Tidal deformation and dynamics of compact bodies

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Outline

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- Newtonian tides
- Relativistic tides
- Relativistic tidal dynamics
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Goal and motivation

Goal

To describe the small deformations of a compact body created by tidal forces, and to explore their dynamical consequences.

Motivation

Tidal interactions are important in extreme mass-ratio inspirals: torquing of the large black hole leads to a gain of orbital angular momentum, with a significant impact on the orbital evolution (self-force effect). [Hughes (2001); Martel (2004); Yunes *et al.* (2010, 2011)]

Tidal deformations of binary neutron stars could be revealed in the gravitational waves generated during the late inspiral. [Flanagan & Hinderer (2008); Postnikov, Prakash, Lattimer (2010); Pannarale *et al* (2011), Lackey *el al* (2012), Read *et al* (2013)]

The relativistic theory of tidal deformation and dynamics should be as complete as the Newtonian theory.

Newtonian tides: Setting and assumptions

We consider a self-gravitating body ("the body") in a binary system with a companion body ("the companion").

The body has a mass M and radius R. It is spherical in isolation.

The companion is far from the body: $r_{\rm orb} \gg R$.

The tidal forces are **weak** and the deformation is **small**.

The orbital period is long compared with the time scale associated with hydrodynamical processes in the body; the tides are **slow**.

We work in the body's moving frame, with its origin at the centre-of-mass.

We focus attention on a neighbourhood that does not extend too far beyond the body; it does not include the companion.

Gravitational potential

External potential

$$U_{\mathrm{ext}}(t,\boldsymbol{x}) = U_{\mathrm{ext}}(t,\mathbf{0}) + g_a(t)x^a - \frac{1}{2}\mathcal{E}_{ab}(t)x^a x^b + \cdots$$

$$g_a(t) = \frac{\partial}{\partial x^a}U_{\mathrm{ext}}(t,\mathbf{0}) = \text{CM acceleration}$$

$$\boldsymbol{\mathcal{E}_{ab}(t)} = -\frac{\partial^2}{\partial x^a \partial x^b}U_{\mathrm{ext}}(t,\mathbf{0}) = \text{tidal tensor}$$

Body potential

$$U_{\mathrm{body}}(t,\boldsymbol{x}) = \frac{GM}{r} + \frac{3}{2}GQ_{ab}(t)\frac{x^a x^b}{r^5} + \cdots$$

$$Q_{ab}(t) = \int \rho\Bigl(x_a x_b - \frac{1}{3}r^2\delta_{ab}\Bigr)\, d^3x = \text{quadrupole moment}$$

Tidal deformation

The quadrupole moment Q_{ab} measures the body's deformation.

The central problem of tidal theory is to relate Q_{ab} to the tidal tensor $\mathcal{E}_{ab}$; this requires solving the equations of hydrostatic equilibrium for the perturbed configuration.

$$GQ_{ab} = -\frac{2}{3} k_2 R^5 \mathcal{E}_{ab}$$

$\boldsymbol{k_2}$ = gravitational Love number

$$U = \frac{GM}{r} - \frac{1}{2}\Big[1 + 2k_2 (R/r)^5\Big] \mathcal{E}_{ab} x^a x^b$$

The Love number k_2 encodes the details of internal structure.

Dissipation

The preceding results don't account for dissipation within the body.

Dissipation, such as created by viscosity, produces a short delay in the body's response to the tidal forces.

$$GQ_{ab}(t) \simeq -\frac{2}{3}k_2 R^5 \mathcal{E}_{ab}(t - \boldsymbol{\tau})$$

$\boldsymbol{\tau}$ = viscous delay

The viscous delay depends on the precise mechanism responsible for dissipation and the details of internal structure; it is typically much shorter than the orbital period.

Lag and lead

The delay creates a misalignment between the direction of the tidal bulge and the direction of the companion.

When $\Omega_{\rm body} < \Omega_{\rm orbit}$

Orbital motion is **faster** than body's intrinsic rotation.

Tidal bulge **lags** behind the companion.

Tidal torquing **increases** the body's angular momentum.

When $\Omega_{\rm body} > \Omega_{\rm orbit}$

Orbital motion is **slower** than body's intrinsic rotation.

Tidal bulge **leads** in front of the companion.

Tidal torquing **decreases** the body's angular momentum.

