Scalar Green function in Kerr space-time: Branch cut contribution

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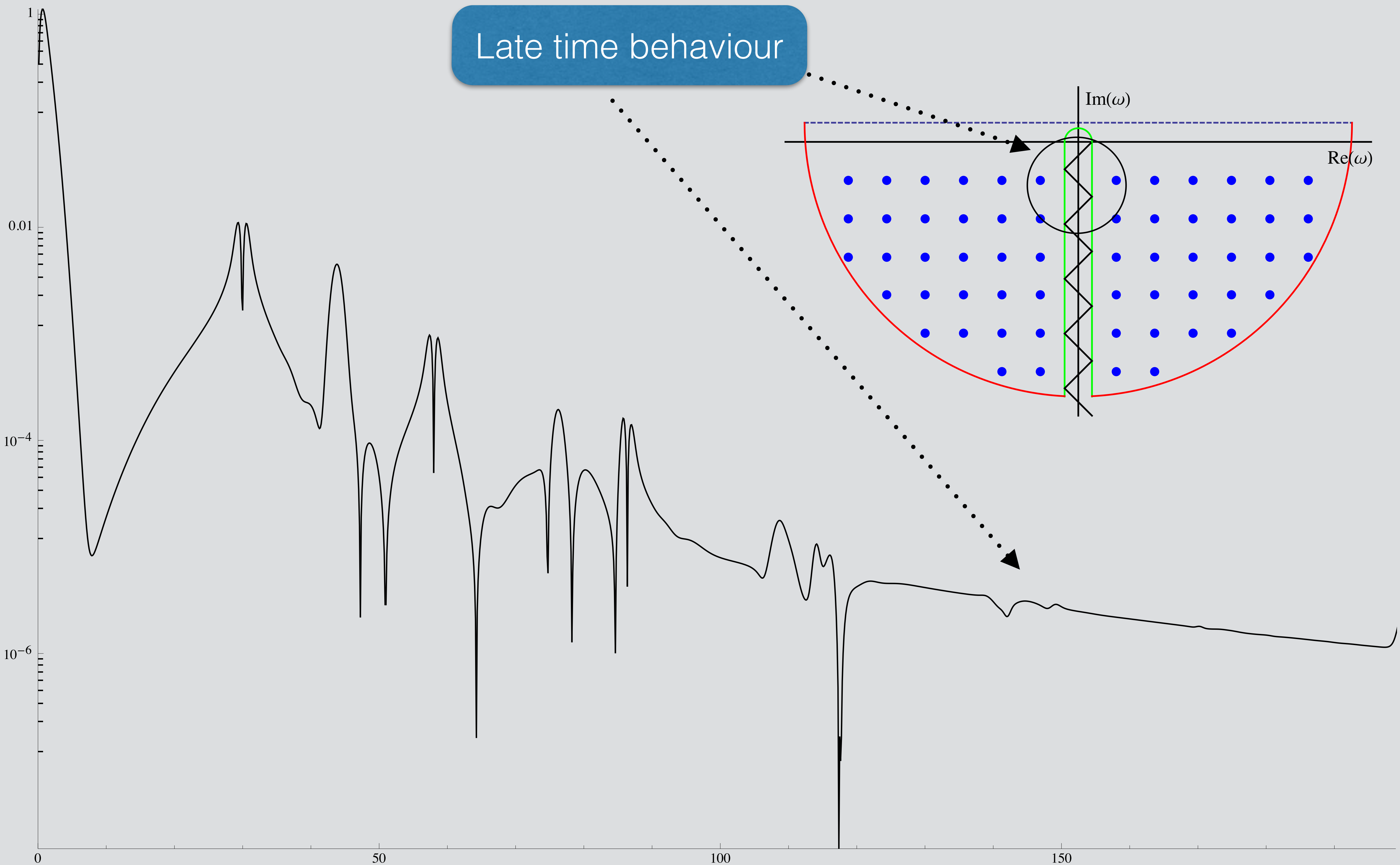
Leaver picture and Green fn pic

