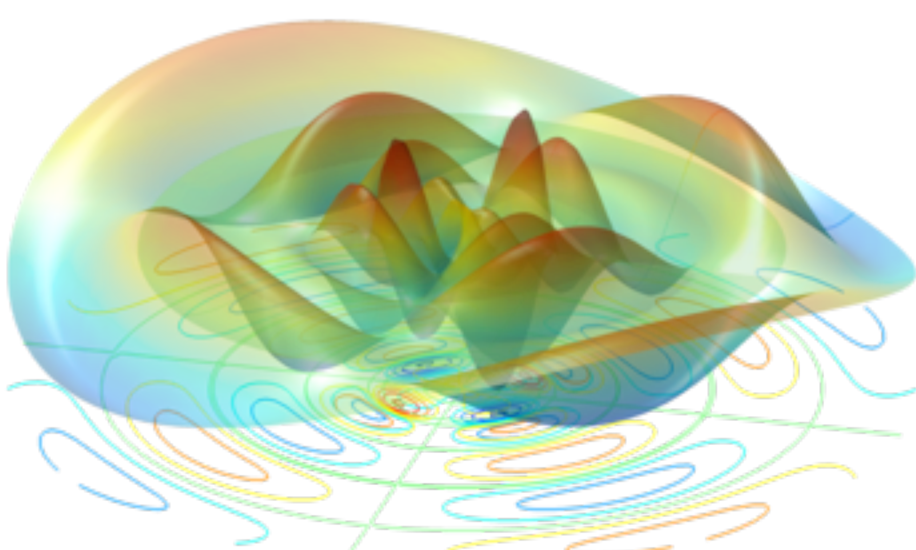
Time-domain solution of gravitational perturbations of Kerr and waveforms from large-mass-ratio mergers

S. Bernuzzi
TPI-FSU Jena / SFB-TR7

05.06.2014 Caltech
17th Capra



with
E.Harms, A.Nagar, A.Zenginoglu

Time-domain 2+1 Teukolsky equation

- Scalar case under control *[Krivan+ 1996; Scheel+ 2004; Lehner+ 2004; Dorband+ 2006; Gleiser+ 2008;Tiglio+ 2008; Zenginoglu&Kidder 2010; Zenginoglu+ 2009; Jasiulek 2012; Racz&Toth 2012; ...]*
- Gravitational case (|S|=2) only *[Krivan+ 1996,1997]* *(used in [Lopez-Aleman+ 2003, Khanna 2004, Pazos-Avalos&Lousto 2005])*
- Time-domain wanted/efficient for specific problems, e.g.
  - Tails
  - Particle sources and generic orbits
  - Large-mass-ratio mergers (our BBH Lab...)
- This talk based on:

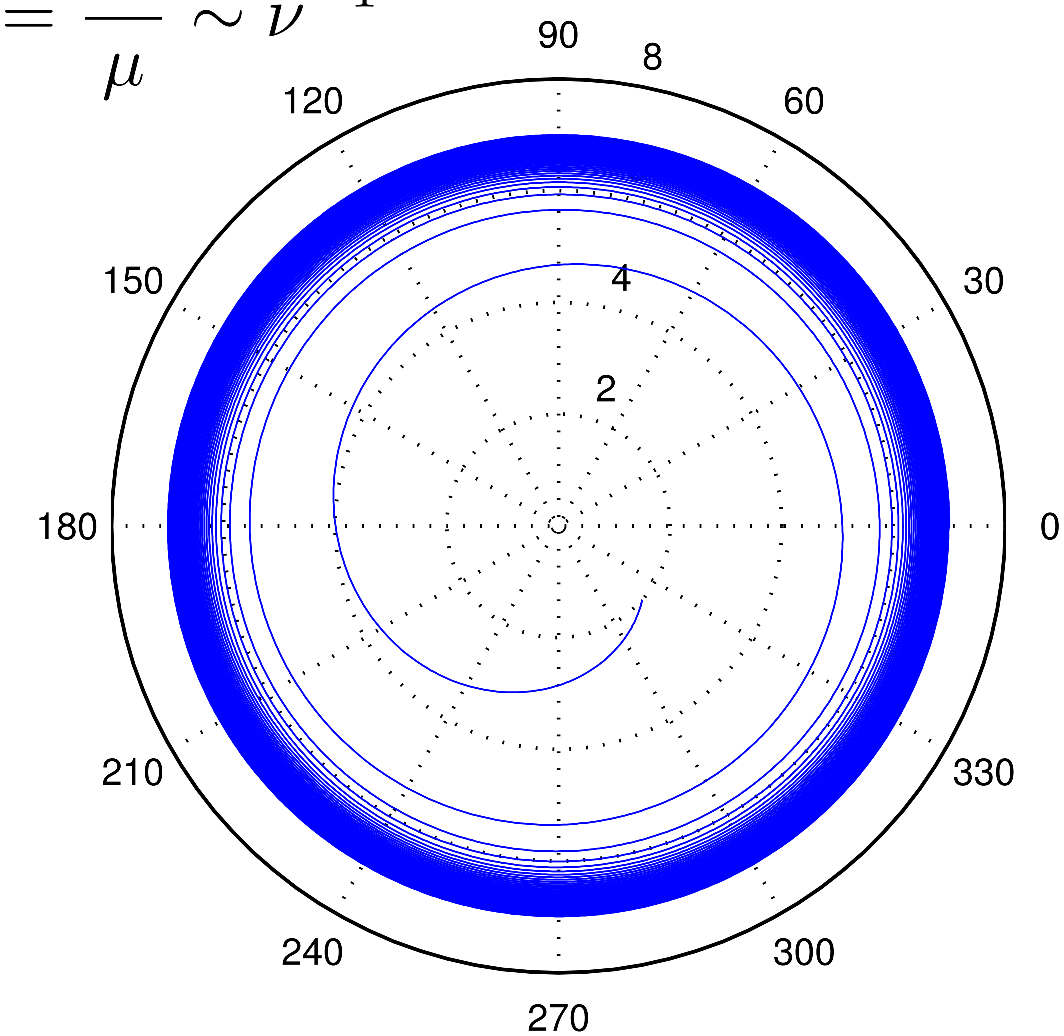
[Harms, SB, Bruegmann 2013]

[Harms, SB, Nagar, Zenginoglu 2014]

Large-q BBH Lab: EOB + GWs

[Nagar+ 2007, Damour&Nagar 2008]

$q = \frac{M}{\mu} \sim \nu^{-1}$



- Hamiltonian (geodesic) dynamics
- Linear in ν Radiation Reaction (EOB resum waveform)

[Damour+ 2008; Fujita&Iyer 2010; Pan+ 2011]

Large-q BBH Lab: EOB + GWs

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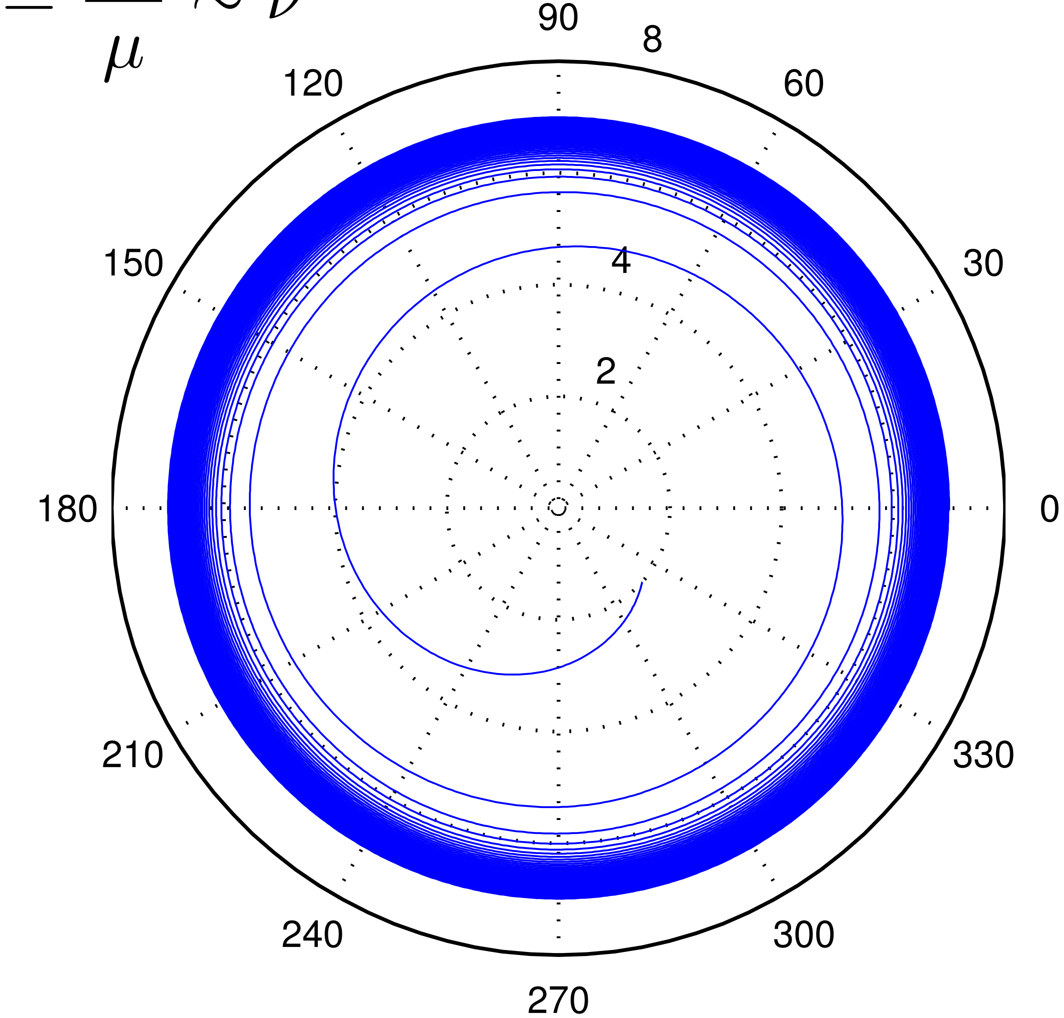
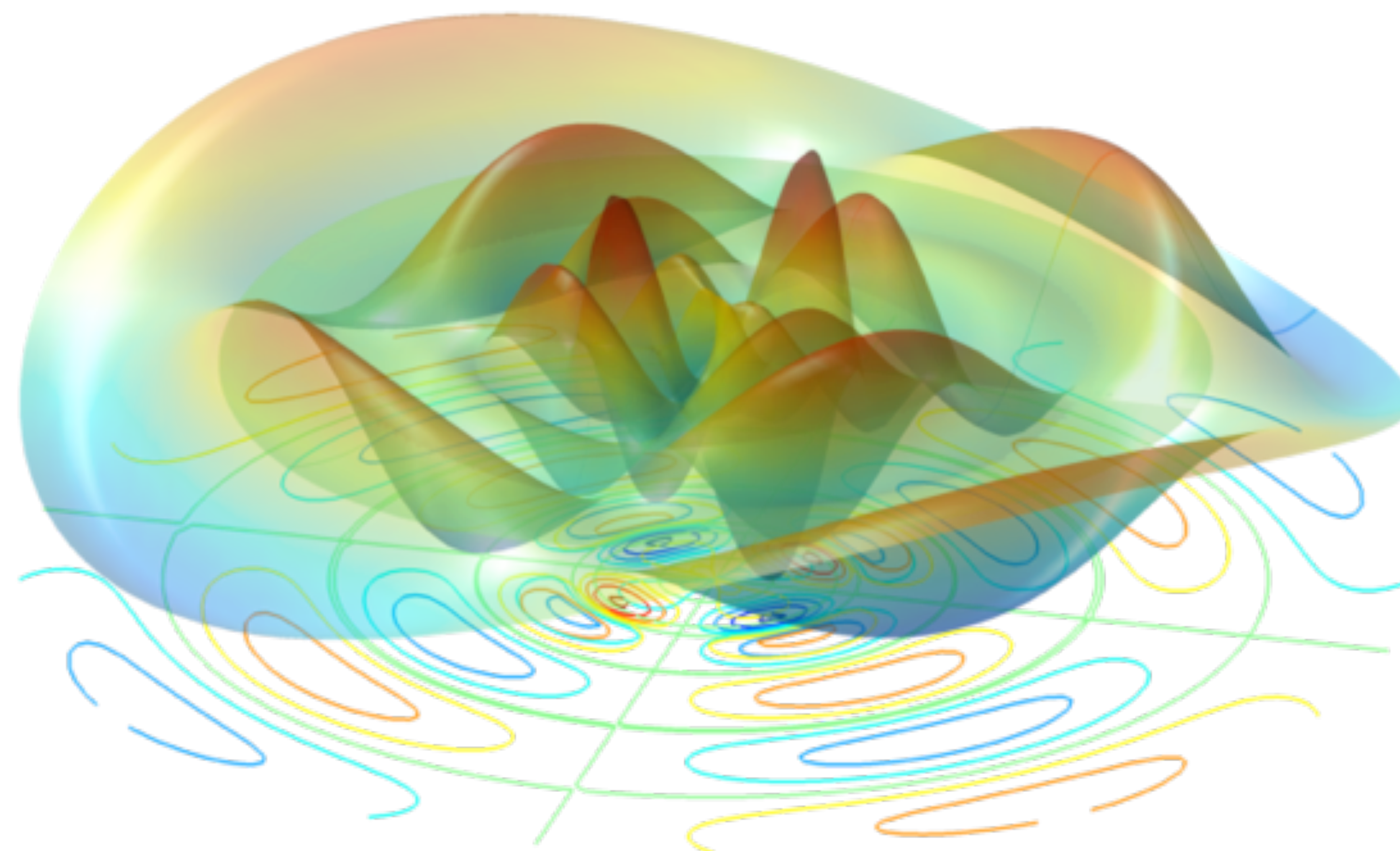




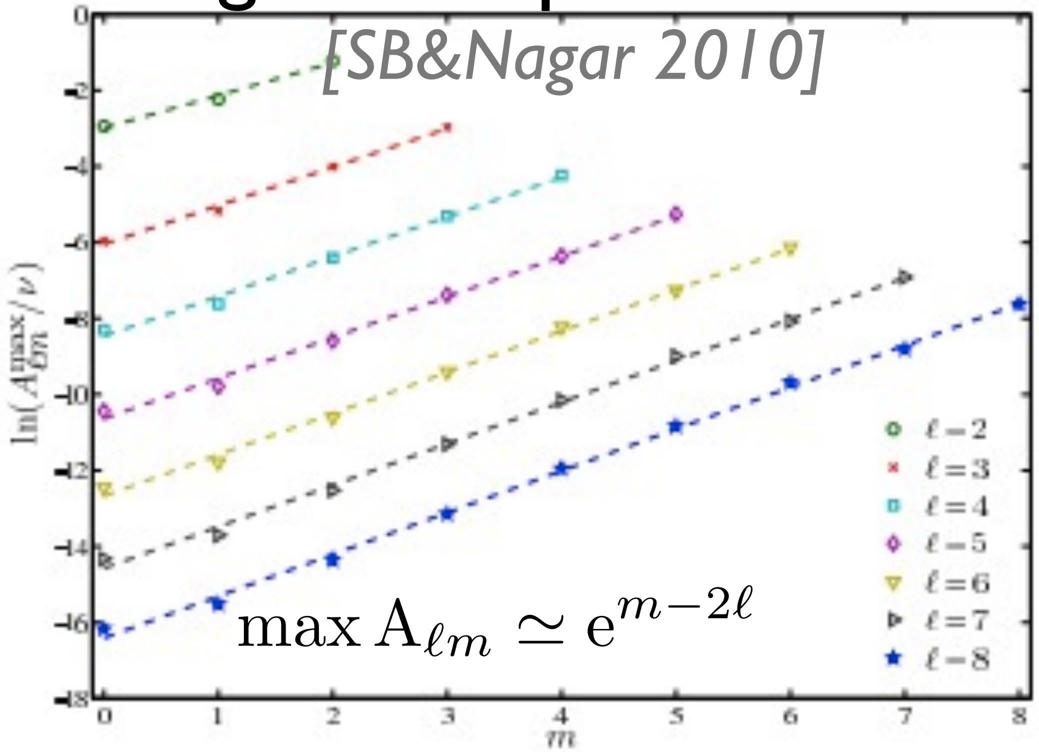
- Hamiltonian (geodesic) dynamics
- Linear in ν Radiation Reaction (EOB resum waveform)
- GW generation algorithm (RWZ/Teukolsky)
- Point-particle sources

[Damour+ 2008; Fujita&Iyer 2010; Pan+ 2011]

Nonrotating background results

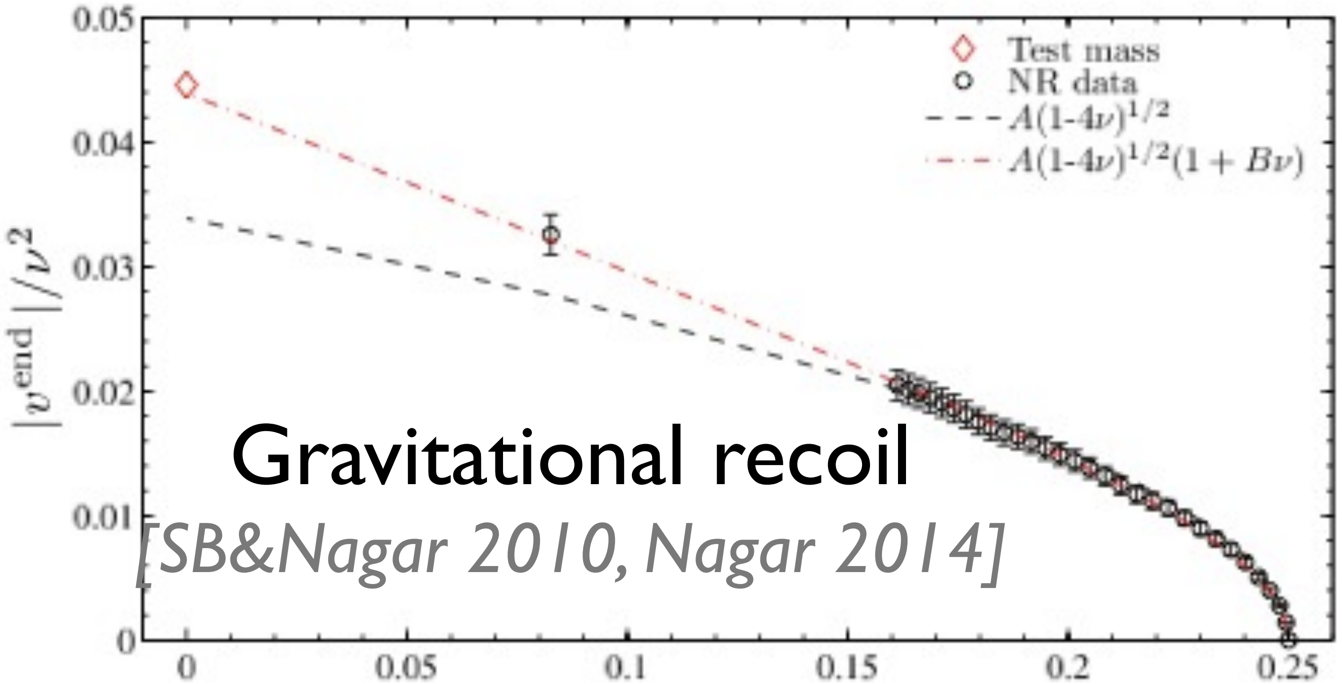
Merger multipolar structure

[SB&Nagar 2010]



