Key idea

Manifold

$p \prec q \iff$ you can travel from p to q without going faster than the speed of light

Manifold $\implies (\prec \quad \leftrightarrow \quad g_{\mu\nu}/|g|)$

General

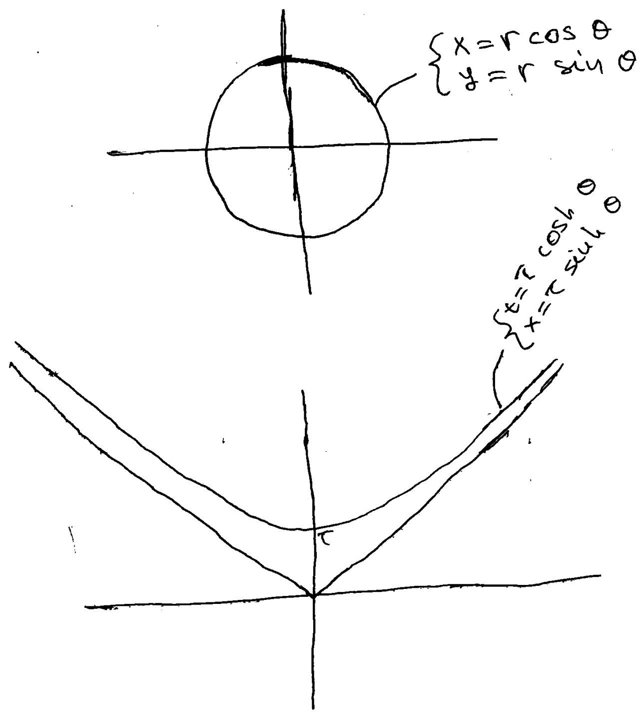
$\prec$ well defined, $g_{\mu\nu}$ is not

(Some conditions on $\prec$) $\implies$ Manifold $\implies$ Existence of $g_{\mu\nu}$

Above conditions on $\prec$ are unknown!

Key motivation : Manifold structure breaks down on small scales due to quantum fluctuations

Non-locality of Lorentz neighborhood



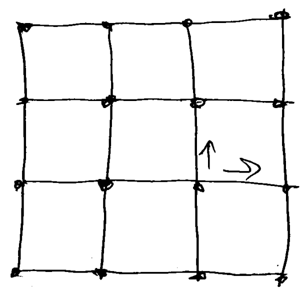
$$t = \sqrt{\tau^2 + r^2} = r\sqrt{1 + \frac{\tau^2}{r^2}} = r\left(1 + \frac{\tau^2}{2r^2}\right) + 0(\tau^2) = r + \frac{\tau^2}{2r} + 0(\tau^2) \tag{1}$$

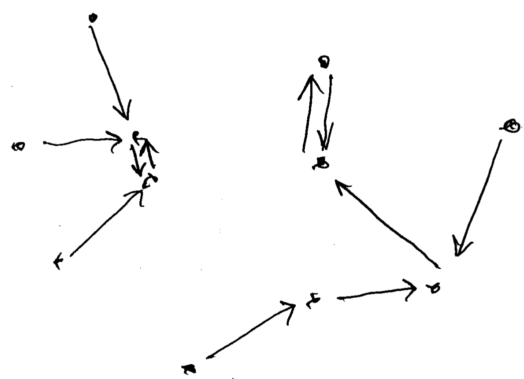
$$t > r + (1 - \epsilon)\frac{\tau^2}{2r} \tag{2}$$

$$\int (t - x)dx > \frac{(1 - \epsilon)\tau^2}{2} \int \frac{dx}{x} = \infty \tag{3}$$

$$\int (t - r)4\pi r^2 dr > \frac{4\pi(1 - \epsilon)\tau^2}{2} \int r dr = \infty \tag{4}$$

"Local" discrete theories result in "preferred frame"





In Lorentzian case "can't choose" nearest neighbor

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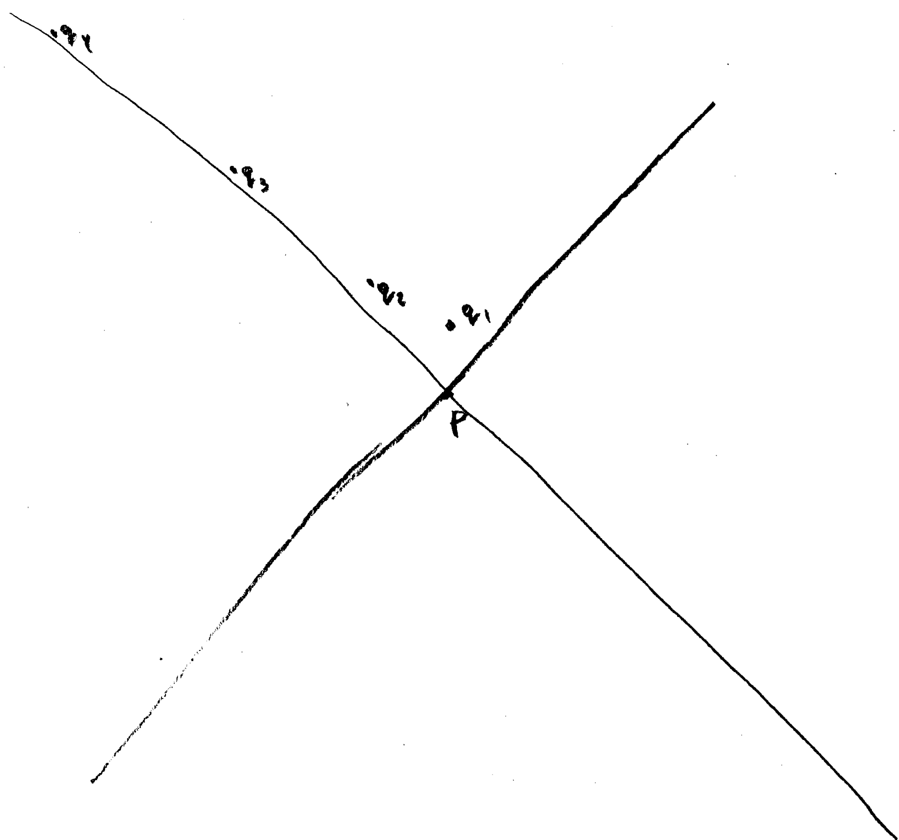


