Status and challenges of binary black hole simulations

Mark A. Scheel
Caltech

Simulation of Extreme Spacetimes (SXS) Collaboration:
www.black-holes.org

Capra Meeting, June 25, 2014

Summary

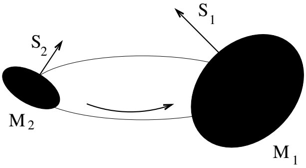
What can NR BBH simulations now do?
What science can we do with them?

1. Introduction: black-hole binaries and numerical relativity (NR)
2. Sources of error in BBH simulations
3. Capabilities and challenges
    - Precession
    - Large spins
    - Large mass ratios
    - Large number of orbits
    - Parameter space coverage

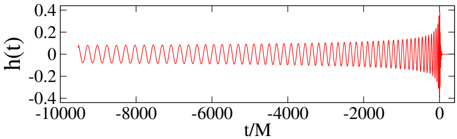
Introduction

Numerical Relativity (NR) and the BBH problem

Some problems, like the binary black hole problem, are difficult to treat with approximation methods in the general case.



$\Longleftrightarrow$



The only way to solve **nonlinear, dynamical, strong-field** Einstein equations is **numerically**.

$\Rightarrow$ **Numerical Relativity**

Numerical relativity (NR)

- Write Einstein's field equations as an initial value problem for $g_{\mu\nu}$.

$$\boxed{G_{\mu\nu} = 8\pi T_{\mu\nu}} \Rightarrow \begin{cases} \text{Constraints} & (\text{like } \nabla \cdot B = 0) \\ \text{Evolution eqs.} & (\text{like } \partial_t B = -\nabla \times E) \end{cases}$$

- Choose 'free' initial data at $t = 0$
- Choose gauge (=coordinate) conditions

Get yourself a computer cluster, and

- Solve constraints at $t = 0$
  (4 (+1) coupled nonlinear 2nd-order elliptic PDEs)
- Use evolution eqs. to advance in time
  (50 coupled nonlinear 1st-order hyperbolic PDEs)
- Use constraints to check quality of evolution.

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- First successful black-hole binary computation: Pretorius 2005
- Today several research groups worldwide have NR codes.

NR Codes for BHBH binaries

About a dozen in existence

Form of Equations:	BSSN	Z4c	Generalized Harmonic
Singularity Treatment:	Moving Punctures		Excision
Numerical Methods:	Finite Differencing		Spectral

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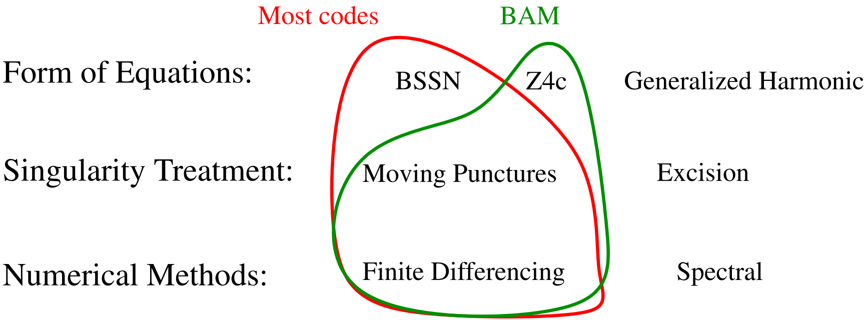

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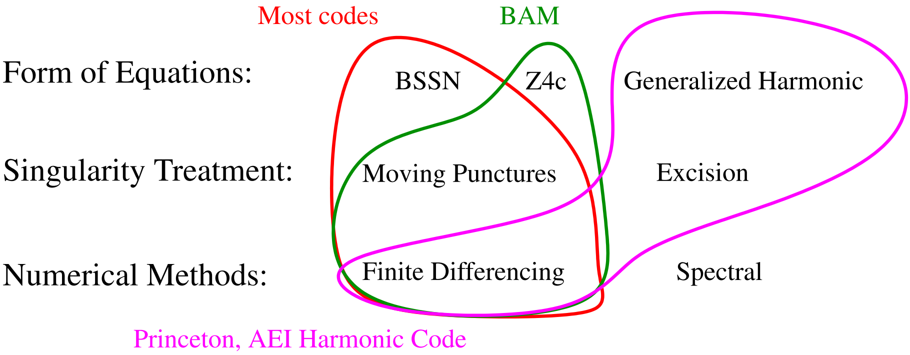

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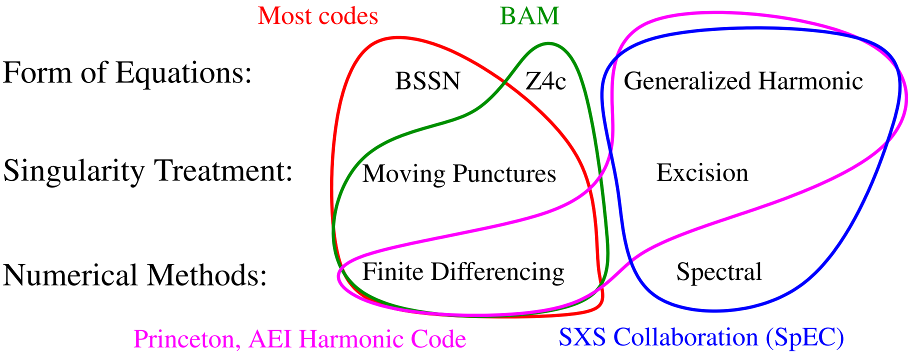