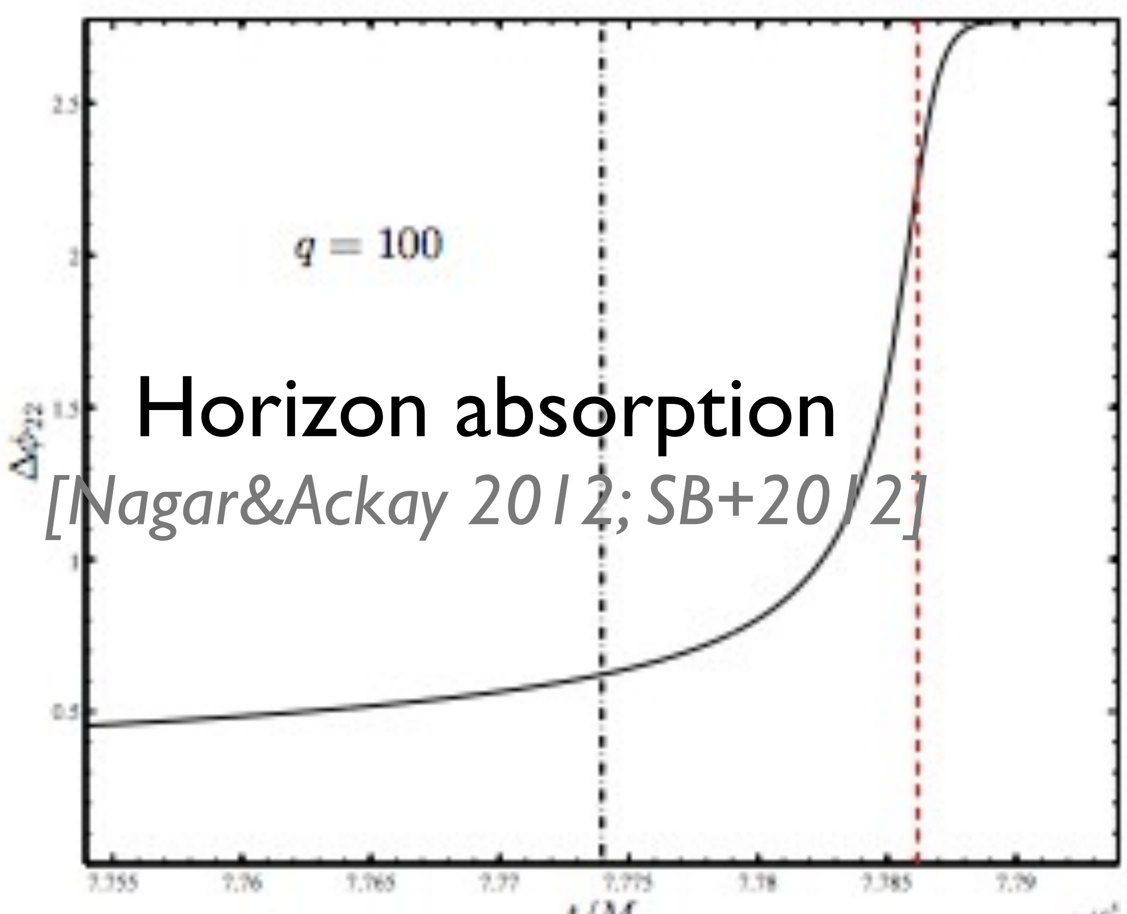
Gravitational recoil

[SB&Nagar 2010, Nagar 2014]



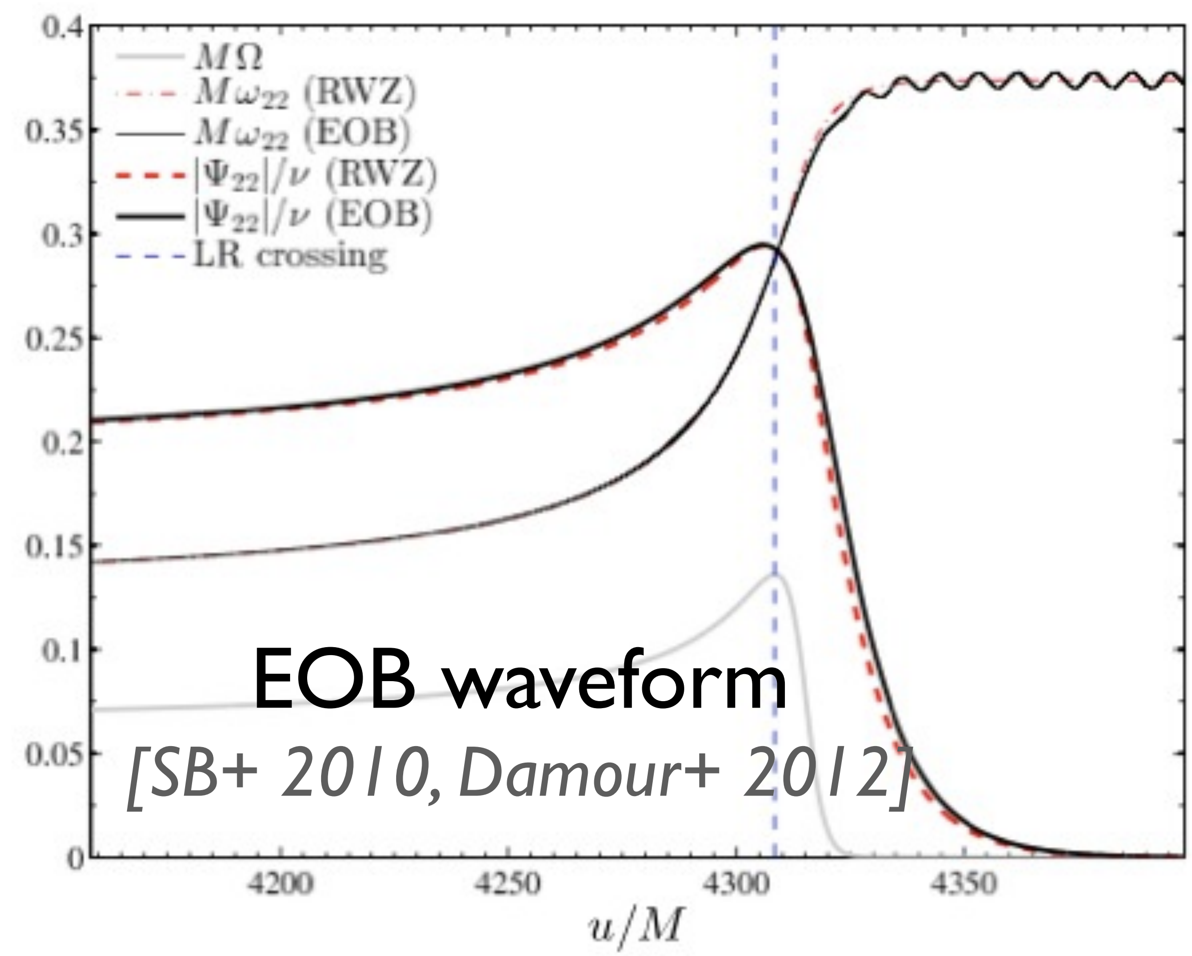
Horizon absorption

[Nagar&Ackay 2012; SB+2012]



EOB waveform

[SB+ 2010, Damour+ 2012]



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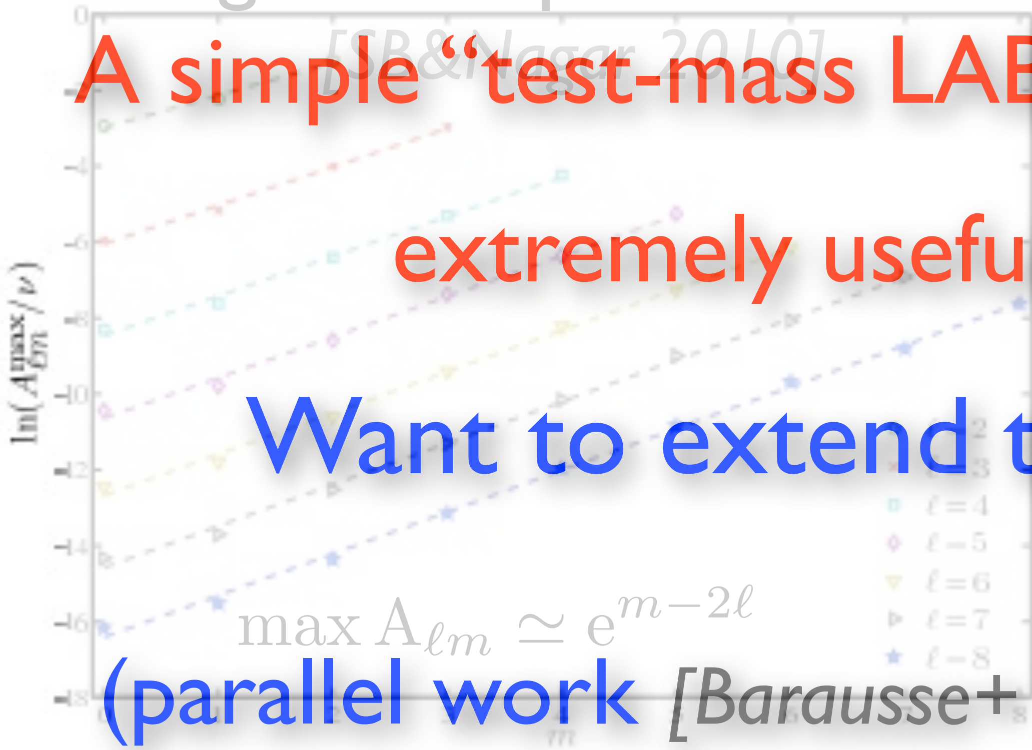
[SB&Nagar 2010]



$\max A_{\ell m} \simeq e^{m-2\ell}$

A simple "test-mass LAB" to study BBH merger physics

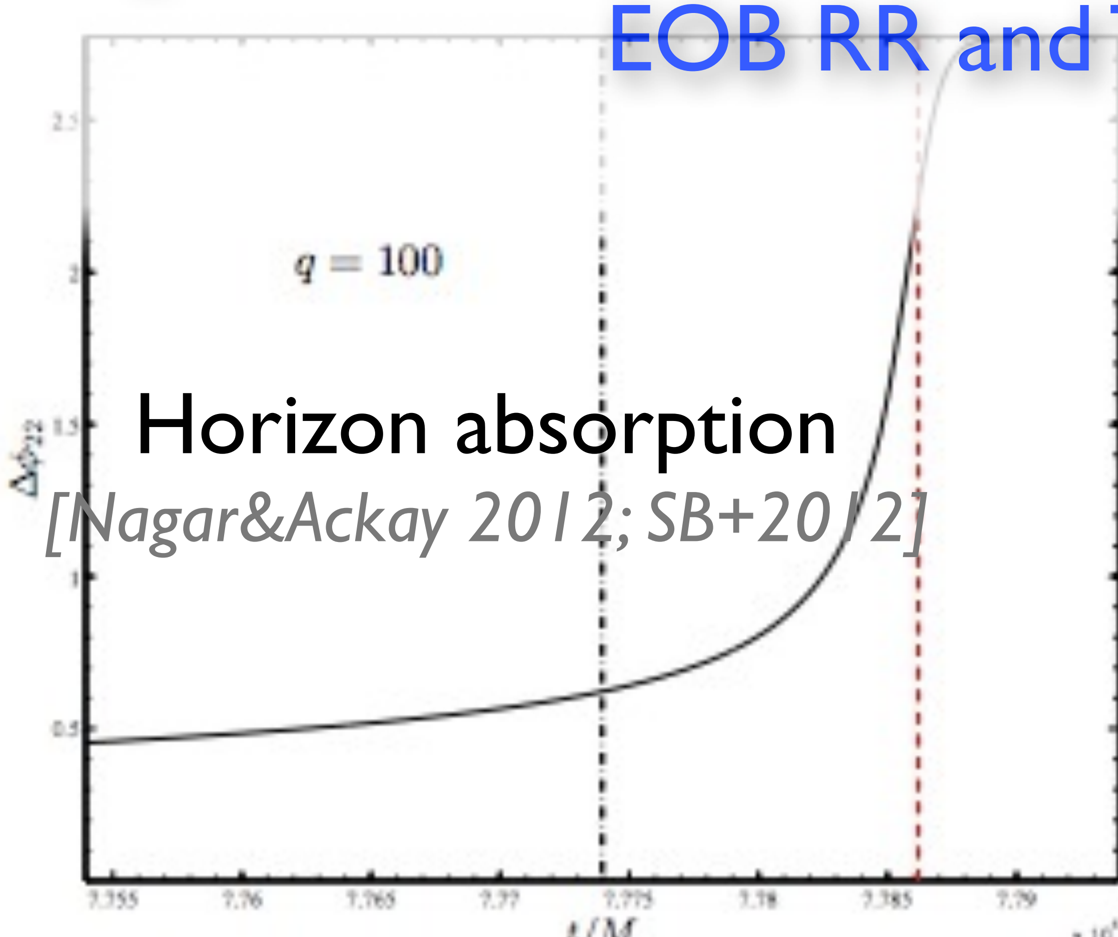
extremely useful for analytical modeling

Want to extend to spinning background !

Gravitational recoil

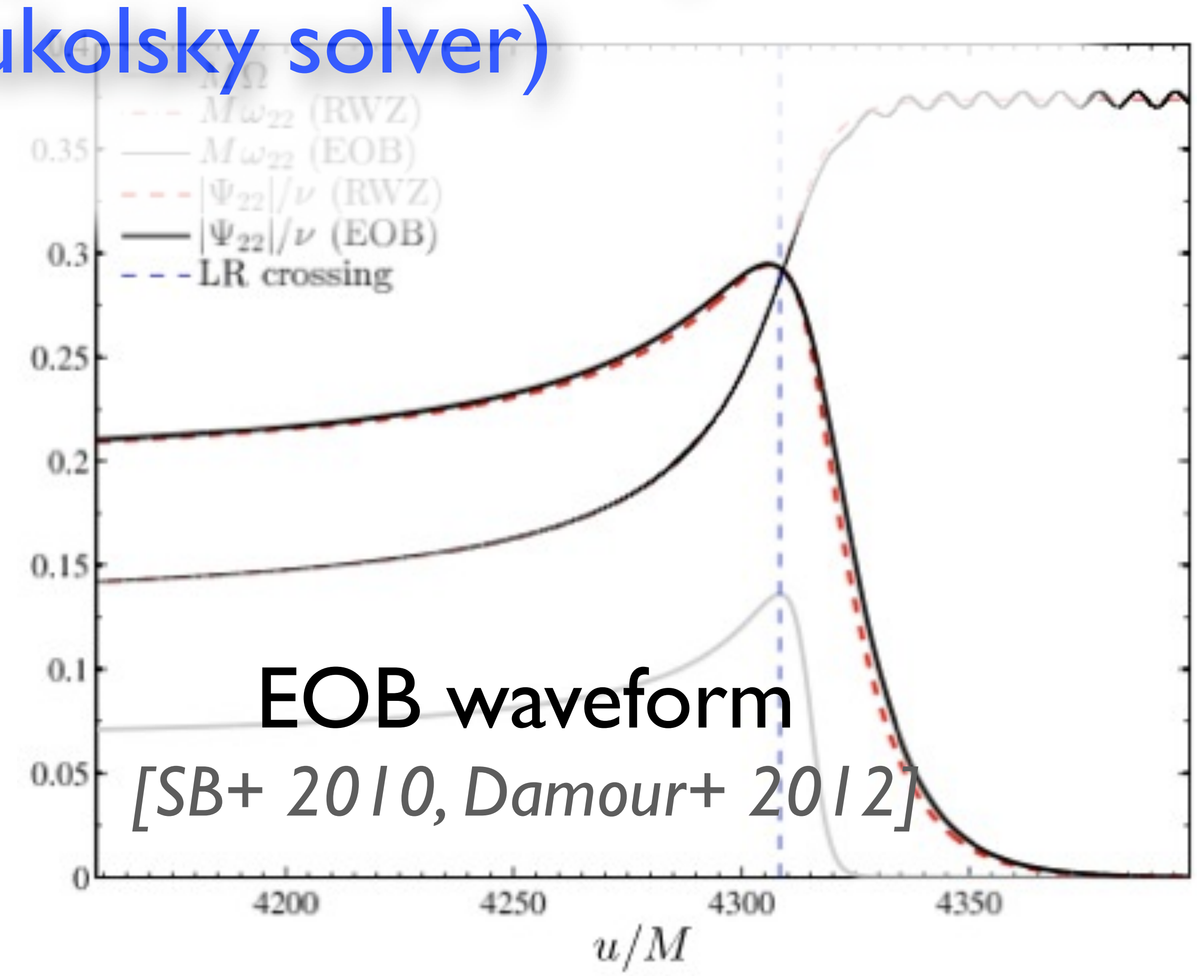
[SB&Nagar 2010, Nagar 2014]

(parallel work *[Barausse+ 2012]* different prescriptions for EOB RR and Teukolsky solver)



Horizon absorption

[Nagar&Ackay 2012; SB+2012]



EOB waveform

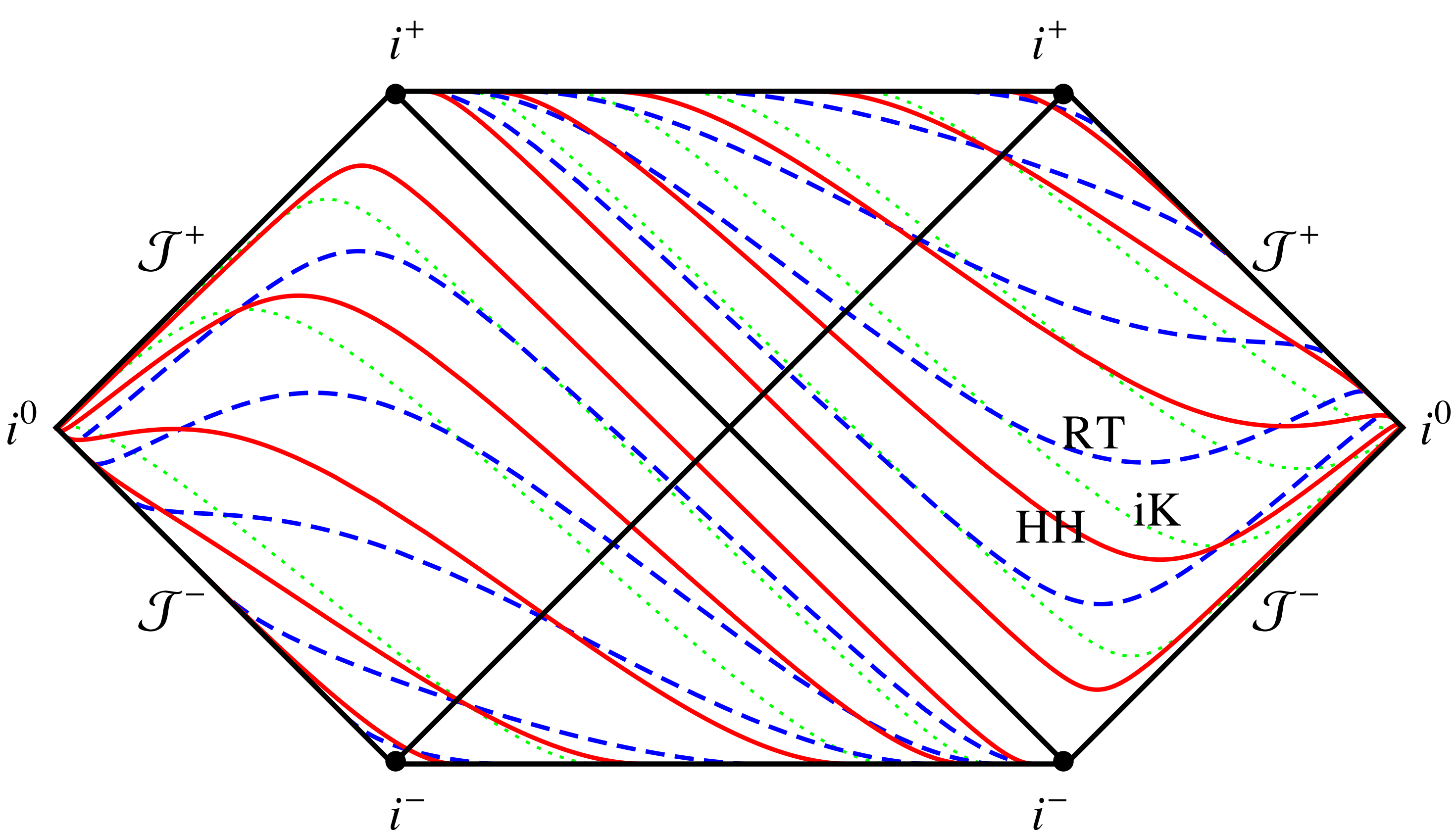
[SB+ 2010, Damour+ 2012]

