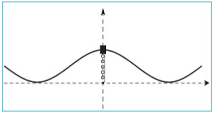
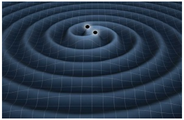
How are Gravitational Waves in any dimension like a bead on a string?



Ofek Birnholtz
with S. Hadar and B. Kol



O.B., S.H. & B.K., Phys. Rev. D 88, 104037 (2013)
O.B. & S.H., Phys. Rev. D 89, 045003 (2014)
O.B., S.H. & B.K., arXiv:1402.2610 (2014)

CAPRA 17, CalTech

26 June 2014

Outline

"Everything should be made as simple as possible, but not simpler" - Albert Einstein

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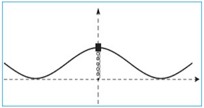
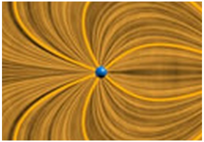
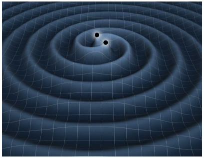


Bead + string (elastic field)



Electric charge + EM field



2-Body system + GR field

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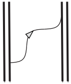
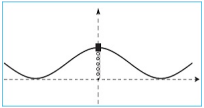
$\hat{S} =$  $= \sum \left(\hat{Q}\, G\, Q\right)$



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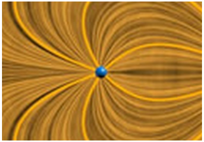

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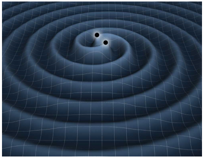

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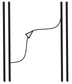
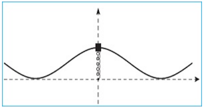
$\hat{S} =$  $= \sum \left(\hat{Q}\, G\, Q \right) \quad , \quad F = \frac{\delta \hat{S}}{\delta \hat{x}}$



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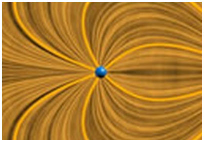

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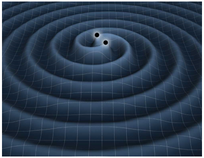

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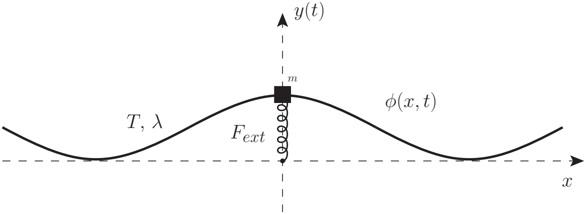

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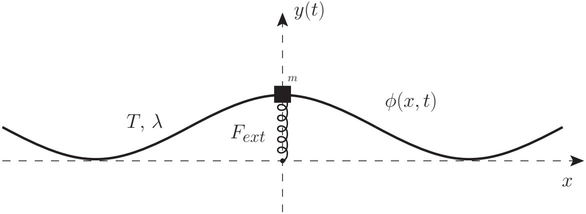

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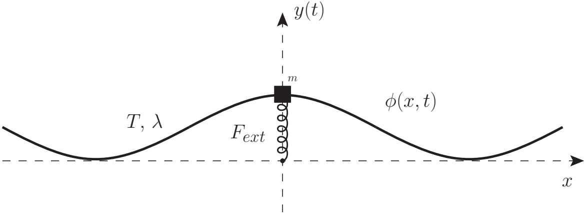

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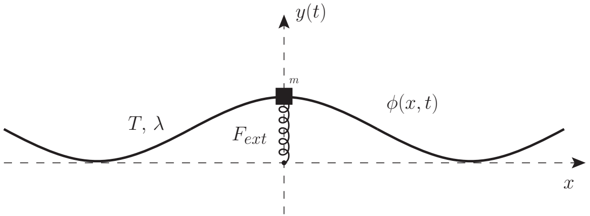

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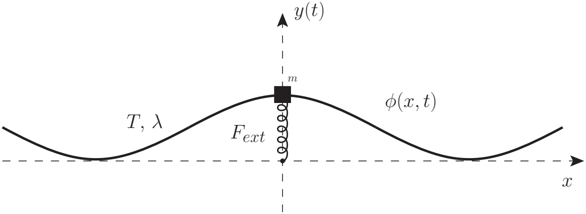

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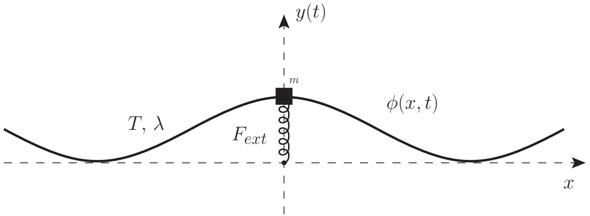

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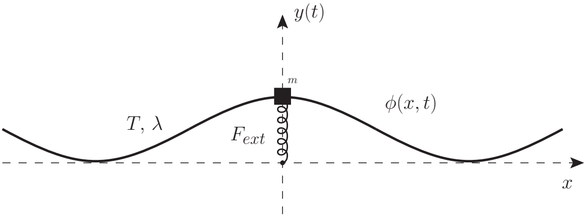

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Field doubling

An effective action for ϕ (kinetic term & source term):

$$S = \frac{T}{2}\int dtdx\left[\frac{1}{c^2}\dot{\phi}^2 - \phi'^2\right] + 2\,Z\int dt\,\dot{y}\,\phi(x=0)$$

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Double the field and the source (Schwinger, Galley) :

- These reflect directed propagation
- Interpretation: "pulling" mirror / radiation "sink"

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Doubled action:

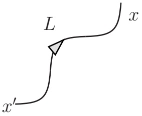
$$\hat{S}\left[\phi, \hat{\phi}; Q, \hat{Q}\right] = \int d^d x \left[\frac{\delta S}{\delta \phi}\,\hat{\phi} + \frac{\delta S}{\delta Q}\,\hat{Q}\right]$$

$$= \int \frac{d\omega}{2\pi}\left[\frac{Z}{c}\int dx\,\hat{\phi}\left(\omega^2 + \partial_x^2\right)\phi - 2i\omega Z\left(y\hat{\phi}(0) + \hat{y}\,\phi(0)\right)\right]$$

EOM found by varying w.r.t. $\hat{\phi}$

Feynman rules

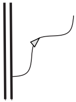
Directed propagator (non-hatted → hatted)



$$= G_\omega(x', x) = -c \, \frac{e^{i \frac{\omega}{c} |x - x'|}}{i \, \omega \, Z}$$