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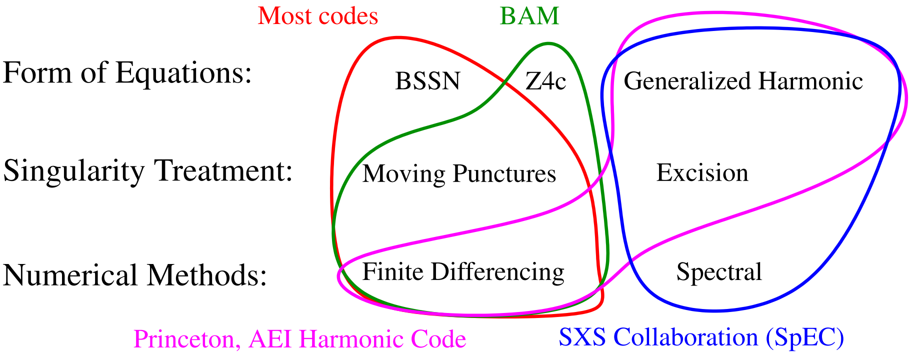


- Comparing different codes improves confidence in results.
- Most examples I will show will be from SpEC.

SpEC - Spectral Einstein Code

http://www.black-holes.org/SpEC.html

Parallel computer code developed at Caltech, Cornell, CITA (Toronto), Washington State, UC Fullerton, plus several contributors at other institutions.

- Spectral methods (exponential convergence for smooth problems).
- Generalized Harmonic form of Einstein's equations.
- Black hole excision.
- Relativistic hydrodynamics (neutron stars,...).

Credits

Kevin Barkett
Thomas Baumgarte
Jon Blackman
Andy Bohn
Mike Boyle
Jeandrew Brink
Luisa Buchman
Darius Bunandar
Yanbei Chen
Tony Chu
Michael Cohen
Greg Cook
Brett Deaton
Matt Duez
Francois Foucart
Byrant Garcia
Matt Giesler
Jason Grigsby
Roland Haas
Kate Henriksson
Frank Herrmann
Francois Hebert
Dan Hemberger
Jeff Kaplan
Drew Keppel
Larry Kidder
Stephen Lau
Francois Limousin
Jonas Lippuner
Lee Lindblom
Geoffrey Lovelace
Ilana MacDonald
Robert McGehee
Abdul Mroué
Curran Muhlberger
David Nichols
Fatemah Nouri
Christian Ott
Serguei Ossokine
Rob Owen
Harald Pfeiffer
Keith D. Matthews
Christian Reisswig
Oliver Rinne
Olivier Sarbach
Dierdre Shoemaker
Mark Scheel
Béla Szilágyi
Nicholas Taylor
Nick Tacik
Saul Teukolsky
Kip Thorne
Will Throwe
Manuel Tiglio
Anil Zenginoglu
Fan Zhang
Aaron Zimmerman

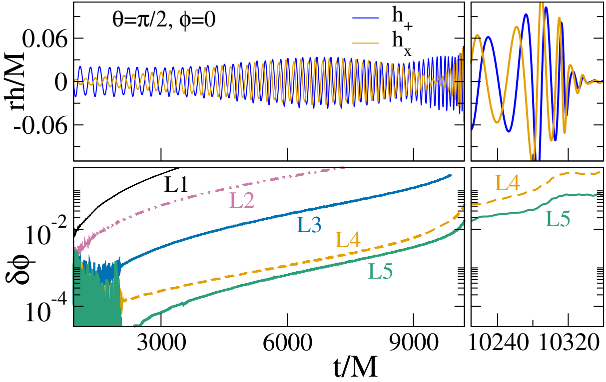
SpEC run time

Run time: depends on masses, spins,
number of orbits.

- Days to months wallclock time running on ~ 50 cores.
- Spectral methods: small memory usage, run 'fits' on laptop.
- Can run many simulations at once on large machines.

Sources of Error

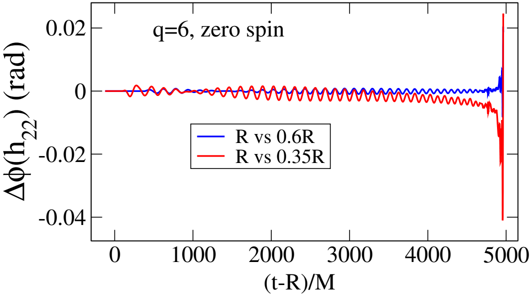
Numerical truncation error



Mass ratio=3
Spin 0.5 on large BH
31 orbits, precession

Mroue, MAS, et al.,
PRL 111:241104 (2013)

Outer boundary error



Buchman et al., PRD 79:124028 (2009)

- Most simulations use a finite outer boundary.
- Constraint-preserving, “transparent” BCs not perfect.