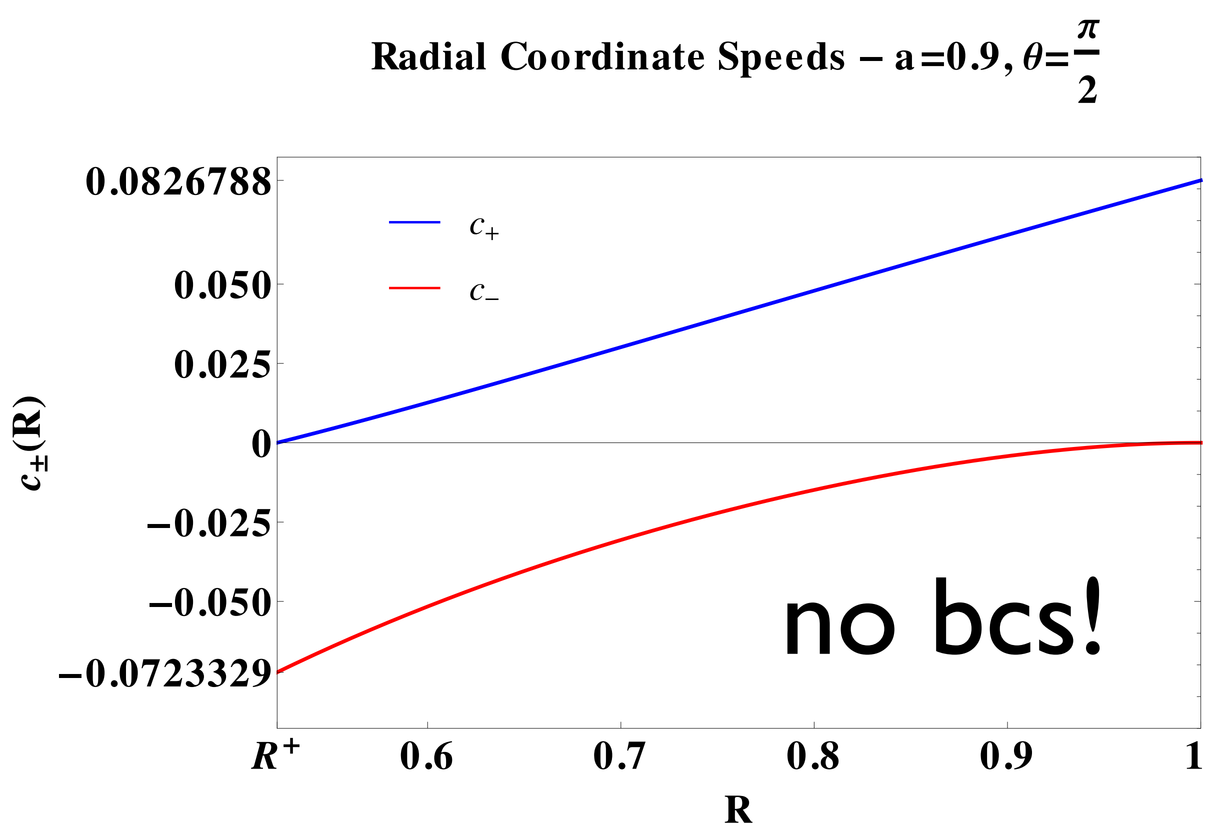
Hyperboloidal Horizon-penetrating coords

$$t = \tau - h(\rho)\ , \quad r = \frac{\rho}{\Omega(\rho)}$$

$$\Omega(\rho) = 1 - \frac{\rho}{S}$$

$$t - (r + 4M \ln r) \stackrel{!}{=} \tau - (\rho + 4M \ln \rho) \ \Rightarrow \ h(\rho) = \ldots$$

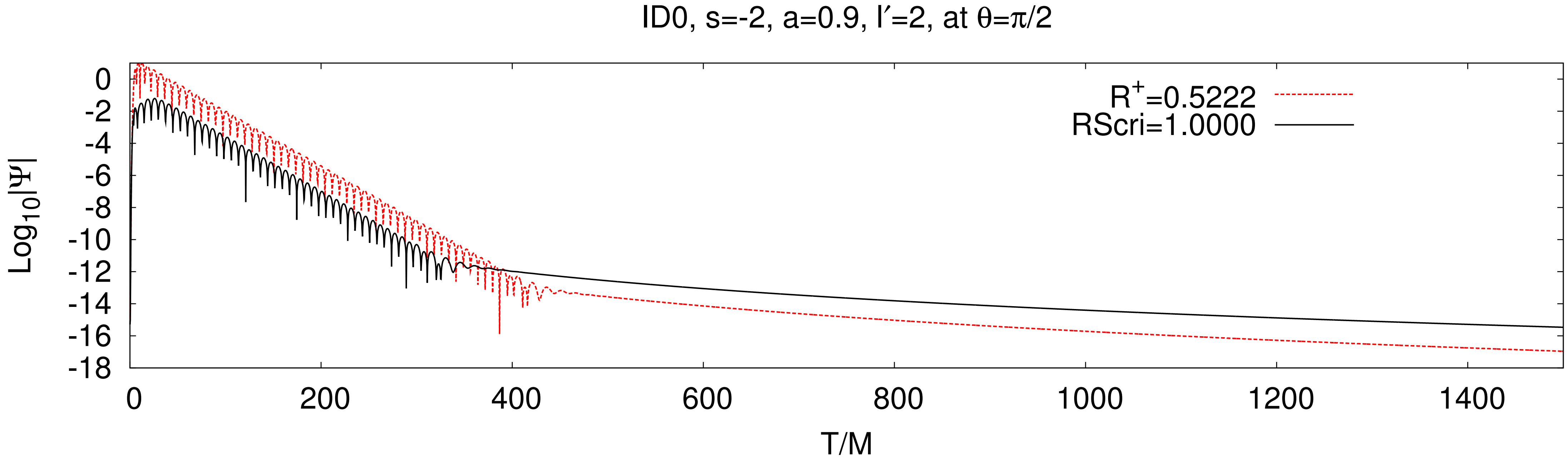


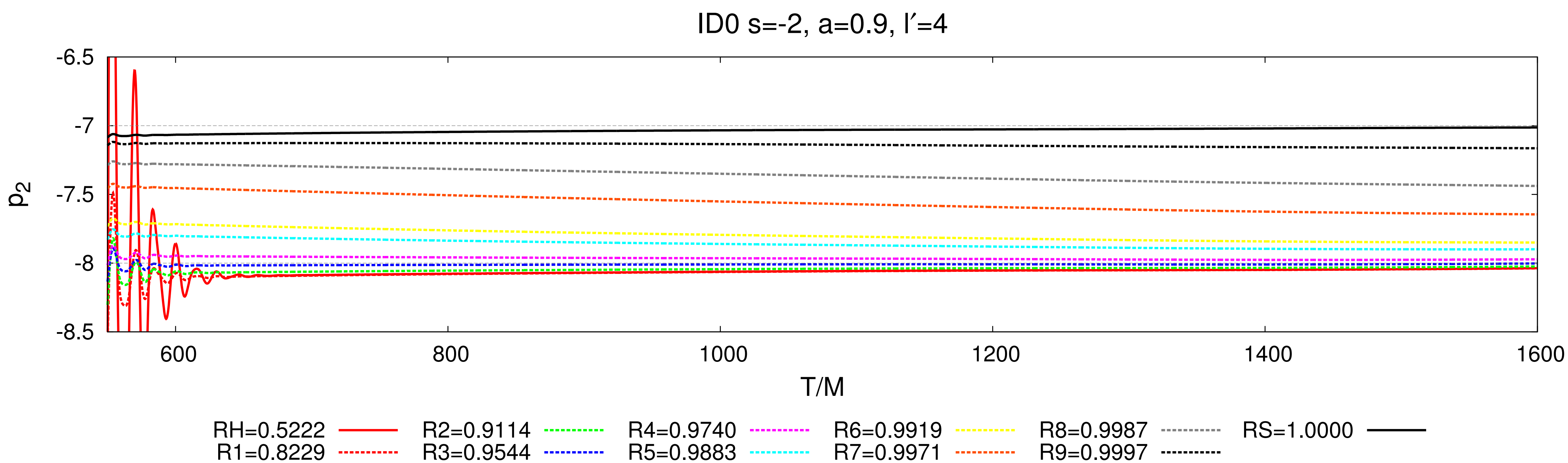


[Moncrief 2000; Zenginoglu 2008,2010; Racz&Toth 2011; SB+ 2012, Harms+ 2014]

Kerr tails *[Harms+ 2013]*

- Axisym/nonaxisym tails for various spin fields (incl. s=-2) @ scri and H
- Local power index (LPI) analysis
- Analytics *[Barack 1999,2000; Hod 2000; Poisson 2002]*





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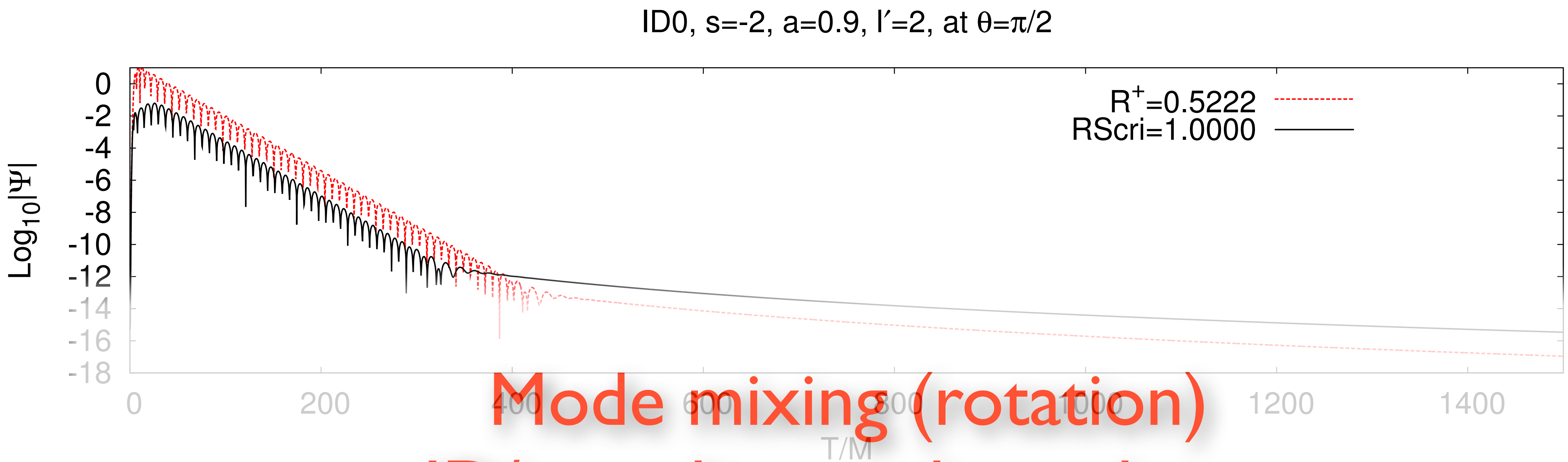


Mode mixing (rotation)

ID/coordinates dependency

Extraction radius dependency

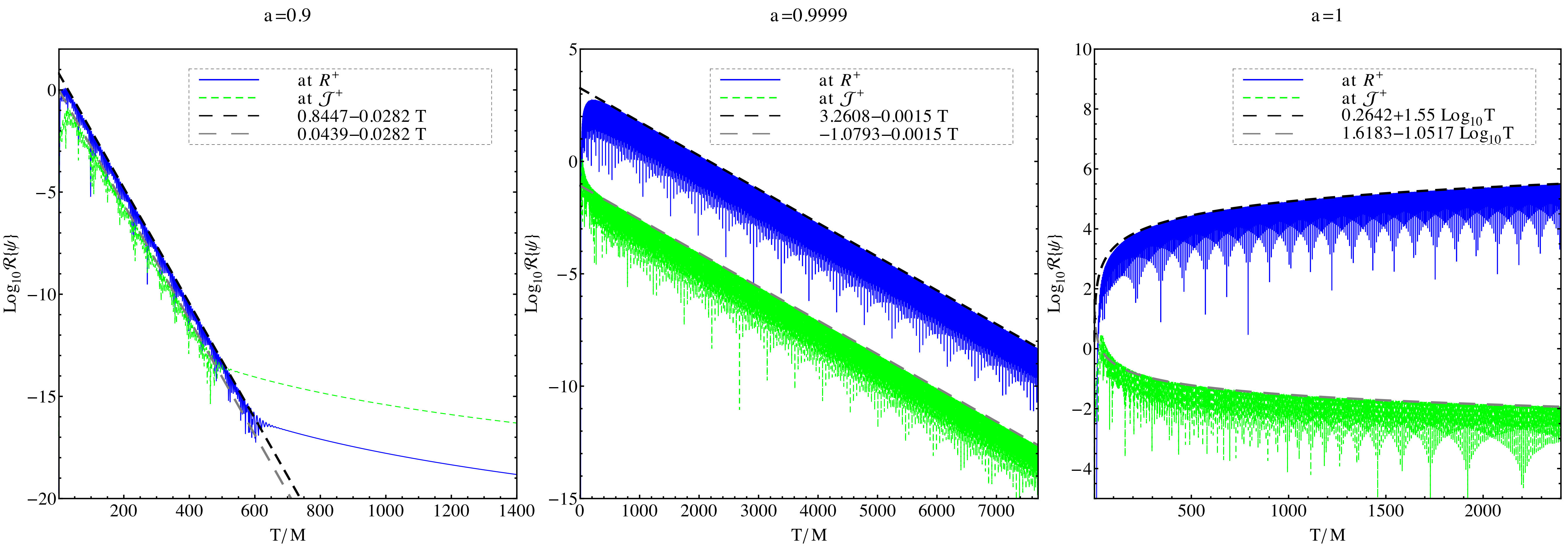
LPI splitting

Empirical formulas

Extend to |s|=2, |s|=1/Confirm s=0 results, e.g.

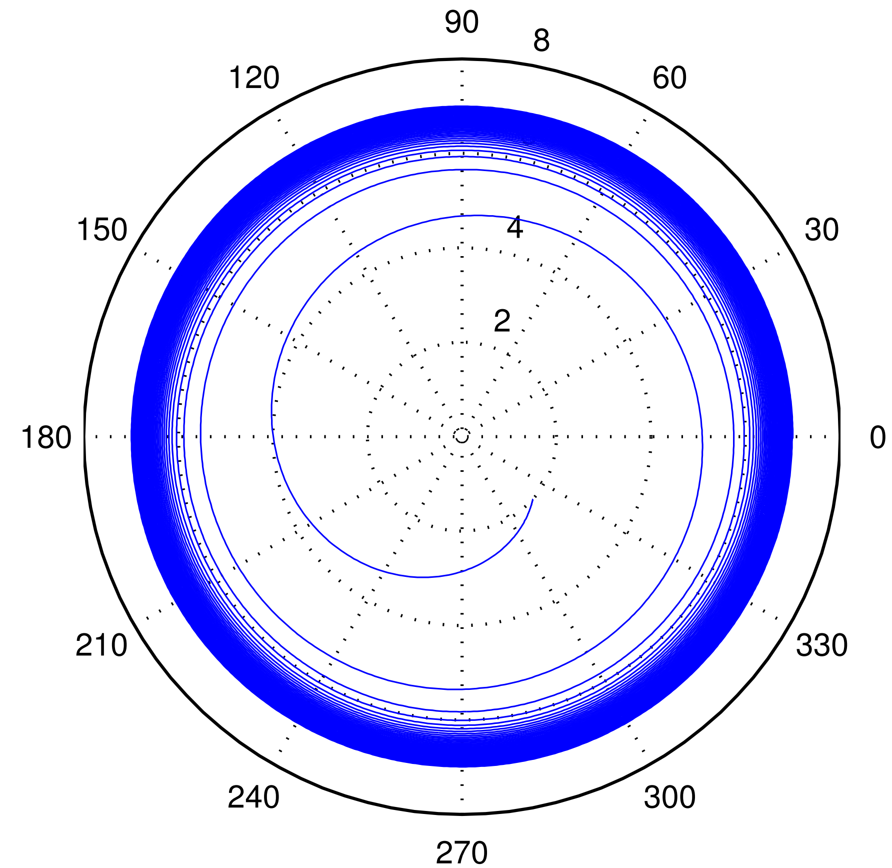
[Krivan+ 1996,1997; Scheel+ 2004; Gleiser+ 2008;Tiglio+ 2008; Burko&Khanna 2003,2009,2011; Zenginoglu+ 2009; Jasiulek 2012; Racz&Toth 2012]

Extreme Kerr tails

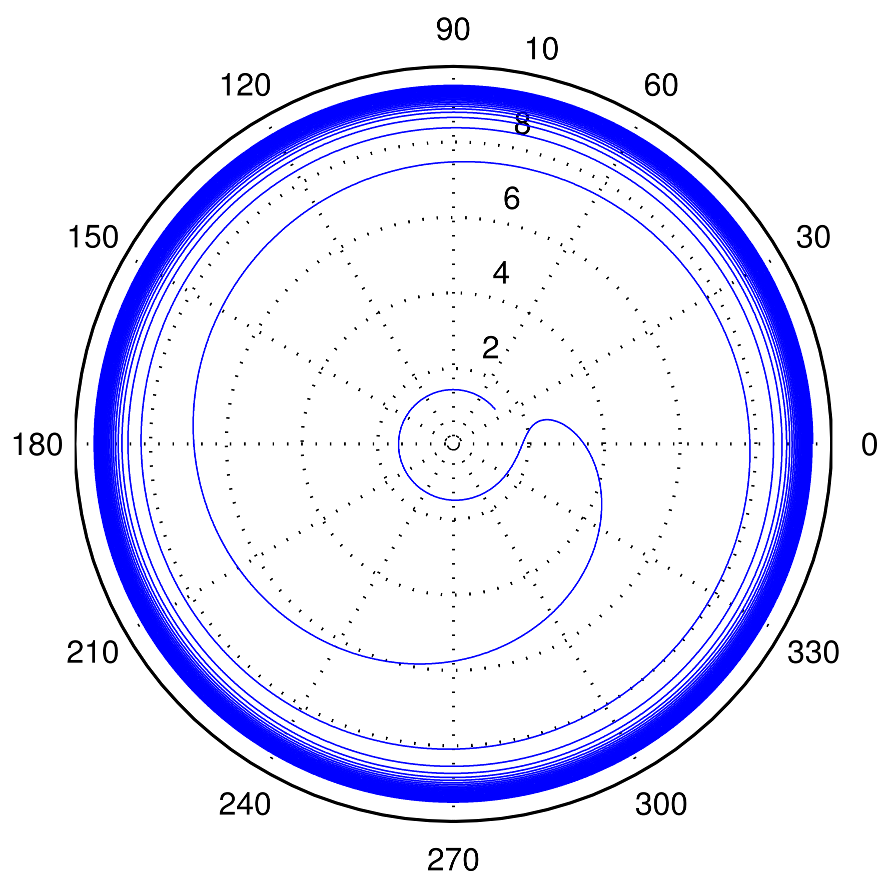


- oscillatory slowly-damped behaviour *[Yang+ 2013]*
- a=1 scri ~1/T *[Andersson&Glampedakis 2001, Yang+ 2013]*
- a=1horizon instability ? *[Aretakis 2012; Lucietti&Reall 2012]*

