Gauge Invariant Quantities for the Self Force

Circular Orbits in Schwarzschild Spacetime

arXiv:1406.4890

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Overview

Solve Einstein's Field Equations:

- Write down field equations, make suitable choice of gauge (Regge-Wheeler Gauge)
- Perform tensor harmonic decomposition to simplify equations
- Approximate boundary conditions by suitable series expansions, numerically integrate
- Include source terms to find suitable inhomogenous fields

Construct Gauge Invariant Quantities:

- Gauge invariant quantities constructed directly from perturbed metric and its derivatives
- Plot Quantities over range of r; Extract Post-Newtonian parameters for large-r data, light ring divergences from small r data

Field Equations

- Model particle's self interaction as a perturbation to Schwarzschild metric
- Describe perturbed metric via Einstein Field $\Box h^{ret}_{\mu\nu} - R_{\mu\nu}{}^{\alpha\beta} h^{ret}_{\alpha\beta} = 4\pi J_{\mu\nu}$
- decompose fields into tensor spherical harmonics, use Regge Wheeler Gauge, gives just two independent scalar fields

$$\partial_{r_\star}\phi^{e/o} + (\omega_m^2 - V^{e/o}(l,r))\phi^{e/o} = 4\pi S_{\mu\nu}$$

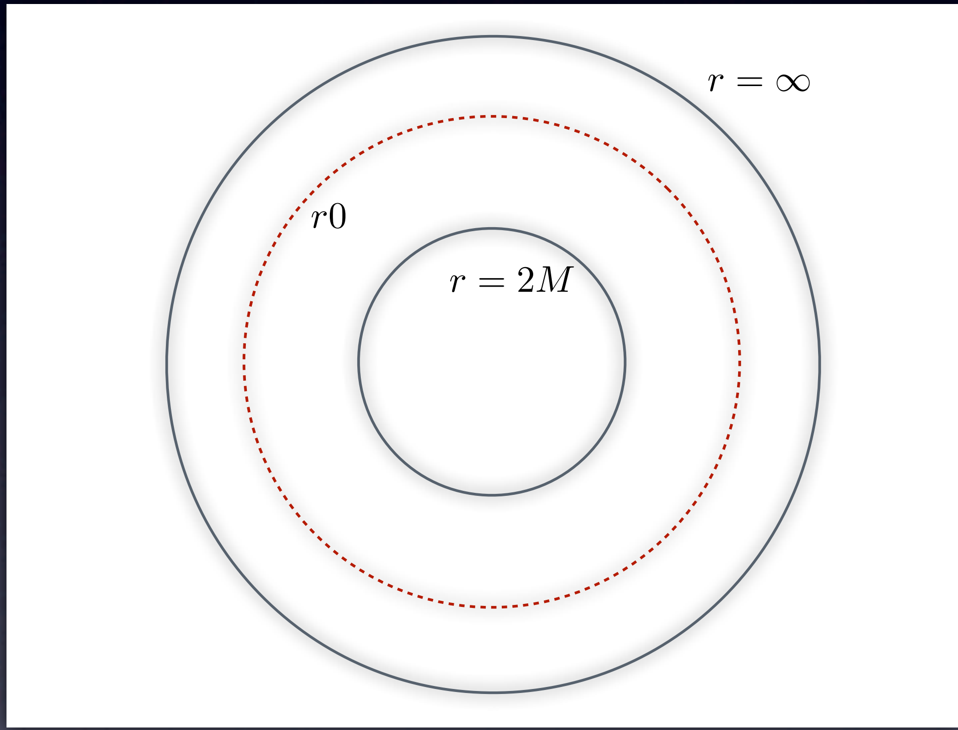
- Construct all of $h_{\mu\nu} = h_{\mu\nu}(\phi^e, \phi^o)$ from these

Solving for the Fields

- We use a series expansion to approximate the fields at the boundaries:

INNER: $$e^{-i\omega r_*}\sum_{n=0}^{n_H} b_n^i (r-2M)^n$$

OUTER: $$e^{i\omega r_*}\sum_{n=0}^{n_\infty} \frac{a_n^i}{r^n}$$

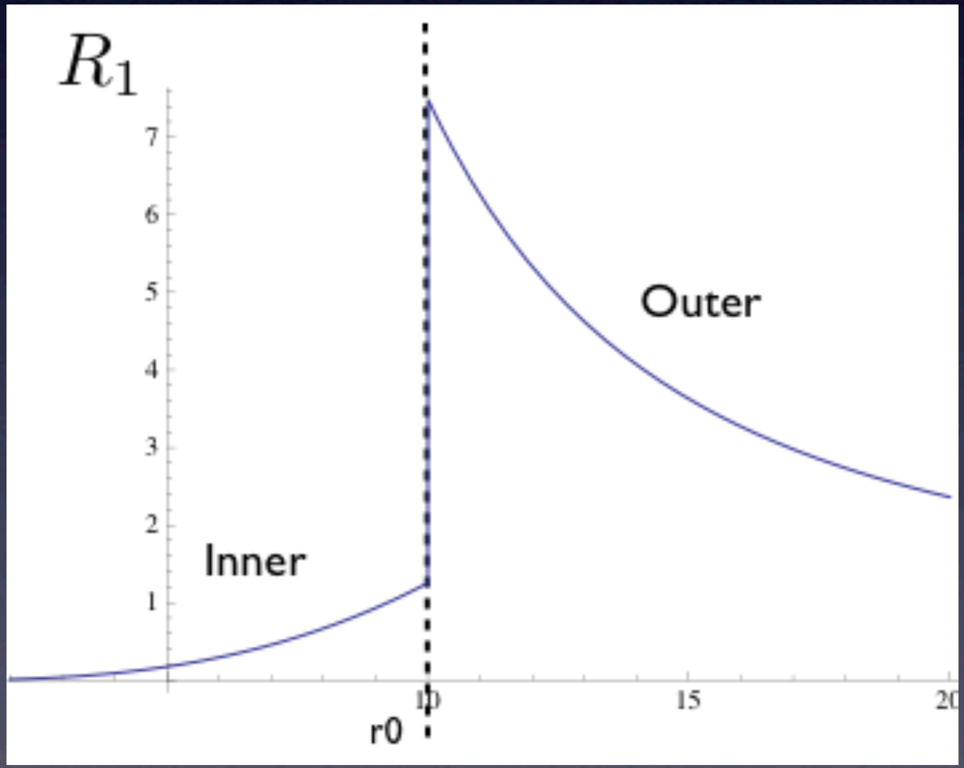


- Using these boundary conditions, numerically integrate towards the particle's orbit

Inhomogeneous Solutions

- Match solutions at the particle's orbit by imposing jump conditions, given by new source terms

$$J_{\mu\nu} = F_{\mu\nu}(\vec{x})\delta(\vec{x}-\vec{x_0}) + G_{\mu\nu}(\vec{x})\delta'(\vec{x}-\vec{x_0})$$



- In RW, source terms are proportional to derivatives of delta functions
- Integrating over these produces a jump in derivatives across orbit