Source vertex

$$-Q_\omega \equiv$$



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Hatted source vertex

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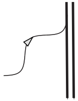
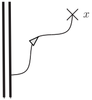


$$= +\, i\omega \, Z \, \hat{y}^*_\omega$$

A bead on a string - results

Radiation $(x > 0)$

$$\phi_\omega(x) =$$



$$= c\, y_\omega\, e^{i\omega x/c} \implies \phi(t, x) = y\left(t - \frac{x}{c}\right)$$

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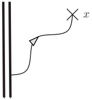

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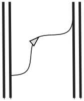
$$\hat{S}_{RR} =$$



$$+ \text{ c.c. } = \int \frac{d\omega}{2\pi}\, \hat{Q}^*_\omega\, G_\omega(0,0)\, Q_\omega + \text{c.c.}$$

$$= \int \frac{d\omega}{2\pi}\, i\omega\, Z\, \hat{y}^*_\omega\, y_\omega + \text{ c.c.} = -2\, Z \int \hat{y}\, \dot{y}\, dt$$

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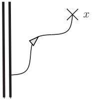

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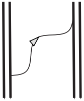

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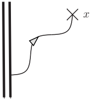
$$\phi_\omega(x) =$$ 

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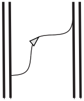
$$\hat{S}_{RR} =$$ 
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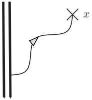

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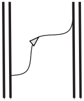

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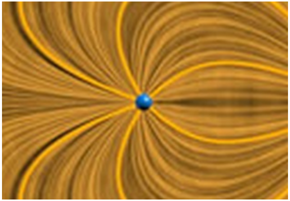
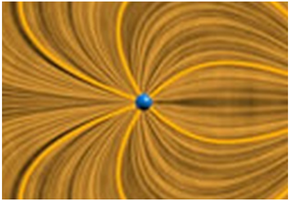
Electromagnetism in d dimensions



$$S = -\frac{1}{4\Omega_{\hat{d}+1}} \int F_{\mu\nu}F^{\mu\nu}\, d^d x - \int A_\mu J^\mu \, d^d x$$

$$(D := d-1\ ,\ \hat{d} := d-3\ ,\ c = 1)$$

Electromagnetism in d dimensions



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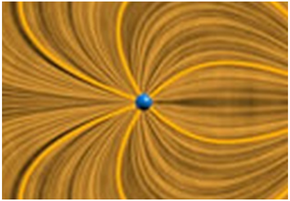
Different, more, or both?

Electromagnetism in d dimensions



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Different, more, or both?

In the right basis, only a little more, and a little different!

Electromagnetism - intermediate basis

Multipoles = (solid) spherical harmonics, scalar & vector

$$A_{t/r} = \int \frac{d\omega}{2\pi} \sum_L A^{L\omega}_{t/r} x_L e^{-i\omega t} \ , \ A_\Omega = \int \frac{d\omega}{2\pi} \sum_L \left(A^{L\omega}_S \partial_\Omega x_L + A^{L\omega}_{V\aleph} x^L_{\aleph\,\Omega} \right) e^{-i\omega t}$$

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$$L = (k_1 k_2 \cdots k_\ell)^{STF} \quad , \quad x^L = \left(x^{k_1} x^{k_2} \cdots x^{k_\ell} \right)^{STF} \sim r^\ell \, Y_{\ell \vec{m}}(\Omega)$$

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$$S = \frac{1}{2} \int \frac{d\omega}{2\pi} \sum_L N_{\ell,\hat{d}} S_{L\omega}$$

$$\begin{aligned} S_{L\omega} = \int dr\, r^{2\ell+\hat{d}+1} \Bigg\{ \Bigg[\left| i\omega A_r^{L\omega} - \frac{1}{r^\ell}(r^\ell A_t^{L\omega})' \right|^2 + \frac{c_s}{r^2} \left| i\omega A_S^{L\omega} - A_t^{L\omega} \right|^2 - \frac{c_s}{r^2} \left| \frac{1}{r^\ell}(r^\ell A_S^{L\omega})' - A_r^{L\omega} \right|^2 \\ + c_s \left(\frac{\omega^2}{r^2} - \frac{c_v}{r^4} \right) \left| A^{L\omega}_{V\aleph} \right|^2 - \frac{c_s}{r^2} \left| \frac{1}{r^\ell}(r^\ell A^{L\omega}_{V\aleph})' \right|^2 \Bigg] \\ - \Omega_{\hat{d}+1} \left[A_r^{L\omega} J^{r*}_{L\omega} + A_t^{L\omega} J^{t*}_{L\omega} + c_s A_S^{L\omega} J^{S*}_{L\omega} + c_s A^{L\omega}_{V\aleph} J^{V\aleph *}_{L\omega} + c.c. \right] \Bigg\} \end{aligned}$$

$$N_{\ell,\hat{d}} = \frac{\Gamma(1+\hat{d}/2)}{2^\ell\, \Gamma(\ell+1+\hat{d}/2)} = \frac{\hat{d}!!}{(2\ell+\hat{d})!!} \ , \quad c_s := \ell(\ell+\hat{d}) \ , \quad c_v = c_s + \hat{d} - 1$$

Electromagnetism - intermediate basis

Multipoles = (solid) spherical harmonics, scalar & vector

$$A_{t/r} = \int \frac{d\omega}{2\pi} \sum_L A^{L\omega}_{t/r} x_L e^{-i\omega t} \ , \quad A_\Omega = \int \frac{d\omega}{2\pi} \sum_L \left(A^{L\omega}_S \partial_\Omega x_L + A^{L\omega}_{V\aleph} x^L_{\aleph\,\Omega} \right) e^{-i\omega t}$$

$$J^{t/r} = \int \frac{d\omega}{2\pi} \sum_L J^{t/r}_{L\omega} x^L e^{-i\omega t} \ , \quad J^\Omega = \int \frac{d\omega}{2\pi} \sum_L \left(J^S_{L\omega} \partial^\Omega x^L + J^{V\aleph}_{L\omega} x^{\aleph\,\Omega}_L \right) e^{-i\omega t}$$

$$S = \frac{1}{2} \int \frac{d\omega}{2\pi} \sum_L N_{\ell,\hat{d}} S_{L\omega}$$

