Higher-Order Self-Forces For Use in Two-Timescale Analyses

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Outline

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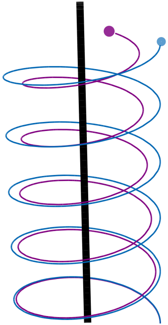
Gravitational Self-Force in Decaying Systems

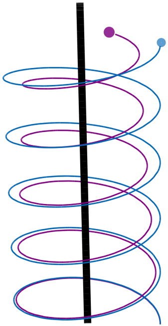
Self Forces: First Order — Second Order

Waveforms: Adiabatic — Post-1/2 (Resonances) — Post-1 Adiabatic



Two-Timescale Yields Good Approximation at Late Times

- Near-time expansions accurate for short spans of time - becomes inaccurate after a de-phasing time
- Two-Timescale expansion captures long-time secular behavior
  - parametrizes decaying behavior and slowly evolving frequency
- Eanna Flanagan and Tanja Hinderer - Two-timescale Worldline, application to Kerr
- Future evolution of this work will attempt to apply this construction to Einstein equations



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An Illustrative example

- Take the differential equation for a weakly nonlinear oscillator

$$\frac{d^2y}{dt^2} + 2\epsilon\frac{dy}{dt} + y = 0$$

- This possesses an exact solution to check against:

$$y = \frac{\exp(-\epsilon t)}{\sqrt{1-\epsilon^2}} \sin(\sqrt{1-\epsilon^2}t)$$

- Unique valid expansion given assumptions of periodicity and slow-scale behavior

$$y = e^{-\epsilon t}\sin(t) + e^{-\epsilon t}\sin\left(1 - \frac{\epsilon^2}{2}\right)t + (...)$$

Assumptions Required for Two-Timescale

- $y(t) = \mathcal{Y}(\varphi(t, \epsilon), \epsilon t)$
- Smooth in both variables, 2π periodic in φ
- φ obeys:

$$\frac{d\varphi}{dt} = \omega(\epsilon, \epsilon t)$$

  similarly smooth
- two-variable function determined

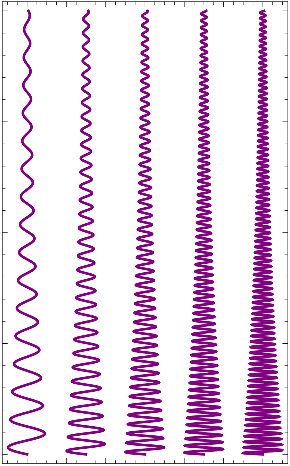

Figure: ϵ limit at constant $\tilde{t}$

Two-Timescale Demands Higher Orders

- At each order, get an equation for a driven oscillator ...

$$\left[\frac{\partial^2}{\partial\varphi^2}+1\right]\mathcal{Y}^{(0)} = 0$$
$$\left[\frac{\partial^2}{\partial\varphi^2}+1\right]\mathcal{Y}^{(1)} = 2\left(\frac{\partial A_0}{\partial\tilde{t}}+A_0\right)\sin\varphi - 2\left(\frac{\partial B_0}{\partial\tilde{t}}+B_0\right)\cos\varphi$$

- ... and an equation from our constraint on secular evolution of next lower order:

$$\frac{\partial A_0}{\partial\tilde{t}}+A_0 = 0 \tag{1}$$
$$\frac{\partial B_0}{\partial\tilde{t}}+B_0 = 0 \tag{2}$$

- This gives the evolution we expect from the lowest order
- An extra order is needed to fix secular behavior

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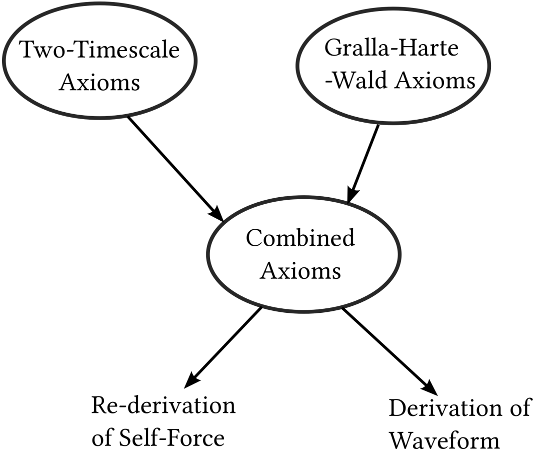
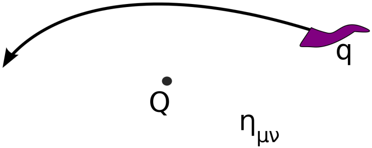
Approach to Self-Consistent Computation of Inspiral for Scalar Charged Body