Constructing Fields, Gauge Invariants

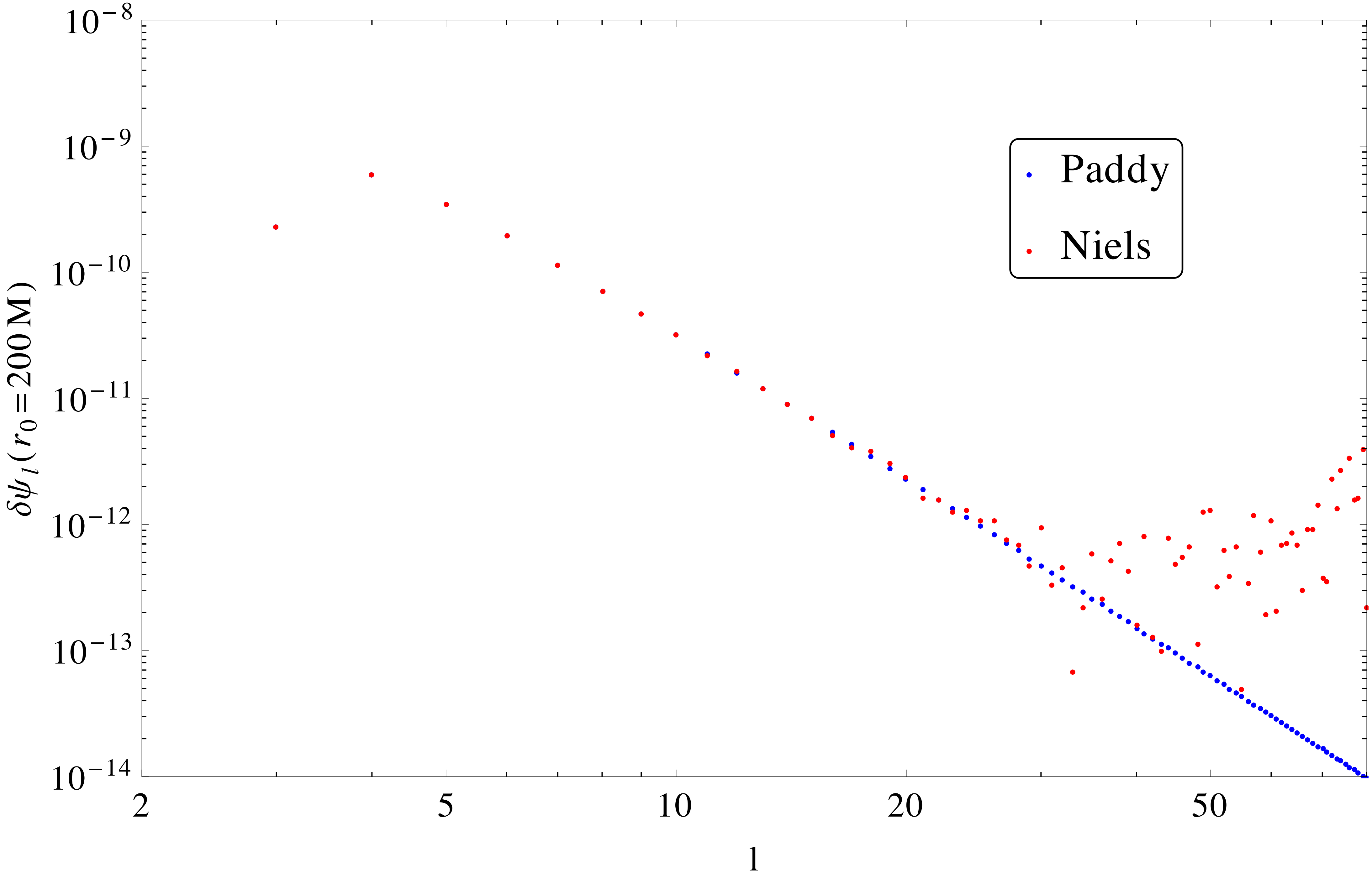
- Once inhomogeneous scalars have been found, can construct perturbed metric $h_{\mu\nu}$
- Gauge invariants such as H, $\delta\psi$, and $\delta\lambda_i$ constructed from $h_{\mu\nu}$ and its derivatives
- Comparing with similar high accuracy calculations for H in Lorenz gauge, see agreement to at least one part in 10^{-20}