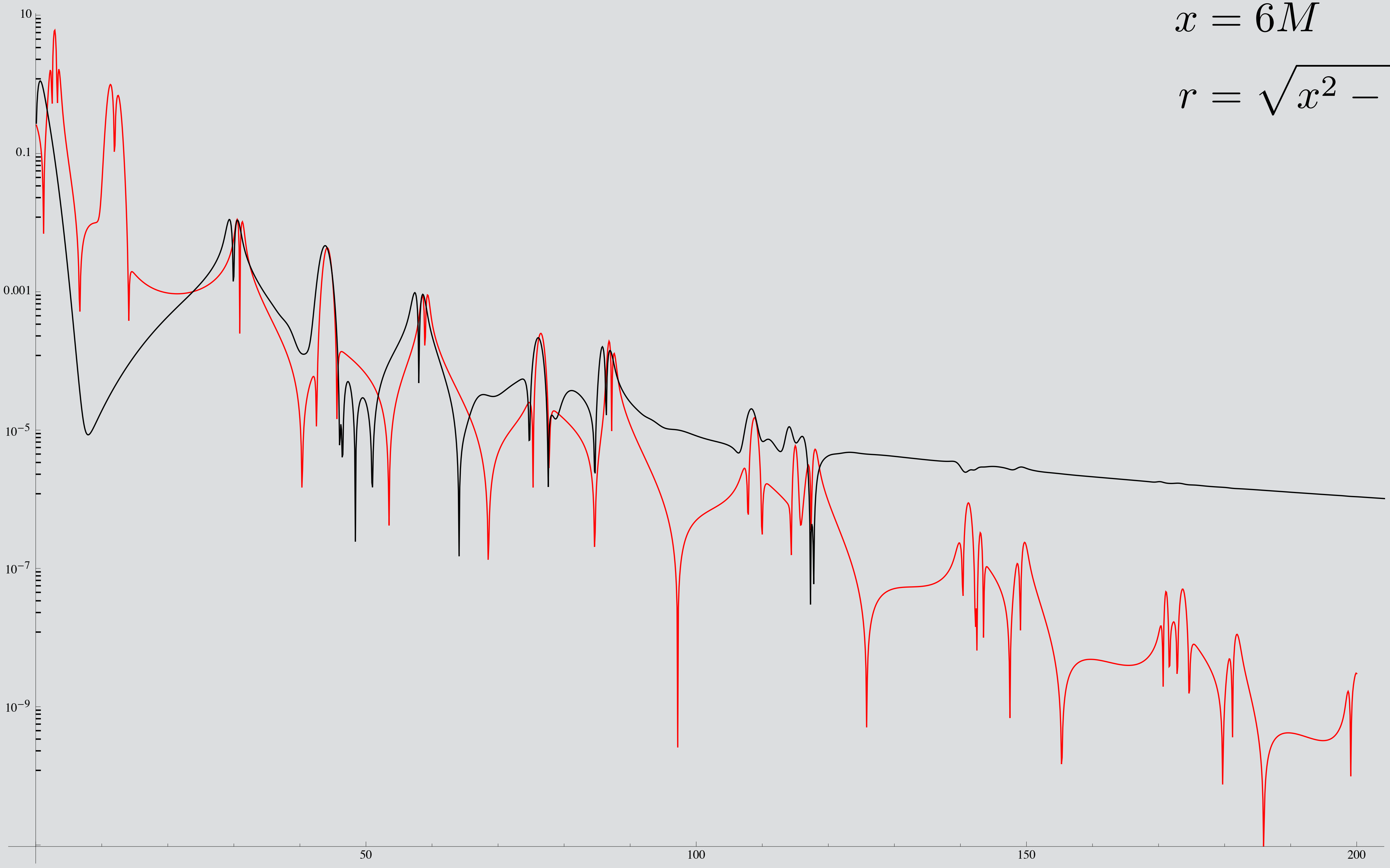


QNM sum with numerics

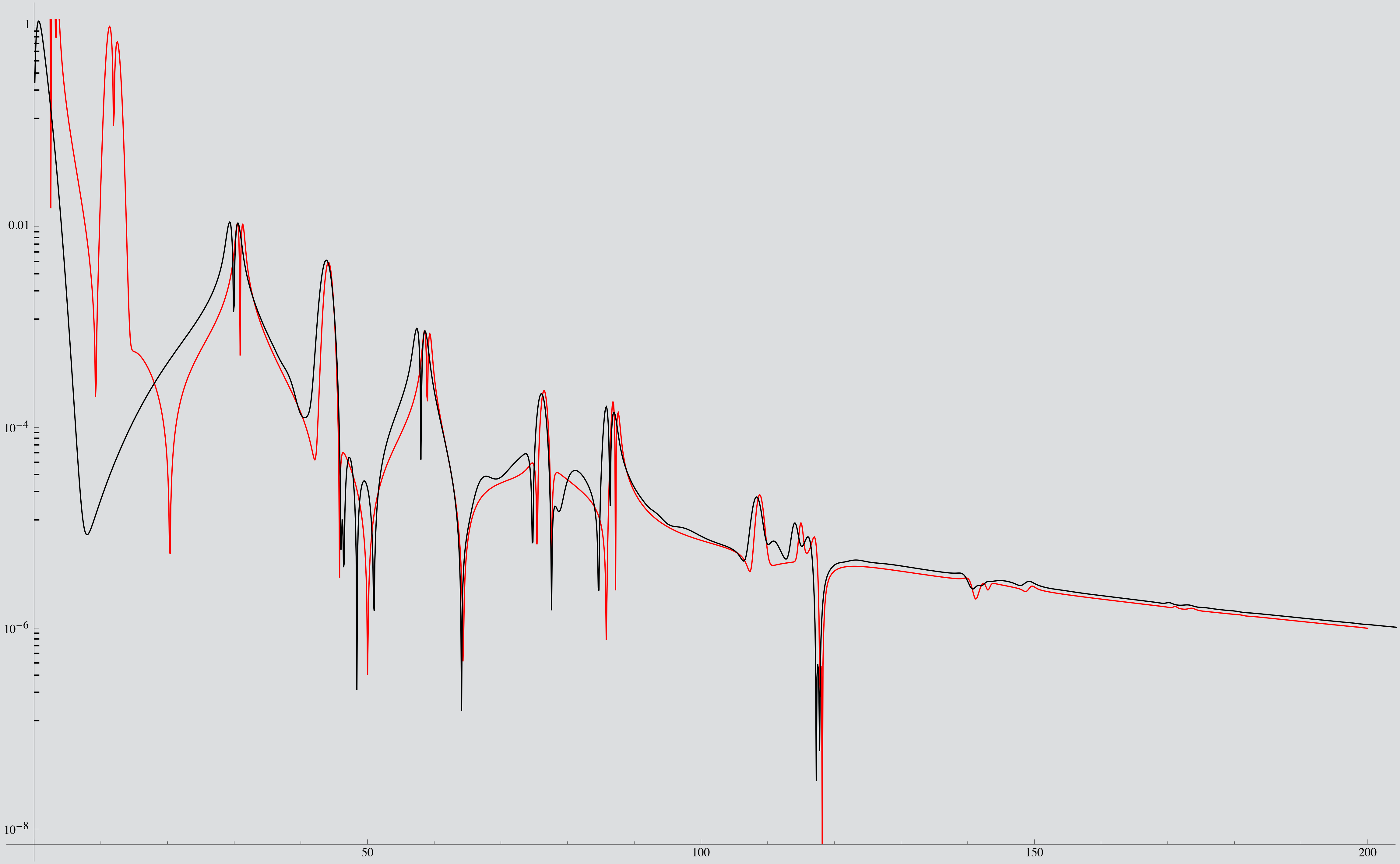
$$a = 0.6$$

$$x = 6M$$

$$r = \sqrt{x^2 - a^2}$$

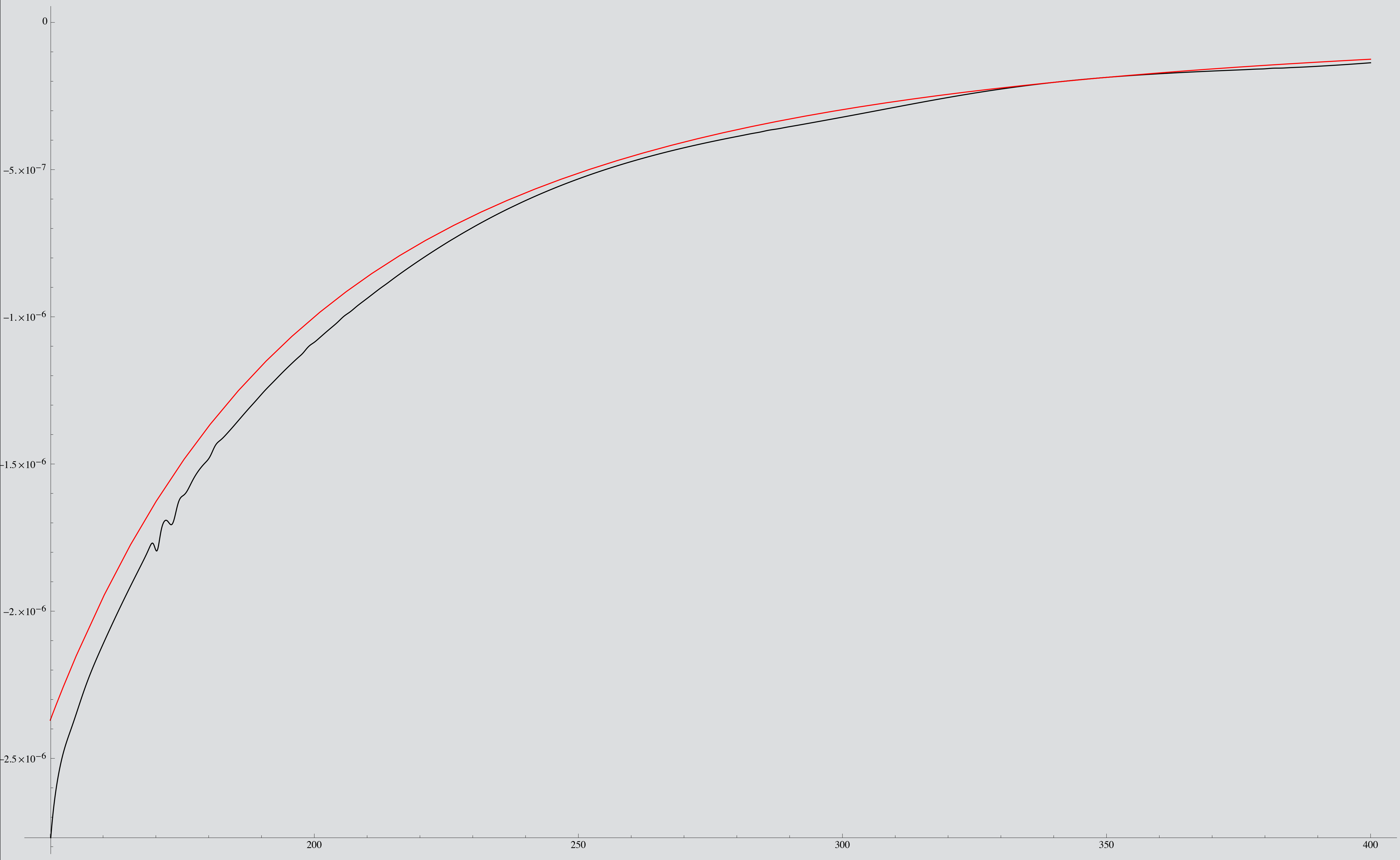


QNM+leading order branch cut



We need higher order terms-systematic way of calculating them

Leading order branch cut



We need higher order terms-systematic way of calculating them

MST Green function

MST- Solutions as a series of hypergeometric functions and irregular confluent hypergeometric functions,

R_{lm}^{up} ⋯⋯⋯▶ Confluent hypergeometric, outer solution

R_{lm}^{in} ⋯⋯⋯▶ Hypergeometric, inner solution

From these, we can construct the Green function modes!

$$G_{lm} = -\frac{\hat{R}^{in}(r_<, \omega)\hat{R}^{up}(r_>, \omega)}{W(\omega)}$$

Branch cut

Leaver: The branch cut comes entirely from the outer solution. This can be traced to the regular confluent hypergeometric function, $\Psi(a,b,z)$

$$\Psi(a,b;ze^{2\pi i}) = e^{-2\pi ib}\Psi(a,b,z) - \tilde{q}(a,b)\Phi(a,b;z)$$

where $z = \omega(r - r_-)$.