Problem with discrete theories

Finite (1) Lorentz (2) neighborhood is non-local (NO 3)

Therefore

Discreteness (1), Lorentz covariance (2) and Locality (3) can NOT co-exist

Continuum QFT: Keep 2 and 3; discard 1

— Continuity $\Longrightarrow$ No nearest neighbor $\Longrightarrow$ no preferred frame

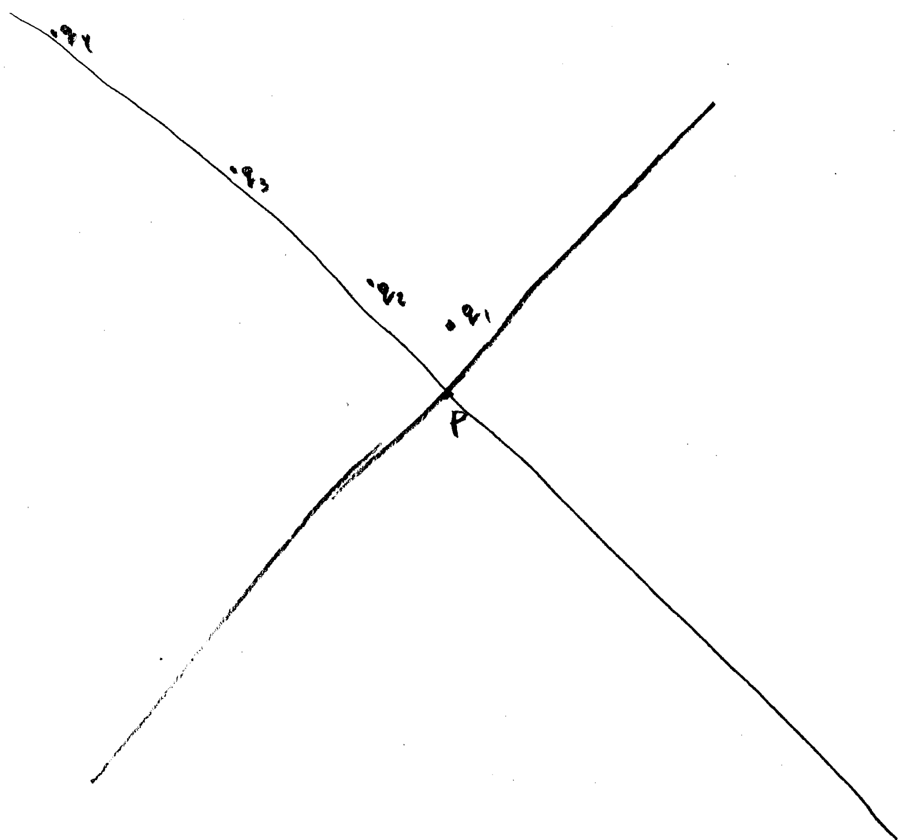
Lattice theory: Keep 1 and 3; discard 2

— Nearest neighbor defines preferred frame

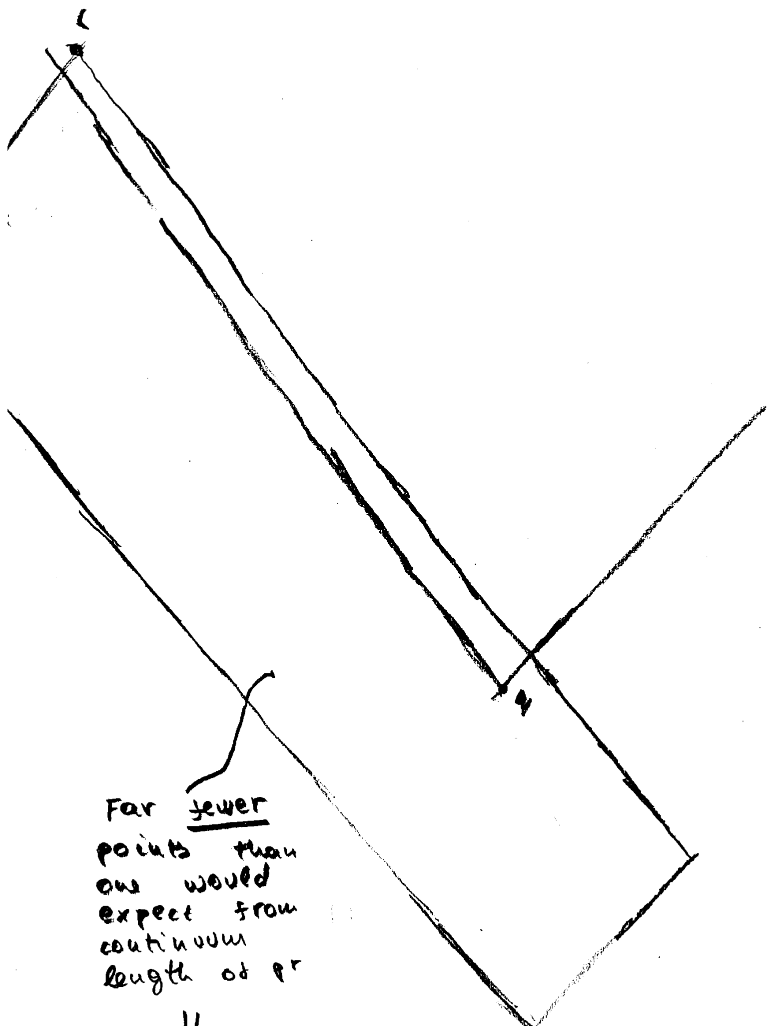
Causal set theory: Keep 2 and 3; discard 1