More about this later…

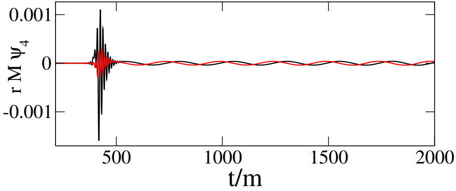
Initial data "error"

Astrophysical initial data

- Initial data should be 'snapshot' of inspiral from $t = -\infty$
- Tidal distortion, initial gravitational radiation are not correct.

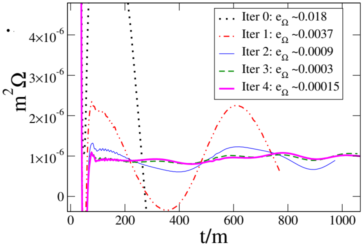
$\Rightarrow$ "Junk radiation" spoils beginning of simulation.

$\Rightarrow$ Masses & spins relax during junk epoch.



Eccentricity

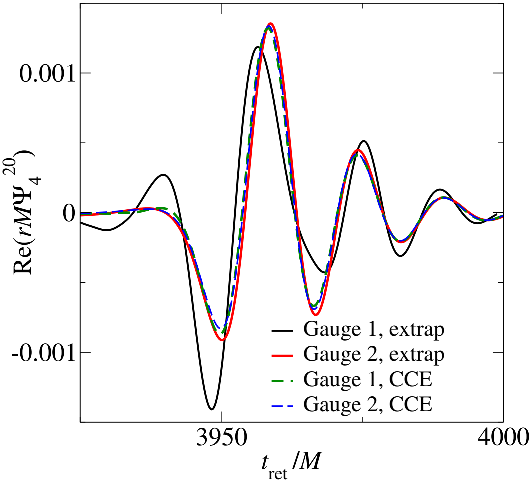
- Cannot *a priori* choose initial data to get desired eccentricity.
- Can produce small eccentricity via iterative scheme: expensive.



Mroue & Pfeiffer, arXiv:1210.2958

Wave extraction error

- Waves are computed at finite distance from source.
  - $\Rightarrow$ Contaminated by gauge, near-field terms.



- Cauchy-characteristic extraction (CCE) mostly solves this problem—gauge invariant.
- Still some error w/ CCE (free data on initial null slice).

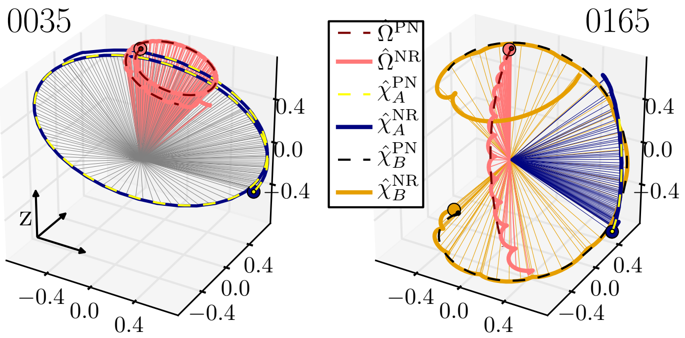
Taylor et al., PRD 88, 124010 (2013)

Capabilities and Challenges

Precession

Precession allows us to study

- Spin and orbit dynamics.



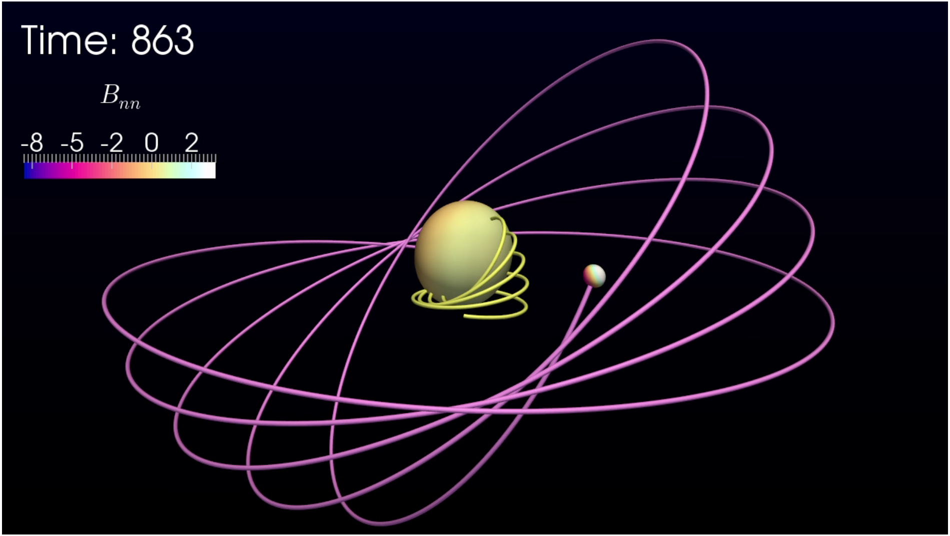
Mroue, MAS, et al., PRL 111:241104 (2013)
Followup: Ossokine et al., in prep

