Gravitational self-force and the cosmic censorship conjecture

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Overview

Cosmic censorship conjecture

The weak cosmic censorship conjecture

(Penrose, 1969) All curvature singularities in classical GR are hidden behind an event horizon (no “naked” singularities exist).

- No proof attained so far $\rightarrow$ consistency checks devised to
  - corroborate the conjecture
  - better understand its extent of validity

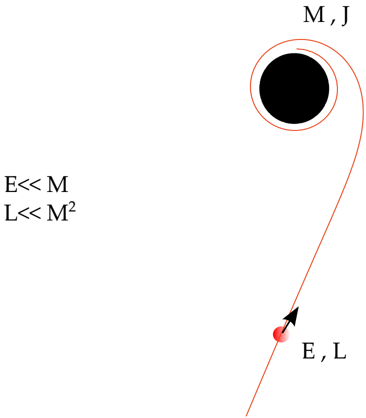
Review of previous works: extremal and charged BH

- **Kerr-Newman**
  Wald, 1972: no violation of cosmic censorship when BH is *extremal*.
  Two cases:
  1. non-spinning, equatorial (conjecture saved by electrostatic/centrifugal repulsion)
  2. spinning, dropped along the symmetry axis (conjecture saved by spin-spin interaction)
- **Reissner-Nordström**

Hubeny, 1999	overcharging possible when BH is *nearly* extremal, neglecting back-reaction
Isoyama *et al.*, 2011	Include back-reaction: no overcharging assuming captured orbits go through a quasi-equilibrium state
Zimmerman *et al.*, 2013	Full SF computation, but neglect back-reaction from grav perturb

Overspinning a Kerr BH

- Problem is cleaner in Kerr (no coupling between electromagnetic and grav perturbation);
- Focus on **equatorial** orbits, **non-spinning** bodies



The overspinning scenario

Two conditions have to be met to form a naked singularity

1. The body has to be captured (*capture condition*)
2. The final state must satisfy

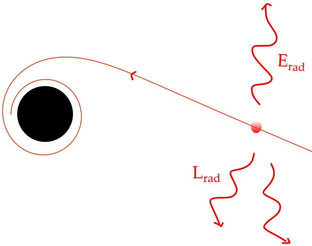
$$(M+E)^2 < aM + L \quad \textit{over-extremality condition}$$

- Back-reaction affects both 1) and 2)
- In order to work in perturbation theory need to consider nearly extremal BH

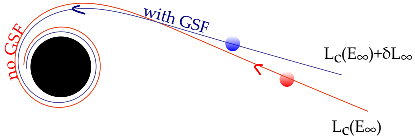
$$\frac{a}{M} = 1 - \epsilon^2, \quad \epsilon \ll 1$$

The GSF effect

1. The small body radiates energy and angular momentum



2. Shift in the parameters of the "critical" orbits (defining the separatrix between scatter and plunge)



Review of previous works: Kerr BH

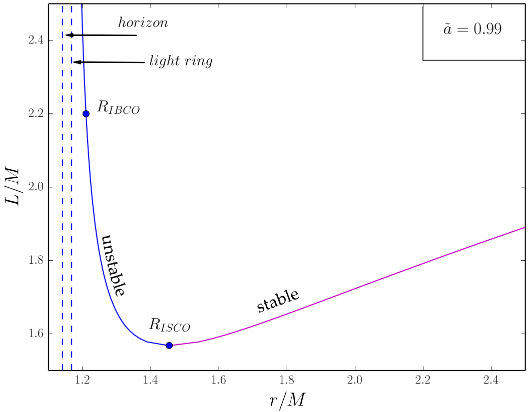
Jacobson and Sotiriou, 2009	overspinning allowed in the geodesic approx
Issues:	1) criterion includes deeply bound orbits; 2) no analytic expression for OS domain
Barausse *et al.*, 2011	Add dissipative effects, UR limit: still $\exists$ overspinning orbits
Conclusions:	Need to consider conservative effects

Work plan

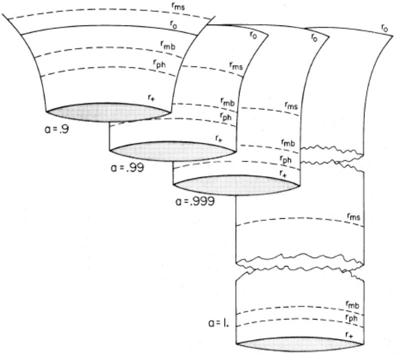
- Re-examine the geodesic case to give analytical expression of the domain of overspinning orbits $\Rightarrow$ better grasp of the problem.
- Study the effects of back-reaction, both dissipative (see Barausse *et al.*) and conservative.