— Lightcone nonlocality $\Longrightarrow$ No nearest neighbor $\Longrightarrow$ no preferred frame

Non-locality in causal set



Violation of Lorentzian (!!!) distance itself!



$\Downarrow$

$$\left.\begin{array}{ll} r \in n_\tau(p) & \underline{\text{discrete}} \\ r \notin n_\tau(p) & \underline{\text{continuum}} \end{array}\right\} \Rightarrow \{r_1, r_2, \ldots\} \subset n_\tau(p) \ \underline{\text{discrete}}$$

Non-transitivity is the key

Conventional (transitive) causality:

Manifold case:

$a \prec b$ if and only if you can go from a to b without going faster than the speed of light

General case:

1. If $a \prec b$ and $b \prec c$ then $a \prec c$
2. There is NO element a satisfying $a \prec a$
3. For any $a \prec b$, the number of elements c satisfying $a \prec c \prec b$ is finite

Non-transitive causality:

Manifold case:

$a \prec b$ if and only if TWO things SIMULTANEOUSLY hold:

(i) One can go from a to b without going faster than the speed of light

(ii) If you take $c \prec d$ where $c \in \alpha(a,b)$ and $d \in \alpha(a,b)$ then

$$(1-\epsilon)v_0 \sharp\alpha(c,d) < Vol(\alpha(c,d)) < (1+\epsilon)v_0 \sharp\alpha(c,d) \tag{5}$$

where $\sharp$ is defined as

$$\sharp(A \subset \mathcal{M}) = \sharp\{p \in S | \mu(p) \in A\} \tag{6}$$

General case:

1. If $a \prec b$ and $b \prec c$ then $a \prec c$
2. Transitivity does NOT hold!
3. For any $a \prec b$, the number of elements c satisfying $a \prec c \prec b$ is finite

Fields on a causal set

General causal set

Scalar field: $\phi\colon S \mapsto \mathbb{R}$

Electromagnetic field: $a\colon S \times S \mapsto \mathbb{R}$

Lagrangian: $\mathcal{L}\colon \{\phi, a; p\} \mapsto \mathbb{R}$

Manifold-like case

Mapping: $\mu\colon S \mapsto \mathcal{M}$ where $\mathcal{M}$ is a 4-manifold

Scalar field: $\phi_S(p) = \phi_{\mathcal{M}}(\mu(p))$

Electromagnetic field: For any p and q, let $\Gamma(p,q)$ be the geodesic connecting them and let $a(p,q)$ be given as

$$a(p,q) = \int_{\Gamma(p,q)} g_{\mu\nu} A^{\mu} dx^{\nu} \tag{7}$$

Goal: $\mathcal{L}_S(\phi_S, a) \approx \mathcal{L}_{\mathcal{M}}(\phi_{\mathcal{M}}, A^{\mu})$ *if* $\phi_{\mathcal{M}}$ and A^{μ} are linear. Here,

$$\mathcal{L}_{\mathcal{M}}(\phi_{\mathcal{M}}, A^{\mu}) = \frac{1}{2}\partial^{\mu}\phi_{\mathcal{M}}\partial_{\nu}\phi_{\mathcal{M}} - \frac{m^2}{2}\phi_{\mathcal{M}}^2 + \frac{1}{4}(\partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu})(\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}) \tag{8}$$

NOTE:

1. A^{μ} and ∂_{μ} are defined ONLY for manifold case

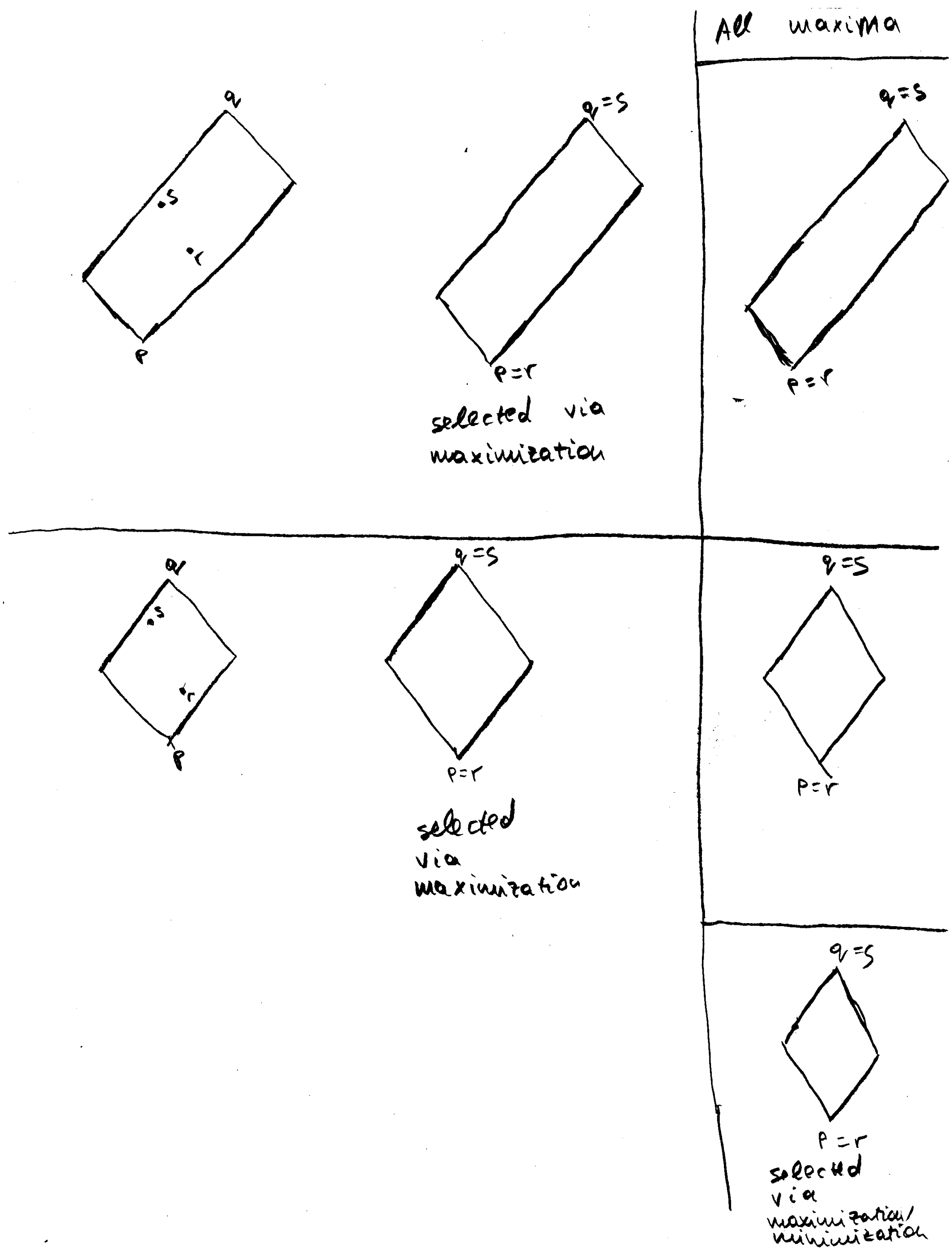
2. $a(p,q)$ is defined for all cases

3. Eq 8 holds only if ϕ and A^{μ} are approximately linear

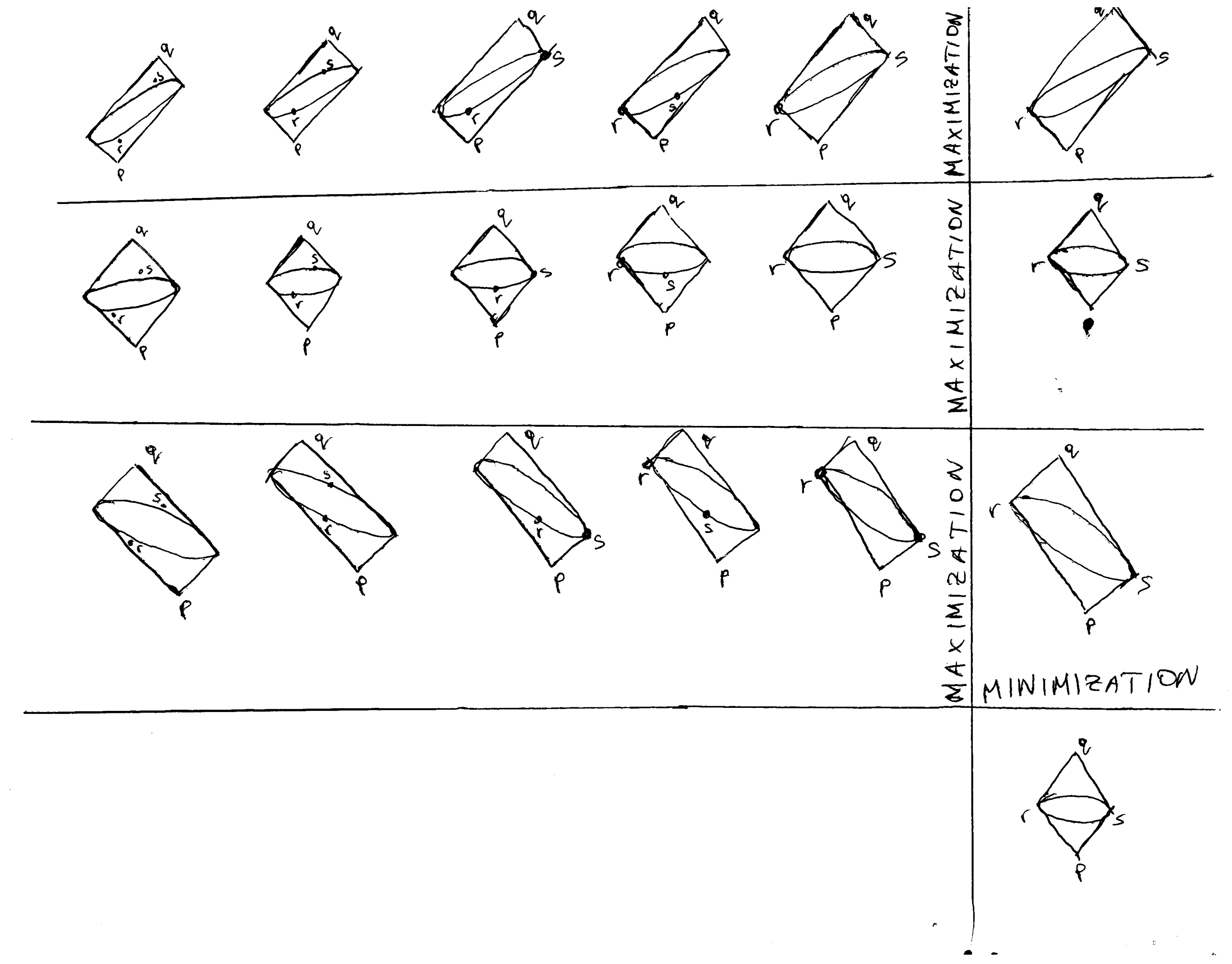
4. Both sides of Eq 8 hold even in non-linear case, it is simply that we can no longer put approximation sign between two sides (THIS IS WHERE SELF-INTERACTION COMES IN)