Regularisation

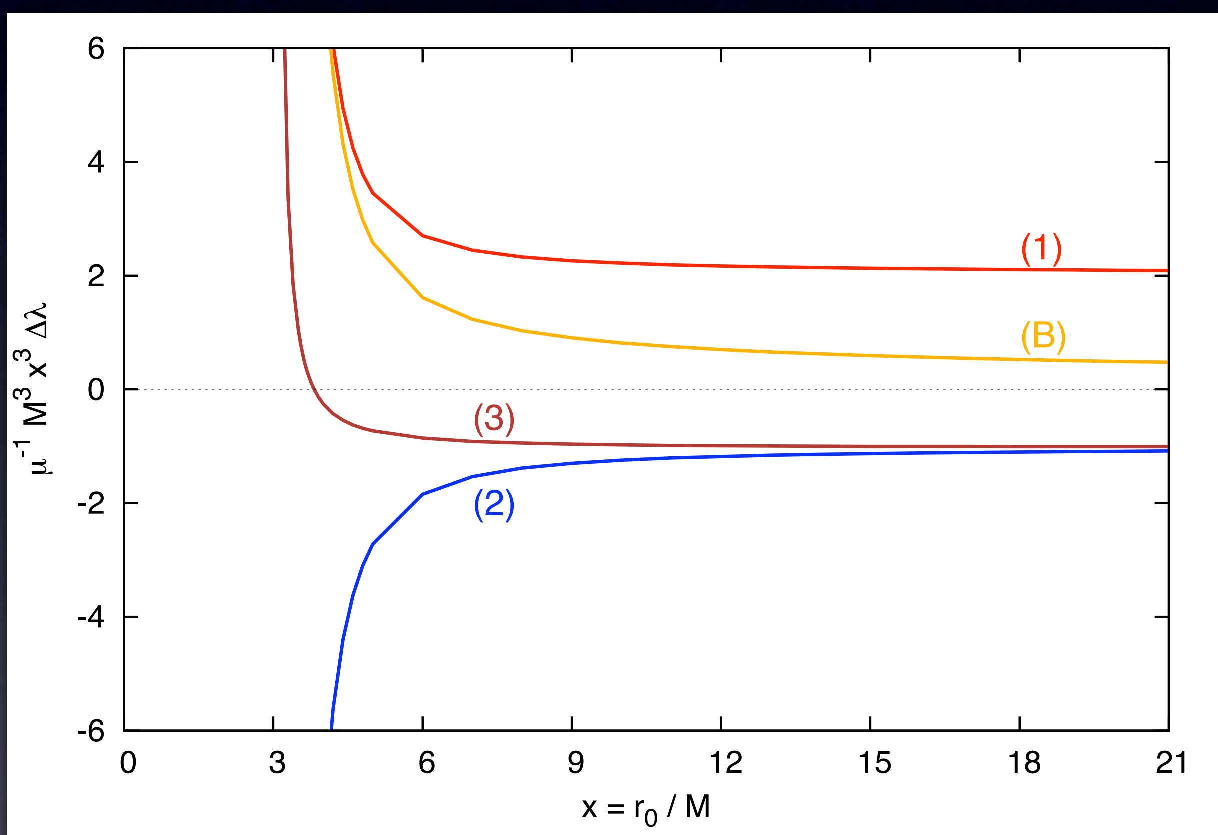
- Gauge Invariants constructed from $h^{ret}_{\alpha\beta}$, still need to be regularised.
- Metric derivatives increase divergence, $\Delta U, \Delta\psi, \Delta\lambda_i^{E/B}$ diverge as L^0, L^1, L^2