- Calibration of analytical waveform models.
  Taracchini et al., PRD 89:061502 (2014)

Precession

Large hole spin $\sim$ 0.91
Small hole spin $\sim$ 0.3

Color = Vorticity
(a measure of spin)
Mass ratio 6



Event Horizon: Andy Bohn Movie: Curran Muhlberger

Capabilities and Challenges

Large spins

Numerical simulations of large spins allow us to study

- The effect of high BH spin on BH/NS binaries.
  Lovelace et al, CQG 30:135004 (2013)
- Remnant properties as a function of initial parameters.
  Hemberger et al., PRD 88:064014 (2013)
- Calibration of analytical waveform models.
  Taracchini et al., PRD 89:061502 (2014)
- How high a spin can LIGO distinguish? Hemberger et al., in prep
- Initial spin of merged horizon. Lovelace et al., in prep

Why is it difficult to simulate large spins?

- Initial data
  - Standard conformally flat data limited to $\chi(\equiv S/M^2) < 0.93$.
  - Superposed Kerr-Schild data breaks this barrier.
    Lovelace et al., PRD 78:084017 (2008)

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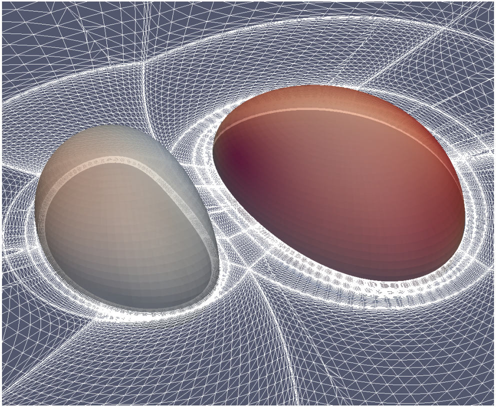
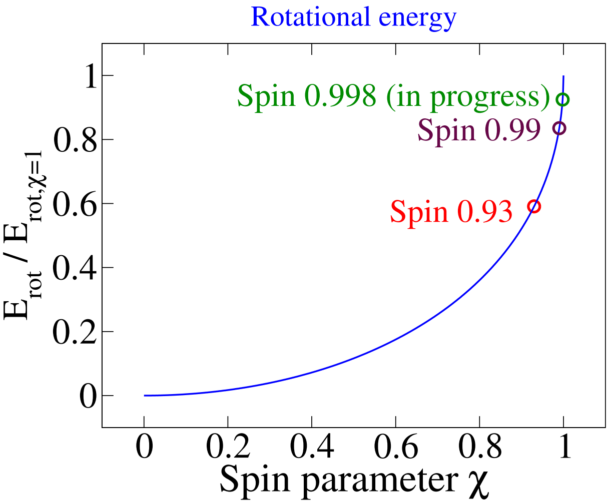
    Lovelace et al., PRD 78:084017 (2008)
- Evolution
  - Need high accuracy.
  - Need precise excision.



$\chi = 0.2$ (left BH), $\chi = 0.991$ (right BH)

Isn't $\chi = 0.93$ large enough already?



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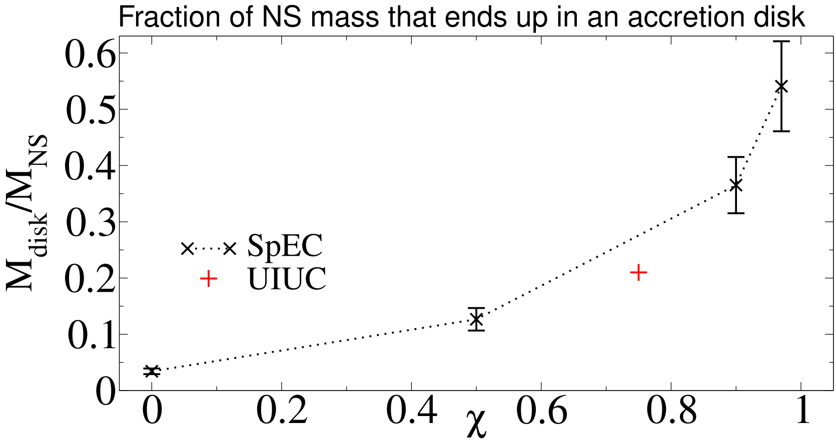
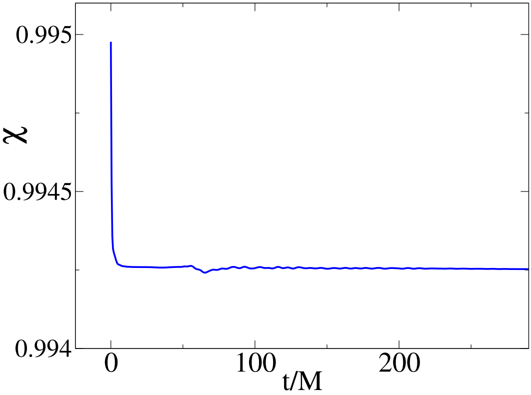
Black hole / neutron star binary:



Lovelace et al, CQG 30:135004 (2013)

New simulation: Equal mass, aligned spins, $\chi = 0.994$



Remnant properties:

Radiated energy: 11.35%

Final spin: $\chi = 0.9499$

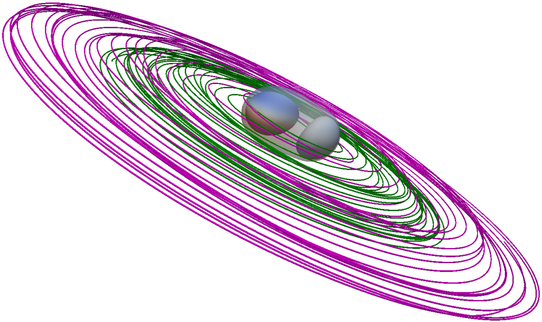
Hemberger et al. 2013 predicts:

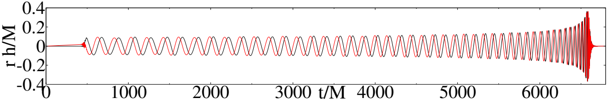
Radiated energy 11.3%

Final spin $\chi = 0.950$

Run pushed by Matt Giesler

New simulation: high spin with (mild) precession





Mass ratio 1.5

Large BH: $\chi = 0.991$

Small BH: $\chi = 0.2$

Horizon colors: Vorticity

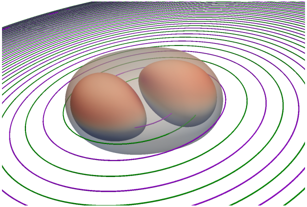
Remnant properties:

Radiated energy: 7.8%

Final spin: $\chi = 0.897$

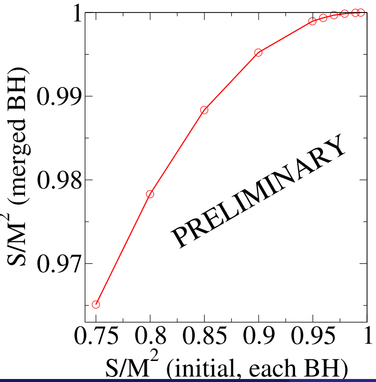
Initial spin of the merged apparent horizon

preliminary results



Measure spin of common AH at formation, as function of initial spin.

Equal spins, aligned with $L_{\rm orb}$



$S/M^2_{\rm (merged)}$ = 0.9999
for $S/M^2_{\rm (init)}$=0.989

Initial spin of the merged apparent horizon

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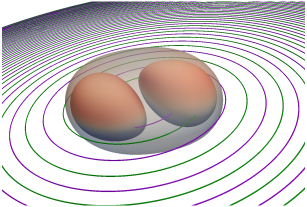

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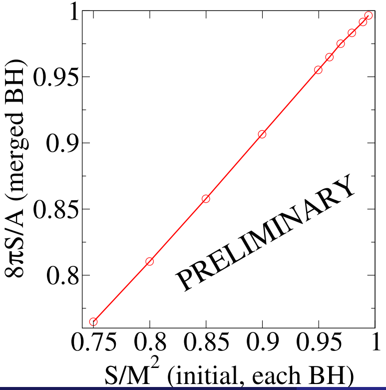


$S/M^2 \leq 1$ trivially, by construction.
$M^2 = M_{\rm irr}^2 + S^2/4M_{\rm irr}^2$

Instead plot extremality parameter $8\pi S/A = S/2M_{\rm irr}^2$.

We find $8\pi S/A$ always < 1.
Unclear if $\lim_{\chi_{\rm init}\to 1}(8\pi S/A) < 1$.

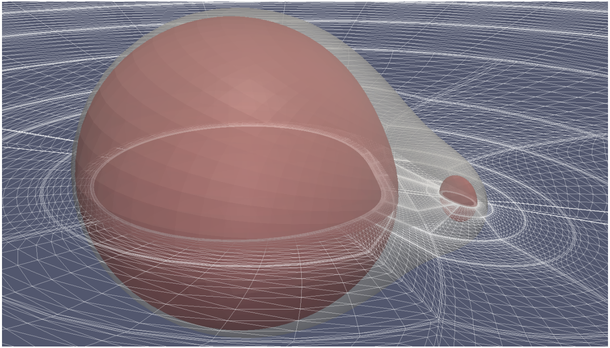
Capabilities and Challenges

Large mass ratios

Large mass ratios

Why is it difficult?

- Time scale of orbit $\sim M_1 + M_2$
- Size of time step $\sim M_{\mathrm{small}}$
- Need high resolution near smaller BH.