5. The approximate linearity stops holding once we move close to the speed of light

Maximization/minimization if gradient is timelike



Maxiomizatioin/minimization if gradient is spacelike



GRADIENT TIMELIKE:

In Reference frame of Alexandrov set,

$$|\partial_0\phi| \approx \min_{\tau(p,q)\geq\tau_0} \max r, s \in \alpha(p,q) \frac{|\phi(r) - \phi(s)|}{\tau(p,q)} \tag{9}$$

IMPLICATION OF MAXIOMUM: $r \approx p$ and $s \approx q$

IMPLICATIONS OF MINIMUM

IMPLICATION OF MINIMUM:

1. Segment pq almost coincides with gradient
2. Reference frame is selected in which the field is well behaved

In general reference frame,

$$\partial^\mu\phi\partial_\mu\phi \approx \min_{\tau(p,q)\geq\tau_0} \max r, s \in \alpha(p,q) \left(\frac{|\phi(r) - \phi(s)|}{\tau(p,q)}\right)^2 \tag{10}$$

GRADIENT SPACELIKE:

In Reference frame of Alexandrov set,

$$|\vec{\nabla}\phi| \approx \min_{\tau(p,q)\geq\tau_0} \max r, s \in \alpha(p,q) \frac{|\phi(r) - \phi(s)|}{\tau(p,q)} \tag{11}$$

IMPLICATION OF MAXIOMUM: r and s lie on "equator" of causal set

IMPLICATIONS OF MIMINUM

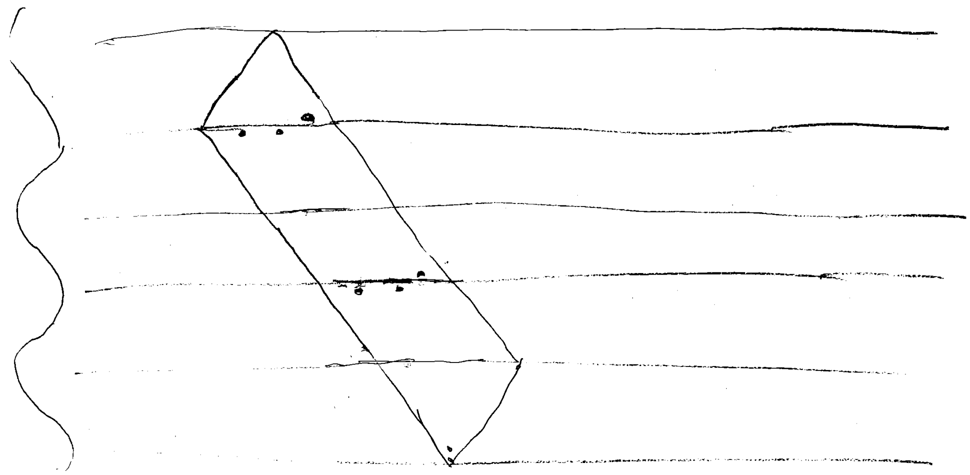
IMPLICATION OF MINIMUM:

1. Segment pq almost coincides with gradient
2. Reference frame is selected in which the field is well behaved

In general reference frame,

$$\sqrt{\partial^\mu\phi}\partial_\mu\phi \approx -\min_{\tau(p,q)\geq\tau_0} \max r, s \in \alpha(p,q) \left(\frac{|\phi(r) - \phi(s)|}{\tau(p,q)}\right)^2 \tag{12}$$

Low fluctuation "by accident"



ACCIDENT GUARANTEED BY LARGE NUMBER OF FRAMES

How to avoid selecting the above "accident"

Problem:

