Gravity waves from plunges into rapidly rotating black holes

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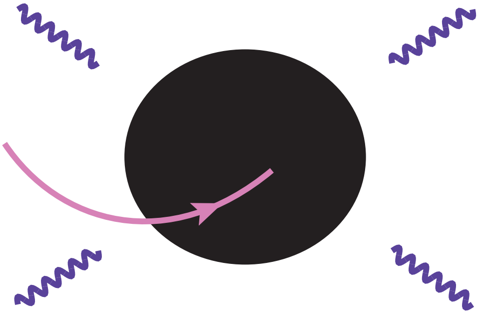
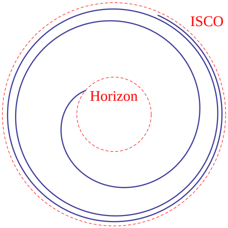
The problem



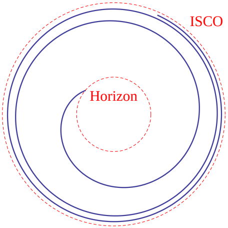
Goal: compute plunge & ringdown GW signal

Background: plunge & ringdown



- EMRI stages:
  - Adiabatic inspiral
  - Plunge & ringdown

Background: plunge & ringdown



- EMRI stages:
  - Adiabatic inspiral
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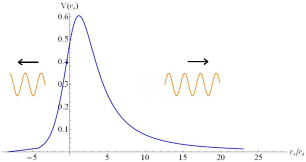
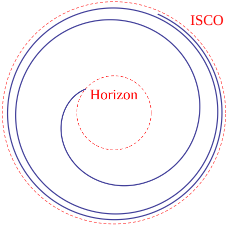


- 1D-reduction (RW/Z)

$$\left[\partial_{r_*}^2 + \omega^2 + V\right]\phi = S$$

Background: plunge & ringdown



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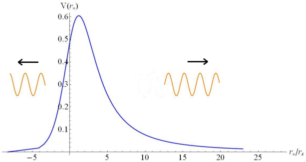
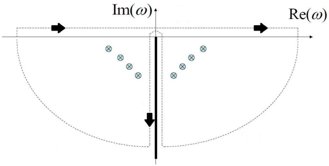




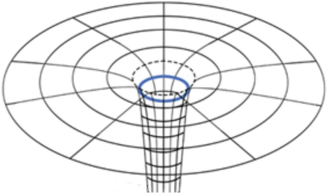
- Sum over QNMs

$$\phi = \int G\,S = \sum A_{n\ell m} e^{-i\omega_{n\ell} t}$$

Background: Kerr/CFT

- Holography & AdS/CFT
- Near horizon decoupled limit of black branes

- Near-extremal near-horizon Kerr throat (NHEK). Enhanced symmetry
- Kerr/CFT conjecture: BH dual to $1+1$ D CFT
- Checks include: scattering, entropy



Motivation

- Analytical GW signatures with new methods
- Illuminate & test Kerr/CFT

Outline

Outline

The near-horizon near-extremal limit

- Kerr BH: $\mathrm{J} \leq \mathrm{M}^2$
- $\mathrm{J} \sim \mathrm{M}^2 \;\Rightarrow$ new small parameter: $\kappa = \sqrt{1 - \left(\frac{\mathrm{J}}{\mathrm{M}^2}\right)^2}$
- Zoom in & extremize. $\hat{\mathrm{r}}, \hat{\mathrm{t}}, \hat{\phi} \Leftrightarrow$ Boyer-Lindquist coordinates

$$\mathrm{r} = \frac{\hat{\mathrm{r}} - \mathrm{r}_+}{\lambda \mathrm{r}_+} \quad ; \quad \mathrm{t} = \lambda \frac{\hat{\mathrm{t}}}{2\mathrm{M}} \quad ; \quad \phi = \hat{\phi} - \frac{\hat{\mathrm{t}}}{2\mathrm{M}}$$

$$\kappa, \lambda \rightarrow 0$$

NHEK/nearNHEK geometry

- Take $\kappa = \alpha\lambda \to 0$ $\qquad\qquad (\alpha = \text{const})$

$$\mathrm{ds}^2 \quad = \quad 2\mathrm{M}^2\Gamma\left(-\mathrm{r}(\mathrm{r}+2\alpha)\mathrm{dt}^2 + \frac{\mathrm{dr}^2}{\mathrm{r}(\mathrm{r}+2\alpha)} + \mathrm{d}\theta^2 + \Lambda^2\left(\mathrm{d}\phi + (\mathrm{r}+\alpha)\mathrm{dt}\right)^2\right)$$

$$\Gamma(\theta) \quad = \quad \frac{1+\cos^2\theta}{2} \quad ; \quad \Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta}$$