Equatorial geodesics in nearly extremal Kerr

Radial motion can be described in terms of an effective potential whose stationary points correspond to stable/unstable circular orbits



Stretching of spatial geometry on $t = const$ slices



Credit: *Bardeen, J. M. et al., Astrophys.J.* **178** *(1972) 347*

When $a = 1 - \epsilon^2, \epsilon \ll 1$, all unstable circular orbits sit at

$$R = 1 + \frac{2\sqrt{2}E}{\sqrt{3E^2 - 1}}\epsilon + o(\epsilon)$$

Overspinning orbits in the geodesic approximation

Fix E. Overspinning orbits

1. must clear the peak of the effective potential (discard deeply bound orbits): $\eta L < \eta L_c(E)$
2. must satisfy the over-extremality cond: $\eta L > \epsilon^2 + 2\eta E + \eta^2 E^2$

Then

- the maximum value of the width of the range in ηL where OS is allowed reads

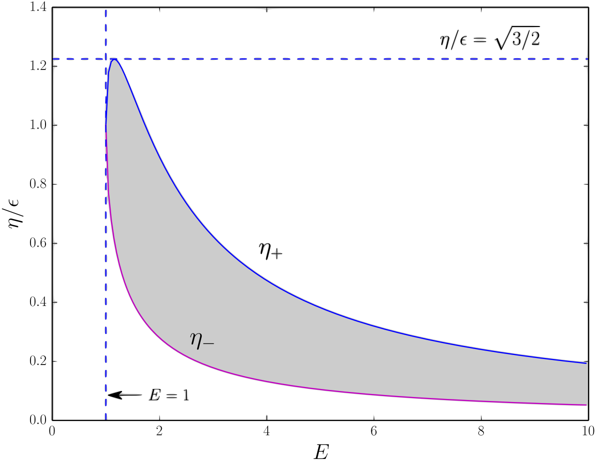
$$\max_{\eta} \eta \Delta_L = \frac{\epsilon^2 (E^2 - 1)}{2E^2}$$

  $\rightarrow$ no overspinning when $E < 1$

- $\forall E > 1$ overspinning achieved ($\Delta_L > 0$) in the range

$$\epsilon \eta_-(E) < \eta < \epsilon \eta_+(E)$$

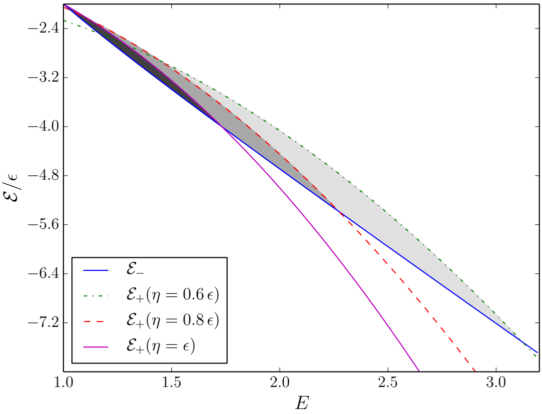
Visualisation of overspinning domain



Visualisation of overspinning domain

$$\epsilon\mathcal{E}_-(E) < \mathcal{E} < \epsilon\mathcal{E}_+(E, \frac{\eta}{\epsilon})$$

where $\mathcal{E} := E - L/2$



Equations of equatorial motion with GSF

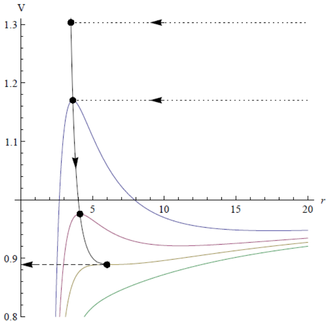
- μ undergoes accelerated motion in the background ST, with a GSF $\propto \mu^2$ acting on it
$$\mu \hat{u}^\beta \nabla_\beta \hat{u}^\alpha = F^\alpha$$
- t and ϕ components determine the evolution of $\hat{E} := -\hat{u}_t$ and $\hat{L} := \hat{u}_\phi$
$$\hat{E}(\tau) - E_\infty = -\int_{-\infty}^{\tau} \frac{F_t}{\mu} d\tau := \Delta E(\tau)$$
$$\hat{L}(\tau) - L_\infty = \int_{-\infty}^{\tau} \frac{F_\phi}{\mu} d\tau := \Delta L(\tau)$$

Capture condition with GSF: critical orbits

- Fix E_∞. There will be a critical orbit (on the separatrix between scatter and plunge) such that

$$\hat{E}_c(\tau \to -\infty; E_\infty) = E_\infty \quad \hat{L}_c(\tau \to -\infty, E_\infty) = L_c^{SF}(E_\infty)$$

- When GSF is switched on, all the critical orbits join a global attractor (infinitely finely tuned orbit evolving from the light ring to the ISCO)



Credit: *Gundlach, C. et al., Phys.Rev.D* **86** *(2012) 084022*

Capture condition with GSF: critical orbits

- Gauge-dependent SF-correction to the geodesic functional relation $L_c(E_\infty)$:

$$\delta L_c(\tau; E_\infty) := \hat{L}_c(\tau; E_\infty) - L_c(E_\infty)$$

Note: Not small during evolution along the global attractor

- Gauge invariant effect of the GSF:

$$\delta L_\infty(E_\infty) := \hat{L}_c(\tau \to -\infty, E_\infty) - L_c(E_\infty)$$

