} \left(B_{\psi,a} + B_{\psi,t} v^a \right) dt'. \tag{35}$$

This part changes when comparing with the contribution for $f^a_{\phi B}$, since in this case the contribution cannot be neglected

- For the potential ψ, the evaluation of the two point function B_ψ gives:

$$B_\psi\left(x, x'\right) = \frac{2Q^2\delta\left(u\right)}{rr'\left(r^2+r'^2\right)^2}\left[\frac{1}{12}\triangle t^3 - \frac{5}{3}e^2\triangle t\right], \tag{36}$$

with $u \equiv \Delta t - r - r'$, and $\Delta t = t - t'$.

- This must be compared with the contribution from ϕ [4]

$$B_\phi = \frac{M}{rr'}\delta\left(u\right) \rightarrow f^a_{\phi B} = q^2\frac{M}{r^3}r_{,a} \tag{37}$$

- The contribution asociated with the integral of the spatial derivative is given by

$$
\begin{aligned}
\frac{1}{2}q^2\int_{-\infty}^{t} B_{\psi,a}dt' = & \frac{1}{2}q^2\left[\frac{1}{12}\left(r+r'\right)^3-\frac{5}{3}e^2\left(r+r'\right)\right]\times \qquad (38)\\
& \left(\begin{array}{c}\frac{2Q^2r_{,a}v_r'}{rr'^2\left(r^2+r'^2\right)^2}+\frac{8Q^2r_{,a}}{r\left(r^2+r'^2\right)^3}-\frac{16Q^2r_{,a}v_r'}{r\left(r^2+r'^2\right)^3}\\ +\frac{2Q^2r_{,a}a_r'v_r'}{rr'\left(r^2+r'^2\right)^2}\end{array}\right)\\
& -\frac{Q^2r_{,a}\left(r+r'\right)^2v_r'}{2rr'\left(r^2+r'^2\right)^2}+\frac{10Q^2r_{,a}e^2v_r'}{3rr'\left(r^2+r'^2\right)^2}\\
& +\frac{5Q^2r_{,a}v_r'}{3rr'\left(r^2+r'^2\right)^2}\left(r'-r\cos\alpha\right)\left(r+r'\right)
\end{aligned}
$$

- While the contribution considering the integral of the time derivative is:

$$
\begin{aligned}
\frac{1}{2}q^2\int_{-\infty}^{t} B_{\psi,t}v^a dt' \quad = \quad & \frac{q^2Q^2\left(r+r'\right)^2 v^a}{2rr'\left(r^2+r'^2\right)^2} - \frac{10q^2Q^2e^2v^a}{3rr'\left(r^2+r'^2\right)^2} \\
& - \frac{q^2Q^2\left(r+r'\right)^3 v^a}{6r^2r'\left(r^2+r'^2\right)^2} + \frac{10q^2Q^2e^2\left(r+r'\right)v^a}{3r^2r'\left(r^2+r'^2\right)^2} \\
& - \frac{2q^2Q^2\left(r+r'\right)^3 v^a}{3r'\left(r^2+r'^2\right)^3} + \frac{40q^2Q^2e^2\left(r+r'\right)v^a}{3r'\left(r^2+r'^2\right)^3} \\
& + \frac{5Q^2\left(r+r'\right)v^a}{3r'\left(r^2+r'^2\right)^2} - \frac{5Q^2\left(r+r'\right)v^a}{3r\left(r^2+r'^2\right)^2}\cos\alpha
\end{aligned} \tag{39}
$$

The Weak Field Limit

The Radiation Reaction Force

- After adding both contributions, integrating, using a Taylor series expansion over v, and that $v' = v_r + O\left(a\right)$, $r' = r\left(1 - 2v_r\right) + O\left(v_r^2, a\right)$, then the contribution to the self force is:

$$
\begin{aligned}
f^a_{\psi B} &= \frac{4Q^2q^2r_{,a}v_r}{3r^4} - \frac{Q^2q^2r_{,a}}{3r^4} + \frac{125Q^2q^2e^2r_{,a}v_r}{12r^6} + \frac{5Q^2q^2e^2r_{,a}}{3r^6} \\
&+ \frac{5Q^2q^2r_{,a}v_r}{12r^4}\cos\alpha - \frac{q^2Q^2v^a}{3r^4} - \frac{25q^2Q^2e^2v^a}{6r^6} \\
&+ \frac{5Q^2q^2v^a}{6r^4}\cos\alpha
\end{aligned}
\tag{40}
$$