NHEK/nearNHEK geometry

- Take $\kappa = 0,\ \lambda \to 0$ (or equivalently $\alpha = 0$)

$$\mathrm{ds}^2 = 2\mathrm{M}^2\Gamma\left(-\mathrm{R}^2\,\mathrm{dT}^2 + \frac{\mathrm{dR}^2}{\mathrm{R}^2} + \mathrm{d}\theta^2 + \Lambda^2\left(\mathrm{d}\Phi + \mathrm{R}\,\mathrm{dT}\right)^2\right)$$

$$\Gamma(\theta) = \frac{1+\cos^2\theta}{2} \quad ; \quad \Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta}$$

NHEK/nearNHEK geometry

$$ds^2 \quad = \quad 2M^2\Gamma\left(-R^2\,dT^2 + \frac{dR^2}{R^2} + d\theta^2 + \Lambda^2\,(d\Phi + R\,dT)^2\right)$$

- Kerr symmetry: $\mathbb{R} \times U(1)$

$$\begin{aligned} Q_0 &= \partial_{\hat{\phi}} \\ H_0 &= \partial_{\hat{t}} \end{aligned}$$

NHEK/nearNHEK geometry

$$ds^2 = 2M^2\Gamma\left(-R^2\,dT^2 + \frac{dR^2}{R^2} + d\theta^2 + \Lambda^2\,(d\Phi + R\,dT)^2\right)$$

- Enhanced symmetry: $\mathbb{R} \times U(1) \;\Rightarrow\; SL(2,\mathbb{R}) \times U(1)$

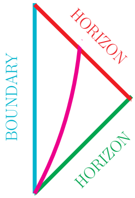
$$\begin{aligned} Q_0 &= \partial_\Phi \\ H_0 &= \partial_T \\ H_1 &= T\,\partial_T - R\,\partial_R \\ H_2 &= \frac{1}{2}\left(\frac{1}{2R^2} + T^2\right)\partial_T - TR\,\partial_R - \frac{1}{R}\partial_\Phi \end{aligned}$$

- NHEK, nearNHEK - nonsingular, $\det g \neq 0$
- Not asymptotically flat

Outline

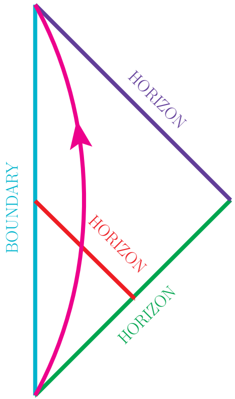
Symmetry at work

$$l = \frac{2M}{\sqrt{3}} \quad ; \quad e = 0$$



$$\begin{aligned} t(r) &= \frac{1}{2\alpha} \ln \frac{1}{r(r+2\alpha)} + t_0 \\ \phi(r) &= \frac{3r}{4\alpha} + \frac{1}{2} \ln \frac{r}{r+2\alpha} + \phi_0 \end{aligned}$$

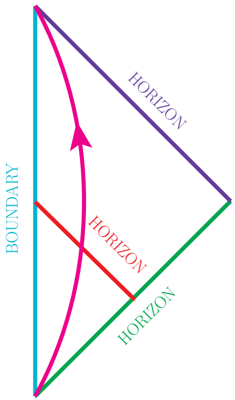
Symmetry at work



$$
\begin{aligned}
t(r) &= \frac{1}{2\alpha}\ln\frac{1}{r(r+2\alpha)} + t_0 \\
\phi(r) &= \frac{3r}{4\alpha} + \frac{1}{2}\ln\frac{r}{r+2\alpha} + \phi_0 \\
&\downarrow \\
R &= R_0 \\
\Phi(r) &= -\frac{3}{4}R_0 T + \Phi_0
\end{aligned}
$$