$$S_{L\omega} = \int dr\, r^{2\ell+\hat{d}+1} \left\{ \left[\left| i\omega A^{L\omega}_r - \frac{1}{r^\ell} (r^\ell A^{L\omega}_t)' \right|^2 + \frac{c_s}{r^2} \left| i\omega A^{L\omega}_S - A^{L\omega}_t \right|^2 - \frac{c_s}{r^2} \left| \frac{1}{r^\ell} (r^\ell A^{L\omega}_S)' - A^{L\omega}_r \right|^2 \right.\right.$$
$$\left. + c_s \left(\frac{\omega^2}{r^2} - \frac{c_v}{r^4} \right) \left| A^{L\omega}_{V\aleph} \right|^2 - \frac{c_s}{r^2} \left| \frac{1}{r^\ell} (r^\ell A^{L\omega}_{V\aleph})' \right|^2 \right]$$
$$\left. - \Omega_{\hat{d}+1} \left[A^{L\omega}_r J^{r*}_{L\omega} + A^{L\omega}_t J^{t*}_{L\omega} + c_s A^{L\omega}_S J^{S*}_{L\omega} + c_s A^{L\omega}_{V\aleph} J^{V\aleph*}_{L\omega} + c.c. \right] \right\}$$

$$N_{\ell,\hat{d}} = \frac{\Gamma(1+\hat{d}/2)}{2^\ell\, \Gamma(\ell+1+\hat{d}/2)} = \frac{\hat{d}!!}{(2\ell+\hat{d})!!} \ , \quad c_s := \ell(\ell+\hat{d}) \ , \quad c_v = c_s + \hat{d} - 1$$

Electromagnetism - intermediate basis

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$$J^{t/r} = \int \frac{d\omega}{2\pi} \sum_L J_{L\omega}^{t/r} x^L e^{-i\omega t} \ , \quad J^\Omega = \int \frac{d\omega}{2\pi} \sum_L \left(J_{L\omega}^{S} \partial^\Omega x^L + J_{L\omega}^{V\aleph} x_L^{\aleph\,\Omega} \right) e^{-i\omega t}$$

$$S = \frac{1}{2} \int \frac{d\omega}{2\pi} \sum_L N_{\ell,\hat{d}} S_{L\omega}$$

$$S_{L\omega} = \int dr\, r^{2\ell+\hat{d}+1} \Bigg\{ \Bigg[\left| i\omega A_r^{L\omega} - \frac{1}{r^\ell}(r^\ell A_t^{L\omega})' \right|^2 + \frac{c_s}{r^2} \left| i\omega A_S^{L\omega} - A_t^{L\omega} \right|^2 - \frac{c_s}{r^2} \left| \frac{1}{r^\ell}(r^\ell A_S^{L\omega})' - A_r^{L\omega} \right|^2$$

$$+ c_s \left(\frac{\omega^2}{r^2} - \frac{c_v}{r^4} \right) \left| A_{V\aleph}^{L\omega} \right|^2 - \frac{c_s}{r^2} \left| \frac{1}{r^\ell}(r^\ell A_{V\aleph}^{L\omega})' \right|^2 \Bigg]$$

$$- \Omega_{\hat{d}+1} \left[A_r^{L\omega} J_{L\omega}^{r*} + A_t^{L\omega} J_{L\omega}^{t*} + c_s A_S^{L\omega} J_{L\omega}^{S*} + c_s A_{V\aleph}^{L\omega} J_{L\omega}^{V\aleph *} + c.c. \right] \Bigg\}$$

$$N_{\ell,\hat{d}} = \frac{\Gamma(1+\hat{d}/2)}{2^\ell\, \Gamma(\ell+1+\hat{d}/2)} = \frac{\hat{d}!!}{(2\ell+\hat{d})!!} \ , \quad c_s := \ell(\ell+\hat{d}) \ , \quad c_v = c_s + \hat{d} - 1$$

Electromagnetism - reduction to gauge invariant fields 1

Scalar field & source:

$$\begin{aligned} A^r_{L\omega} &= -\frac{1}{\omega^2 - \frac{c_s}{r^2}}\left[\frac{i\omega}{r^\ell}(r^\ell A^t_{L\omega})' + \frac{c_s}{r^{\ell+2}}(r^\ell A^S_{L\omega})' - \Omega_{\hat{d}+1} J^r_{L\omega}\right] \\ \tilde{A}_S^{L\omega} &:= A_t^{L\omega} - i\omega A_S^{L\omega} \\ \rho^S_{L\omega} &:= -J^t_{L\omega} + \frac{i}{\omega r^{\ell+\hat{d}+1}}\left(r^{\ell+\hat{d}+1}\frac{\Lambda}{\Lambda - 1} J^r_{L\omega}\right)' \quad , \quad \Lambda := \frac{\omega^2 r^2}{c_s} \end{aligned}$$

Vector field & source:

$$A^{L\omega}_{V\aleph} \quad , \quad \rho^{V\aleph}_{L\omega} := J^{V\aleph}_{L\omega}$$

Electromagnetism - reduction to gauge invariant fields 2

- Scalar ("Electric") field

$$
\begin{aligned}
A_E^{L\omega} &= \left(\ell\, r^{\ell-1}(1-\Lambda)\right)^{-1} \left(r^{\ell}(A_t^{L\omega} - i\omega A_S^{L\omega})\right)' \\
\rho_{L\omega}^{A_E} &= \int d\Omega_{\hat{d}+1} \frac{r^{\hat{d}} x_L}{\ell+\hat{d}} \left[\frac{i}{\omega\, r^{\hat{d}-1}}\left(r^{\hat{d}+1}\frac{\Lambda}{\Lambda-1}\vec{J}_w(\vec{r})\cdot\vec{n}\right)' - r^2\rho_\omega(\vec{r})\right]' \\
0 &= N_{\ell,\hat{d}}\, r^{2\ell+\hat{d}+1}\frac{\ell}{\ell+\hat{d}}\left(\omega^2+\partial_r^2+\frac{2\ell+\hat{d}+1}{r}\partial_r\right) A_E - \rho_{L\omega}^{A_E}
\end{aligned}
$$

Electromagnetism - reduction to gauge invariant fields 2

- Scalar ("Electric") field

$$
\begin{aligned}
A_E^{L\omega} &= \left(\ell\, r^{\ell-1}(1-\Lambda)\right)^{-1}\left(r^{\ell}(A_t^{L\omega} - i\omega A_S^{L\omega})\right)' \\
\rho_{L\omega}^{A_E} &= \int d\Omega_{\hat{d}+1} \frac{r^{\hat{d}} x_L}{\ell+\hat{d}}\left[\frac{i}{\omega\, r^{\hat{d}-1}}\left(r^{\hat{d}+1}\frac{\Lambda}{\Lambda-1}\vec{J}_w(\vec{r})\cdot\vec{n}\right)' - r^2\rho_\omega(\vec{r})\right]' \\
0 &= N_{\ell,\hat{d}}\, r^{2\ell+\hat{d}+1}\frac{\ell}{\ell+\hat{d}}\left(\omega^2+\partial_r^2+\frac{2\ell+\hat{d}+1}{r}\partial_r\right)A_E - \rho_{L\omega}^{A_E}
\end{aligned}
$$