This will propagate into our Green function, giving a discontinuity along the negative imaginary axis

$$\begin{aligned}\Delta G_{lm} &= G_{lm}(i\omega e^{2\pi i}) - G_{lm}(i\omega) \\ &= -2\mu\frac{q(\mu)}{|W|^2}\hat{R}^{in}(r_<, -i\mu)\hat{R}^{in}(r_>, -i\mu)\end{aligned}$$

where $q(\mu)$ is the “branch cut strength” of $\hat{R}^{up}_{lm}$, now a complex function.

$\mu = -i\omega$

Late time tail

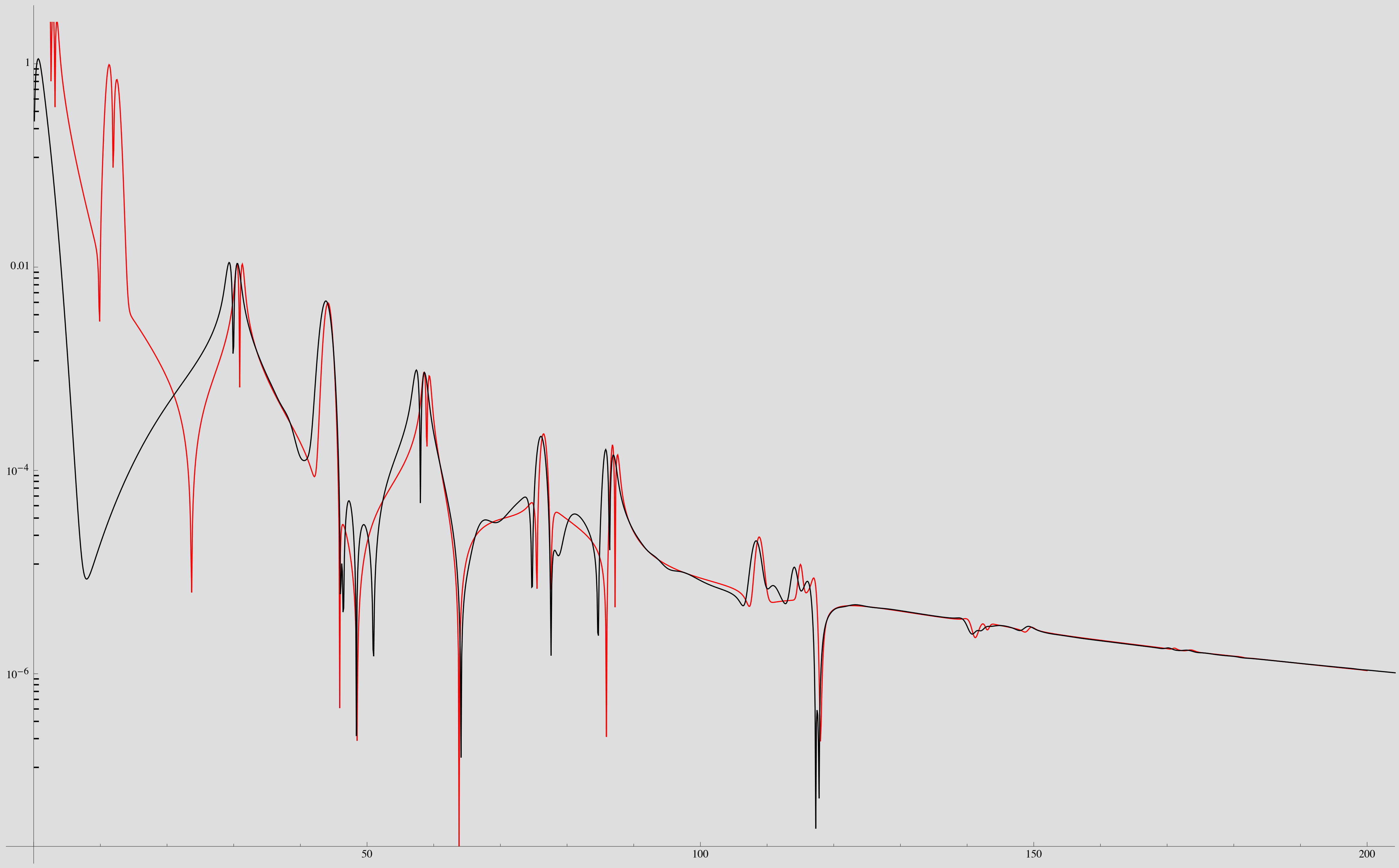
- We now have everything in terms of MST quantities, all of which are readily expandable for small-ω.
- Calculating the higher order branch cut terms is now a computational problem.