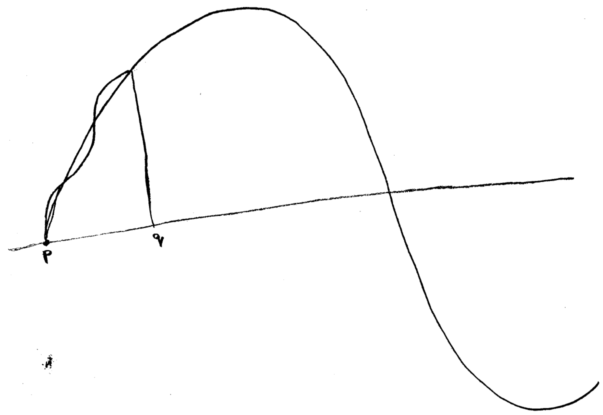
If we move fast enough to speed of light, we can find a frame in which the variation is arbitrary small

Solution:

Non-transitivity $\Longrightarrow$ Locality around the "edge" $\Longrightarrow$ Replace $\mathcal{L}(\phi, a; p)$ with $\mathcal{L}(\phi, a; p_1, p_2)$ $\Longrightarrow$ "velocity dependence" of Lagrangian which is trivial unless velocity is close to the speed of light

NOTE: "Close to the speed of light" is in reference to the reference frame of the "edge" $\Longrightarrow$ Lorentz covariance is preserved, in analogy with "translational covariance"

High frequency/ low amplitude is suppressed



The way waves are being suppressed

1. If the time gradient of ”small frequency” wave is much larger than the one of ”large frequency” wave, then the former cominates

2. If the small frequency wave moves very fast relative to large frequency wave, the above will be the case in the reference frame of large frequency wave

3. Gradient setup $\Longrightarrow$ large frequency wave contributes in its own reference frame

4. (2, 3) $\Longrightarrow$ if the large frequency wave moves fast with respect to small frequency wave, then the former doesn't contribute much

5. 4 $\Longrightarrow$ There is a velocity cutoff set around the reference frame of small frequency wave

6. 5 $\Longrightarrow$ If many small frequency waves move very fast relative to each other, then NONE of large frequency waves contribute!

7. 5,6 $\Longrightarrow$ The histories that contribute tend to single out preferred frames

8. The reason 7 doesn't violate relativity is that we are integrating over all possible preferred frames

9. A problem in quantum foundations: when fields are being measured, do they create preferred frame?

$$|\vec{E}|^2 - |\vec{B}|^2 = F^{\mu\nu}F_{\mu\nu} \tag{13}$$

$$\vec{E} \cdot \vec{B} = \frac{1}{k}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta} \tag{14}$$

$$\vec{E} \cdot \vec{B} = |\vec{E}||\vec{B}|\cos\theta \tag{15}$$

Case 1: $|\vec{E}| > |\vec{B}| \iff F^{\mu\nu}F_{\mu\nu} > 0$

$$\frac{1}{k}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta} = |\vec{B}|\sqrt{F^{\mu\nu}F_{\mu\nu} + |\vec{B}|^2}\cos\theta = |\vec{B}|\sqrt{|F^{\mu\nu}F_{\mu\nu}| + |\vec{B}|^2}\cos\theta \tag{16}$$

Case 2: $|\vec{E}| < |\vec{B}| \iff F^{\mu\nu}F_{\mu\nu} < 0$

$$\frac{1}{k}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta} = |\vec{E}|\sqrt{-F^{\mu\nu}F_{\mu\nu} + |\vec{E}|^2}\cos\theta = |\vec{E}|\sqrt{|F^{\mu\nu}F_{\mu\nu}| + |\vec{E}|^2}\cos\theta \tag{17}$$

Common feature:

$$\text{Minimize } |\vec{E}| \iff \text{Minimize } |\vec{B}| \iff \text{Maximize } |\cos\theta| \implies \theta \in \{0, \pi\} \tag{18}$$

Covariant expressions for "electric" and "magnetic" fields

$$E^2 - B^2 = F^{\mu\nu}F_{\mu\nu} \tag{19}$$

$$EB = \frac{1}{k}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta} \tag{20}$$