- Vector ("Magnetic") field

$$
\begin{aligned}
A_{M\aleph}^{L\omega} &= \ell\, A_{V\aleph}^{L\omega}/r \\
\rho_{L\omega}^{A_{M\aleph}} &= \frac{1}{\ell} r^{\hat{d}+2}\int \vec{J}_w(\vec{r})\cdot\left(*(\vec{r}\wedge\vec{\nabla})\right)_\aleph x_L\, d\Omega_{\hat{d}+1} \\
0 &= N_{\ell,\hat{d}}\, r^{2\ell+\hat{d}+1}\frac{\ell+\hat{d}}{\ell}\left(\omega^2+\partial_r^2+\frac{2\ell+\hat{d}+1}{r}\partial_r\right)A_{M\aleph} - \rho_{L\omega}^{A_{M\aleph}}
\end{aligned}
$$

Electromagnetism - reduction to gauge invariant fields 2

More $(2 \times L \times \omega)$ fields, but 1d ; Different wave equations

- Scalar ("Electric") field

$$A_E^{L\omega} = \left(\ell\, r^{\ell-1}(1-\Lambda)\right)^{-1} \left(r^{\ell}(A_t^{L\omega} - i\omega A_S^{L\omega})\right)'$$

$$\rho_{L\omega}^{A_E} = \int d\Omega_{\hat{d}+1} \frac{r^{\hat{d}} x_L}{\ell+\hat{d}} \left[\frac{i}{\omega\, r^{\hat{d}-1}} \left(r^{\hat{d}+1} \frac{\Lambda}{\Lambda-1} \vec{J}_w(\vec{r})\cdot\vec{n}\right)' - r^2 \rho_\omega(\vec{r})\right]'$$

$$0 = N_{\ell,\hat{d}}\, r^{2\ell+\hat{d}+1} \frac{\ell}{\ell+\hat{d}} \left(\omega^2 + \partial_r^2 + \frac{2\ell+\hat{d}+1}{r}\partial_r\right) A_E - \rho_{L\omega}^{A_E}$$

- Vector ("Magnetic") field

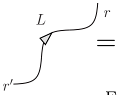
$$A_{M\aleph}^{L\omega} = \ell\, A_{V\aleph}^{L\omega}/r$$

$$\rho_{L\omega}^{A_{M\aleph}} = \frac{1}{\ell} r^{\hat{d}+2} \int \vec{J}_w(\vec{r}) \cdot \left(*(\vec{r}\wedge\vec{\nabla})\right)_{\aleph} x_L\, d\Omega_{\hat{d}+1}$$

$$0 = N_{\ell,\hat{d}}\, r^{2\ell+\hat{d}+1} \frac{\ell+\hat{d}}{\ell} \left(\omega^2 + \partial_r^2 + \frac{2\ell+\hat{d}+1}{r}\partial_r\right) A_{M\aleph} - \rho_{L\omega}^{A_{M\aleph}}$$

EM Feynman rules in general d

Directed propagator, from solution of wave equation

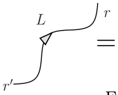


$$= \mathrm{G}_{\mathrm{ret}}^{\mathrm{A_E}/\mathrm{A_M}\aleph}(\mathrm{r}',\mathrm{r}) = -\mathrm{i}\,\omega^{2\ell+\hat{\mathrm{d}}}\,\mathrm{M}_{\ell,\hat{\mathrm{d}}}\,\mathrm{R}_1^{\mathrm{E}/\mathrm{M}}\,\tilde{\mathrm{j}}_\alpha(\omega \mathrm{r}_<)\,\tilde{\mathrm{h}}_\alpha^+(\omega \mathrm{r}_>)\delta_{\mathrm{LL}'}\ ,$$

$$\mathrm{R}_1^{\mathrm{E}} = \frac{\ell+\hat{\mathrm{d}}}{\ell},\ \mathrm{R}_1^{\mathrm{M}} = 1/\mathrm{R}_1^{\mathrm{M}},\ \alpha = \ell+\frac{\hat{\mathrm{d}}}{2},\ \mathrm{M}_{\ell,\hat{\mathrm{d}}} = \frac{\pi}{2^{2\alpha+1}\mathrm{N}_{\ell,\hat{\mathrm{d}}}\,\Gamma^2(\alpha+1)}$$

EM Feynman rules in general d

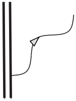
Directed propagator, from solution of wave equation



$$= G^{A_E/A_{M\aleph}}_{\rm ret}(r', r) = -i\,\omega^{2\ell+\hat{d}}\, M_{\ell,\hat{d}}\, R_1^{E/M}\, \tilde{j}_\alpha(\omega r_<)\, \tilde{h}^+_\alpha(\omega r_>)\delta_{LL'}\ ,$$

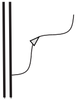
$$R_1^E = \frac{\ell+\hat{d}}{\ell},\ R_1^M = 1/R_1^M,\ \alpha = \ell + \frac{\hat{d}}{2},\ M_{\ell,\hat{d}} = \frac{\pi}{2^{2\alpha+1} N_{\ell,\hat{d}}\, \Gamma^2(\alpha+1)}$$

“Electric” & “Magentic” source vertices



$$\equiv -Q^{(E)}_{L\omega} = -\int dr' \tilde{j}_\alpha(\omega r')\rho^{A_E}_{L\omega}(r')$$

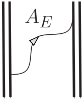
$$= \frac{1}{\ell+\hat{d}} \int d^D x' x'_L \left[i\omega\, \tilde{j}_\alpha(\omega r') \vec{J}_\omega(\vec{x}\,') \cdot \vec{x}' - \frac{1}{r'^{\ell+\hat{d}-1}} \left(r'^{\ell+\hat{d}}\, \tilde{j}_\alpha(\omega r') \right)' \rho_\omega(\vec{x}\,') \right]$$

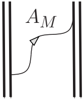


$$\equiv -Q^{(M,\aleph)}_{L\omega} = -\int d^D x \tilde{j}_\alpha(\omega r) \left(*(\vec{r} \wedge \vec{J}_w(\vec{r})) \right)^{\aleph}_{(k_\ell} x_{L-1)}$$

EM in general d - results

Radiation-Reaction effective action:

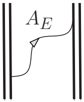
$\hat{S}_{RR} =$ 

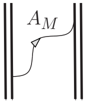
$+$ 

$= \sum \hat{Q}\ G\ Q^*$

EM in general d - results

Radiation-Reaction effective action:

$\hat{S}_{RR} =$ 

$+$ 

$= \sum \hat{Q}\ G\ Q^*$

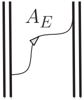
$$= \frac{1}{2}\int\frac{d\omega}{2\pi}\sum_{L,L'}\left[\hat{Q}^{(E)}_{L\omega}\, G^{A_E}_{ret}(0,0)\, Q^{(E)*}_{L'\omega} + \hat{Q}^{(M,\aleph)}_{L\omega}\, G^{A_{M\aleph}}_{ret}(0,0)\, Q^{(M,\aleph)*}_{L'\omega}\right] + \text{c.c.}$$

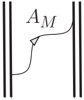
In even d:

$$= \int dt \sum_{L}\frac{(-)^{\ell+\frac{\hat{d}+1}{2}}}{\hat{d}!!(2\ell+\hat{d})!!}\left[\frac{\ell+\hat{d}}{\ell}\hat{Q}^{(E)}_{L}\cdot\partial_t^{2\ell+\hat{d}}Q^{L}_{(E)} + \frac{\ell^2\hat{d}\ \hat{Q}^{(M)}_{L}\cdot\partial_t^{2\ell+\hat{d}}Q^{L}_{(M)}}{(\ell+1)(\ell+\hat{d}-1)}\right]$$

EM in general d - results

Radiation-Reaction effective action:

$\hat{S}_{RR} =$ 

$+$ 

$= \sum \hat{Q}\ G\ Q^*$

$$= \frac{1}{2}\int \frac{d\omega}{2\pi} \sum_{L,L'} \left[\hat{Q}^{(E)}_{L\omega}\, G^{A_E}_{ret}(0,0)\, Q^{(E)*}_{L'\omega} + \hat{Q}^{(M,\aleph)}_{L\omega}\, G^{A_{M\aleph}}_{ret}(0,0)\, Q^{(M,\aleph)*}_{L'\omega} \right] + c.c.$$

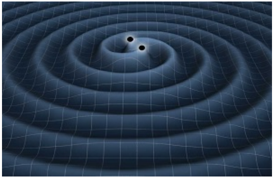
In even d:

$$= \int dt \sum_L \frac{(-)^{\ell+\frac{\hat{d}+1}{2}}}{\hat{d}!!(2\ell+\hat{d})!!} \left[\frac{\ell+\hat{d}}{\ell} \hat{Q}^{(E)}_L \cdot \partial_t^{2\ell+\hat{d}} Q^L_{(E)} + \frac{\ell^2 \hat{d}\ \hat{Q}^{(M)}_L \cdot \partial_t^{2\ell+\hat{d}} Q^L_{(M)}}{(\ell+1)(\ell+\hat{d}-1)} \right]$$

Radiation-Reaction (Self-) Force (on a point-charge in d dimensions)

$$\vec{F}_{ALD} = \frac{\delta \hat{S}_{RR}}{\delta \hat{\vec{x}}} = q^2 \frac{(-)^{\frac{d}{2}}(d-2)}{(d-1)!!(d-3)!!} \partial_t^{d-1} \vec{x} + (6\ \text{'1PN' terms}) + \cdots$$

Gravitation - GW from a 2-body system

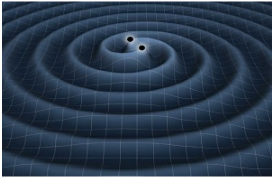


$$S = \frac{1}{16\pi G_d}\int\sqrt{-g}\,R\,d^dx - \frac{1}{2}\int h_{\mu\nu}T^{\mu\nu}d^dx\,,$$

$$\nabla_\mu T^{\mu\nu} = 0\ ,\ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$(\ c = 1,\ \text{linearized source}\ )$$

Gravitation - GW from a 2-body system



$$S = \frac{1}{16\pi G_d}\int \sqrt{-g}\, R\, d^d x - \frac{1}{2}\int h_{\mu\nu} T^{\mu\nu} d^d x\,,$$

$$\nabla_\mu T^{\mu\nu} = 0\;,\;\; g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$(\; c = 1,\;\; \text{linearized source}\;)$$

The same as EM fields and sources, but different

- Radiation zone: $A^{(E/M)}_{L\omega}(r) \to h^{(E/M/T)}_{L\omega}(r)$ (gauge-invariant fields)
- System zone: $h_{\mu\nu} \leftrightarrow (\phi, A_i, \sigma_{ij})$ (NRG fields, Kol & Smolkin '07)

these highlight spatial tensor structure, hierarchy in terms of PN

$$ds^2 = e^{2\phi}\left(dt - A_i\, dx^i\right)^2 - e^{-2\phi}\gamma_{ij}\, dx^i\, dx^j$$

Gravitation - 1D reduced action & Feynman rules

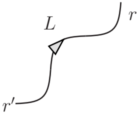
Goal - like EM:

$$S_{(E/M/T)} = \frac{1}{2}\int\frac{d\omega}{2\pi}\sum_{L}\int dr\left[N_{\ell,\hat{d}}\frac{r^{2\ell+\hat{d}+1}}{R^{\epsilon}_{\ell,\hat{d}}}h^{L\omega*}_{\epsilon}\left(\omega^2+\partial_r^2+\frac{2\ell+\hat{d}+1}{r}\partial_r\right)h^{\epsilon}_{L\omega} - \left(h^{L\omega*}_{\epsilon}\mathcal{T}^{\epsilon}_{L\omega}+c.c.\right)\right]$$

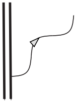
Gravitation - 1D reduced action & Feynman rules

Goal - like EM:

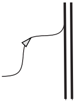
$$S_{(E/M/T)} = \frac{1}{2}\int\frac{d\omega}{2\pi}\sum_{L}\int dr\left[N_{\ell,\hat{d}}\frac{r^{2\ell+\hat{d}+1}}{R^{\epsilon}_{\ell,\hat{d}}}h^{L\omega *}_{\epsilon}\left(\omega^2+\partial_r^2+\frac{2\ell+\hat{d}+1}{r}\partial_r\right)h^{\epsilon}_{L\omega} - \left(h^{L\omega *}_{\epsilon}\mathcal{T}^{\epsilon}_{L\omega}+c.c.\right)\right]$$



$$= G^{\epsilon}_{ret}(r',r) = -i\,G\,\omega^{2\ell+\hat{d}}\,M_{\ell,\hat{d}}\,R^{\epsilon}_{\ell,\hat{d}}\,\tilde{j}_{\alpha}(\omega r_<)\,\tilde{h}^{+}_{\alpha}(\omega r_>)\delta_{LL'}$$



$$= -Q^{\epsilon}_{L\omega}$$,



$$= -\hat{Q}^{\epsilon *}_{L\omega}$$

Gravitation - special case d = 4

No tensor modes!