Tidal dynamics

Dissipation permits an exchange of angular momentum between the body and the orbit.

Tidal torquing

$$\frac{dJ}{dt} = 6(\boldsymbol{k_2 \tau})\frac{GM_{\rm com}^2 R^5}{r_{\rm orb}^6}\left(\Omega_{\rm orbit} - \Omega_{\rm body}\right)$$

This eventually leads to tidal locking: $\Omega_{\rm body} = \Omega_{\rm orbit}$.

Dissipation is accompanied by the generation of heat.

Tidal heating

$$\frac{dQ}{dt} = 6(\boldsymbol{k_2 \tau})\frac{GM_{\rm com}^2 R^5}{r_{\rm orb}^6}\left(\Omega_{\rm orbit} - \Omega_{\rm body}\right)^2$$

Relativistic theory

A relativistic theory of tidal deformation and dynamics must provide meaningful generalizations of the Newtonian quantities

$$\mathcal{E}_{ab} = -\partial_{ab} U_{\rm ext} = \text{tidal tensor}$$

$$k_2 = \text{gravitational Love number}$$

$$U = \frac{GM}{r} - \frac{1}{2}\Big[1 + 2k_2(R/r)^5\Big]\mathcal{E}_{ab}x^a x^b$$

A tidal tensor $\mathcal{E}_{ab}$ can be defined in terms of the Weyl tensor evaluated at a distance r such that $R \ll r \ll r_{\rm orb}$.

The deformation of the gravitational field from a spherical configuration can be described as a perturbative expansion.

The construction provides a relativistic, gauge-invariant definition for k_2. [Damour & Nagar (2009); Binnington & Poisson (2009)]

Metric of a tidally deformed body

$$g_{tt} = -1 + 2U_{\text{eff}}/c^2$$

$$U_{\text{eff}} = \frac{M}{r} - \frac{1}{2}\Big[A(r) + 2k_2(R/r)^5 B(r)\Big]\mathcal{E}_{ab}x^a x^b$$

$$A(r) = \text{simple polynomial in } 2M/r$$

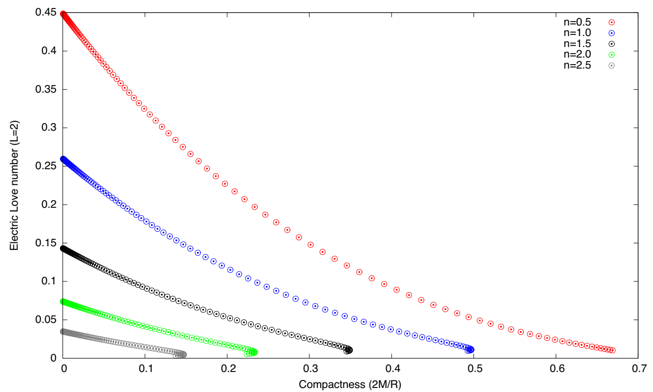
$$B(r) = \text{complicated function of } 2M/r$$

The relativistic Love number k_2 is calculated by solving the equations of hydrostatic equilibrium for the body's perturbed configuration.

The results depend on the body's equation of state $p(\rho)$ and its compactness M/R.

Relativistic Love numbers

k_2 for polytropes: $p = K\rho^{1+1/n}$



[Damour & Nagar (2009)]

For black holes, $k_2 = 0$ [Binnington & Poisson (2009)]

Tidal environment

Applications require the computation of the tidal tensor $\mathcal{E}_{ab}$.

This involves inserting the black hole in a larger spacetime that contains additional bodies, and matching the metrics.

[Taylor & Poisson (2008); Johnson-McDaniel *et al* (2009)]

For a binary system in post-Newtonian circular orbit,

$$\mathcal{E}_{12} = -\frac{3M_{\rm com}}{2r_{\rm orb}^3}\left[1 - \frac{3M + 4M_{\rm com}}{2(M + M_{\rm com})}\,v^2 + O(v^4)\right]\sin(2\omega t)$$

$$\omega = \Omega\left[1 - \frac{MM_{\rm com}}{(M + M_{\rm com})^2}\,v^2 + O(v^4)\right]$$

The angular frequency ω differs from the orbital angular velocity Ω: the transformation from the global inertial frame to the black hole's moving frame involves time dilation and rotation.