- Subtract analytical regularisation parameters, numerically extrapolate higher terms. Final gauge invariants are given by

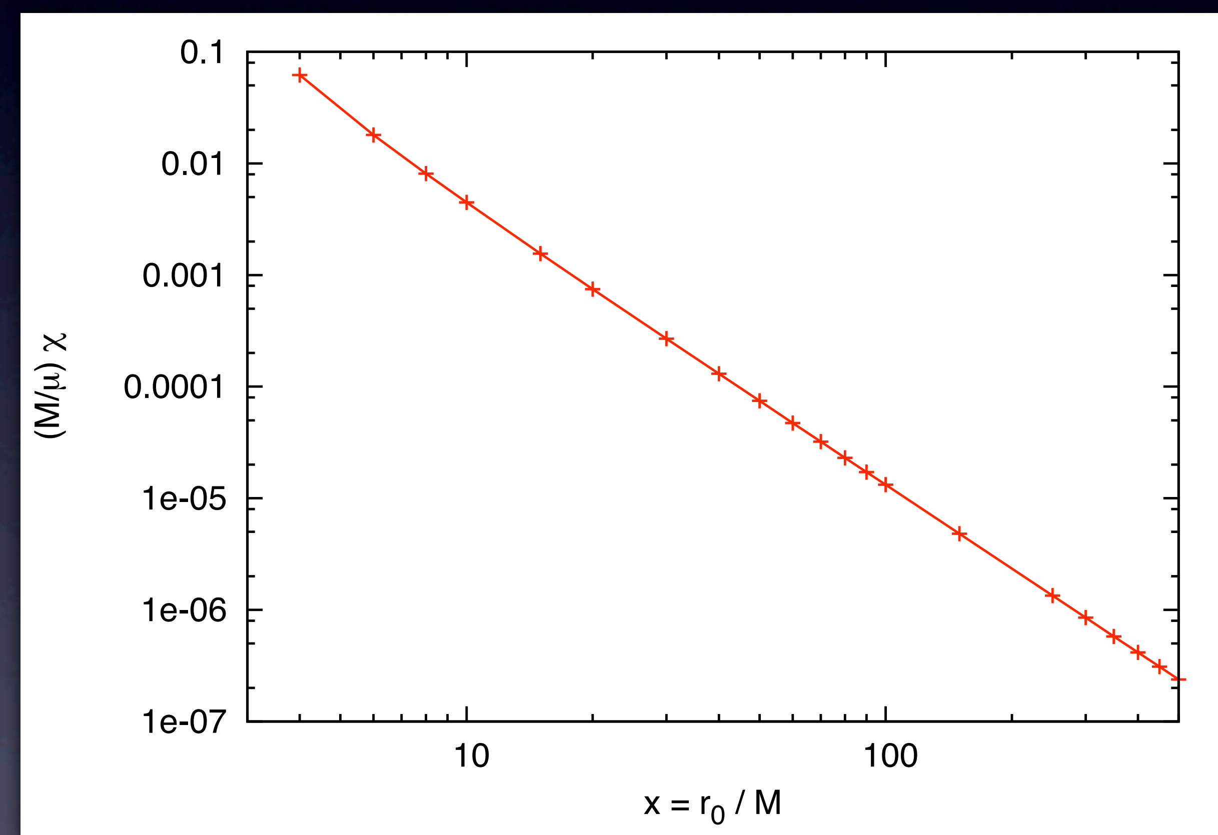
$$\Delta G^R = \sum_{l=0}^{\infty}\left(\Delta G - Z_G L^2 - A_G L - B_G - \frac{D_G}{L} - \dots\right)$$