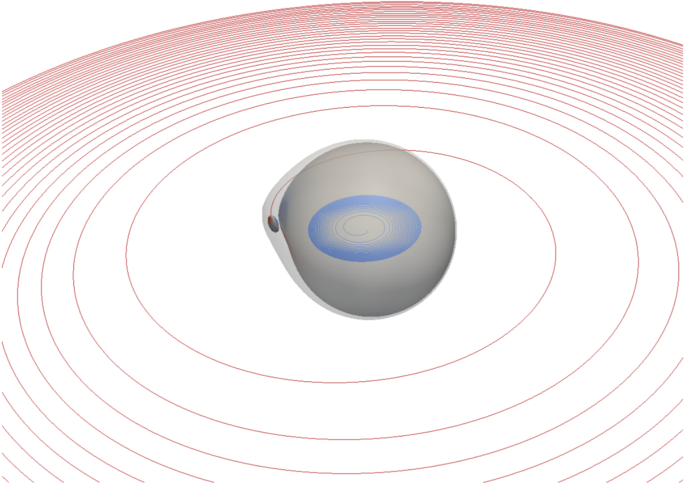


Mass ratio 8:1 Mroue, MAS, et al., PRL 111:241104 (2013)

Large mass ratios allow us to study

- Comparison with PN.
  MacDonald et al., PRD 87, 024009 (2013)
- Calibration of analytical waveform models.
  Taracchini et al., PRD 89:061502 (2014)
- Periastron advance
  Le Tiec et al., PRL 107:141101 (2011)
  Hinderer et al., PRD 88:084005 (2013)
  Le Tiec et al., PRD 88:124027 (2013)
- Early BH/NS inspirals, using BHBH as a proxy
  Barkett et al., in prep

Large mass ratios



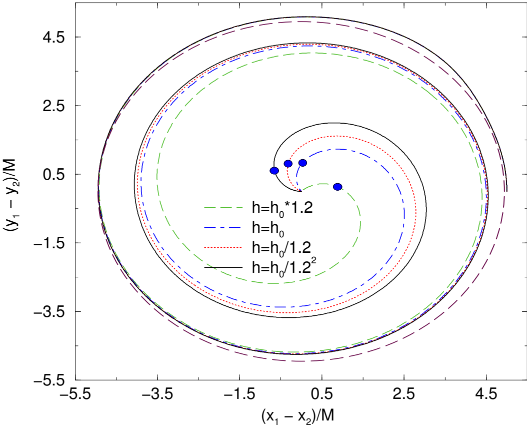
10:1, no spin
30 orbits
$e \sim 10^{-3}$

Largest mass ratio done w/ SpEC

20:1 in progress; difficulties remain.

Run pushed by S. Ossokine, B. Szilagyi

Large mass ratios

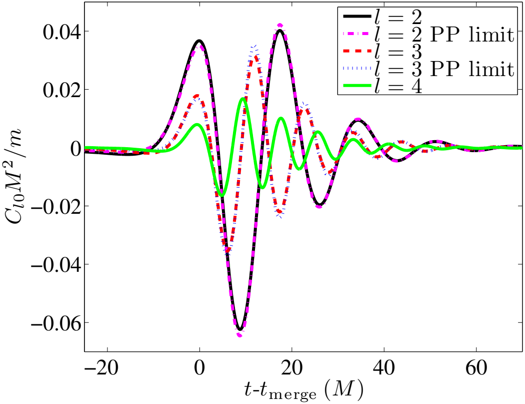


100:1, no spin
most extreme to date

Medium res: 0.5M CPU-hr

Lousto & Zlochower, PRL 106:041101 (2011)

Large mass ratios



$q = 10^6$

star falling radially into BH

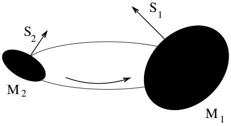
Method assumes known background + perturbation

East & Pretorius, PRD 87:043004 (2013)

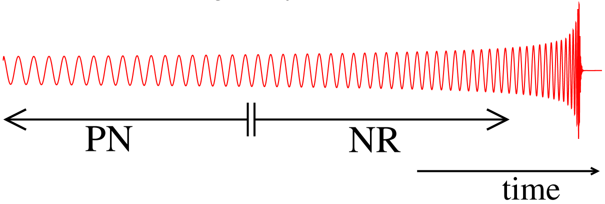
Capabilities and Challenges

Number of orbits

Why do we want many orbits? PN matching

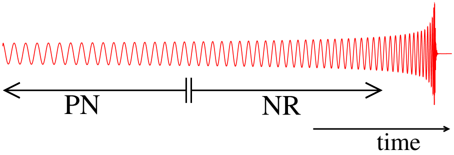


Waveform visible to LIGO has 100s of binary orbits.
Problem: Following many orbits with NR is difficult.



Solution: Match NR simulation to PN, *before PN becomes inaccurate.*

How do you know when PN is accurate?