Dissipation

There is currently no relativistic theory of tidal interactions that includes dissipation, except for black holes. [On my todo list]

The event horizon of a black hole naturally provides a dissipation mechanism: the black hole absorbs whatever crosses its boundary.

The horizon's null generators can be thought of as the streamlines of an effective fluid. [Membrane paradigm]

This fluid possesses an effective viscosity.

The absorbing properties of the event horizon lead to the tidal torquing and heating of the black hole.

Tidal dynamics of black holes (1)

Black-hole dynamics

$$\frac{dJ}{dt} = \frac{32}{5}\frac{M_{\rm com}^2 M^6}{r_{\rm orb}^6}\left(\Omega_{\rm orbit} - \Omega_{\rm hole}\right)$$

$$\frac{dQ}{dt} = \frac{32}{5}\frac{M_{\rm com}^2 M^6}{r_{\rm orb}^6}\left(\Omega_{\rm orbit} - \Omega_{\rm hole}\right)^2$$

Newtonian dynamics

$$\frac{dJ}{dt} = 6(\boldsymbol{k_2\tau})\frac{GM_{\rm com}^2 R^5}{r_{\rm orb}^6}\left(\Omega_{\rm orbit} - \Omega_{\rm body}\right)$$

$$\frac{dQ}{dt} = 6(\boldsymbol{k_2\tau})\frac{GM_{\rm com}^2 R^5}{r_{\rm orb}^6}\left(\Omega_{\rm orbit} - \Omega_{\rm body}\right)^2$$

Tidal dynamics of black holes (2)

Comparison produces $R \propto GM/c^2$ and $(k_2\tau) \propto GM/c^3$.

More precisely

$$(k_2\tau)R^5 = \frac{16}{15}\frac{(GM)^6}{c^{13}}$$

The quantity $(k_2\tau)$ is assigned a nonzero value in spite of the fact that $k_2 = 0$ for a black hole!

The tidal dynamics of black holes is in near-quantitative agreement with the tidal dynamics of viscous Newtonian bodies.

Relativistic corrections: [Taylor & Poisson (2008); Chatziioannou *et al* (2013)]

$$\frac{dJ}{dt} = -\frac{32}{5}\frac{M_{\rm com}^2 M^6}{r_{\rm orb}^6}\Omega_{\rm hole} \times \frac{1}{2}\left(1+\sqrt{1-\chi^2}\right)\left\{1 + 3\chi^2 - \frac{1}{4}\left[12 + 51\chi^2 - \frac{(4+12\chi^2)M}{M+M_{\rm com}}\right]v^2 + O(v^3)\right\}$$

Conclusion

The Newtonian theory of tidal deformations and dynamics is undergoing a generalization to relativistic gravity.

A meaningful description of the tidal deformation of a relativistic body has been achieved; the gravitational Love numbers have been ported to general relativity.

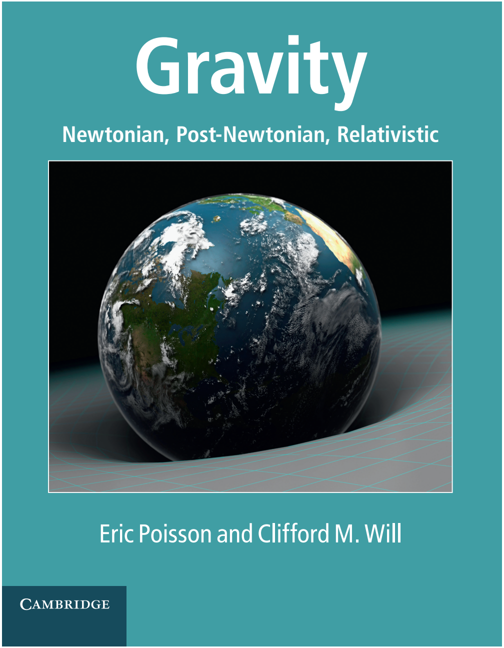
The tidal dynamics of black holes is well developed; it displays a remarkable similarity with the Newtonian theory of viscous bodies.

The tidal dynamics of material bodies remains to be developed; this requires the incorporation of viscosity.

Recent work: Surficial love numbers [Landry & Poisson (2014)]

Current work: Slowly rotating bodies.

Future work: Higher dimensions.


Gravity
Newtonian, Post-Newtonian, Relativistic
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