$$l = 0 : \qquad \frac{q}{|W|^2} = 2\pi\bar{\mu} + \frac{1}{3}\pi\left(6\sqrt{1-a^2} + 6\log\left(1-a^2\right) + 17\right)\bar{\mu}^2 + O\left(\bar{\mu}^3\right)$$

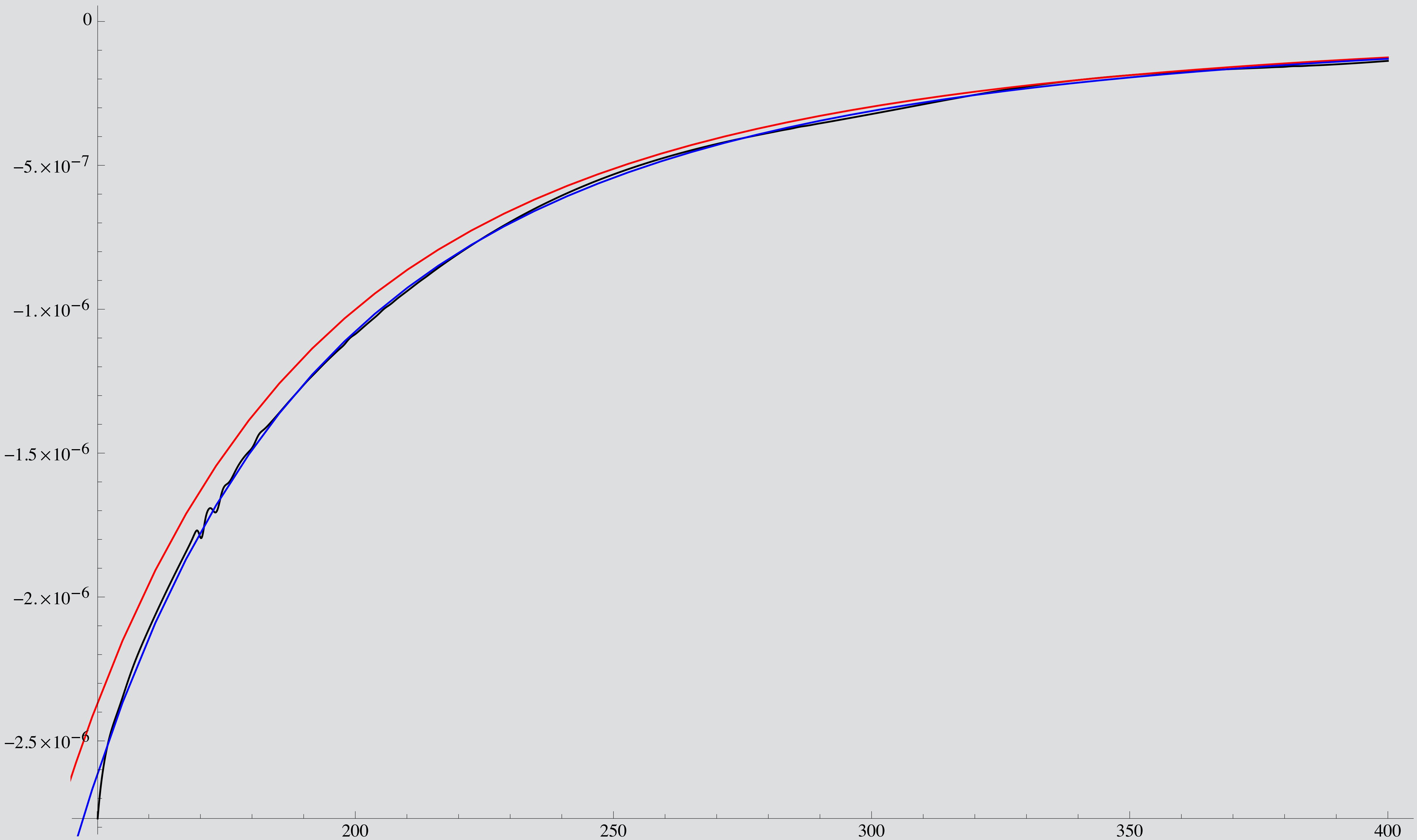
We can now update our tail:

$$\Delta G = -\frac{8.}{t^3} - \frac{123.07419418036814}{t^4} - \frac{1408.\log\left(\frac{1}{t}\right) + 7252.8218387085235}{t^5}$$