Convergence

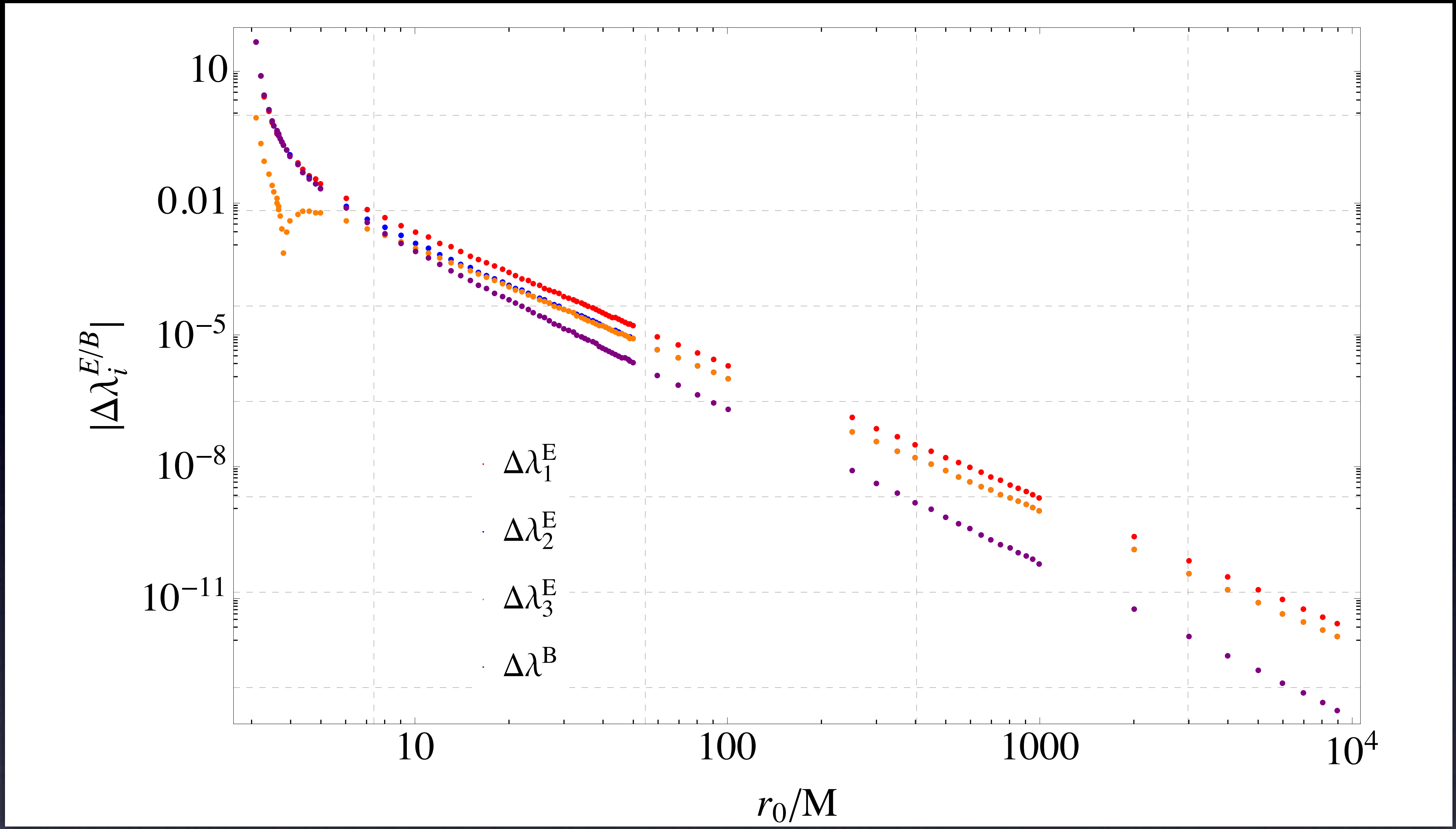


Plots of Gauge Invariants





Large r Data



We compare large r data with PN expansions of the form

$$\Delta\lambda_i(y << 1) = \frac{\mu}{M^3}\sum_{n=3}^{\infty}(a_n^i + b_n^i \ln(y))y^n$$

$$\Delta\lambda_B(y << 1) = \frac{\mu}{M^3}\sum_{n=3}^{\infty}(a_n^B + b_n^B \ln(y))y^{n+1/2}$$

$$n = 3, 4, 5, 5.5, 6, 6.5, \ldots$$

$$y = \frac{M}{r_0}$$

PN results

From data, can extract parameters up to 5PN :

	a_n^1	a_n^2	a_n^3	a_n^B	b_n^1	b_n^2	b_n^3	b_n^B
$n=3$	2	-1	-1	2	0	0	0	0
$n=4$	2	$-3/2$	$-1/2$	2.9999999(5)	0	0	0	$1^{+7}_{-5} \times 10^{-6}$
$n=5$	$-4.7499(7)$	$-2.8750(4)$	7.6249(5)	14.7499(6)	$6^{+61}_{-87} \times 10^{-6}$	$-1^{+25}_{-26} \times 10^{-5}$	$-5^{+48}_{-49} \times 10^{-6}$	$-3^{+555}_{-562} \times 10^{-7}$

Can extract data up to n=5, data suggests exact results:

$$a_4^B = 3, \;\; a_5^1 = -\frac{19}{4}, \;\; a_5^2 = -\frac{23}{8}, \;\; a_5^3 = \frac{61}{8}, \;\; a_5^B = \frac{59}{4}$$

$$b_4^B = 0, \;\; b_5^1 = 0, \;\; b_5^2 = 0, \;\; b_5^3 = 0, \;\; b_5^B = 0$$

Hence see no log terms, which differs from the $\Delta\psi$ case

Informing EOB

- We can construct quantities of interest to EOB theory from new GIs. Electric Quadrupole moment given by :