$$S_{(E/M)} = \frac{1}{2}\int\frac{d\omega}{2\pi}\sum_{L}\int dr\left[\frac{r^{2\ell+2}}{R^{\epsilon}(2\ell+1)!!}h^{L\omega*}_{(E/M)}\left(\omega^2+\partial_r^2+\frac{2(\ell+1)}{r}\partial_r\right)h^{(E/M)}_{L\omega} - \left(h^{L\omega*}_{(E/M)}\mathcal{T}^{(E/M)}_{L\omega}+c.c.\right)\right],$$

$$R^{\epsilon} := \frac{\ell+2}{\ell-1}\left(\frac{\ell+1}{\ell}\right)^{\epsilon} \quad , \quad \epsilon = 1\ (E)\ ,\ \epsilon = -1\ (M)$$

Gravitation - special case d = 4

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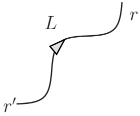
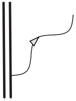
$$S_{(E/M)} = \frac{1}{2}\int\frac{d\omega}{2\pi}\sum_L\int dr\left[\frac{r^{2\ell+2}}{R^\epsilon(2\ell+1)!!}h^{L\omega*}_{(E/M)}\left(\omega^2+\partial_r^2+\frac{2(\ell+1)}{r}\partial_r\right)h^{(E/M)}_{L\omega} - \left(h^{L\omega*}_{(E/M)}\mathcal{T}^{(E/M)}_{L\omega}+c.c.\right)\right],$$

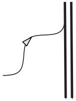
$$R^\epsilon := \frac{\ell+2}{\ell-1}\left(\frac{\ell+1}{\ell}\right)^\epsilon \quad , \quad \epsilon = 1\ (E)\ ,\ \epsilon=-1\ (M)$$



$$= G^\epsilon_{ret}(r',r) = \frac{-iG\omega^{2\ell+1}}{(2\ell+1)!!}\tilde{j}_{\ell+\frac{1}{2}}(\omega r_<)\,\tilde{h}^+_{\ell+\frac{1}{2}}(\omega r_>)\,R^\epsilon$$



$$= -Q^\epsilon_{L\omega}$$,

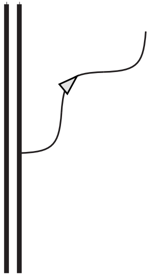


$$= -\hat{Q}^{\epsilon*}_{L\omega}$$

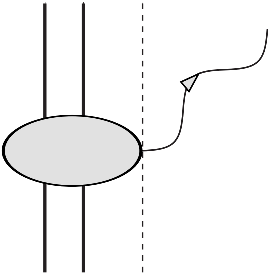
Gravitation - Vertices via zone seperation

- Work in radiation zone ⇒ eliminate system zone
- System zone itself includes non-linear vertices; grouped together (by PN order)
- Radiation-zone vertices defined as

$-Q_L :=$

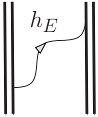


$:=$

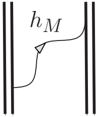


Gravitation (4d) - results

$\hat{S}_{\text{linear}} =$

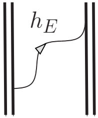


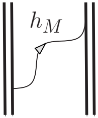
$+$



$$= \int dt \sum_{\ell} \frac{G\,(-)^{\ell+1}\,(\ell+2)}{(2\ell+1)!!\,(\ell-1)} \left[\frac{(\ell+1)}{\ell} \hat{Q}^{L}_{(E)} \partial_t^{2\ell+1} Q_L^{(E)} + \frac{\ell}{(\ell+1)} \hat{Q}^{L}_{(M)} \partial_t^{2\ell+1} Q_L^{(M)} \right]$$

Gravitation (4d) - results

$\hat{\mathrm{S}}_{\mathrm{linear}} =$ 

$+$ 

$$= \int \mathrm{dt} \sum_{\ell} \frac{\mathrm{G}\,(-)^{\ell+1}\,(\ell+2)}{(2\ell+1)!!\,(\ell-1)} \left[\frac{(\ell+1)}{\ell} \hat{\mathrm{Q}}^{\mathrm{L}}_{(\mathrm{E})} \partial_{\mathrm{t}}^{2\ell+1} \mathrm{Q}^{(\mathrm{E})}_{\mathrm{L}} + \frac{\ell}{(\ell+1)} \hat{\mathrm{Q}}^{\mathrm{L}}_{(\mathrm{M})} \partial_{\mathrm{t}}^{2\ell+1} \mathrm{Q}^{(\mathrm{M})}_{\mathrm{L}} \right]$$

As promised.

Gravitation (4d) - results LO

$$\hat{S}_{LO} = -G \int dt \, \frac{1}{5} \hat{Q}_E^{ij} \, \partial_t^5 Q_E^{ij}$$

- Q_E^{ij} : Mass quadrupole $\Longrightarrow$ Burke-Thorne potential & self-force

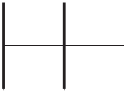
Gravitation (4d) - results LO, NLO

$$\hat{S}_{LO} = -G \int dt \, \frac{1}{5} \hat{Q}_E^{ij} \, \partial_t^5 Q_E^{ij}$$

- Q_E^{ij} : Mass quadrupole $\Longrightarrow$ Burke-Thorne potential & self-force

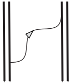
$$\hat{S}_{LO+NLO} = G \int dt \left[-\frac{1}{5} \hat{Q}_E^{ij} \, \partial_t^5 Q_E^{ij} - \frac{4}{45} \hat{Q}_M^{ij} \, \partial_t^5 Q_M^{ij} + \frac{1}{189} \hat{Q}_E^{ijk} \, \partial_t^7 Q_E^{ijk} \right]$$

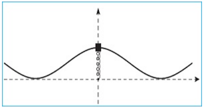
- Q_E^{ij} : Mass quadrupole (+1PN corrected including first system zone nonlinear effect: gravitating potential energy, $\sim -\frac{G m_A m_B}{r}$)



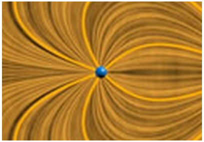
- Q_M^{ij} : Current quadrupole
- Q_E^{ijk} : Mass octupole

“Everything should be made as simple as possible, but not simpler” - Albert Einstein