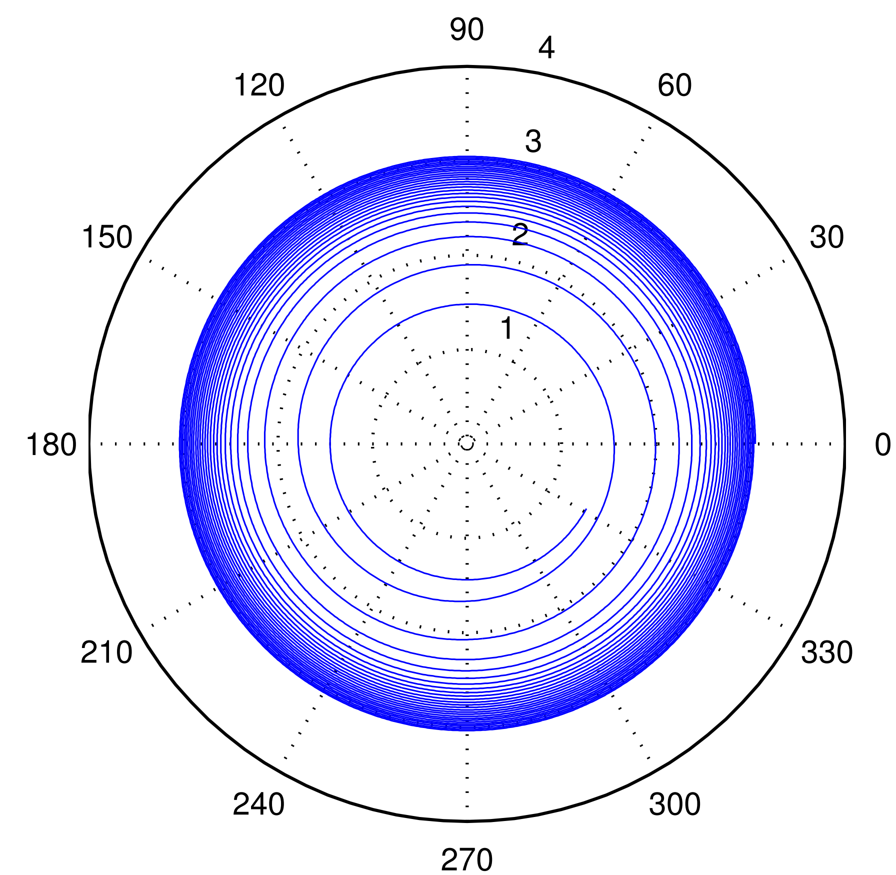
EOB “*insplunge*” trajectories



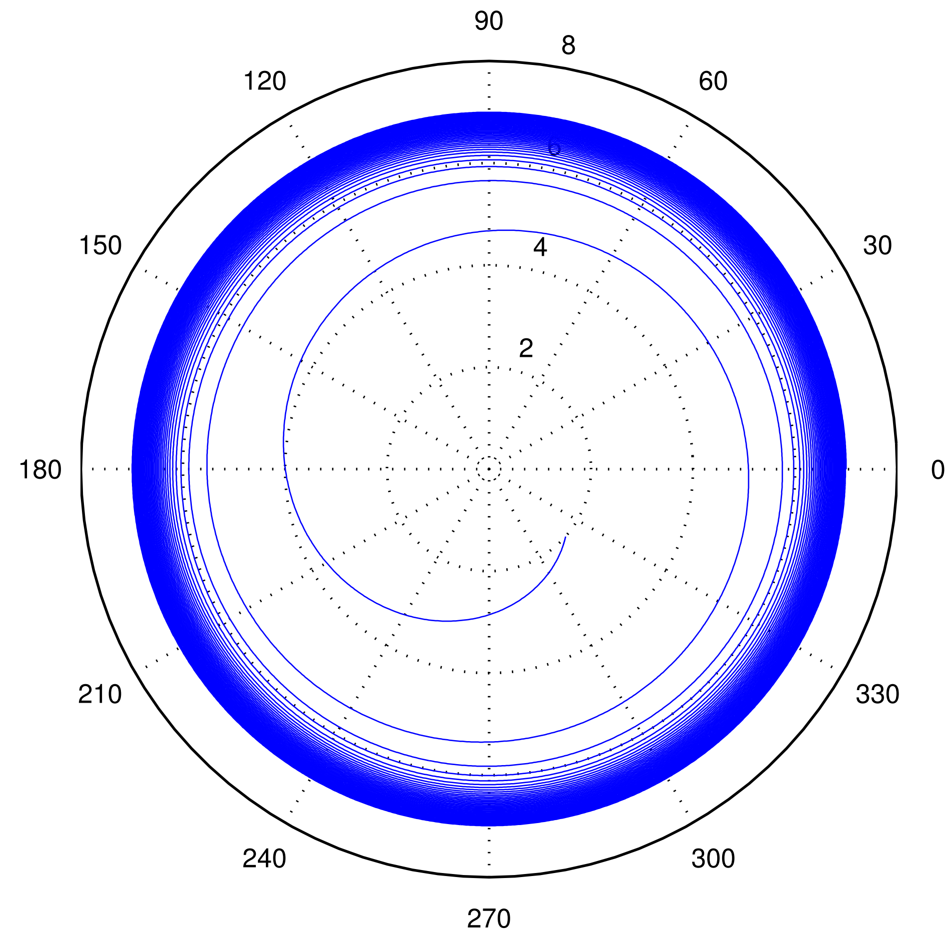
a=0



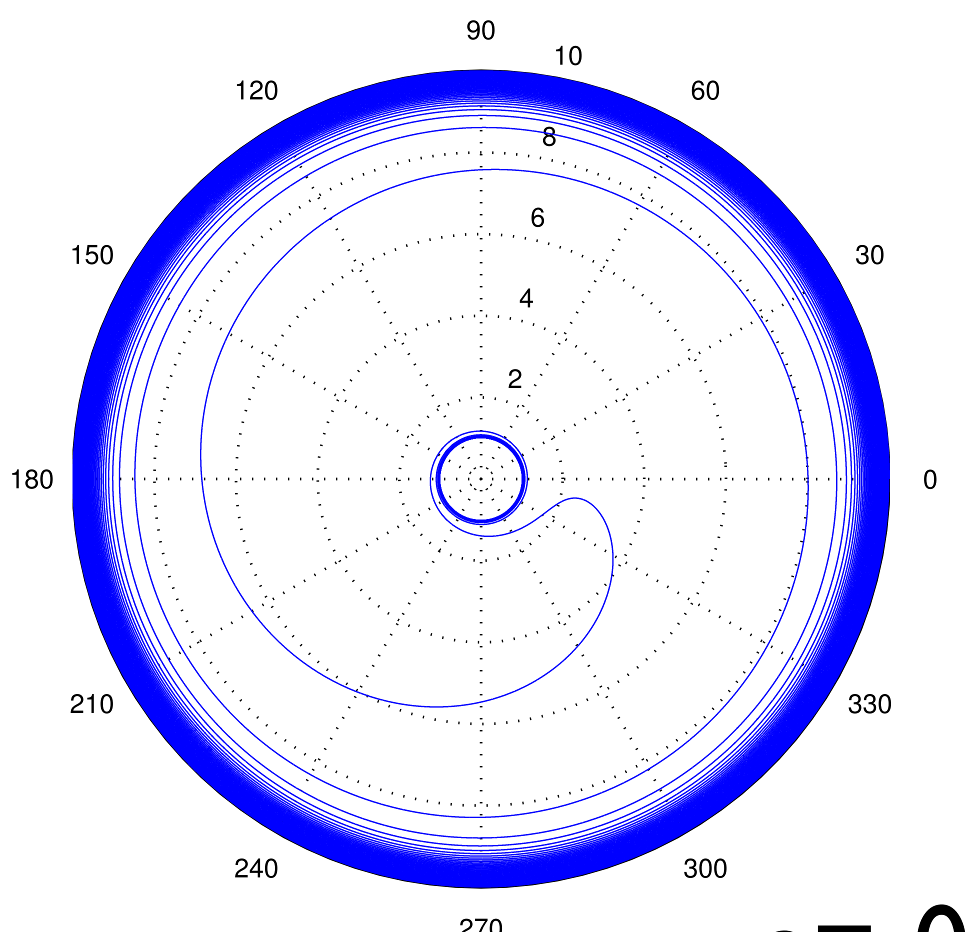
a=-0.9



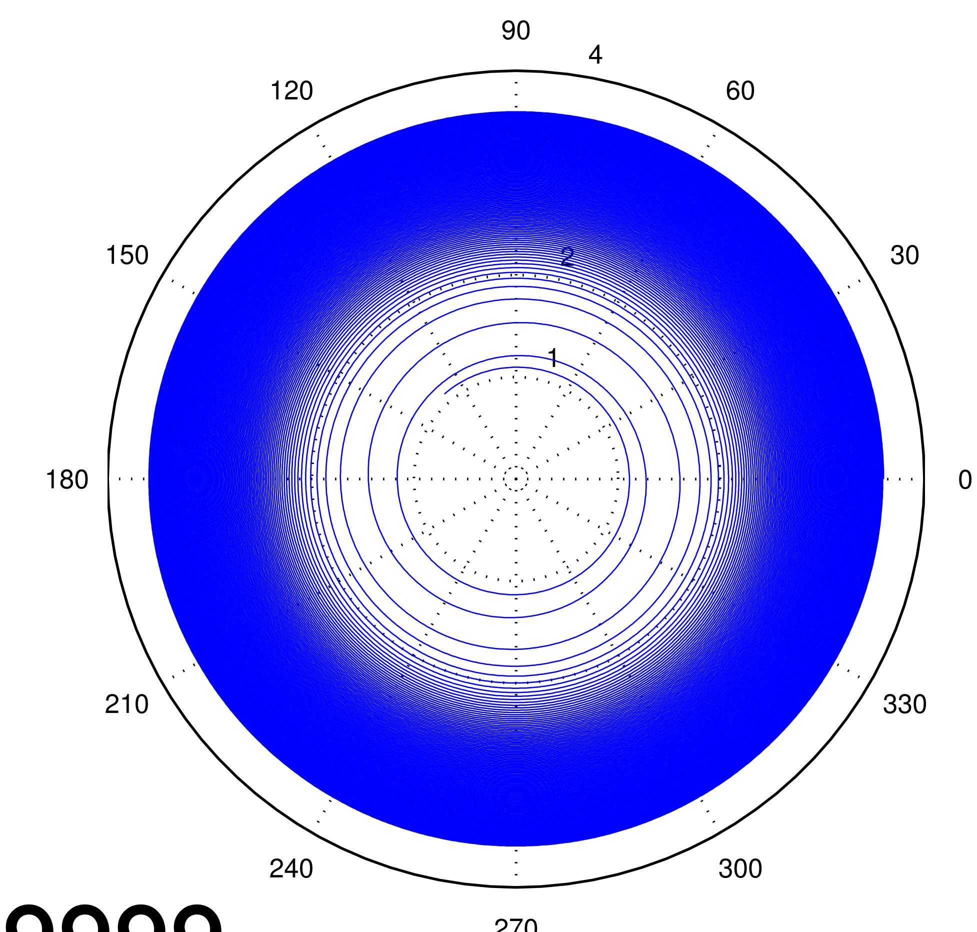
a=+0.9



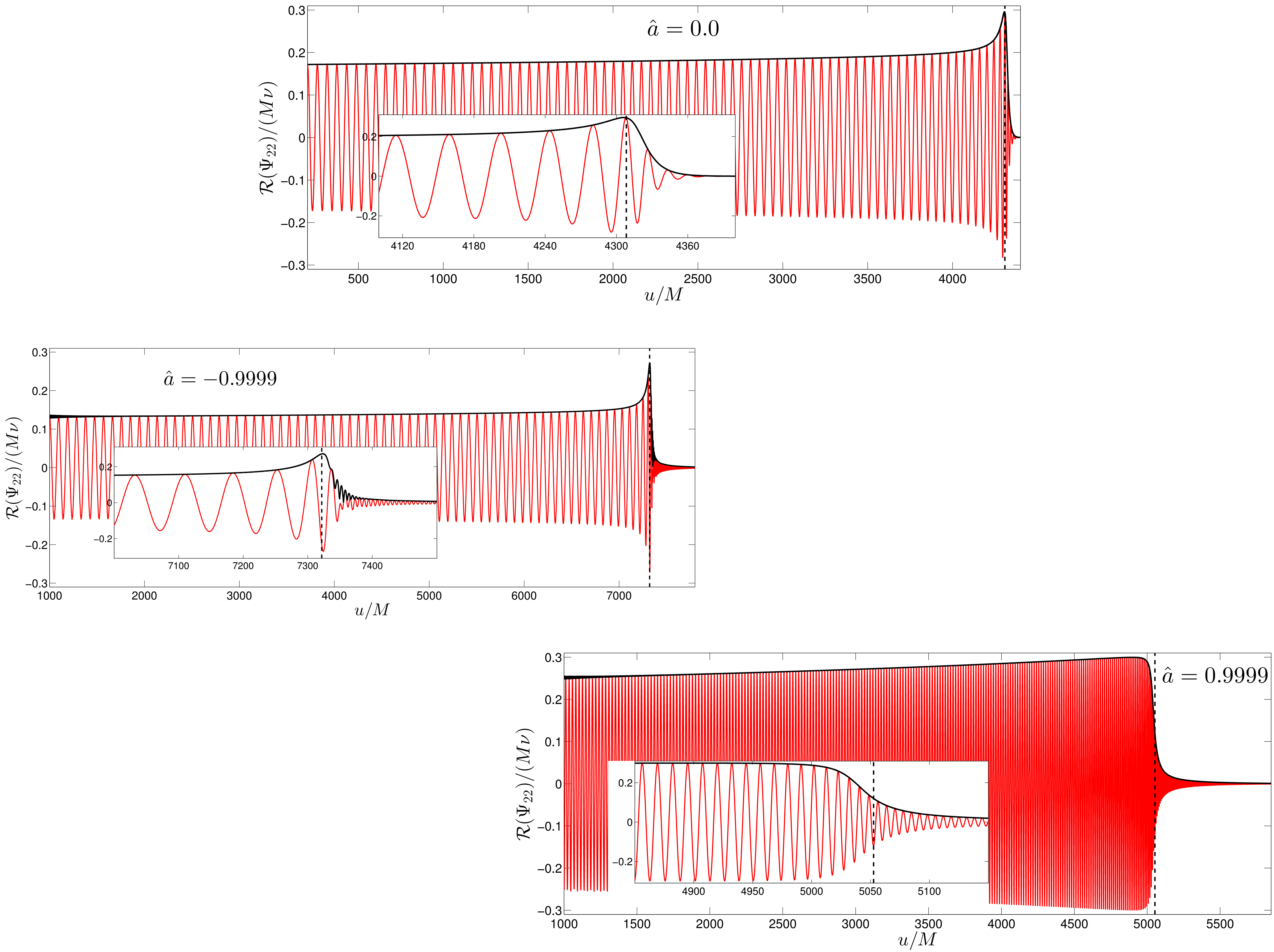
a=0



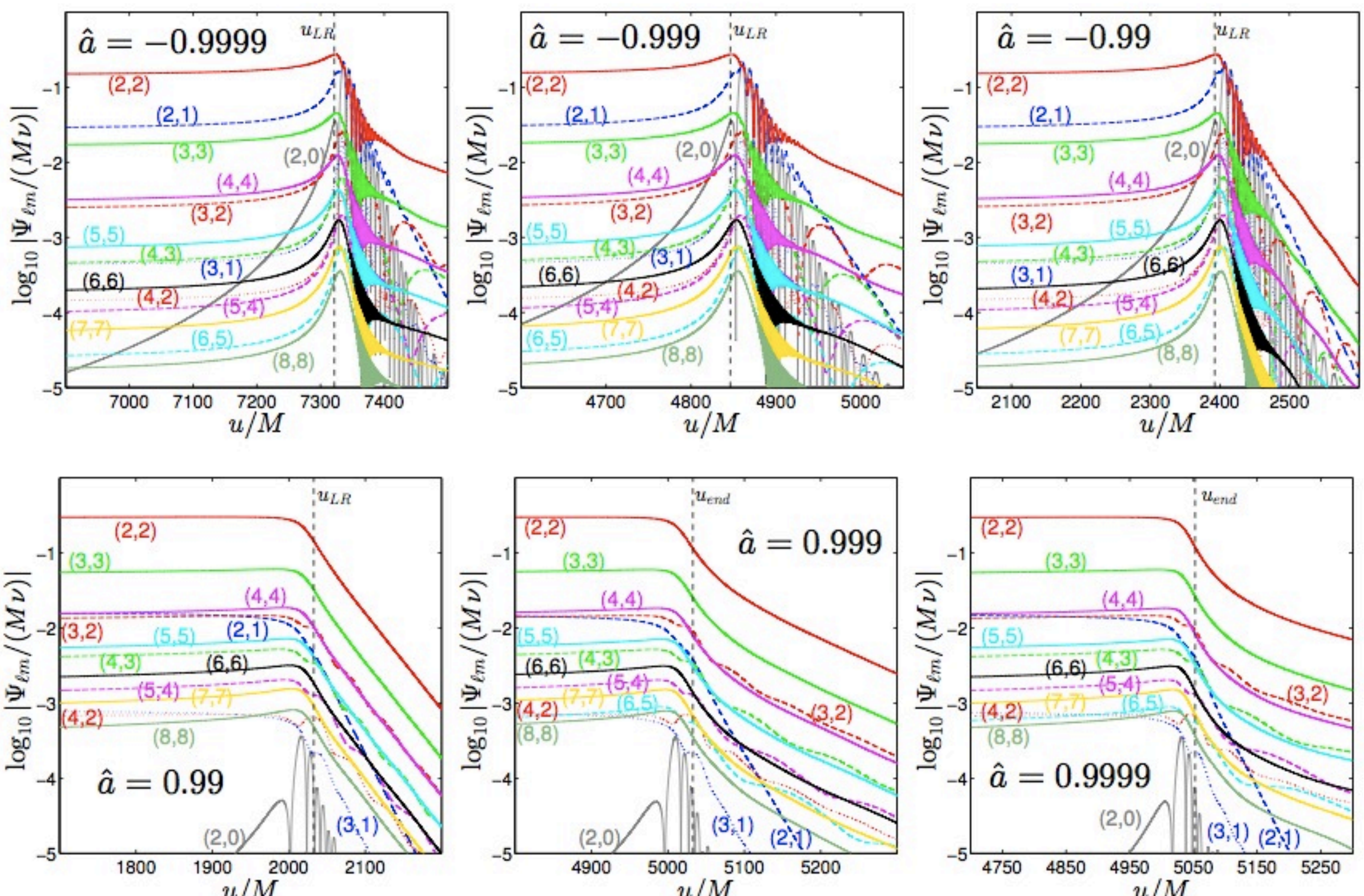
a=-0.9999



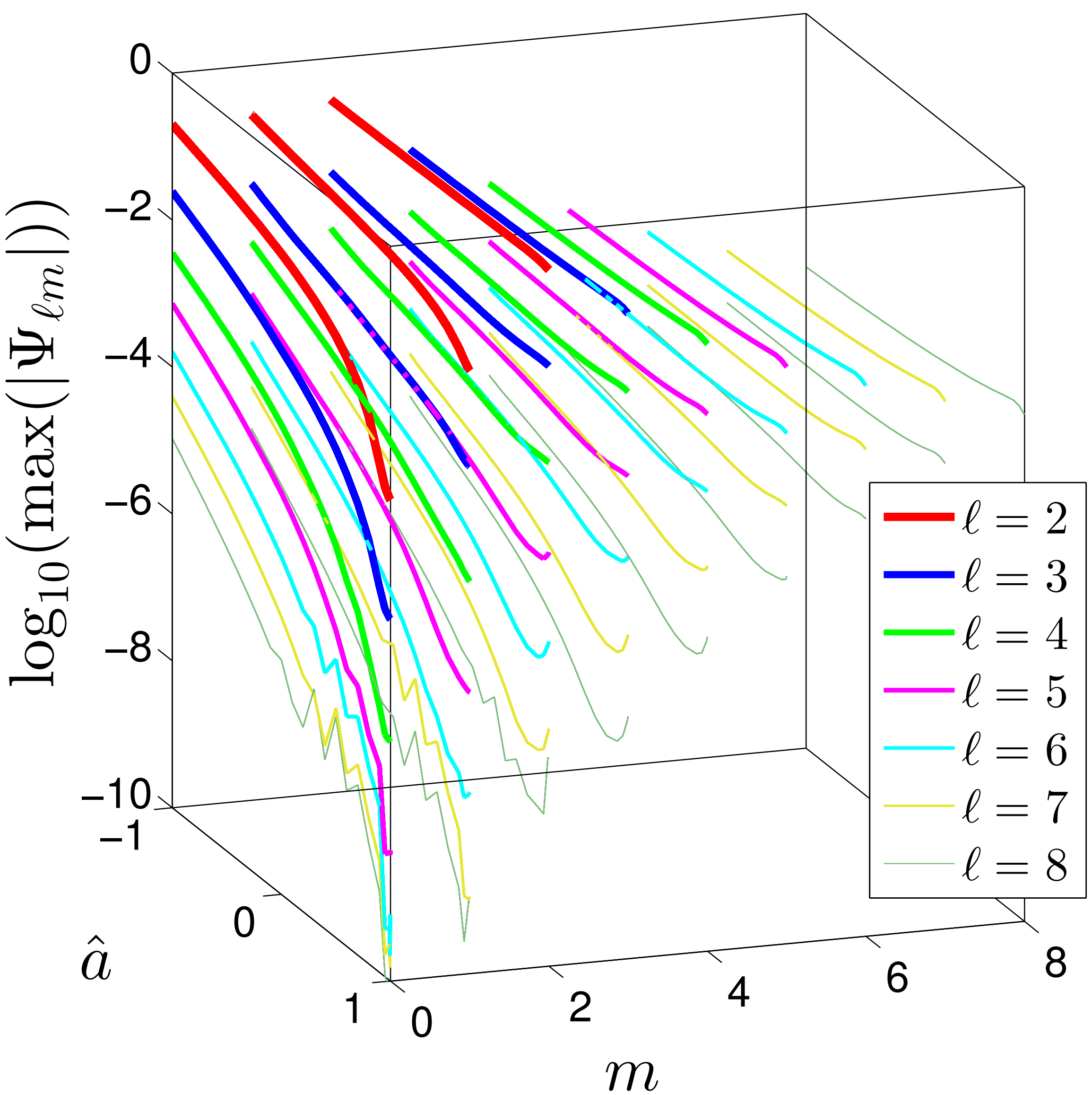
a=+0.9999

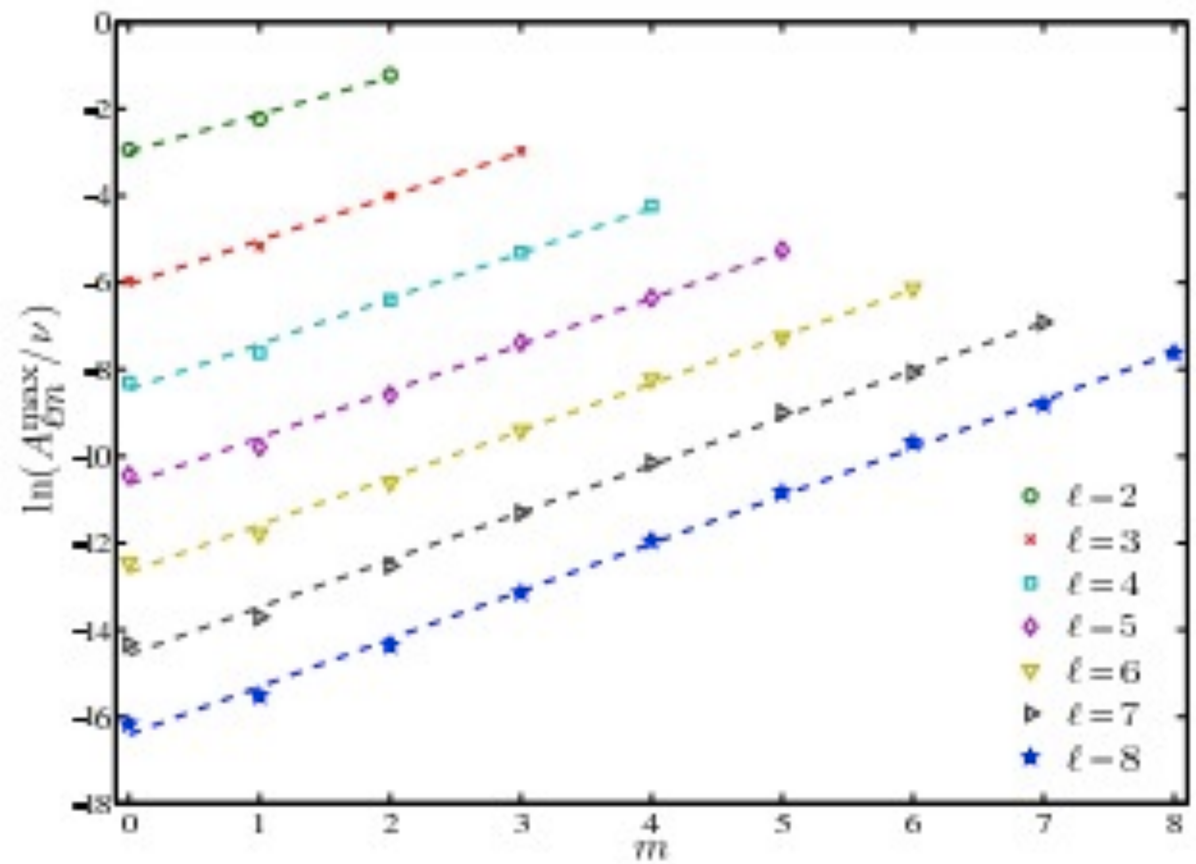

$\hat{a} = 0.0$
$\mathcal{R}(\Psi_{22})/(M\nu)$