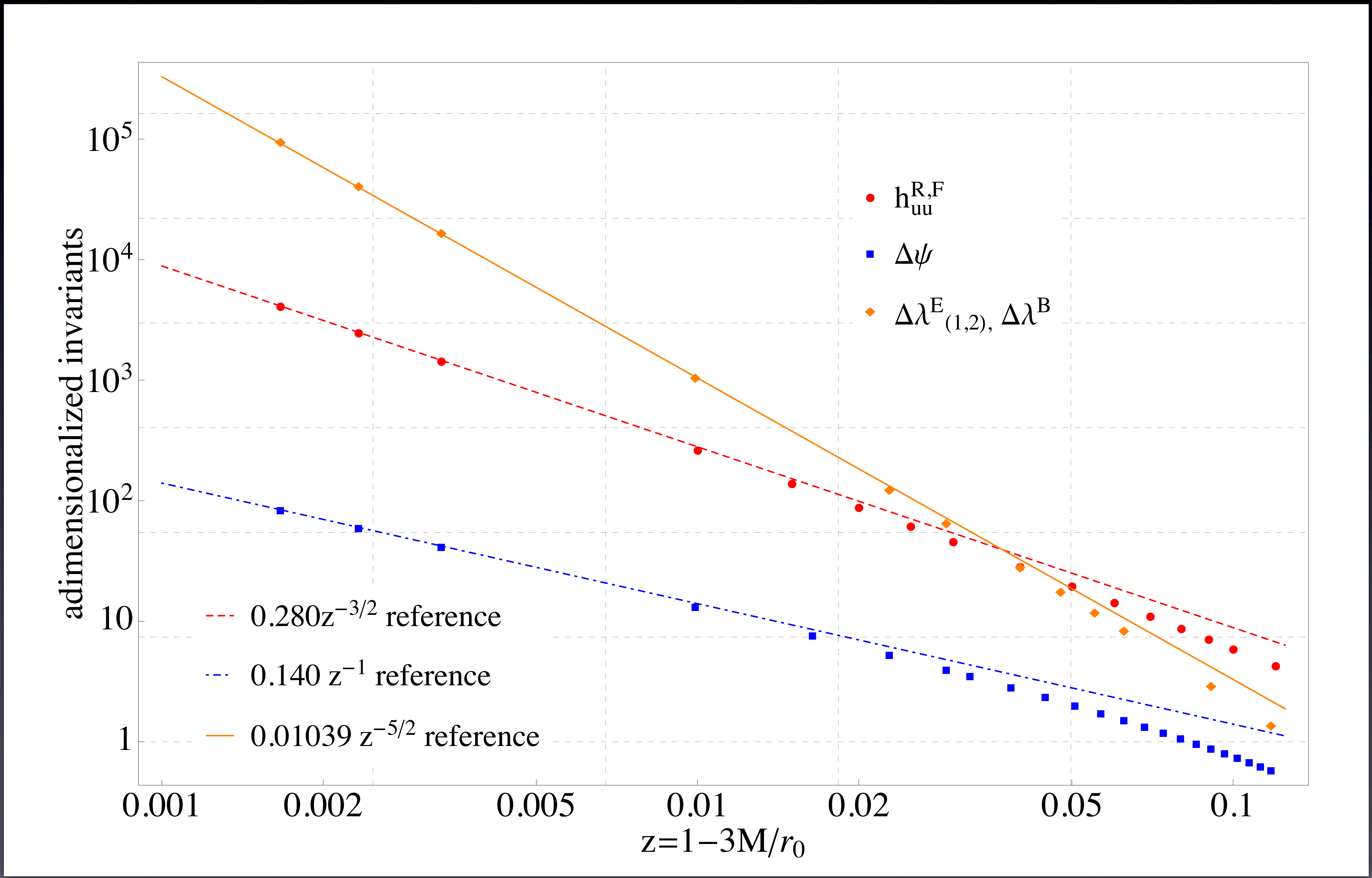
$$\epsilon^2 = \epsilon_{\alpha\beta}\epsilon^{\alpha\beta} = (\lambda_1^E)^2 + (\lambda_2^E)^2 + (\lambda_3^E)^2$$

$$\rightarrow \Delta\epsilon^2 = 2(\lambda_1^E \Delta\lambda_1^E + \lambda_2^E \Delta\lambda_2^E + \lambda_3^E \Delta\lambda_3^E)$$

- Our data implies the following PN expansion for this term:

$$\Delta\epsilon^2 = \underset{(1PN)}{-12y^6} - \underset{(2PN)}{30y^7} - \underset{(3PN)}{\frac{93}{2}y^8} + \ldots$$

Near the Light Ring



Conclusions

- Can extract PN terms for the new gauge invariants from large r data
- Confirm Bini & Damour's prediction of z^{-1} $\Delta\psi$ divergence at light ring
- See divergence of $z^{-5/2}$ for $\Delta\lambda_{1,2,B}$
- Extracted PN terms for electric quadropole from data, useful to EOB. can be repeated for more terms like this