Self-force on a charge outside a five-dimensional black hole

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Smith & Will (1980) calculated the electromagnetic self-force acting on a point charge $q$ held at a fixed position $r$ outside a Schwarzschild black hole of event-horizon radius $R = 2M$.

Working from the exact Copson-Linet solution, they found that the self-force is repulsive:

$$F_{\text{self}} = \frac{q^2 R}{2r^3}$$

The repulsive nature of the self-force is difficult to explain.
Exploration of static self-forces

- Replace the black hole with a material body [Shankar & Whiting (2007); Drivas & Gralla (2011); Isoyama & Poisson (2012)]
- Replace flat asymptotic conditions by de Sitter or anti de Sitter conditions [Kuchar, Poisson & Vega (2013)]
- Examine the problem in higher dimensions

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The electrostatic potential $\Phi$ is decomposed in (generalized) Legendre polynomials.

The modes are given by associated Legendre functions.

The self-force is expressed as an infinite mode sum, which requires regularization.
Regularization

The self-force is regularized by calculating the average of $\partial_a \Phi$ on a surface of constant proper distance $s$ around the particle, and taking the limit $s \to 0$.

Diverging terms proportional to the particle's acceleration can be absorbed into a renormalization of the mass.

The procedure is implemented by introducing a potential $\Phi^S$ that is as singular as $\Phi$ near the particle, and writing

$$\langle \partial_a \Phi \rangle_{\text{ren}} = \partial_a \Phi - \partial_a \Phi^S + \langle \partial_a \Phi^S \rangle_{\text{ren}}$$

The difference $\partial_a \Phi - \partial_a \Phi^S$ is smooth and can be calculated as a convergent mode sum.

The contribution $\langle \partial_a \Phi^S \rangle_{\text{ren}}$ can be calculated analytically; this would vanish in four dimensions, but doesn't in five.
Hadamard regularization

The singular potential $\Phi^S$ is identified with the Hadamard Green's function for the electrostatic potential in the 4D spatial sections of the static, 5D spacetime

$$G_H(x, x_0) = \frac{1}{d-3} \frac{U(x, x_0)}{(2\sigma)^{\frac{1}{2}(d-3)}} + V(x, x_0) \ln(2\sigma) + W(x, x_0)$$

This is decomposed in (generalized) Legendre polynomials to obtain regularization parameters for the mode sum.

This is also used to calculate $\langle \partial_a \Phi^S \rangle_{\text{ren}}$, which contains a term proportional to $\ln s$ that cannot be renormalized away.

Unlike its 4D version, the 5D self-force depends on the averaging radius $s$. 

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Large-$r$ expansion

The self-force modes can be expressed as expansions in powers of $R/r$ (which also include logarithms).

The series expansions is inserted into the regularized mode sum, which can then be summed exactly.

$$F_{self} = \frac{q^2 R^2}{2r^5} \left( F_{poly} + \frac{9}{8} \frac{R^2}{r^2} F_{ln} \ln \frac{sR}{4r^2} \right)$$
Large-$r$ expansion

\[ F_{\text{poly}} = 1 + \frac{45}{32} x + \frac{5}{4} x^2 + \frac{667}{512} x^3 + \frac{711}{512} x^4 + \frac{12159}{8192} x^5 + \frac{8103}{5120} x^6 + \frac{1101771}{655360} x^7 \]
\[ + \frac{1632059}{917504} x^8 + \frac{55051139}{29360128} x^9 + \frac{28914535}{14680064} x^{10} + \frac{7266927967}{3523215360} x^{11} + \frac{3974912613}{1845493760} x^{12} \]
\[ + \frac{13249034609}{5905580032} x^{13} + \frac{33562030445}{14394851328} x^{14} + \frac{2079185041775}{8598524526592} x^{15} + \frac{21523569752673}{8598524526592} x^{16} \]
\[ + \frac{1423581113305233}{550305569701888} x^{17} + O(x^{18}) \]

\[ F_{\text{ln}} = 1 + \frac{2}{3} x + \frac{5}{8} x^2 + \frac{5}{8} x^3 + \frac{245}{384} x^4 + \frac{21}{32} x^5 + \frac{693}{1024} x^6 + \frac{715}{1024} x^7 + \frac{23595}{32768} x^8 \]
\[ + \frac{12155}{16384} x^9 + \frac{600457}{786432} x^{10} + \frac{205751}{262144} x^{11} + \frac{3380195}{4194304} x^{12} + \frac{1300075}{1572864} x^{13} \]
\[ + \frac{28415925}{33554432} x^{14} + \frac{29084535}{33554432} x^{15} + \frac{1903421235}{2147483648} x^{16} + O(x^{17}) \]

with \( x = (R/r)^2 \).
Remarkably, these series can be summed.

**Self-force**

\[
F_{\text{self}} = \frac{q^2 R^2}{2r^5} \frac{\Xi}{f^{3/2}},
\]

\[
\Xi = -\frac{1}{4x} + \frac{5}{8} + \frac{139}{96} x - \frac{281}{192} x^2 + \left(\frac{1}{4x} + \frac{1}{2} - \frac{15}{16} x\right) \sqrt{f}
\]

\[
+ \frac{3}{16} x (6 - 5x) \ln \frac{\tilde{s} x (1 + \sqrt{f})}{8\sqrt{f}}
\]

\[
x = (R/r)^2, \quad f = 1 - (R/r)^2, \quad \tilde{s} = s/R
\]

The self-force approaches \(q^2 R^2/(2r^5)\) when \(r \gg R\); it is repulsive at large distances. It becomes attractive when \(r\) becomes comparable to \(R\), and diverges when \(r \to R\).
Plot

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Summary

We have computed the electromagnetic self-force on a static charge in the 5D Schwarzschild-Tangherlini spacetime. The self-force is attractive at large distances, repulsive when \( r < 5R \), and divergent when \( r \to R \). The regularization method introduces a dependence on the averaging radius \( s \); this is an indication that the self-force cannot be expected to be independent of the particle’s internal structure.

Outlook

- Self-force in six dimensions?
- Self-force for a 5D black string?