Note: $O(\eta)$ *correction*

Overspinning with the full GSF: outline of the analysis

We will proceed as follows

1. With dissipation switched off, we will study the critical orbit and get an analytical expression for the gauge-invariant quantity $\delta L_\infty(E_\infty)$
2. We will argue that the radiative evolution of the critical orbit during the whirl does not alter at the relevant order the above result
3. Radiative effects will appear in the overspinning condition as alterations of the energy and AM falling into the BH
4. The upshot of the analysis will be a GSF-corrected condition to prevent overspinning, where we include the *full* GSF

Conservative correction to $L_c(E_\infty)$

Turn off dissipation. Then for a given (fixed) E_∞, a critical orbit will approach the corresponding unstable CO whirling at a GSF-corrected radius $\hat{R}(E_\infty) = R(E_\infty) + \delta R$.

- Solve the GSF-corrected circularity conditions. One gets

$$\delta L_\infty^{cons}(E_\infty) = -\frac{1}{2\eta}\int_{-\infty}^{+\infty}(2F_t + F_\phi)d\tau := \Delta\mathcal{E}(\infty)$$

- Split $\delta L_\infty^{cons}(E_\infty)$ into "approach" $(-\infty < \tau < \tau_0)$ and "whirl" $(\tau_0 < \tau < +\infty)$ parts, with τ_0 chosen so that $r(\tau_0) - \hat{R} \sim \epsilon \ll 1$.
- Using scaling arguments, one can neglect the whirl contribution, so that

$$\delta L_\infty^{cons}(E_\infty) = 2\Delta\mathcal{E}^{cons}(\tau_0) + O(\eta\epsilon)$$

Adiabatic evolution along the attractor

- Key point is that the radiative evolution's timescale is $O(1/\eta)$ slower than the orbital revolution $\Rightarrow$ evolution along the attractor can be viewed as adiabatic progression along a sequence of CO
- At each orbit along the sequence (reached at some $\tau = \tau_w$), the circularity conditions imply

$$\delta L_\infty(E_\infty) = 2\Delta\mathcal{E}(\tau_w)$$

- One can show that, at the relevant order,

$$\rightarrow \delta L_\infty(E_\infty) = 2\Delta\mathcal{E}(\tau_0)$$

i.e. only the integral of the SF up to the time of approach τ_0 matters

Overspinning domain with the full GSF

- At infinity the particle has parameters E_∞, L_∞.
- During the evolution towards the BH the particle radiates energy and AM$\rightarrow$ when the particle crosses the horizon the overspinning condition reads:

$$\epsilon^2 + 2\eta\left(\mathcal{E}_\infty - \mathcal{E}^+_{rad}\right) + \eta^2 E^2_\infty < 0$$

- The capture condition reads

$$\eta\, L_\infty < \eta\, L_c(E_\infty) + \eta\, \delta L_\infty(E_\infty)$$

How the GSF can save cosmic censorship

- The width of the range in angular momentum where overspinning is allowed is shifted from its geodesic value Δ_L to

$$\hat{\Delta}_L = \Delta_L + 2\mathcal{E}^{+}_{rad} + \delta L_\infty(E_\infty)$$

- $\delta L_\infty(E_\infty) = -2\mathcal{E}^{+}_{rad} - 2\mathcal{E}^{-}_{rad} + \delta L^{\mathsf{cons}}_\infty(E_\infty)$
- Eventually we find that the GSF can *avert* overspinning provided that

$$-2\mathcal{E}^{-}_{rad} + \delta L^{\mathsf{cons}}_\infty(E_\infty) \leq \frac{\eta}{2}\left(1 - E^2_\infty\right)$$

- One expects $\mathcal{E}^{-}_{rad} > 0$ (1st law of BH mechanics, applied to near-extremal Kerr)

Methods to extract the GSF information needed

To decide whether the GSF acts as a cosmic censor we need some numerical input.
Two possible methods

1. Time-domain code to evaluate the integral of the GSF along the whole orbit. Currently available only for circular orbits (Dolan), needs to be extended to infalling orbits.
2. Use first law of binary black-hole mechanics (Le Tiec *et al.*) to evaluate $\delta L_{\infty}^{\mathsf{cons}}(E_{\infty})$. Only use numerical value of $h^{R}_{\mu\nu}\hat{u}^{\mu}\hat{u}^{\nu}$ along circular orbits!

Conclusions and future work

- In the scenario where a particle is absorbed by a nearly-extremal Kerr BH, we presented a necessary and sufficient condition for cosmic censorship to be safe when including the full GSF
- Radiative effects appear to enter the condition only in terms of horizon absorption (in contrast with previous works)
- We will need to numerically integrate some components of the SF along unbound orbits
- We will then in position to check that the two approaches suggested lead to consistent results