u/M
$\hat{a} = -0.9999$
$\mathcal{R}(\Psi_{22})/(M\nu)$
u/M
$\hat{a} = 0.9999$
$\mathcal{R}(\Psi_{22})/(M\nu)$
u/M

Multipolar amplitudes



Hierarchy of multipolar amplitudes





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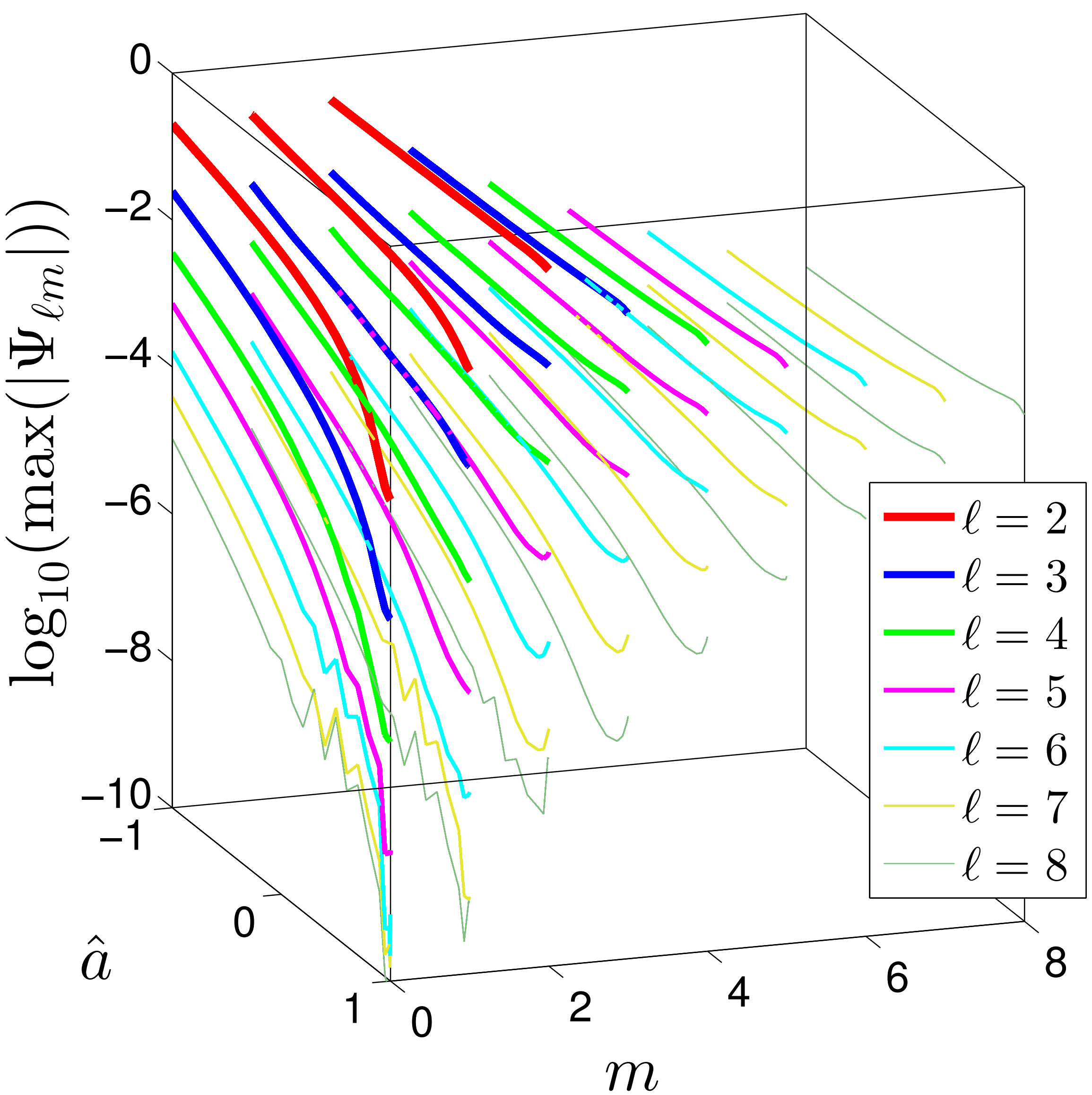
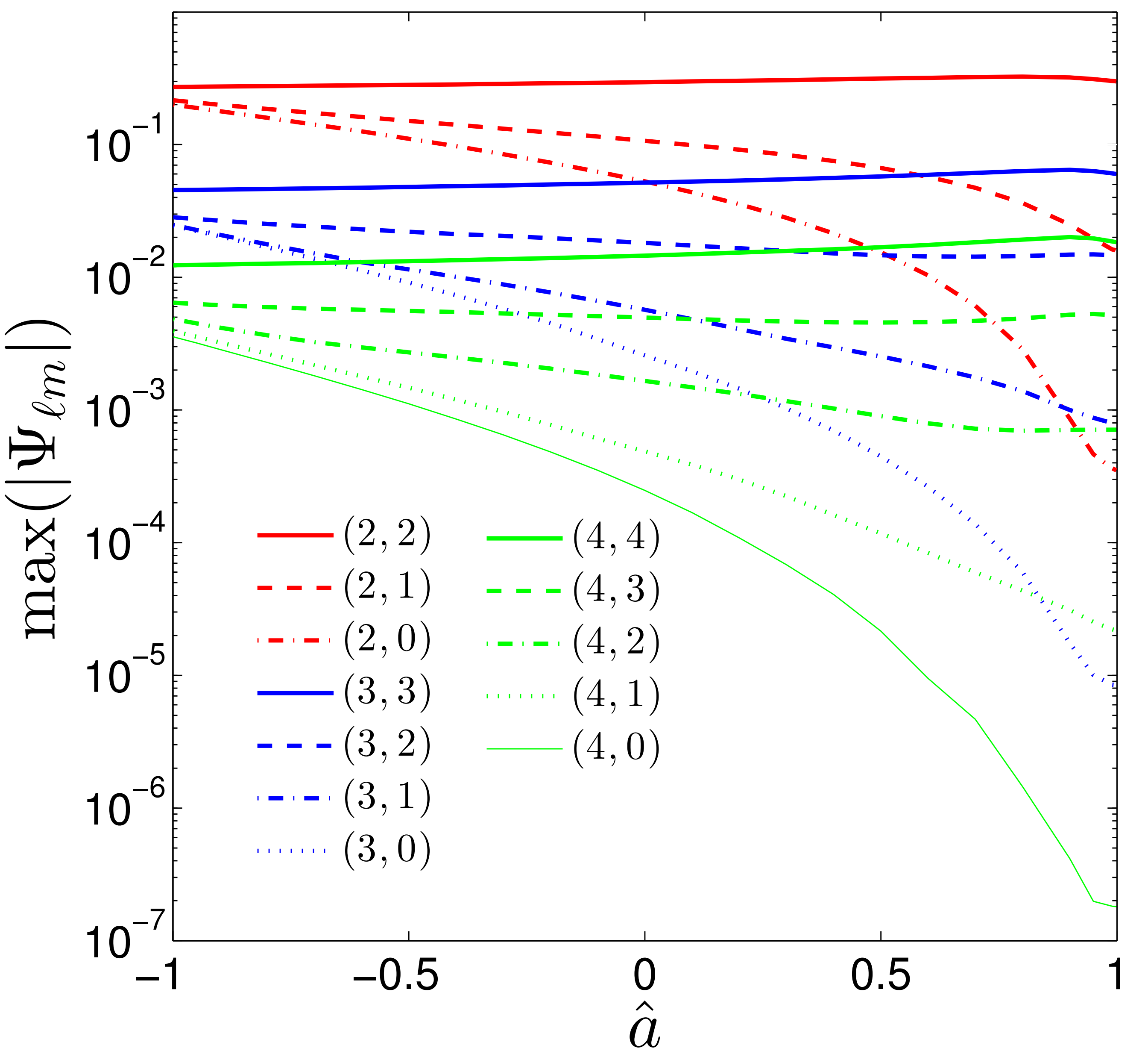
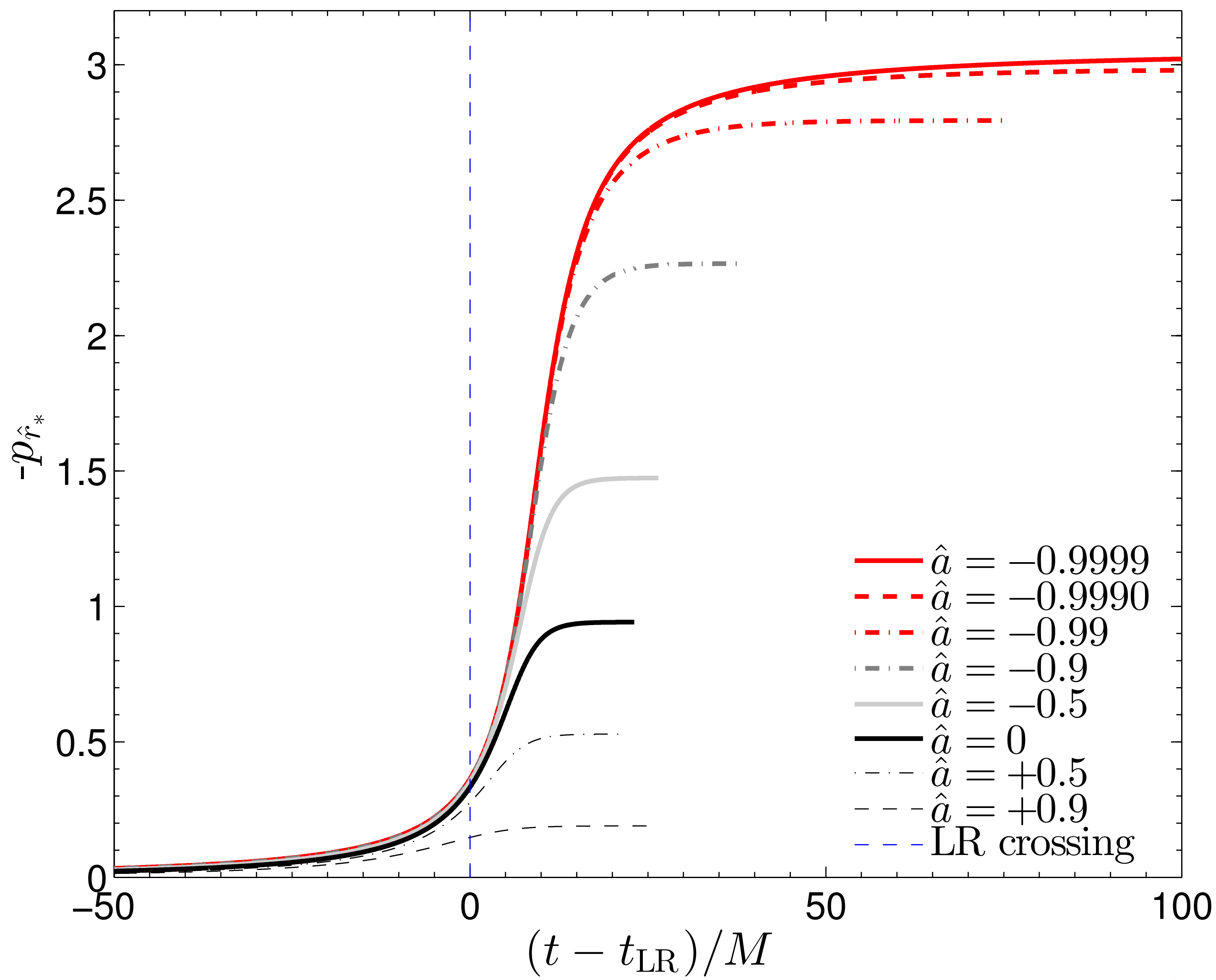
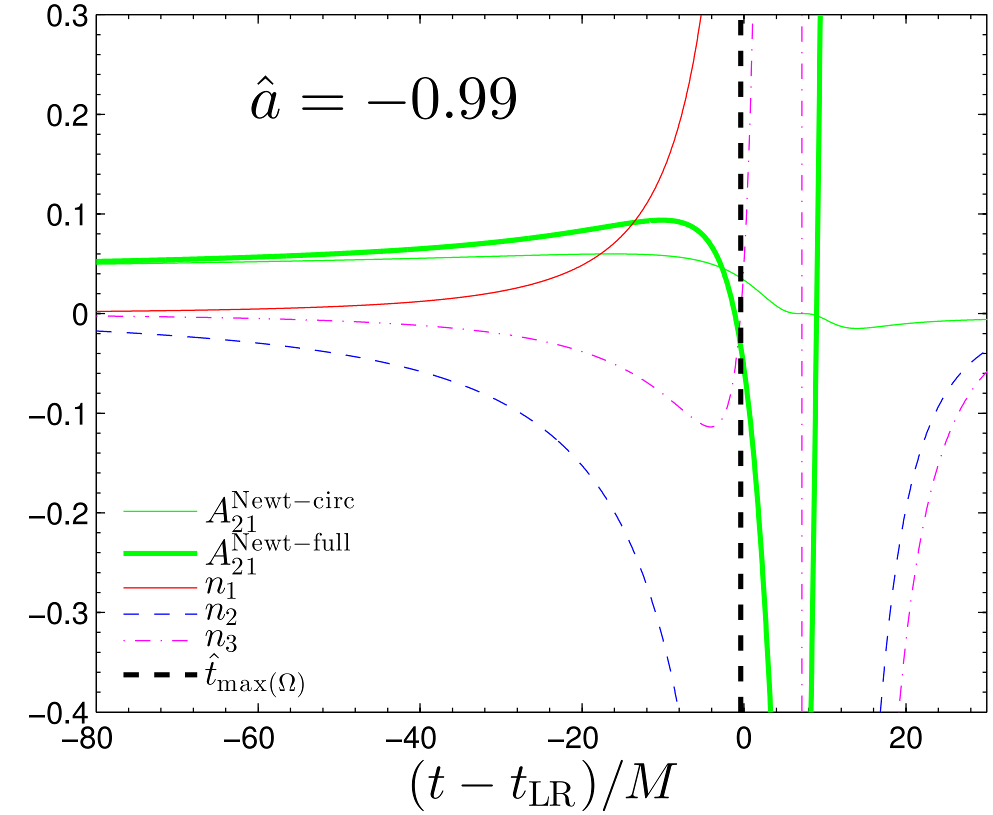
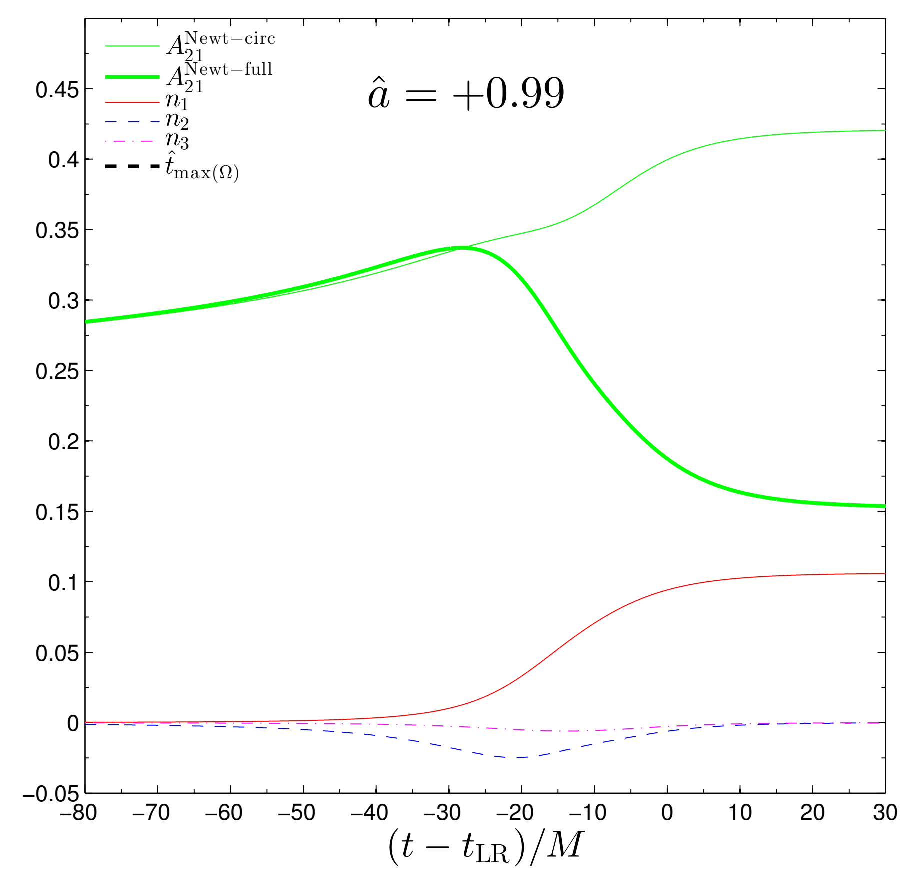




