Self-force on a charge outside a five-dimensional black hole

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Electromagnetic self-force outside a 4D black hole

Smith & Will (1980) calculated the electromagnetic self-force acting on a point charge q held at a fixed position r outside a Schwarzschild black hole of event-horizon radius $R = 2M$.

Working from the exact Copson-Linet solution, they found that the self-force is repulsive:

$$F_{\text{self}} = \frac{q^2 R}{2r^3}$$

The repulsive nature of the self-force is difficult to explain.

Exploration of static self-forces

- Replace the black hole with a material body [Shankar & Whiting (2007); Drivas & Gralla (2011); Isoyama & Poisson (2012)]
- Replace flat asymptotic conditions by de Sitter or anti de Sitter conditions [Kuchar, Poisson & Vega (2013)]
- Examine the problem in higher dimensions

Point charge outside a 5D black hole

Schwarzschild-Tangherlini spacetime

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2\,d\Omega_3^2$$
$$f = 1 - (R/r)^2$$
$$d\Omega_3^2 = d\chi^2 + \sin^2\chi\big(d\theta^2 + \sin^2\theta\,d\phi\big)$$

The electrostatic potential Φ is decomposed in (generalized) Legendre polynomials.

The modes are given by associated Legendre functions.

The self-force is expressed as an infinite mode sum, which requires regularization.

Regularization

The self-force is regularized by calculating the average of $\partial_a \Phi$ on a surface of constant proper distance s around the particle, and taking the limit $s \to 0$.

Diverging terms proportional to the particle's accelation can be absorbed into a renormalization of the mass.

The procedure is implemented by introducing a potential Φ^{S} that is as singular as Φ near the particle, and writing

$$\langle \partial_a \Phi \rangle_{\rm ren} = \partial_a \Phi - \partial_a \Phi^{\mathsf{S}} + \langle \partial_a \Phi_{\mathsf{S}} \rangle_{\rm ren}$$

The difference $\partial_a \Phi - \partial_a \Phi^{\mathsf{S}}$ is smooth and can be calculated as a convergent mode sum.

The contribution $\langle \partial_a \Phi^{\mathsf{S}} \rangle_{\rm ren}$ can be calculated analytically; this would vanish in four dimensions, but doesn't in five.

Hadamard regularization

The singular potential Φ^{S} is identified with the Hadamard Green's function for the electrostatic potential in the 4D spatial sections of the static, 5D spacetime

$$G_{\mathrm{H}}(\boldsymbol{x}, \boldsymbol{x}_0) = \frac{1}{d-3} \frac{U(\boldsymbol{x}, \boldsymbol{x}_0)}{(2\sigma)^{\frac{1}{2}(d-3)}} + V(\boldsymbol{x}, \boldsymbol{x}_0) \ln(2\sigma) + W(\boldsymbol{x}, \boldsymbol{x}_0)$$

This is decomposed in (generalized) Legendre polynomials to obtain regularization parameters for the mode sum.

This is also used to calculate $\langle \partial_a \Phi^{\mathsf{S}} \rangle_{\mathrm{ren}}$, which contains a term proportional to $\ln s$ that cannot be renormalized away.

Unlike its 4D version, the 5D self-force depends on the averaging radius s.

Large-r expansion

The self-force modes can be expressed as expansions in powers of R/r (which also include logarithms).

The series expansions is inserted into the regularized mode sum, which can then be summed exactly.

$$F_{\rm self} = \frac{q^2 R^2}{2r^5}\left(F_{\rm poly} + \frac{9}{8}\frac{R^2}{r^2} F_{\ln} \ln \frac{sR}{4r^2} \right)$$