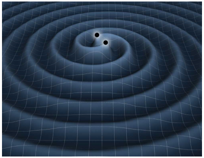
$$\hat{S} =$$  $$= \sum \left(\hat{Q}\, G\, Q \right) \quad , \quad F = \frac{\delta \hat{S}}{\delta \hat{x}}$$



Bead + string (elastic field)

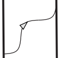


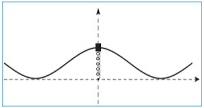
Electric charge + EM field



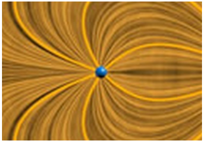
2-Body system + GR field

"Everything should be made as simple as possible, but not simpler" - Albert Einstein

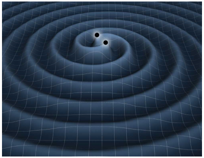
$\hat{S} =$ ‖  ‖ $= \sum \left(\hat{Q}\, G\, Q\right) \quad , \quad F = \frac{\delta \hat{S}}{\delta \hat{x}}$



$$\hat{S} = -2\, Z \int \hat{y}\, \dot{y}\, dt$$



$$\hat{S} = \int dt \sum_L \frac{(-)^{\ell+\frac{\hat{d}+1}{2}}}{\hat{d}!!(2\ell+\hat{d})!!}\left[\frac{\ell+\hat{d}}{\ell}\,\hat{Q}_L^{(E)}\cdot\partial_t^{2\ell+\hat{d}}\,Q^L_{(E)} + \frac{\ell^2\hat{d}\,\hat{Q}_L^{(M)}\cdot\partial_t^{2\ell+\hat{d}}\,Q^L_{(M)}}{(\ell+1)(\ell+\hat{d}-1)}\right]$$



$$\hat{S} = \int dt \sum_L \frac{G\,(-)^{\ell+1}(\ell+2)}{(2\ell+1)!!(\ell-1)}\left[\frac{\ell+1}{\ell}\,\hat{Q}^L_{(E)}\,\partial_t^{2\ell+1}\,Q_L^{(E)} + \frac{\ell}{\ell+1}\,\hat{Q}^L_{(M)}\,\partial_t^{2\ell+1}\,Q_L^{(M)}\right]$$

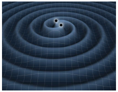
Conclusions

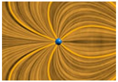
- Unified description of different physical systems
- Joint analytical Action formulation of radiation & reaction
- EM (ALD): New results in general dimension
- GR:
  - Economization of traditional computations
  - Nonlinear effects
  - Coming soon: leading, +1PN, some +2PN in any dimension

Questions?

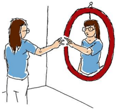
Thank you for your attention

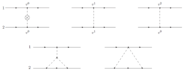
Image credits

 K. Thorne (Caltech) & T. Carnahan (NASA GSFC)

 MIT OpenCourseWare Electrostatic Visualizations

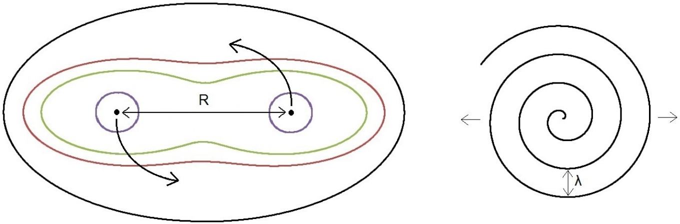
 Flickr's Flood G, (creative commons license)

 Elhanan Nafha

 W. D. Goldberger & I. D. Rothstein, PRD 73 (2006) 104029

Zones & enhanced symmetries

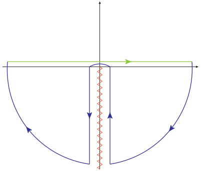
In PN: $\lambda \propto \frac{R}{v} \gg R$.



- System zone: $\sim$ stationary.
- Radiation zone: $\sim$ spherical symmetry.

EM in non-even d

- In frequency domain - SAME.
- Transforming to time domain $\Longrightarrow$ branch-cut $\Longrightarrow$ non-locality



$$\hat{Q}(t)\partial_t^{2\ell+\hat{d}}Q(t) \quad \rightarrow \quad \hat{Q}(t)\left[\left(\frac{1}{2}H(2\ell+\hat{d}) - H(\ell+\frac{\hat{d}}{2})\right)\partial_t^{2\ell+\hat{d}}Q^L(t)\right.$$
$$\left. - \int_{-\infty}^{t}\frac{dt'}{t-t'}\partial_{t'}^{2\ell+\hat{d}}Q(t')\Big|_{Reg}\right]$$

Reg.: Detweiler-Whiting decomposition, generalizes Dirac's odd propagator

Gravitation - Full source vertices

$$Q^L_{(E)} = \frac{1}{(\ell+1)(\ell+2)}\int d^3x\, x^L \Big[r^{-\ell}\left(r^{\ell+2}\,\tilde{j}_{\ell+\frac{1}{2}}(ri\partial_t)\right)''(T^{00}+T^{aa})$$
$$-\frac{4}{r^{\ell+1}}\left(r^{\ell+2}\tilde{j}_{\ell+\frac{1}{2}}(ri\partial_t)\right)'\partial_t T^{0a}x^a + 2\tilde{j}_{\ell+\frac{1}{2}}(ri\partial_t)\partial_t^2 T^{ab}x^a x^b + r^2\tilde{j}_{\ell+\frac{1}{2}}(ri\partial_t)\partial_t^2(T^{00}-T^{aa})\Big]$$
$$= \sum_{p=0}^{\infty}\frac{(2\ell+1)!!}{(2p)!!(2\ell+2p+1)!!}\left(1+\frac{8p(\ell+p+1)}{(\ell+1)(\ell+2)}\right)\left[\int d^3x\,\partial_t^{2p}T^{00}r^{2p}x^L\right]^{STF}$$
$$+\sum_{p=0}^{\infty}\frac{(2\ell+1)!!}{(2p)!!(2\ell+2p+1)!!}\left(1+\frac{4p}{(\ell+1)(\ell+2)}\right)\left[\int d^3x\,\partial_t^{2p}T^{aa}r^{2p}x^L\right]^{STF}$$
$$-\sum_{p=0}^{\infty}\frac{(2\ell+1)!!}{(2p)!!(2\ell+2p+1)!!}\frac{4}{\ell+1}\left(1+\frac{2p}{\ell+2}\right)\left[\int d^3x\,\partial_t^{2p+1}T^{0a}r^{2p}x^{aL}\right]^{STF}$$
$$+\sum_{p=0}^{\infty}\frac{(2\ell+1)!!}{(2p)!!(2\ell+2p+1)!!}\frac{2}{(\ell+1)(\ell+2)}\left[\int d^3x\,\partial_t^{2p+2}T^{ab}r^{2p}x^{abL}\right]^{STF}$$

$$Q^L_{(M)} = \frac{1}{\ell+2}\int d^3x\Big\{ r^{-\ell-1}\left(r^{\ell+2}\,\tilde{j}_{\ell+\frac{1}{2}}(ri\partial_t)\right)'\left(2\,\vec{x}\times\left(\vec{T}^{0a}\right)\right)^{k_\ell}x^{L-1}$$
$$-\tilde{j}_{\ell+\frac{1}{2}}(ri\partial_t)\,2\epsilon^{k_\ell ba}\partial_t T^{ac}x^{bcL-1}\Big\}$$