$$E^2F^{\mu\nu}F_{\mu\nu} = E^2(E^2 - B^2) = E^4 - (EB)^2 \tag{21}$$

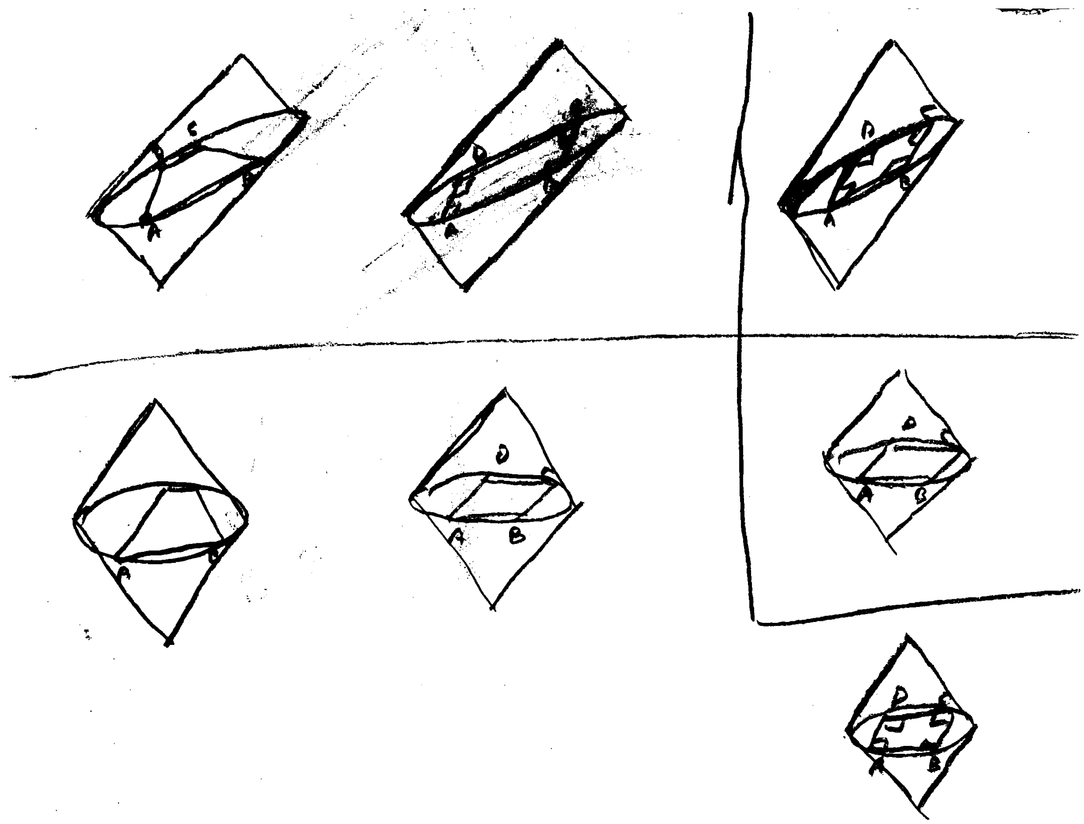
$$0 = E^4 - E^2F^{\mu\nu}F_{\mu\nu} - (EB)^2 \tag{22}$$

$$0 = E^4 - E^2F^{\mu\nu}F_{\mu\nu} - \frac{1}{k^2}(\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta})^2 \tag{23}$$

$$E^2 = \frac{1}{2}\left(F^{\mu\nu}F_{\mu\nu} + \sqrt{(F^{\mu\nu}F_{\mu\nu})^2 + \frac{4}{k^2}(\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta})^2}\right) \tag{24}$$

$$B^2 = \frac{1}{2}\left(-F^{\mu\nu}F_{\mu\nu} + \sqrt{(F^{\mu\nu}F_{\mu\nu})^2 + \frac{4}{k^2}(\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta}F^{\gamma\delta})^2}\right) \tag{25}$$

Maxiomizatioin/minimization if gradient is spacelike



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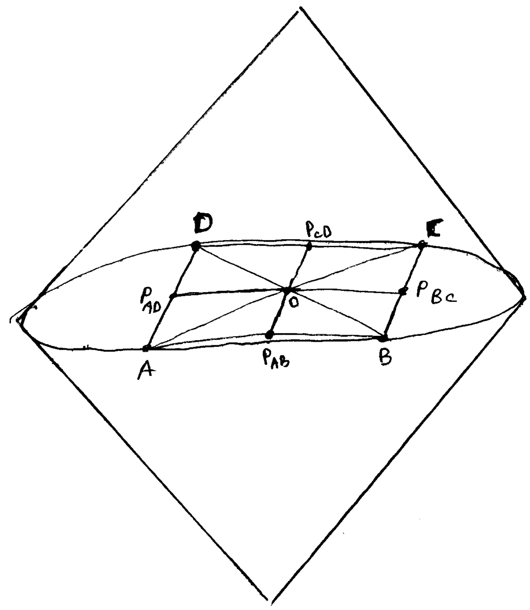
D
P_CD
C
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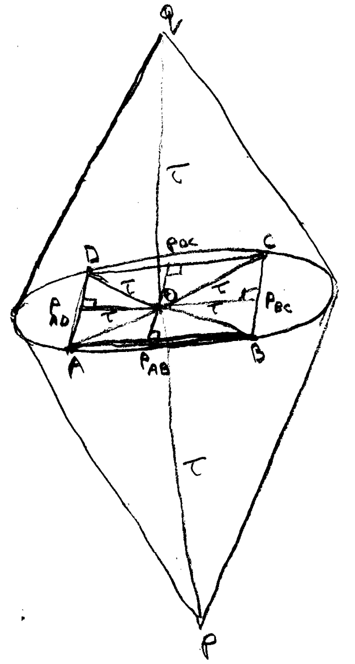
Q
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D
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P_{AD}
τ
τ
P_{BC}
A
P_{AB}
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τ
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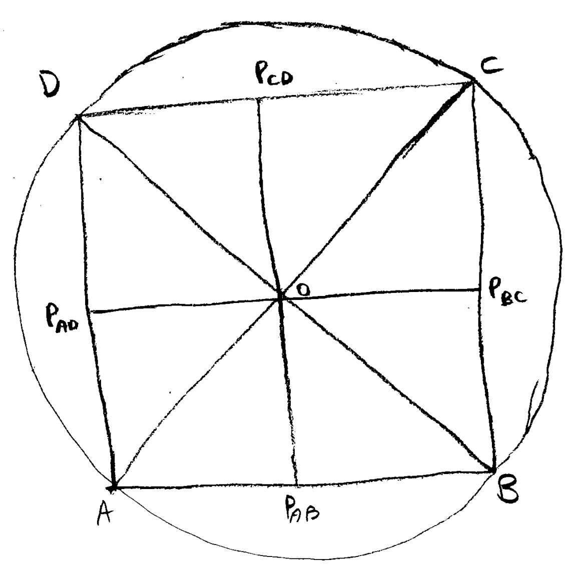
D
P_{CD}
C
O
P_{BC}
P_{AD}
A
P_{AB}
B