$$R_0 = R_0(t_0) \quad ; \quad \Phi_0 = \Phi_0(\phi_0)$$

Mapping



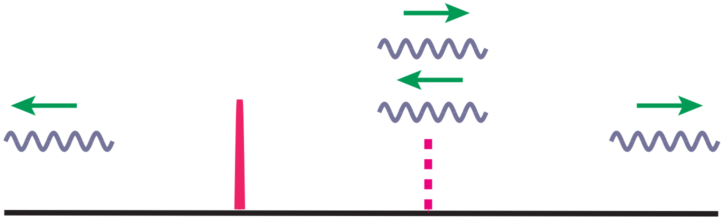
$$\begin{aligned} t(r) &= \frac{1}{2\alpha}\ln\frac{1}{r(r+2\alpha)} + t_0 \\ \phi(r) &= \frac{3r}{4\alpha} + \frac{1}{2}\ln\frac{r}{r+2\alpha} + \phi_0 \\ &\downarrow \\ R &= R_0 \\ \Phi(r) &= -\frac{3}{4}R_0 T + \Phi_0 \end{aligned}$$

$$\begin{aligned} T &= -e^{-\alpha t}\frac{r+\alpha}{\sqrt{r(r+2\alpha)}} \\ R &= \frac{1}{\alpha}e^{\alpha t}\sqrt{r(r+2\alpha)} \\ \Phi &= \phi - \frac{1}{2}\ln\frac{r}{r+2\alpha} \end{aligned}$$

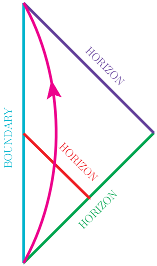
Outline

Solution

- Solution for circular orbit:



- Via homogeneous solutions - Whittaker functions
- Outgoing BC
- Inverse transformation - solve plunge
- Then match to far, asymptotically flat region



Results

- For simplicity discuss results for scalar field (did also GR)

$$S = \int \sqrt{-g}\left[(\partial\psi)^2 + \rho\,\psi\right] d^4x$$

$$\rho = q\int d\tau\,\sqrt{-g}\,\delta^4(x - x_p(\tau))$$

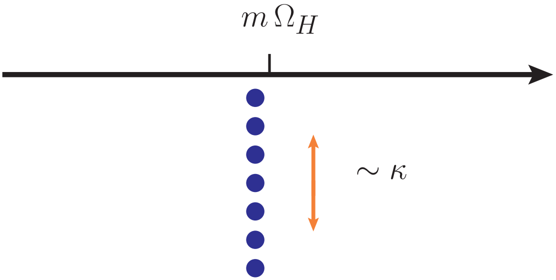
- Radiation at infinity

$$\psi = \sum \hat{R}_{\ell m\hat{\omega}}\, S_\ell(\theta)\, e^{-i(\hat{\omega}\hat{t} - m\hat{\phi})}$$

- Full frequency domain solution for $\hat{R}_{\ell m\hat{\omega}}$

Quasinormal mode decomposition

- $\hat{\mathrm{R}}_{\ell m \hat{\omega}} = \sum \mathrm{A}_{\ell m \mathrm{N}}\, \mathrm{e}^{-\mathrm{i}\hat{\omega}_{\ell m \mathrm{N}} \hat{\mathrm{t}}} \qquad \hat{\omega}_{\ell m \mathrm{N}} = \frac{1}{2\mathrm{M}} (\mathrm{m} - \mathrm{i}\kappa(\mathrm{N} + \mathrm{h}))$



Quasinormal mode decomposition

- $\hat{R}_{\ell m \hat{\omega}} = \sum A_{\ell m N}\, e^{-i\hat{\omega}_{\ell m N}\hat{t}} \qquad \hat{\omega}_{\ell m N} = \frac{1}{2M}\left(m - i\kappa(N+h)\right)$

- QNMs can be summed analytically to give

$$\Psi^{\rm far}(r\to\infty) = \sum_{\ell,m} \frac{3^{h-\frac{1}{2}} q}{2^h M} (-1)^{-h} e^{3im/4} S_\ell(\pi/2) W_{im,h-\frac{1}{2}}(3im/2)$$
$$\times\, (im)^{2h-2+im} e^{\pi m} \frac{(1-2h)\Gamma(h-im)^2}{\Gamma(2h)^2} e^{im(\hat{\phi}-\hat{\phi}_0)} S_\ell(\theta)\, r^{-1+im} e^{imr/2}$$
$$\times \exp\left[-i\frac{m-i\kappa h}{2M}(\hat{t}-\hat{t}_0) - \frac{3im}{4\kappa} e^{-\frac{\kappa}{2M}(\hat{t}-\hat{t}_0)}\right]$$

$$h := \frac{1}{2} + \sqrt{\frac{1}{4} + K_\ell - 2m^2}$$

Quasinormal mode decomposition

$$\Psi_{\ell m}^{\text{far}}(r \to \infty) \propto \exp\left[-i\frac{m - i\kappa h}{2M}(\hat{t} - \hat{t}_0) - \frac{3im}{4\kappa}e^{-\frac{\kappa}{2M}(\hat{t}-\hat{t}_0)}\right]$$