Two ways:

1. Test PN approximants against each other.

   Example: Nitz et al., arXiv:1307.1757 (2013):

   - For BH/NS parameters ($q \sim 7:1$, $\mathrm{spin}_{\mathrm{BH}} \gtrsim 0.2$), PN is **inaccurate** for **most** of the aLIGO frequency band.

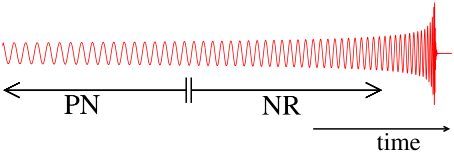
     $\Rightarrow$ There is **no good model** for BH/NS inspiral waveforms.

2. Compare PN with NR.
   (Need **enough NR orbits** to reach realm of PN validity.)

   Example: Boyle et al., PRD 76:124038 (2007)

   - $q = 1$, spin=0: PN accurate $\sim$ 10 orbits before merger.

PN investigations

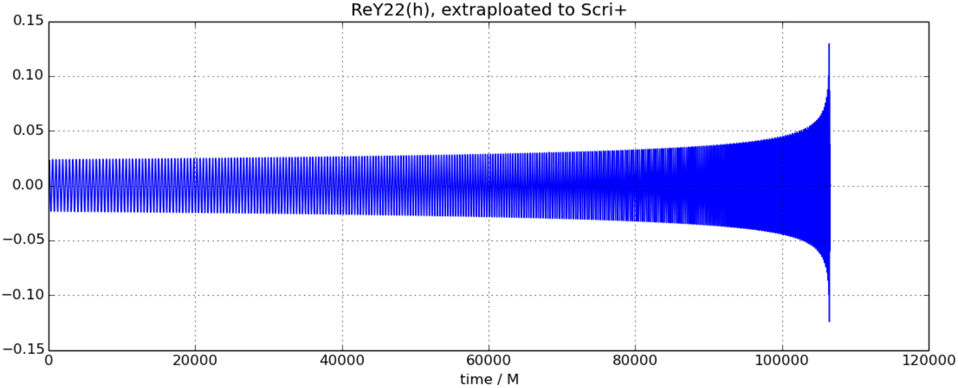


- We would like to
  - Determine how many NR orbits must be simulated for aLIGO.
    Boyle et al., PRD 76:124038 (2007)
  - Investigate PN accuracy.
    MacDonald et al., PRD 87, 024009 (2013)
    Ossokine et al., in prep.
  - Build better waveform models for BH/NS inspirals.
    Barkett et al., in prep.

- Most SpEC simulations now follow 20+ orbits.
  Still **not enough** for spinning binaries.

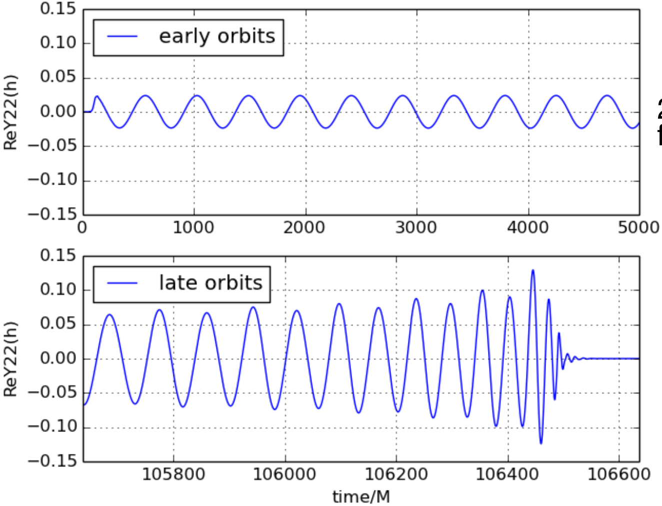
175 orbit simulation

- 7:1 mass ratio
- zero spin
- 175 orbits, merger, ringdown
- 4 numerical resolutions



Szilagyi, Blackman, MAS, et al., in prep.

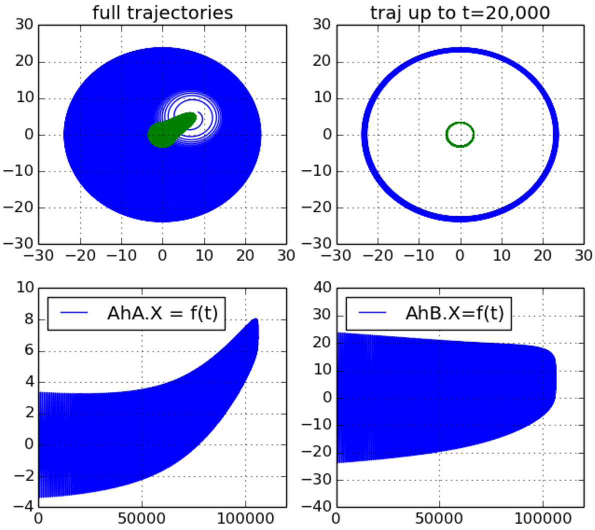
175 orbit simulation



2 orders of magnitude frequency variation.