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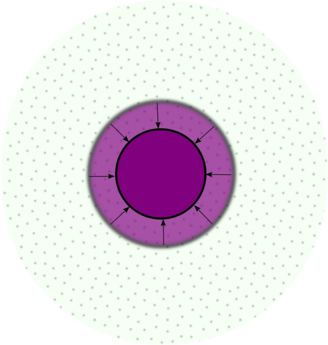
Gralla-Harte-Wald Derivation Method

- Scale parameter : ratio of body size to lengthscale of external field
- This gives two limits
  - $\epsilon \to 0$ at fixed "far zone" coordinates, body shrinks
  - $\epsilon \to 0$ at fixed "near zone" coordinates, background stretches, body remains constant
- Closely related to matched asymptotic expansions (previous work by Poisson; Detweiler; Mino et al; Quinn)



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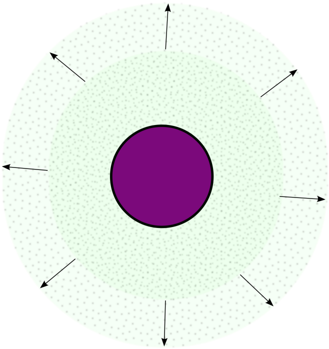

Precise Statement of Assumptions - Common Axioms

- $\exists$ A one-parameter family of fields, $F_{\mu\nu}(\lambda, x^\mu), J^\mu(\lambda, x^\mu)$, and $T^M_{\mu\nu}(\lambda, x^\mu)$, which satisfy the charge conservation and stress-energy conservation equations
- all $F_{\mu\nu}$, J^μ and $T^M_{\mu\nu}$ smooth in λ away from $\lambda = 0$; The latter two have compact support near the worldline
- $F_{\mu\nu}(\lambda) - F^{ret}_{\mu\nu}[J^\mu(\lambda)]$ smooth in λ including $\lambda = 0$

Assumptions - Differing Axioms

GHW Axioms:

- $\exists$ A function $z^i(\lambda, t)$, smooth and timelike
- $\exists$ functions $\tilde{J}^\mu(\lambda, t, X^i)$ and $\tilde{T}_M^{\mu\nu}(\lambda, t, X^i)$ st

$$J^\mu(\lambda, t, x^i) = \lambda^{-2} \tilde{J}^\mu\left(\lambda, t, \frac{x^i - z^i(\lambda, t)}{\lambda}\right)$$
$$T_M^{\mu\nu}(\lambda, t, x^i) = \lambda^{-2} \tilde{T}_M^{\mu\nu}\left(\lambda, t, \frac{x^i - z^i(\lambda, t)}{\lambda}\right)$$

and are jointly smooth in their arguments.

Assumptions - Differing Axioms

Our Modifications:

- $\exists$ A function $z^i(\lambda, \tilde{t}, \varphi)$, jointly smooth; φ 2π periodic
- $\exists$ functions:

$$\tilde{t}(t) = \tilde{t}_0 + \lambda t$$
$$\varphi(t) = \varphi_0 + \frac{1}{\lambda}\int_{\tilde{t}_0}^{\tilde{t}} d\tilde{t}' \omega(\lambda, \tilde{t}')$$

s.t. $z^i(\lambda, \tilde{t}(t), \varphi(t))$ is a timelike worldline

- $\exists$ functions $\tilde{J}^\mu(\lambda, \tilde{t}, \varphi, X^i)$ and $\tilde{T}^{\mu\nu}_M(\lambda, \tilde{t}, \varphi, X^i)$ st

$$J^\mu(\lambda, t, x^i) = \lambda^{-2}\tilde{J}^\mu\left(\lambda, \tilde{t}(t), \varphi(t), \frac{x^i - z^i(\lambda, \tilde{t}(t), \varphi(t))}{\lambda}\right)$$
$$T^{\mu\nu}_M(\lambda, t, x^i) = \lambda^{-2}\tilde{T}^{\mu\nu}_M\left(\lambda, \tilde{t}(t), \varphi(t), \frac{x^i - z^i(\lambda, \tilde{t}(t), \varphi(t))}{\lambda}\right)$$