Newtonian analysis of noncircular effects







$$\hat{A}_{22}^{\rm Newt} = \hat{A}_{22}^{0}\hat{A}_{22}^{\rm NQC} = \sqrt{\frac{16\pi}{30}}v_\phi^2\left(1-\frac{n_1}{2}-\frac{n_2}{2}\right),$$

$$\hat{A}_{21}^{\rm Newt} = \hat{A}_{21}^{0}\hat{A}_{21}^{\rm NQC} = \sqrt{\frac{16\pi}{45}}v_\phi^3\left(1-6n_1-3n_2-6n_3\right),$$

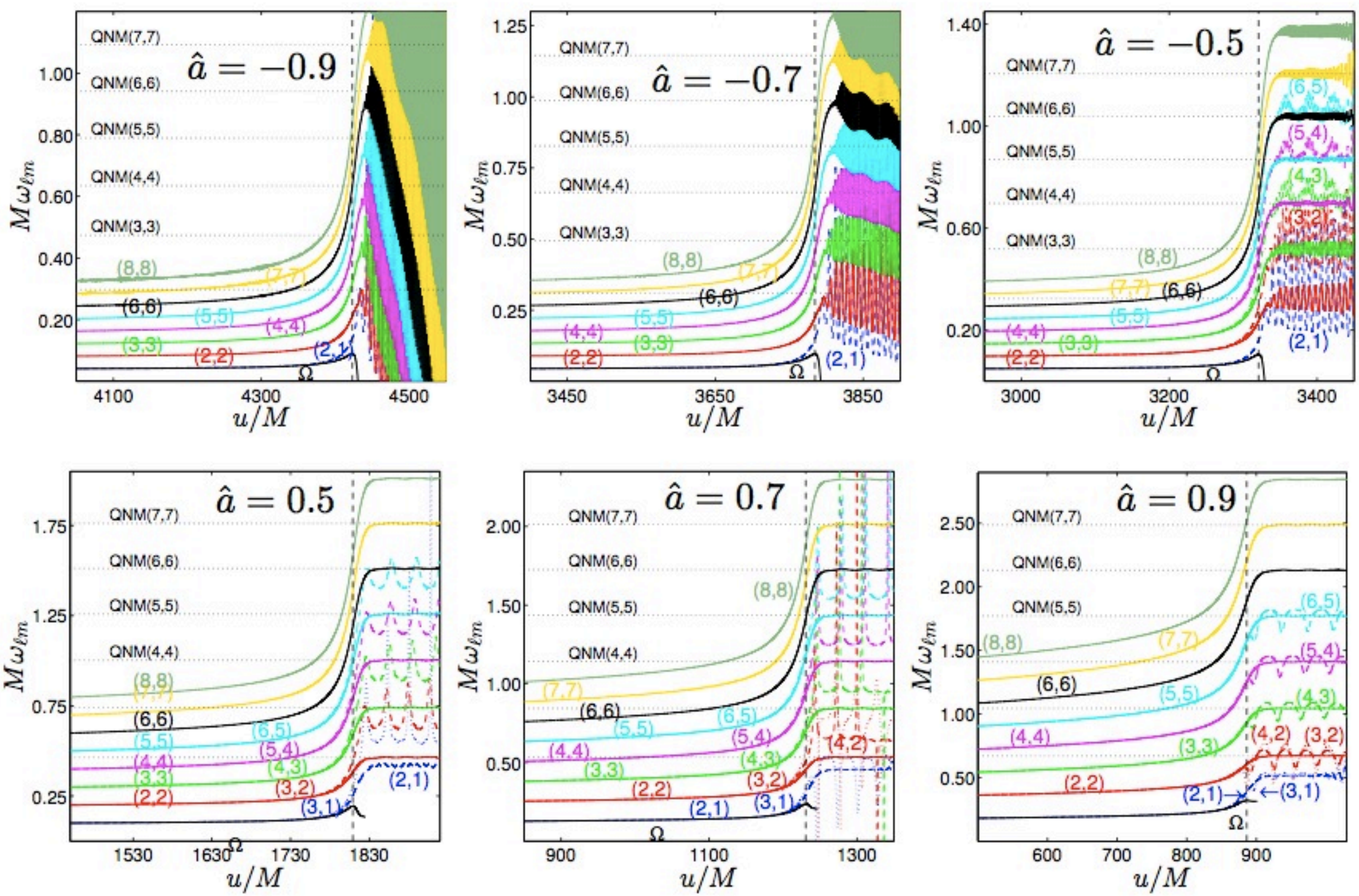
$$\hat{A}_{20}^{\rm Newt} = \sqrt{\frac{4\pi}{15}}\left(\dot{p}_{r_*}^2+r\dot{p}_{r_*}\right).$$

$$n_1 = \frac{p_{r_*}^2}{(\hat{r}\hat{\Omega})^2},$$

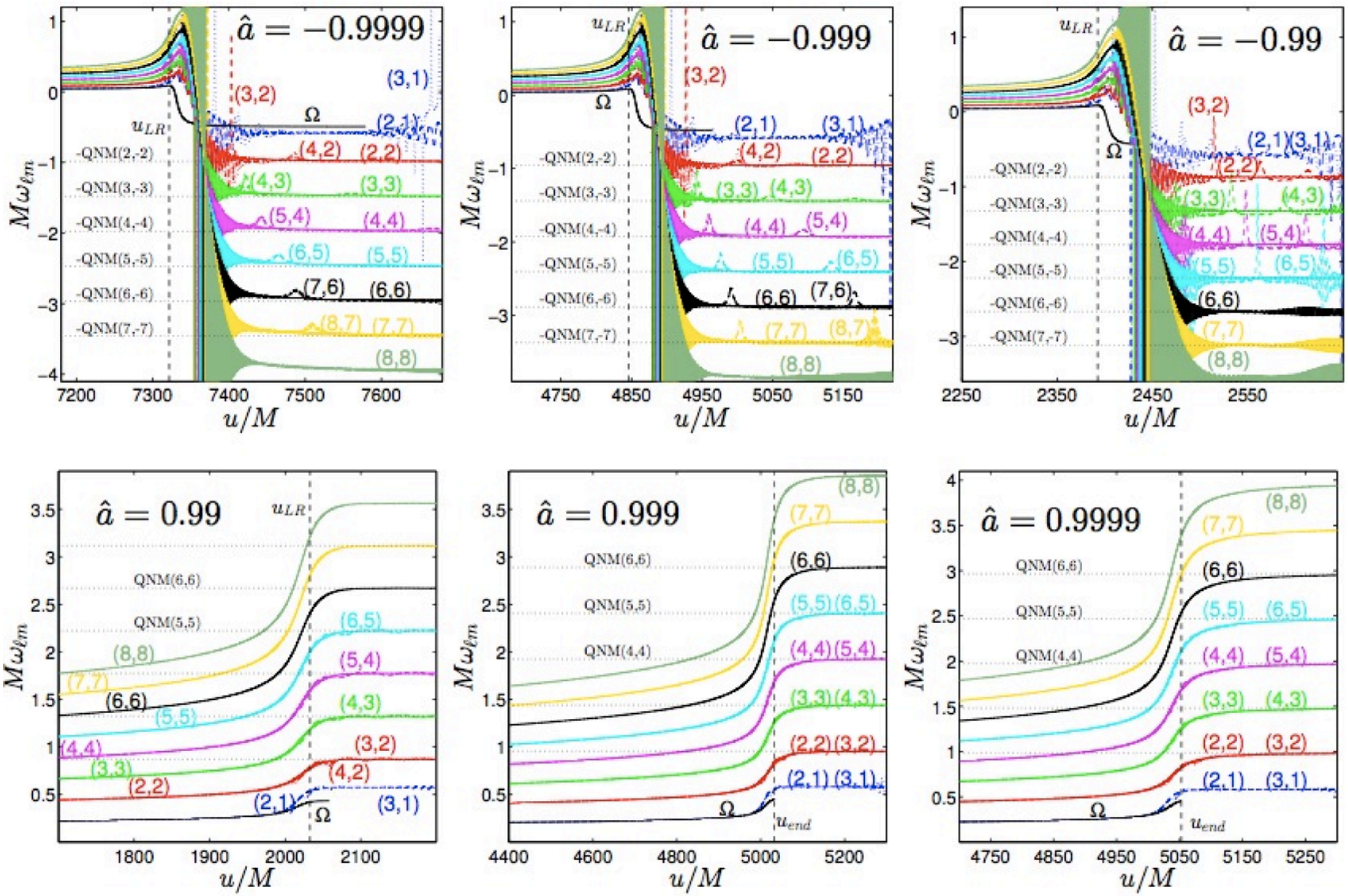
$$n_2 = \frac{\dot{p}_{r_*}}{\hat{r}\hat{\Omega}^2},$$

$$n_3 = \frac{p_{r_*}}{\hat{r}\hat{\Omega}^2}\frac{\dot{\hat{\Omega}}}{\hat{\Omega}}.$$

QNM excitation

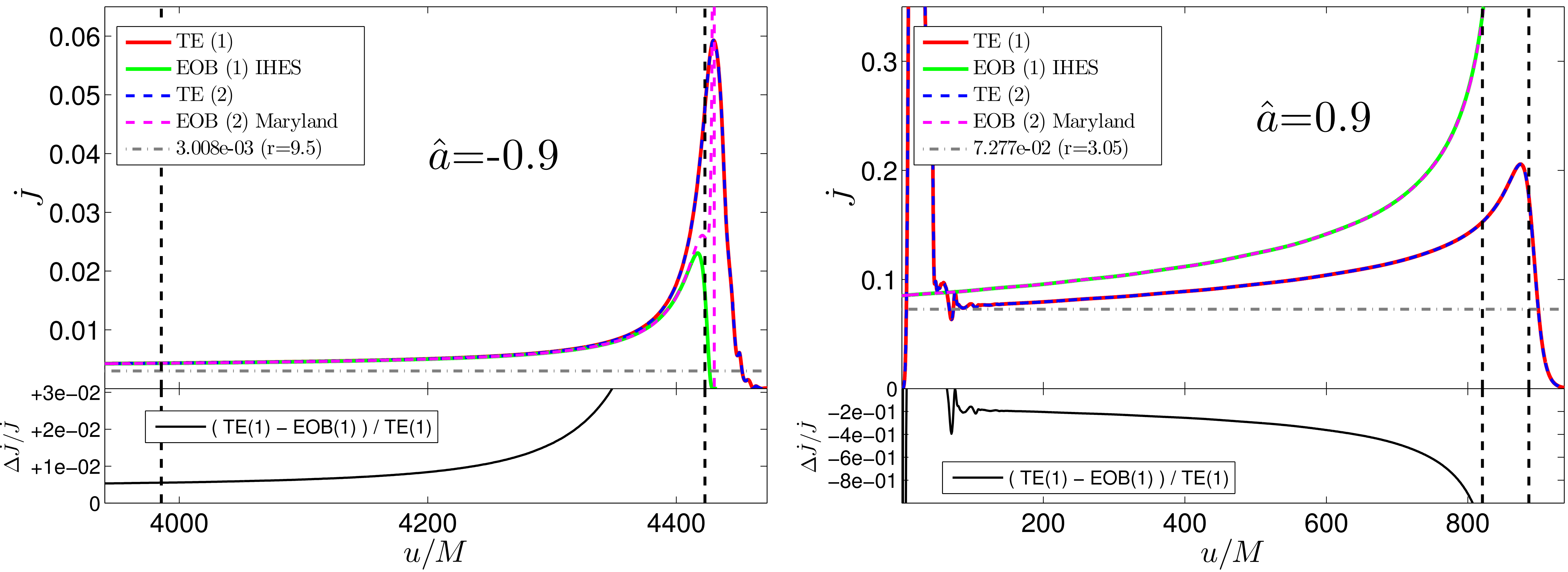


QNM "interference" *[SB&Nagar 2010; Barausse+ 2012;Taracchini+ 2014]*



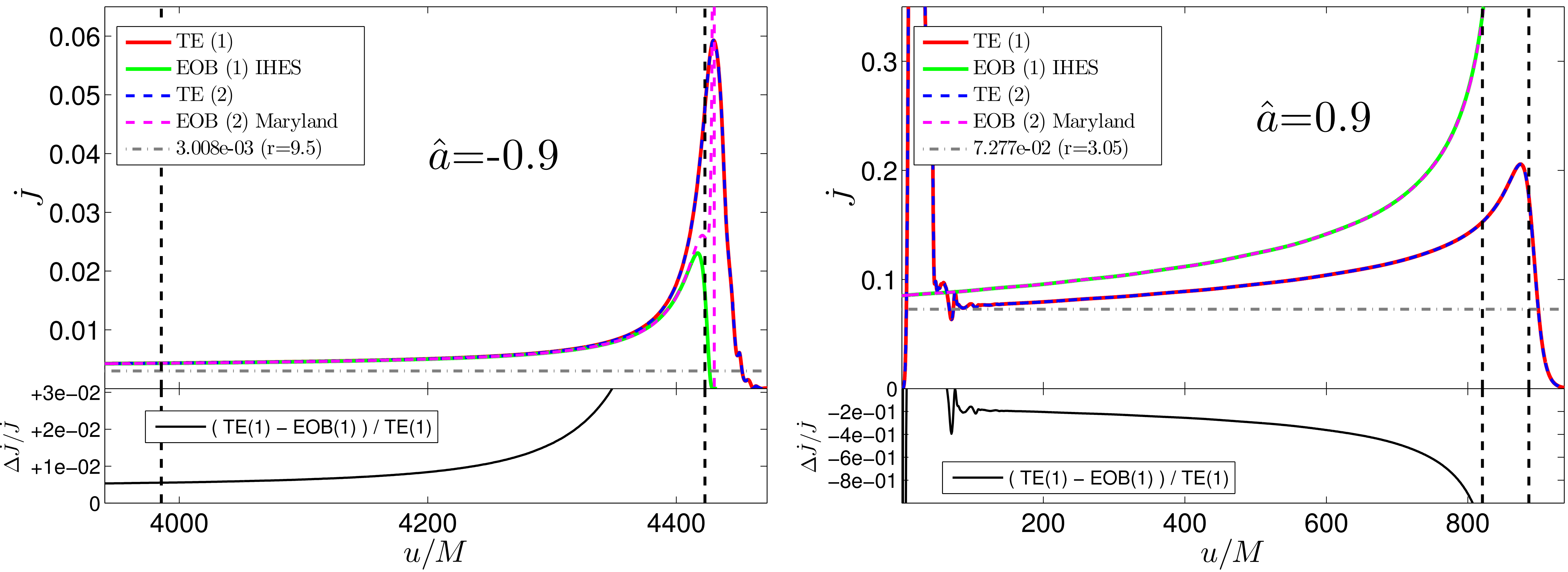
a→-1 QNM negative frequencies (also *[Taracchini+ 2014]*)

Flux consistency



RR: *[Damour+ 2008; Fujita&Iyer 2010; Pan+ 2011]*

Flux consistency

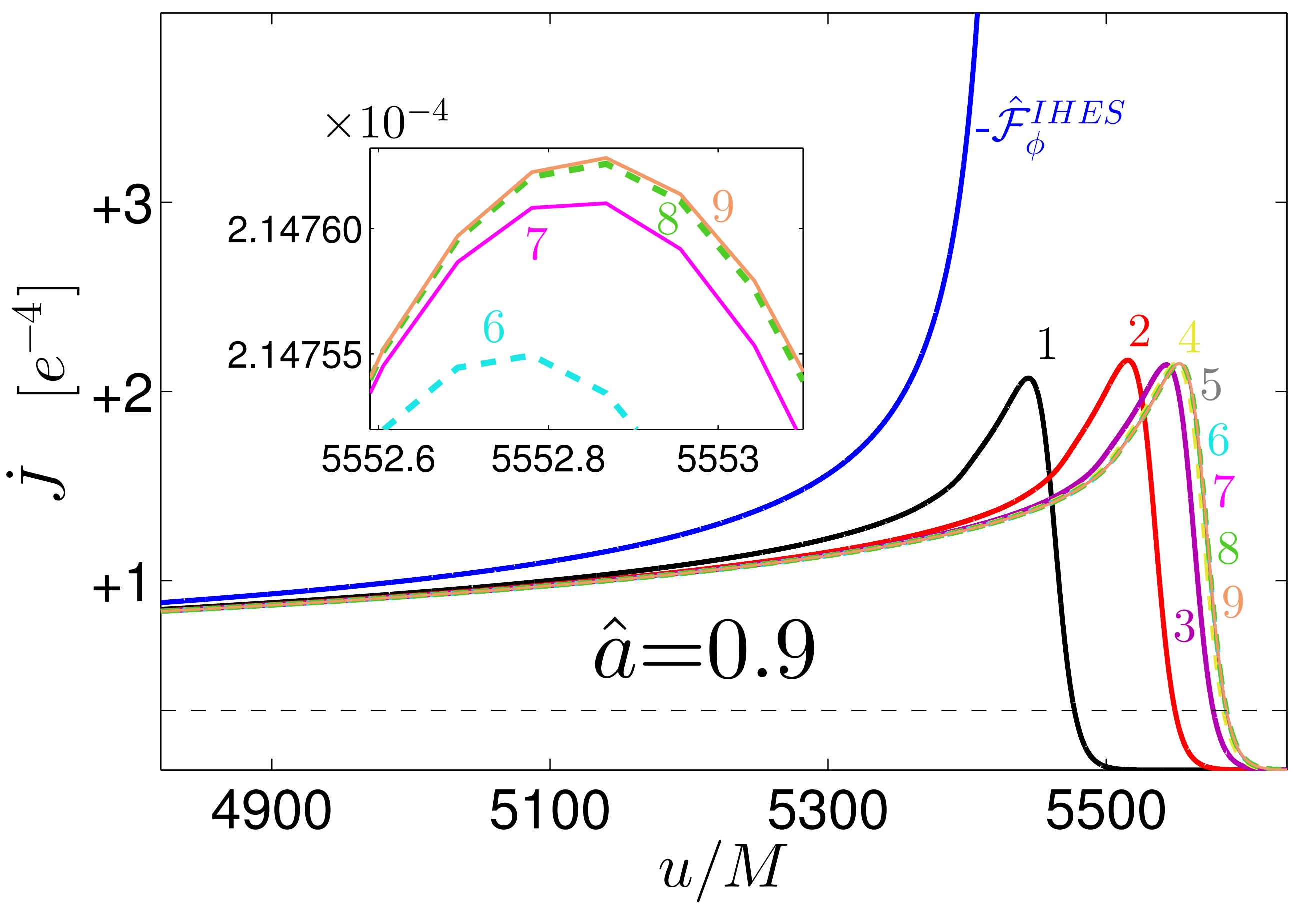


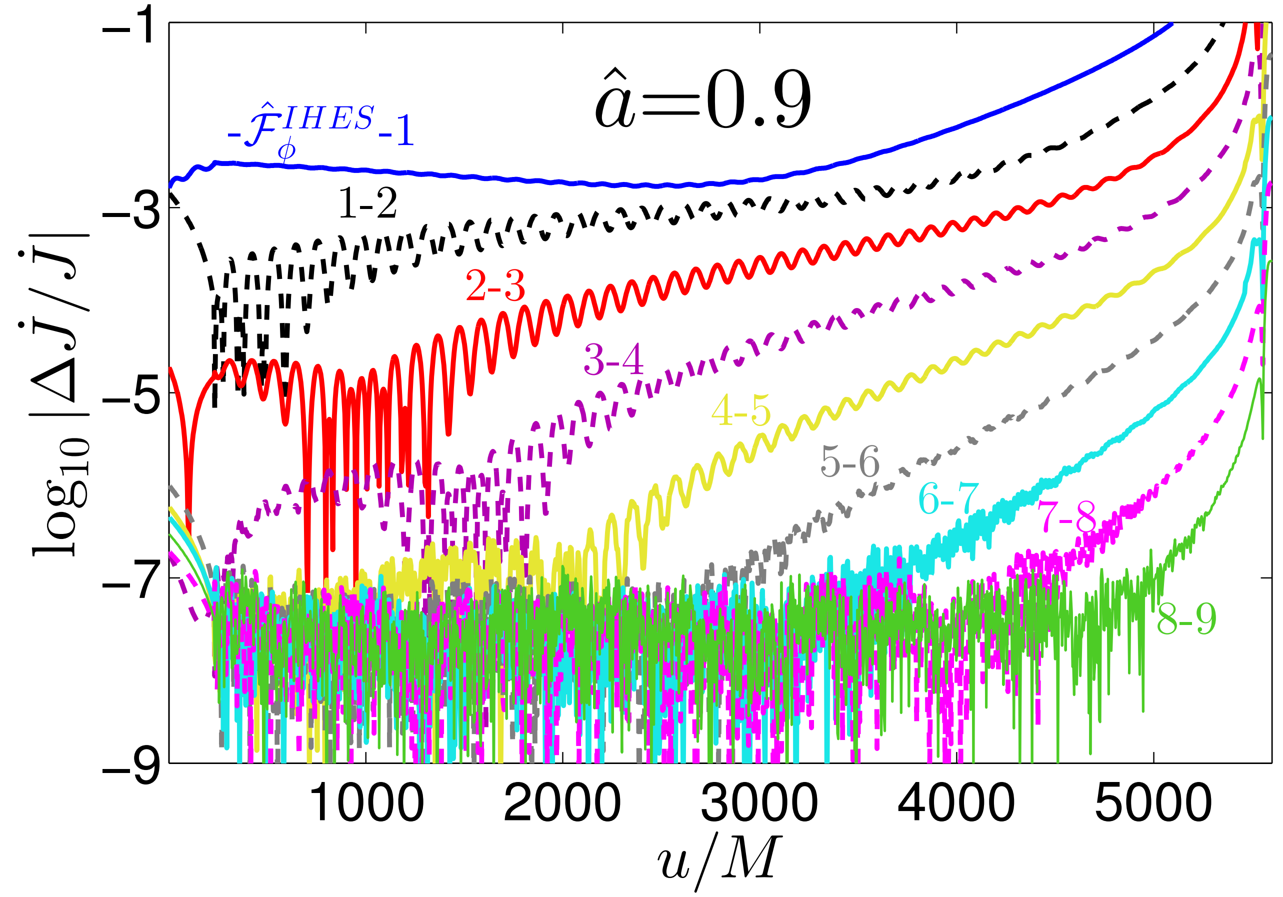
RR: *[Damour+ 2008; Fujita&Iyer 2010; Pan+ 2011]*