175 orbit simulation

Unphysical center of mass motion



- Depends on outer bdry.
- Largely a gauge effect.
- Secondary effect on waveforms under investigation.

Capabilities and Challenges

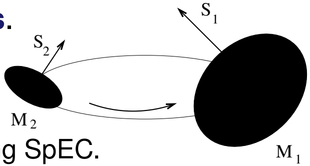
Template banks and parameter space coverage

How to use NR to assist LIGO detection of BBHs?

- Use NR to calibrate **analytical models** of waveforms.
  Example: "Effective One Body" model (fitted to SpEC waveforms).
  Pan et al., PRD 84:124052 (2011)
  Taracchini et al., PRD 86:024011 (2012), PRD 89:061502(R) (2014)

- Use NR as 'signals' to **test LIGO detection algorithms**.
  - Numerical Injection Analysis (NINJA) project.
    8 NR groups+LIGO Scientific Collaboration
    Aasi et al., arXiv:1401.0939 (2014)

- Use NR to **generate waveform templates**.
  - Difficult:
    - 7D parameter space
    - Simulations are expensive.
  - Can do for 1D subspace (mass ratio) using SpEC.
    Kumar et al., PRD 89, 042002 (2014)
  - May be possible in 7D with "reduced basis" methods.
    Blackman et al., arXiv:1401.7038 (2014)



Simulation catalogs

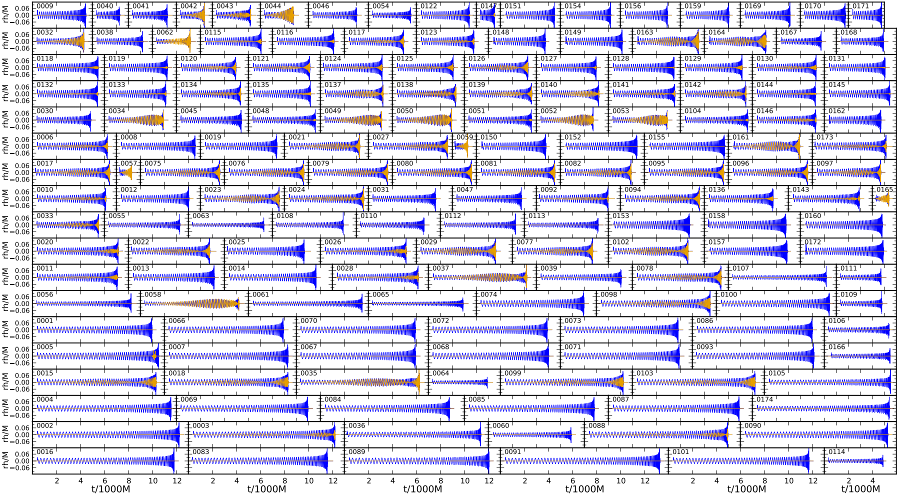
NR simulations for LIGO science

- NRAR (Numerical Relativity/Analytical Relativity) project
  - Goal: Improve analytic waveform models using NR simulations.
  - 9 NR codes.
  - 25 simulations

    Hinder et al., CQG 31 025012 (2014)
- NINJA (Numerical INJection Analysis) collaboration
  - Goal:
    - Add numerical waveforms into LIGO/Virgo detector noise
    - Test how well detection pipelines can detect/identify them
  - 8 NR groups.
  - 60 waveforms, matched to PN.
- Georgia Tech
  - $\sim$ 200 simulations, Pekowsky et al., arXiv:1304.3176

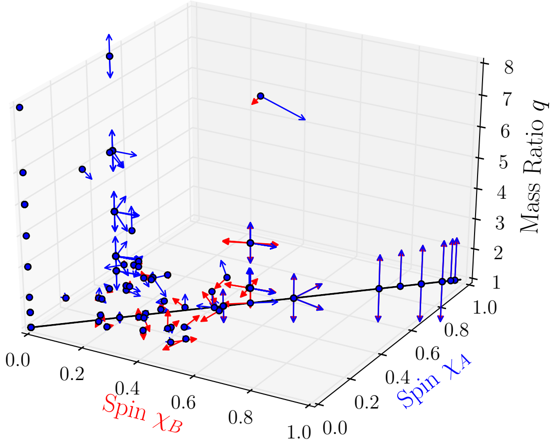
SpEC **public** simulation catalog

174 simulations; `www.black-holes.org/waveforms`



Mroue, MAS, et al., PRL 111:241104 (2013)

SpEC simulation catalog: parameter space coverage



Mroue, MAS, et al., PRL 111:241104 (2013)

- Red/blue arrows = Initial spin directions
- Spins up to 0.97
- Mass ratios up to 8

- Very sparse coverage!

Summary

- Binary black hole simulations are now mature.
- Now being used for LIGO science and studies of strong-field compact object interactions.

In the future . . .

- Push current limitations on mass ratios, spins, number of orbits.
- Enlarge catalog, informed by reduced basis.
- Further comparisons with analytic approximations.
- Improve accuracy to levels necessary for LISA-like missions.
- Comparisons with future events detected by LIGO.