Computation of Scalar Self-Force via Extension of Gralla, Harte, and Wald

- Take near-zone expansion in body-following coordinates
- Expand conservation of stress-energy order-by-order:

$$\nabla_\alpha T^{\alpha\beta}_{(M+SF)} = \rho \Phi^{(ext),\beta}$$

- Results in terms of body parameters defined by by integrating over spacelike 3-surfaces
- Gives expressions for a^α, $\partial_\tau M$
  - e.g. first order general expression:

$$\begin{aligned} M a^{(1)\alpha} = -M^{(1)} a^\alpha + \dot{a}^\beta S^\alpha{}_\beta + (Q\Phi^{(ext)})^{(1)} + \\ a_\beta \Phi^{(ext),\beta} D^\alpha + \frac{1}{3} Q^2 \dot{a}^\alpha + a_\beta \dot{S}^{\alpha\beta} \end{aligned}$$

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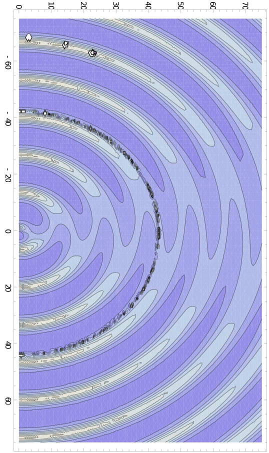
Shortcomings with Mass Definition Apparent at Higher Order

- Mass integral:

$$M = \int_\Sigma (T^{\alpha\beta}_{(m)} + T^{\alpha\beta}_{(S\,S)})\xi_\alpha d^3\Sigma_\beta$$

  - perpetually oscillating charge gives divergent energy

- One approach: adjust splitting into self + external (Harte)



Long Range Contributions to Self-Energy

- Illustrative Exact solution for Self-Energy in constant-acceleration case (away from Rindler Horizon):

$$T^{00}_{(SF)}\Big|_{t=0} = \frac{Q^2}{r^4\pi}\frac{(1+a_0^2r^2+2a\cdot r)}{(4+a_0^2r^2+4a\cdot r)^2}$$

- Gives a distinctly finite mass, when integrated: $\int_0^{\epsilon R} d\Sigma_\alpha T^{\alpha\beta}\xi_\beta$
- This expression indicates that there are non-negligible contributions at $r \propto \mathcal{O}(1/a_0)$ at a certain order (2nd)
- The effect is not a 'true' divergence, but a result of the expansion
- Confounds the desired GHW separation of scale

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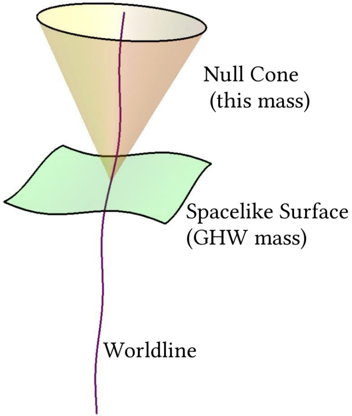
Current Work: Mass Defined by Integral over Future Null Cone (Harte)



- Non-conserved mass - dependence on hypersurface choice
- For flat space, the field on null cone depends only on worldline point
- This mass definition solves superficial problem; fits well with GHW
- Computations far simpler (Poisson has pointed this out in the past)

Summary

- Gralla, Harte, Wald method offers a particularly solid framework on which to build more elaborate approximations
- A combination of GHW axioms with two-timescale axioms should allow rigorous self-consistent computations of inspirals
- Derivation of higher-order self-force requires a modification of the definition of the body parameters, which is under exploration
- Outlook
  - Near future: use of this in Two-Timescale toy model
  - Less Near future: application to EMRI's