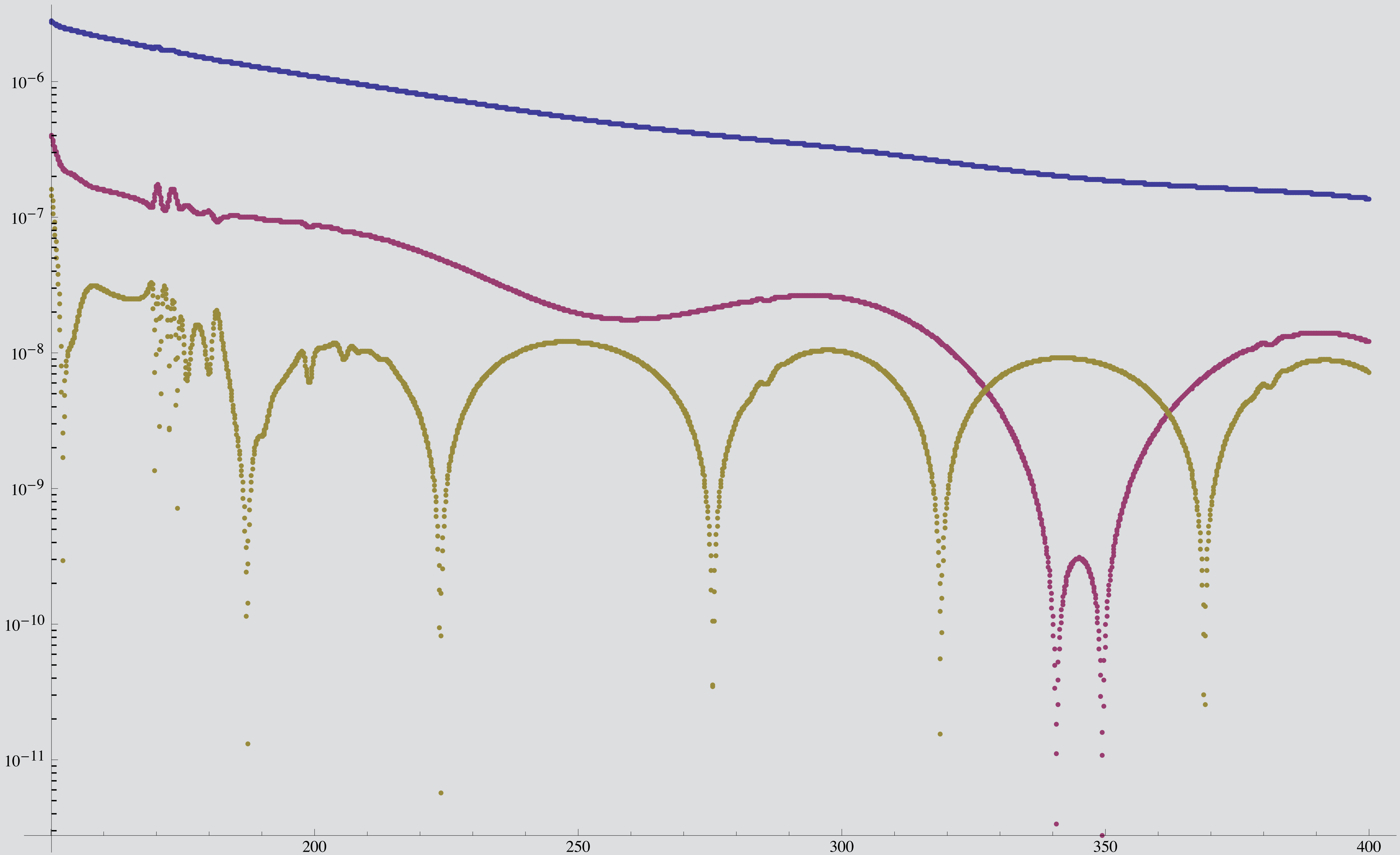
Numerical comparison



Numerical comparison

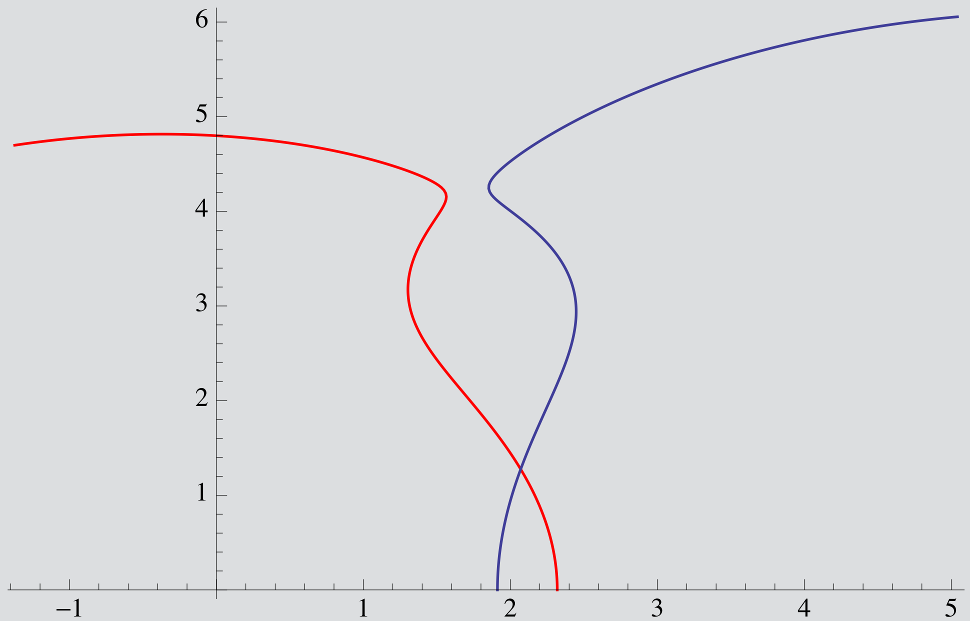


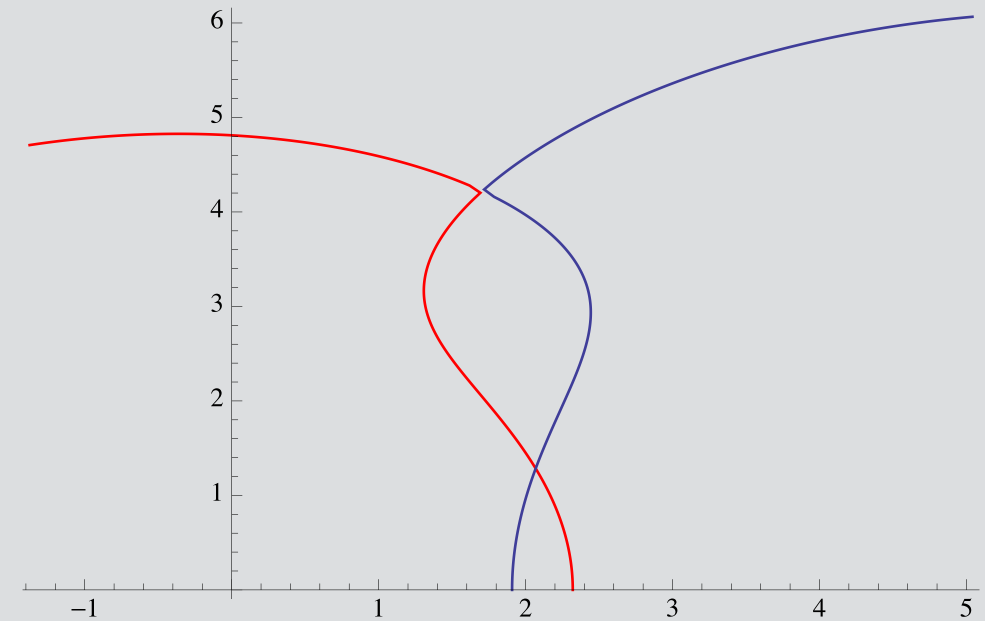
Numerical Comparison



Angular branch cuts?

Across the cuts we have switching of eigenvalues





$$l = 0$$

$$l = 2$$

Is this a labelling issue?