High-PN information exist! *[Fujita 2012; Shah 2014]*

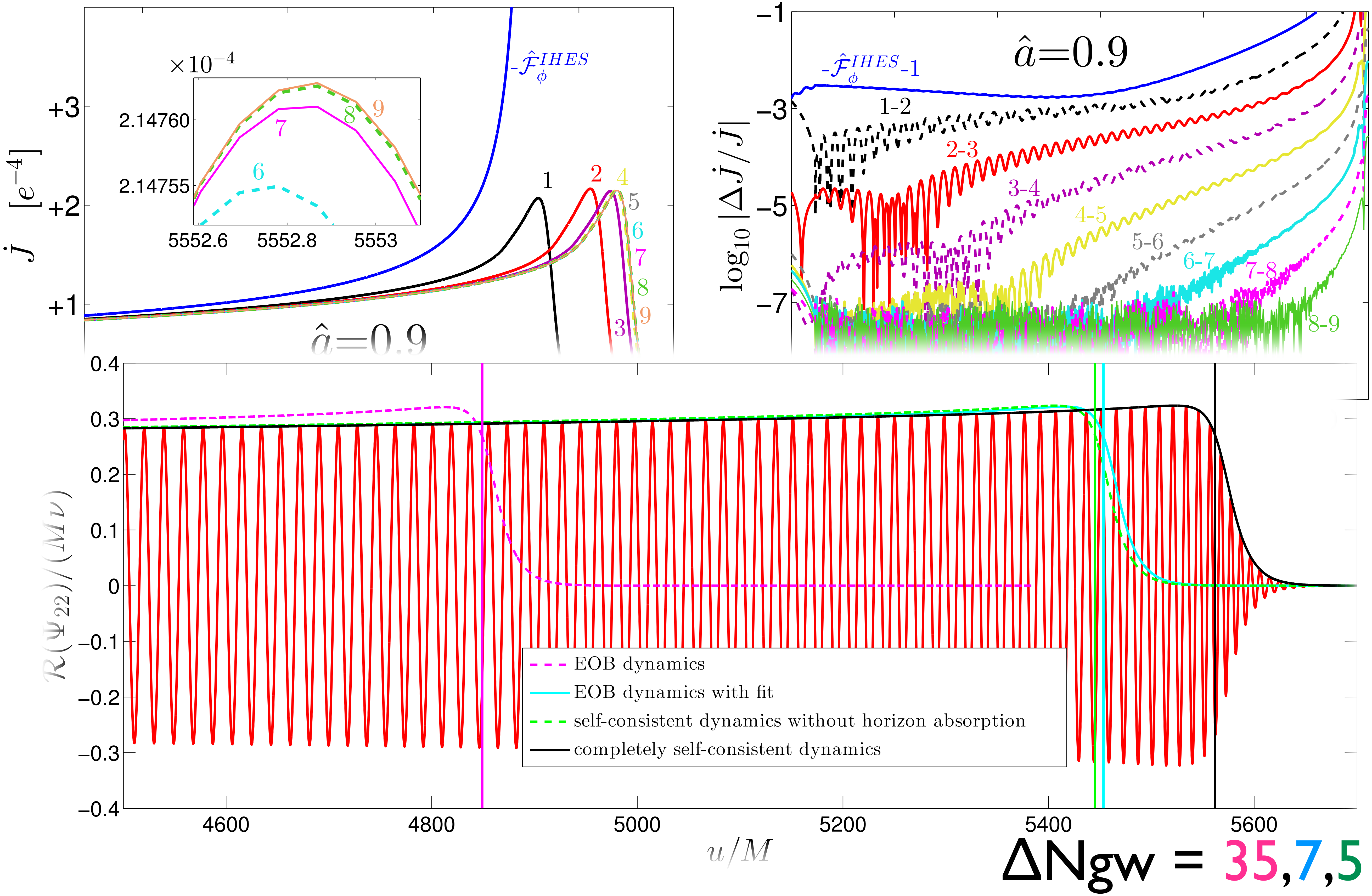
Need to include in the resummed waveform

Consistent flux calculation





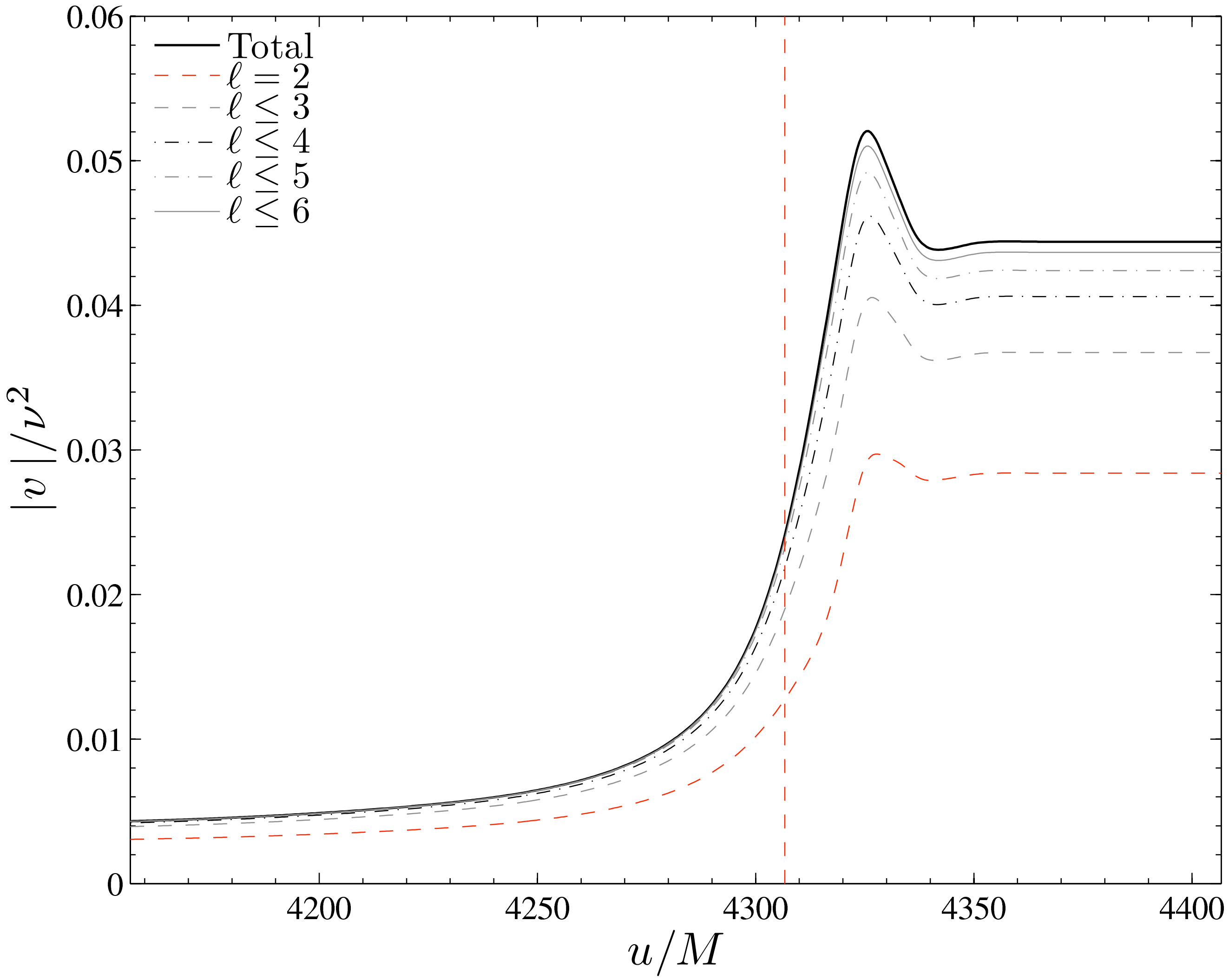
Consistent flux calculation



Summary

- Accurate solver 2+1 gravitational (curvature) perturbation
- Key technical point: hyperboloidal coordinates
- Kerr tails for generic spin perturbation
- Large-q BBH Lab:
  - Multipolar amplitudes and QNMs excitation
  - Performances EOB resummed fluxes
  - Horizon-absorbed fluxes (not in this talk)
  - Waveforms extrapolation (not in this talk)
  - Delta discretization (not in this talk)

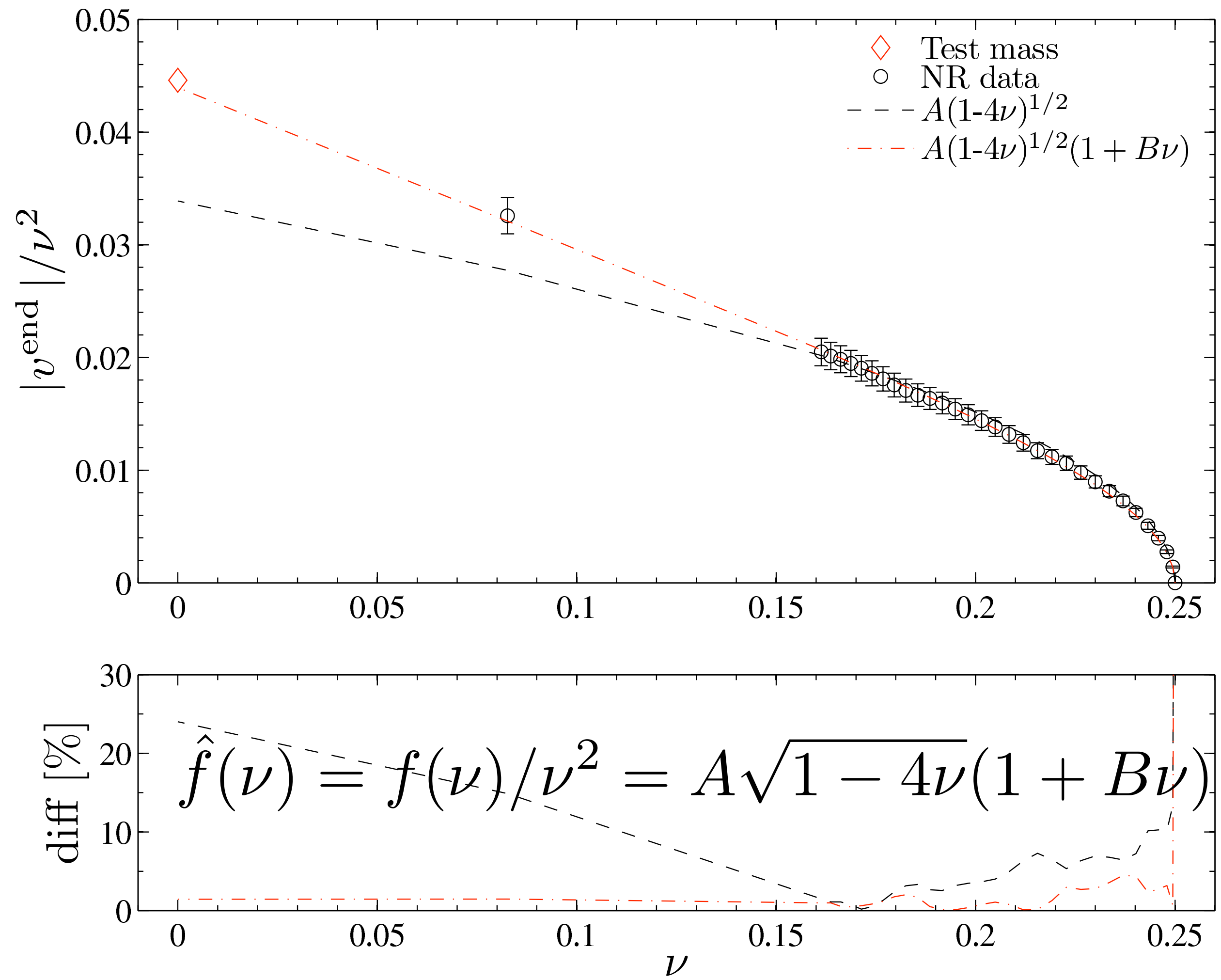
Gravitational Recoil



ν	$\lvert v^{\rm end}\rvert/\nu^2$	$\lvert v^{\rm max}\rvert/\nu^2$
10^{-2}	0.043234	0.050547
10^{-3}	0.044401	0.052058
10^{-4}	0.044587	0.052298
0	**0.0446**	**0.0523**

[SB&Nagar 2010]

with Jena data *[Gonzales+ 07,09]*

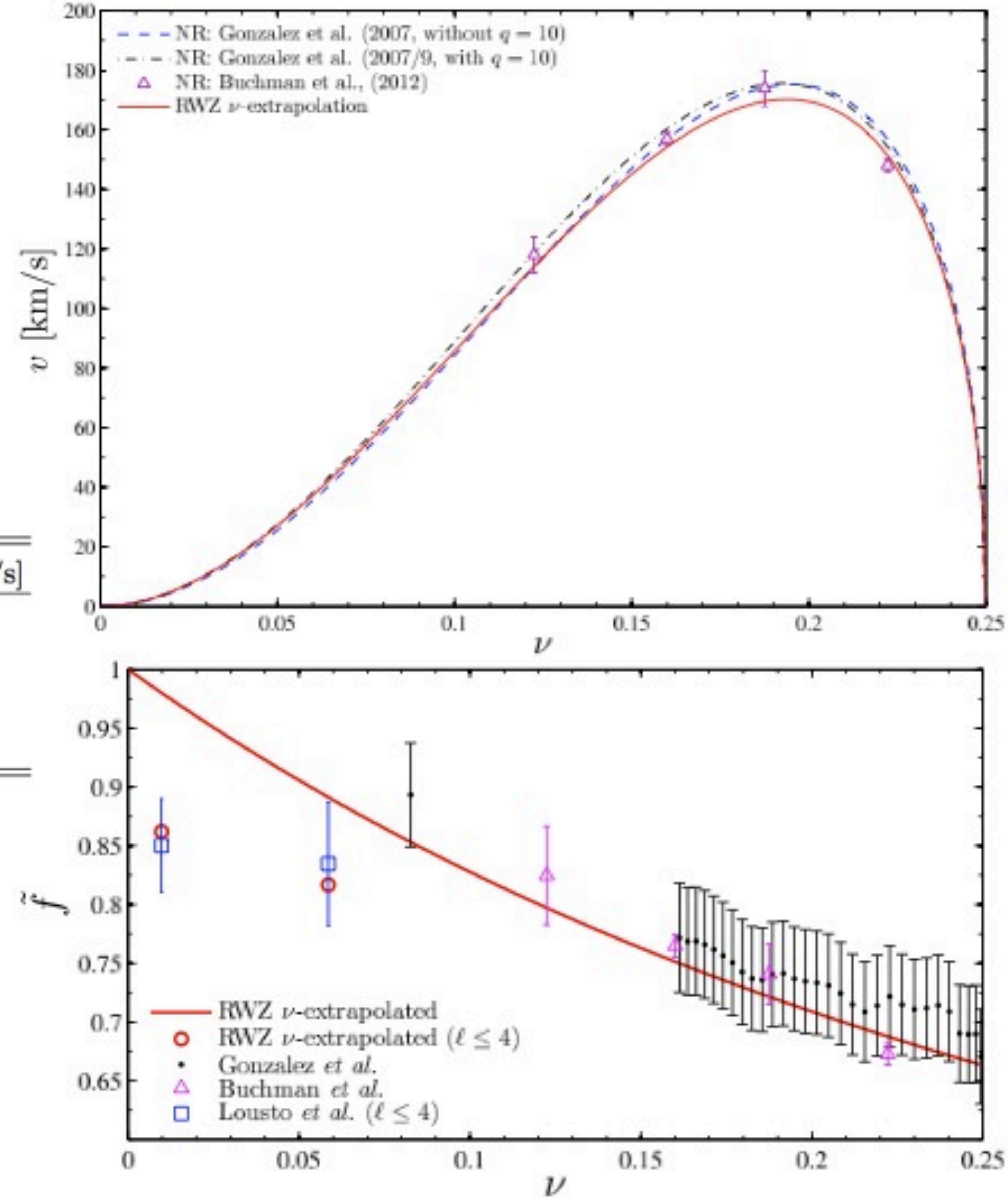


$$\hat{f}(\nu) = f(\nu)/\nu^2 = A\sqrt{1-4\nu}(1+B\nu)$$

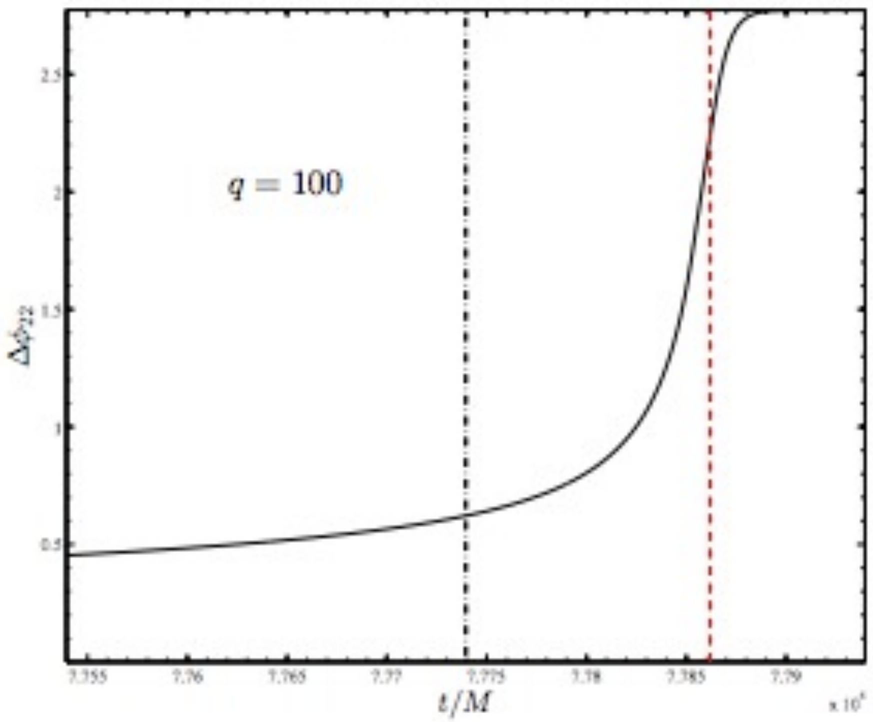
Reference	$\lvert v^{\rm end}\rvert/\nu^2$
Damour and Gopakumar	$[0.010, 0.035]$
Schnittman and Buonanno	$[0.018, 0.041]$
Sopuerta *et al.*	$[0.023, 0.046]$
Le Tiec, Blanchet and Will	0.032
González *et al.*	0.04
Sundararajan *et al.*	0.044
This work	**0.0446**

q	ν	$v_{\rm end}^{\rm NR}$[km/s]	$v_{\rm end}^{{\rm RWZ}_\nu}$[km/s]	$v_{\rm end}^{{\rm RWZ}_{\rm LO}}$[km/s]
2	$0.\bar{2}$	148 ± 2	151.3	219.9
3	0.1875	174 ± 6	169.5	234.8
4	0.1600	157 ± 2	154.2	205.2
6	0.1224	118 ± 6	114.1	143.1