$$\text{Area of OAP}_{\text{AB}} = \frac{|AP_{AB}||OP_{AB}|}{2} = \frac{(\tau \sin AOP_{AB})(\tau \cos AOP_{AB})}{2} =$$
$$= \frac{\tau^2}{2} \sin AOP_{AB} \cos AOP_{AB} = \frac{\tau^2}{4} \sin(2 \times AOP_{AB}) = \frac{\tau^2}{4} \sin AOB \tag{26}$$

$$\text{Area of ABCD} = \frac{\tau^2}{4}(\sin AOB + \sin BOC + \sin COD + \sin DOA) \tag{27}$$

$$0 = \frac{\partial}{\partial AOB}\bigg|_{AOC=\text{const}} (\sin AOB + \sin BOC) =$$
$$= \frac{\partial}{\partial AOB}\bigg|_{AOC=\text{const}} (\sin AOB + \sin(AOC - AOB)) =$$
$$= \cos AOB - \cos(AOC - AOB) = 0 \tag{28}$$

$$\cos AOB - \cos(AOC - AOB) = 0 \Longrightarrow AOB = AOC - AOB \Longrightarrow AOB = BOC \tag{29}$$

$$\text{Same Logic} \Longrightarrow AOB = BOC = COD = DOA = \frac{\pi}{2} \tag{30}$$

$$(\text{Area of ABCD}) = 4(\text{Area of AOB}) = 4\frac{\tau^2}{2} = 2\tau^2 \tag{31}$$

$$(\text{Area of ABCD}) = (\text{Length of AB})^2 = (\tau\sqrt{2})^2 = 2\tau^2 \tag{32}$$

$$a(A, B) + a(B, C) + a(C, D) + a(D, A) = 2\tau^2 B \tag{33}$$

$$a(p, A) + a(A, q) + a(q, C) + a(C, p) = 2\tau^2 E \tag{34}$$

Non-linear corrections to Lagrangian

General setup: arxiv:0807.4709

Non-linear corrections: arXiv:1201.5850

Sources of non-linear corrections:

1. Non-linear behavior of a field itself within "fixed" neighborhood
2. Slight displacement of "poles" due to non-linearity of the field
3. Slight displacement of points on equator due to the same

1 $\Longrightarrow$ what we normally expect

2, 3 $\Longrightarrow$ what we don't expect

Lagrangian generators

Lagrangian generator is $\mathcal{K} = (\mathcal{K}_1, \mathcal{K}_2)$ where

1. $\mathcal{K}_1 \colon \{(p,q;\mathcal{F}) | p \prec q\} \mapsto S^a$ is a function so that if you enter $p \prec q$ it "returns" $(r_1, \cdots, r_a)$ each of which satisfies $p \prec r_k \prec q$. Physically, $r_1 \to r_2 \to \cdots \to r_a \to r_1$ is a "contour of integration" in "magnetic" case and $r_1 \to r_2$ (where $a = 2$) is a means of "discretized differentiation" in "scalar" case.

2. $\mathcal{K}_2 \colon S^a \times \{\mathcal{F}\} \to \mathbb{R}$ "takes" $(r_1, \cdots, r_a)$ and returns "Lagrangian estimate".

We then define Lagrangian to be

$$\mathcal{L}(s;\mathcal{F}) = \min_{p \prec s \prec q; \tau(p,s)=\tau(s,q)=(1/2)\tau(p,q)=\tau_0} \max_{(r_1,\cdots,r_a)\in \mathcal{K}_2(p,q;\mathcal{F})} \mathcal{K}_1(r_1, \cdots, r_a; \mathcal{F}) \tag{35}$$

Multiple Lagrangian generators, $\{(\mathcal{K}_1^{(1)}, \mathcal{K}_2^{(1)}), \cdots, (\mathcal{K}_1^{(n)}, \mathcal{K}_2^{(n)})\}$

$$\mathcal{L}^{(k)}(s) = \min_{p \prec s \prec q; \tau(p,s)=\tau(s,q)=(1/2)\tau(p,q)=\tau_0} \max_{(r_1,\cdots,r_a)\in \mathcal{K}_2^{(k)}(p,q;\mathcal{F})} \mathcal{K}_1^{(k)}(r_1, \cdots, r_a; \mathcal{F}) \tag{36}$$

$$\mathcal{L}^{(k)}(s;\mathcal{F}) = \sum_{k=1}^{n} \mathcal{K}^{(k)}(s;\mathcal{F}) \tag{37}$$

Conclusions

1. Locality, relativity and discreteness can not co-exist

2. To make them co-exist we have to say that relativity is broken NOT apriori BUT by the field itself that "creates" reference frame

3. If field creates reference frame, the theory becomes non-linear since one harmonic will propagate in the reference frame created by the other harmonic

4. Said non-linearity is a "self-force" I was talking about

5. Controversial point: is it okay for preferred frame to be "created" by external field?

a) Does this imply "ether" that was created a thousand years ago?

b) Is there any hope for analytic calculations???