Origin-normalized Bessel functions

From Bessel's functiosn $B_\alpha \equiv \{J_\alpha, Y_\alpha, H_\alpha^\pm\}$,

$$\tilde{b}_\alpha := \Gamma(\alpha+1) 2^\alpha \frac{B_\alpha(x)}{x^\alpha}$$

They satisfy

$$\left[\partial_x^2 + \frac{2\alpha+1}{x}\partial_x + 1\right] \tilde{b}_\alpha(x) = 0$$

$\tilde{j}_\alpha$ in the vicinity of the origin $x = 0$ is given by

$$\tilde{j}_\alpha(x) = \sum_{p=0}^{\infty} \frac{(-)^p \, (2\alpha)!!}{(2p)!!(2p+2\alpha)!!} x^{2p}$$

The outgoing waves at $x \to \infty$ are $\tilde{h}^\pm := \tilde{j} \pm i\tilde{y}$:

$$\tilde{h}_\alpha^\pm(x) \sim (\mp i)^{\alpha+1/2} \frac{2^{\alpha+1/2}\Gamma(\alpha+1)}{\sqrt{\pi}} \frac{e^{\pm ix}}{x^{\alpha+1/2}}$$

Effective Field Theory description of GR

For example, Einstein-Infeld-Hoffmann Lagrangian

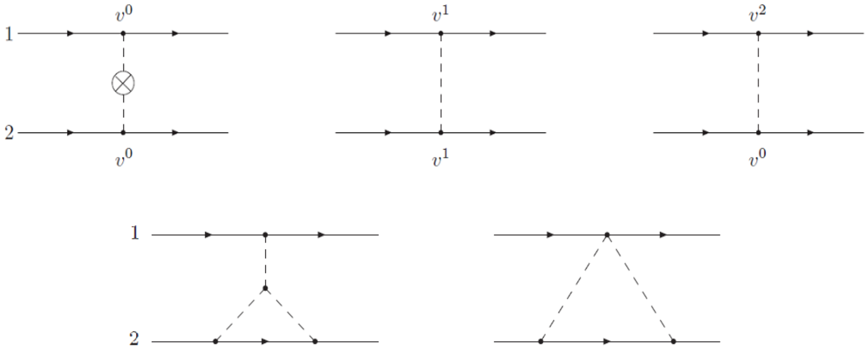
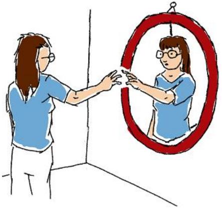


Figure from W. D. Goldberger and I. D Rothstein, PRD 73 (2006) 104029

Hatted variables - interpretation

Doubling the fields effectively "mirrors" them.



Hatted fields "pull" from the mirror's surface, absorbing radiation from the original fields.

$$0 = \frac{\delta \hat{S}}{\delta \phi(x)} = \int dx' \, \frac{\delta EOM_{\phi}(x')}{\delta \phi(x)} \hat{\phi}(x') - \hat{\rho}(x)$$

- $\hat{\phi}$ satisfies an equation for linearized deviations from the background ϕ with a source $\hat{\rho}$ and reversed propagation.
- In the absence of its source $\hat{\phi}$ vanishes, namely

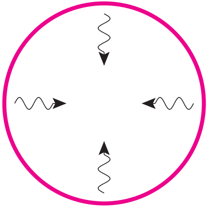
$$\hat{\rho} = 0 \Longrightarrow \hat{\phi} = 0$$

- ϕ, ρ are physical observables, unlike $\hat{\phi}$, $\hat{\rho}$.

Odd/regular propagation

- Problem: field diverges at location of particle.
- Solution: RR force due to $\phi_{\text{odd}} := \phi_{\text{ret}} - \phi_{\text{sym}}$.
- $\Box G_{\text{ret}} = \delta \;, \;\; \Box G_{\text{sym}} = \delta \implies \Box G_{\text{odd}} = 0$

  $\implies \phi_{\text{odd}}$ determined by incoming waves.



$\implies$ Matching regions automatically gives ϕ_{odd}.

2-way multipoles: matching lifted to action

- Problem: match system & radiation zones within action.
- Solution: introduce 2-way multipoles: fields living on matching surface

$$
\begin{aligned}
S &= S_{(S)}[\phi^{(S)}] + S_{(R)}[\phi^{(R)}] \\
&- \int dt \left[Q\, \partial\phi^{(R)}|_{r=0} + P\, \partial\phi^{(S)}|_{r=\infty} + g\, P\, Q \right]
\end{aligned}
$$

- $Q(t,\theta,\phi) \sim Q_{\ell m}(t) \sim Q^{L}(t)$

GR - NLO details

$$\hat{S} = G \int dt \left[-\frac{1}{5} \hat{Q}_E^{ij} \, \partial_t^5 Q_E^{ij} - \frac{4}{45} \hat{Q}_M^{ij} \, \partial_t^5 Q_M^{ij} + \frac{1}{189} \hat{Q}_E^{ijk} \, \partial_t^7 Q_E^{ijk} \right]$$

where

$$\begin{aligned}
Q_E^{ij} &= \sum_{A=1}^{2} m_A \left[1 + \frac{3}{2} v^2 - \frac{m_B}{r} - \frac{4}{3} \partial_t \left(\vec{v} \cdot \vec{x} \right) + \frac{11}{42} \partial_t^2 x^2 \right]_A \left(x^i x^j - \frac{1}{3} \delta^{ij} x^2 \right)_A \\
Q_M^{ij} &= \sum_{A=1}^{2} \left[m \, \left(\vec{x} \times \vec{v} \right)^i \, x^j \right]_A^{TF} \\
Q_E^{ijk} &= \sum_{A=1}^{2} \left(m \, x^i x^j x^k \right)_A^{TF}
\end{aligned}$$