[Nagar 2013]



Horizon absorption (no spin)

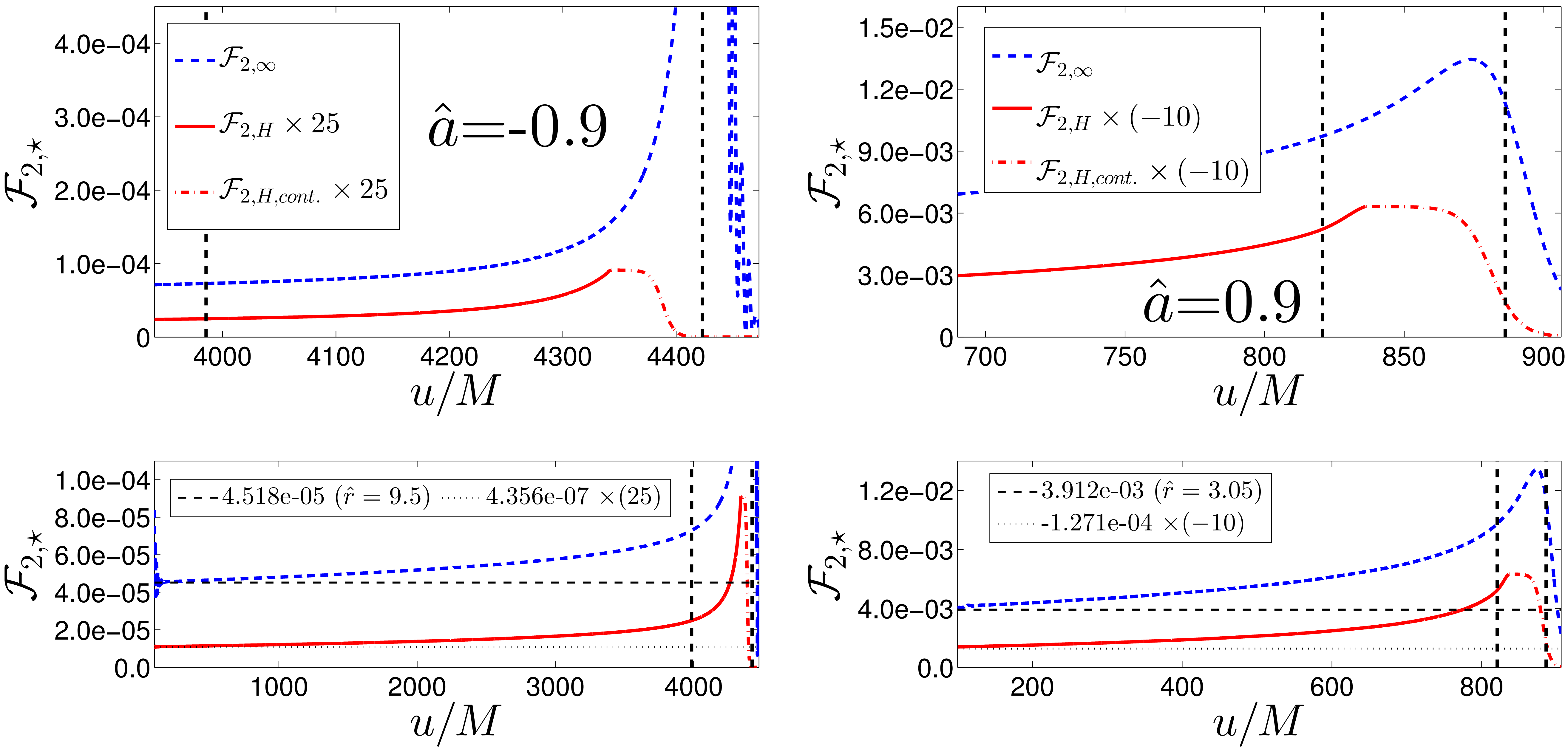


q	$M_A + M_B [M_\odot]$	$f_{\rm merger}$ [Hz]	$\mathcal{A}_{\rm aLIGO}$	$\mathcal{A}_{\rm ET}$
10	$10 + 100$	89.16	0.9999	0.9998
50	$10 + 500$	17.92	0.9991	0.9995
71.43	$1.4 + 100$	89.21	0.9991	0.9983
100	$1.4 + 140$	63.63	0.9992	0.9970

- Dynamics: resummed horizon-absorbed fluxes *[Nagar&Akcay 2012]*
- Consistency analytical / RWZ absorbed flux beyond LSO
- Significant dephasing, but <u>not</u> relevant for detection
- LO (non-resum) PN flux underestimate effect (< /2)

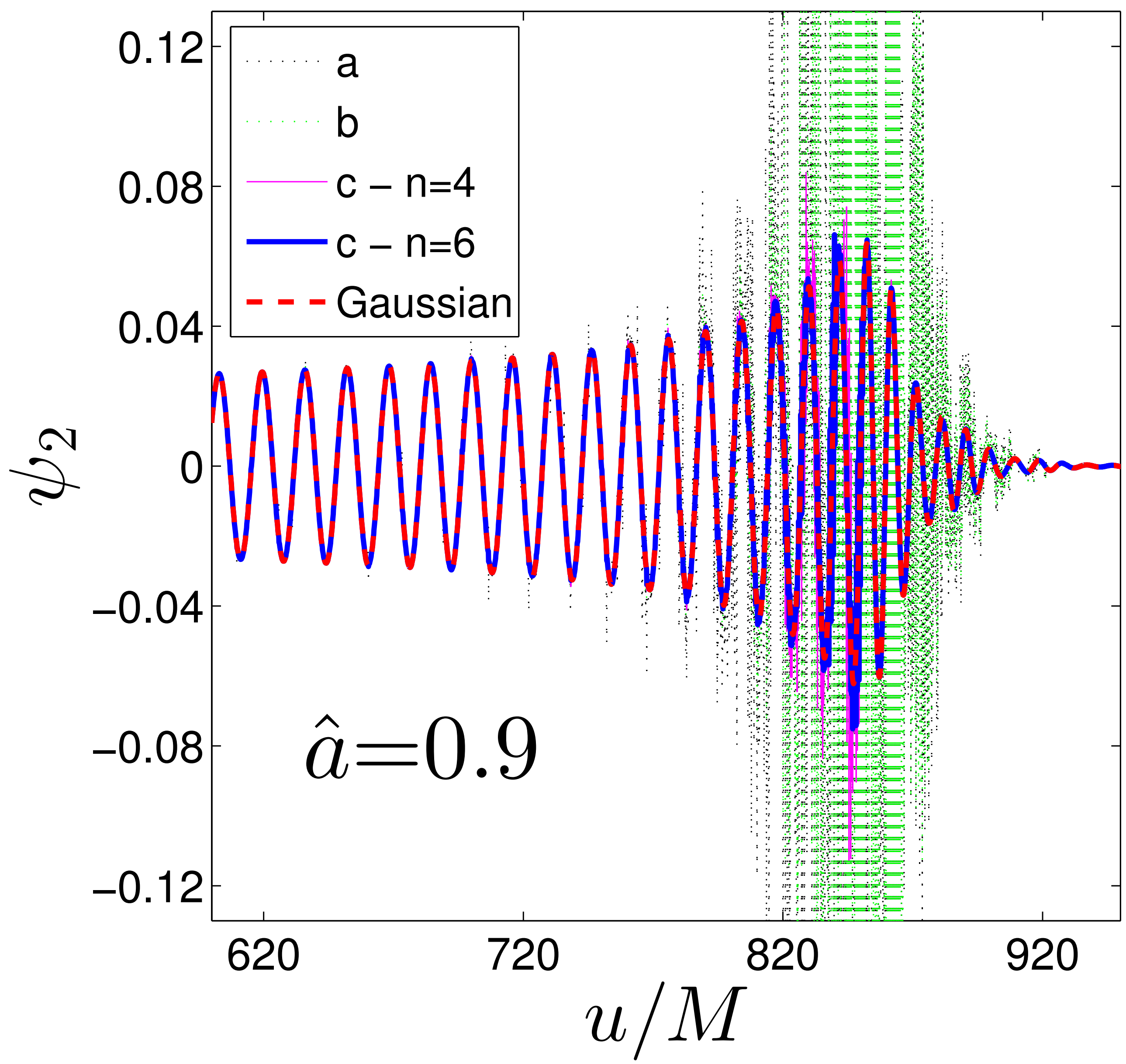
q	ν	r_0	$N_{\rm orb}$	$\Delta\phi_{22}^{\rm LSO}$ [rad]	$\Delta\mathcal{N}^{\rm LSO}$	$\Delta\phi_{22}^{\rm LR}$ [rad]	$\Delta\mathcal{N}^{\rm LR}$	$\Delta^{\rm 1PN}\phi_{22}^{\rm LSO}$ [rad]	$\Delta^{\rm 1PN}\phi_{22}^{\rm LR}$ [rad]
1	0.250000	15	15	0.003289	0.000523	0.005475	0.000871	0.002849	0.004547
4	0.160000	15	21	0.028725	0.004572	0.104712	0.016665	0.012320	0.020246
10	0.082645	15	38	0.064372	0.010245	0.220496	0.035093	0.052834	0.199428
50	0.019223	15	153	0.312210	0.049690	1.115319	0.177508	0.230220	0.765105
100	0.009803	15	296	0.620662	0.098781	2.217042	0.352853	0.458168	1.549226
1000	0.000998	7	41.2	0.129978	0.020687	1.453992	0.231410	...	...
1002	0.000996	7	40.9	0.129023	0.020535	1.563971	0.248914	...	...

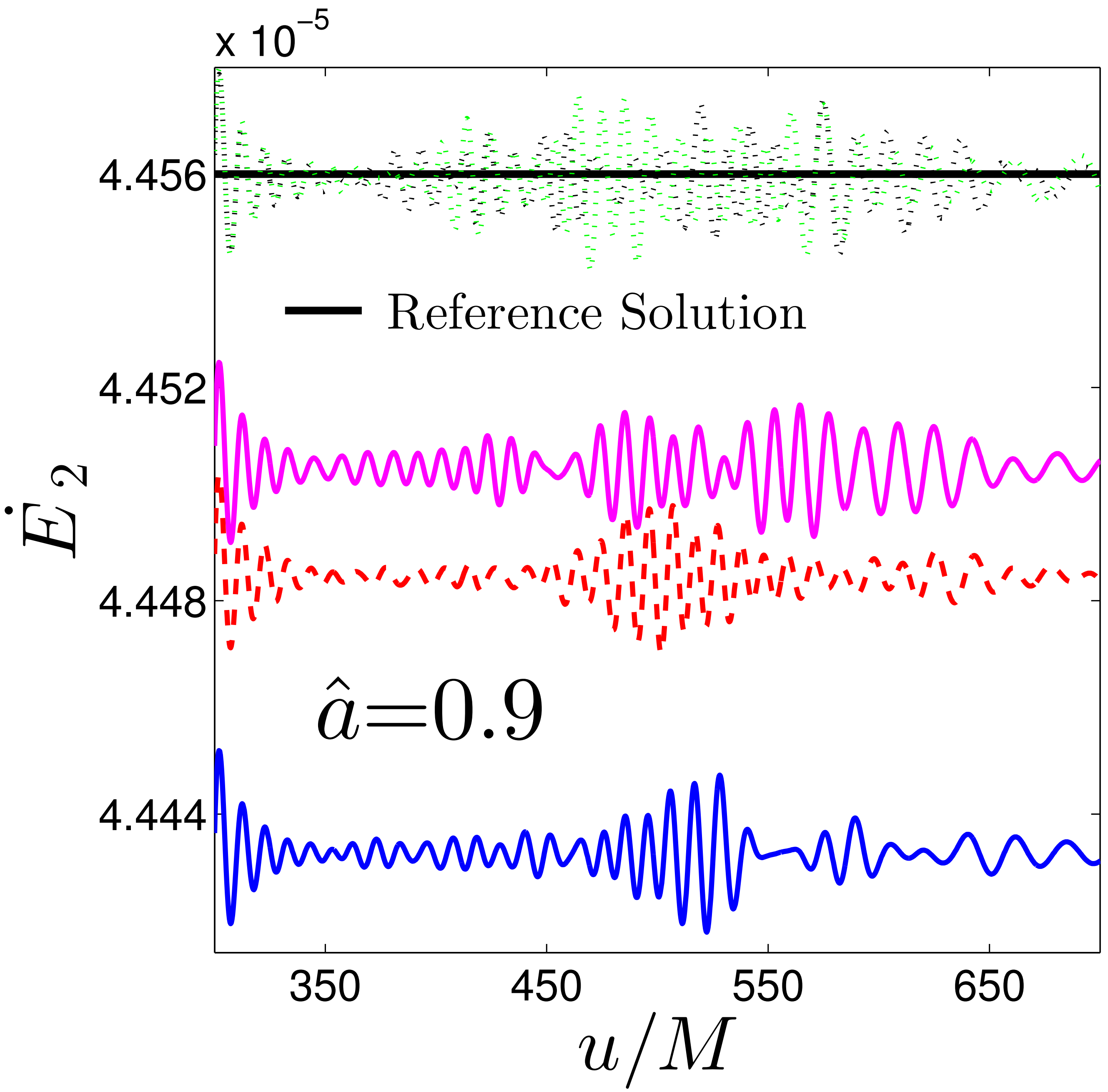
Horizon absorption (spin)



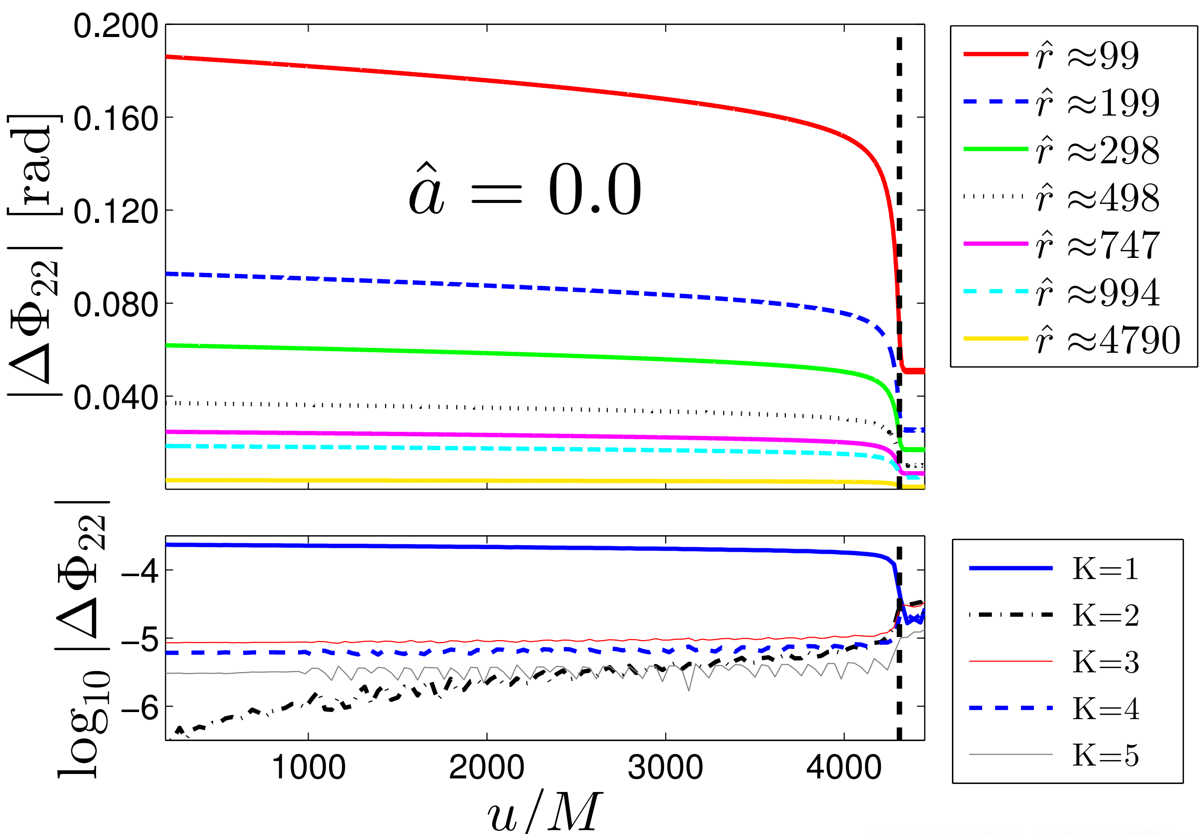
Formalism: *[Poisson 2004]*

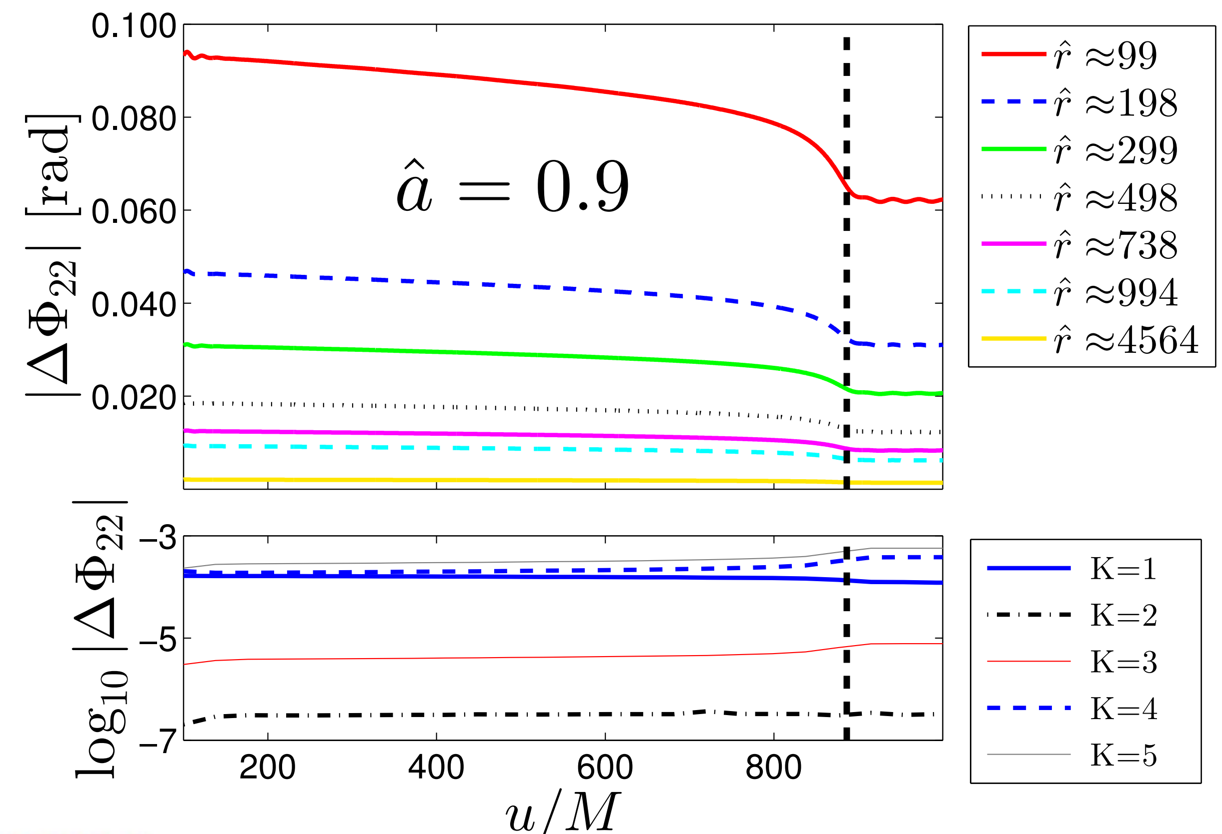
Discrete delta's





Waveform extrapolation





$$f(u,r) = f^{(0)}(u) + \sum_{k=1}^{K} r^{-k} f^{(k)}(u)$$