Approximating the ellipticity as $e^2 = \frac{r^2}{8}$, and considering that the angle between $\mathbf{x}$ and $\mathbf{x}'$ is very small, and then $\cos\alpha \sim 1$, it is obtained

$$f_B^a = \frac{293 Q^2 q^2 r_{,a} v_r}{96 r^4} - \frac{Q^2 q^2 r_{,a}}{8 r^4} - \frac{q^2 Q^2 v^a}{48 r^4} \tag{41}$$

In its vectorial form, the self-force is

$$\overrightarrow{f}_B = \frac{97 Q^2 q^2 v_r}{32 r^4}\widehat{r} - \frac{Q^2 q^2}{8 r^4}\widehat{r} - \frac{q^2 Q^2}{48 r^3}\frac{d\widehat{r}}{dt} \tag{42}$$

The Weak Field limit

The complete electromagnetic self-force

- The radiation reaction force that acts over the charged particle q will be:

$$\overrightarrow{f}_{self} = q^2 \frac{M}{r^3}\hat{\mathbf{r}} + \frac{2}{3} q^2 \frac{d\mathbf{g}}{dt} - \frac{2}{3} q^2 Q^2 \frac{d\nabla \frac{1}{r^2}}{dt} + \frac{97 Q^2 q^2 v_r}{32 r^4}\hat{r} - \frac{Q^2 q^2}{8r^4}\hat{r} - \frac{q^2 Q^2}{48 r^3}\frac{d\hat{r}}{dt} \quad (43)$$

- The complete equation of motion will be, the Ford- O'Conell equation in flat space-time given by

$$m\overrightarrow{a} = \overrightarrow{f}_{emext} + \frac{2}{3}\frac{e^2}{m}\frac{d}{dt}\overrightarrow{f}_{emext} \quad (44)$$

plus the gravitational force, and the radiation reaction force $\overrightarrow{f}_{self}$.

- Then the equation of motion is:

$$
\begin{aligned}
m\vec{a} &= \vec{f}_g + \vec{f}_E + \frac{2}{3}\frac{q^2}{m}\frac{d}{dt}\vec{f}_E + q^2\frac{M}{r^3}\hat{r} + \frac{2}{3}q^2\frac{d\vec{g}}{dt} \\
&\quad - q^2 Q^2 \frac{2}{3}\frac{d\nabla\frac{1}{r^2}}{dt} + \frac{97Q^2q^2v_r}{32r^4}\hat{r} - \frac{Q^2q^2}{8r^4}\hat{r} - \frac{q^2Q^2}{48r^3}\frac{d\hat{r}}{dt}
\end{aligned} \tag{45}
$$

What is next?

- Since now we have an expression for the self-force then we will try:
  - To use the classical orbit for the charge and obtain an approximated trajectory for the radiating charge
  - To compare the trajectories for the case of the charged particle in the Reissner-Nordström space time with or without radiation
  - To use the mode sum method and compare the results
  - To compare with the solutions for the Cubic Quintic Duffing Oscillators, since the equations have almost the same form, if it is considered the change of variable $u = \frac{1}{r}$.
    - Note that the equation for an undamped cubic-quintic Duffing oscillator is:

$$x'' + \alpha x + \beta x^3 + \gamma x^5 = 0$$

References

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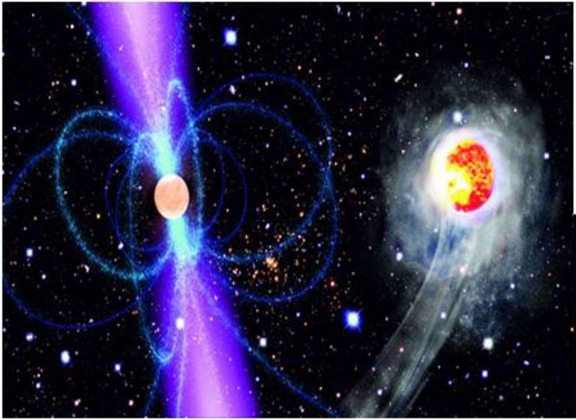
Thank you!



Artist's impression shows the speedy companion (right) as it races around the pulsar PSR J1311-3430 (left).

(http://www.space.com/18218-fastest-orbiting-pulsar-neutron-star.html)