$$h := \frac{1}{2} + \sqrt{\frac{1}{4} + K_\ell - 2m^2}$$

Outline

The dual CFT

- GW source is dual to CFT source

$$S = S_{CFT} + \int \mathcal{O}\, J$$

- $J_{\ell m} \propto \int M_{(\ell m)}\, \rho_{(\ell m)}\, dr \sim$ multipole moments
- Radiation flux down the horizon is dual to transition rate in the CFT, given by Fermi's golden rule

$$\mathcal{F} = \int dT\, d\Phi\, |J|^2\, G_{CFT}$$

- 2-point function in CFT fixed solely by symmetry, up to a multipicative constant
- $J^{circular}$, source for circular case, known

The dual CFT

- The diffeo reduces to conformal transformation on boundary

$$\begin{aligned} \Phi &= \phi \\ \mathrm{T} &= -\mathrm{e}^{-\kappa \mathrm{t}} \end{aligned}$$

- Transform source J according to conformal weight:

$$\mathrm{J}^{\mathrm{plunge}} = \left(\kappa\, \mathrm{e}^{-\kappa \mathrm{t}}\right)^{1-\mathrm{h}} \mathrm{J}^{\mathrm{circular}}$$

- Bulk & boundary agree

Summary

- Analytical GW signatures from enhanced symmetry
- Incorporation of sources into Kerr/CFT
- To do (until) next (Capra): generalize orbit

Appendices

Kerr & zoom

- Kerr geometry

$$
\begin{aligned}
ds^2 &= -\frac{\Delta}{\hat\rho^2}\left(d\hat t - a\sin^2\theta d\phi\right)^2 + \frac{\sin^2\theta}{\hat\rho^2}\left((\hat r^2 + a^2)d\hat\phi - a\,d\hat t\right)^2 \\
&+ \frac{\hat\rho^2}{\Delta}d\hat r^2 + \hat\rho^2 d\theta^2
\end{aligned}
$$

$$\hat\rho := \hat r^2 + a^2\cos^2\theta \quad ; \quad \Delta := \hat r^2 - 2M\hat r + a^2 = (\hat r - r_+)(\hat r - r_-)$$

- $J \le M^2$. New small parameter: $\kappa = \sqrt{1 - \left(\frac{a}{M}\right)^2}$
- Zoom in & extremize (Bardeen & Horowitz '98)

$$r = \frac{\hat r - r_+}{\lambda r_+} \quad ; \quad t = \lambda \frac{\hat t}{2M} \quad ; \quad \phi = \hat\phi - \frac{\hat t}{2M} \quad ; \quad \kappa, \lambda \to 0$$

Results

- Radiation (frequency domain) at infinity

$$\psi = \sum \hat{R}_{\ell mn}\, S_\ell(\theta)\, e^{-i(\hat{\omega}\hat{t} - m\hat{\phi})}$$

$$\hat{R}_{\ell mn} = 4q\,(-)^{-h}\,(2\kappa)^{h-1} \left(\frac{3R_0}{4}\right)^{i(n-m)} \left(\frac{\hat{r}}{2M}\right)^{-1+im} e^{im\frac{\hat{r}}{2M}}$$

$$\times \frac{S_\ell(\frac{\pi}{2}) W_{im,h-\frac{1}{2}}\left(\frac{3im}{2}\right)(im)^{h-2+im} e^{\pi m}(1-2h)\frac{\Gamma^2(h-im)}{\Gamma^2(2h)}}{\frac{1}{\Gamma(h-i(n-m))} - (-2im\kappa)^{2h-1}\frac{\Gamma^2(1-2h)}{\Gamma^2(1-2h)}\frac{\Gamma(h-im)}{\Gamma(1-h-im)}\frac{1}{\Gamma(1-h-i(n-m))}}$$

$$h := \frac{1}{2} + \sqrt{\frac{1}{4} + K_\ell - 2m^2} \quad ; \quad n := 2M\frac{\hat{\omega} - m\Omega_H}{\kappa}$$

CFT propagator

$$G = \# (-)^{h_L + h_R} \left(\frac{\pi T_L}{\sinh(\pi T_L t^+)} \right)^{2h_L} \left(\frac{\pi T_R}{\sinh(\pi T_R t^-)} \right)^{2h_R} e^{i q_R \Omega_R t^-}$$