Large-r expansion

$$F_{\rm poly} = 1 + \frac{45}{32}x + \frac{5}{4}x^2 + \frac{667}{512}x^3 + \frac{711}{512}x^4 + \frac{12159}{8192}x^5 + \frac{8103}{5120}x^6 + \frac{1101771}{655360}x^7 + \frac{1632059}{917504}x^8 + \frac{55051139}{29360128}x^9 + \frac{28914535}{14680064}x^{10} + \frac{7266927967}{3523215360}x^{11} + \frac{3974912613}{1845493760}x^{12} + \frac{13249034609}{5905580032}x^{13} + \frac{33562030445}{14394851328}x^{14} + \frac{20791850417775}{8598524526592}x^{15} + \frac{21523569752673}{8598524526592}x^{16} + \frac{14235811133305233}{5503055697018888}x^{17} + O(x^{18})$$

$$F_{\rm ln} = 1 + \frac{2}{3}x + \frac{5}{8}x^2 + \frac{5}{8}x^3 + \frac{245}{384}x^4 + \frac{21}{32}x^5 + \frac{693}{1024}x^6 + \frac{715}{1024}x^7 + \frac{23595}{32768}x^8 + \frac{12155}{16384}x^9 + \frac{600457}{786432}x^{10} + \frac{205751}{262144}x^{11} + \frac{3380195}{4194304}x^{12} + \frac{1300075}{1572864}x^{13} + \frac{28415925}{33554432}x^{14} + \frac{29084535}{33554432}x^{15} + \frac{1903421235}{2147483648}x^{16} + O(x^{17})$$

with $x = (R/r)^2$.

Summing the series

Remarkably, these series can be summed.

Self-force

$$F_{\text{self}} = \frac{q^2 R^2}{2r^5} \frac{\Xi}{f^{3/2}},$$

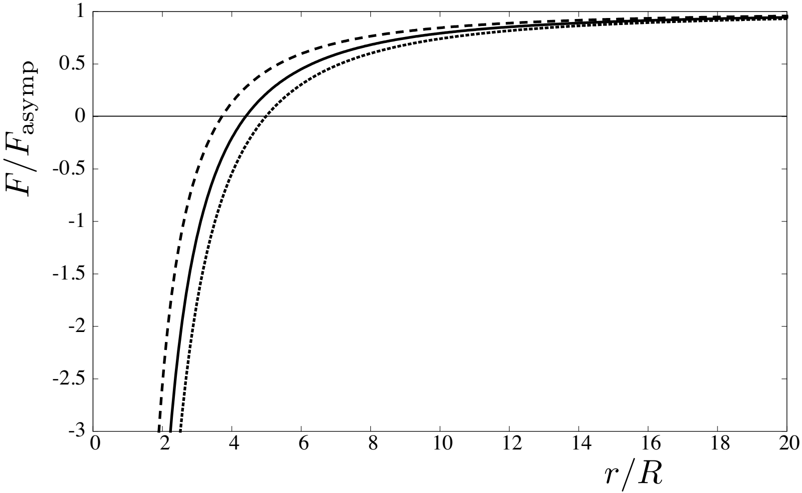
$$\Xi = -\frac{1}{4x} + \frac{5}{8} + \frac{139}{96}x - \frac{281}{192}x^2 + \left(\frac{1}{4x} + \frac{1}{2} - \frac{15}{16}x\right)\sqrt{f} + \frac{3}{16}x(6-5x)\ln\frac{\tilde{s}x(1+\sqrt{f})}{8\sqrt{f}}$$

$$x = (R/r)^2, \qquad f = 1-(R/r)^2, \qquad \tilde{s} = s/R$$

The self-force approaches $q^2R^2/(2r^5)$ when $r \gg R$; it is repulsive at large distances.

It becomes attractive when r becomes comparable to R, and diverges when $r \to R$.

Plot



Summary and outlook

Summary

- We have computed the electromagnetic self-force on a static charge in the 5D Schwarzschild-Tangherlini spacetime.
- The self-force is attractive at large distances, repulsive when $r < 5R$, and divergent when $r \to R$.
- The regularization method introduces a dependence on the averaging radius s; this is an indication that the self-force cannot be expected to be independent of the particle's internal structure.

Outlook

- Self-force in six dimensions?
- Self-force for a 5D black string?