Scalar self-force for highly eccentric orbits in Kerr spacetime

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in collaboration with

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Outline

Goals, overall plan of the computation

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Brief review of effective-source (puncture-function) regularization

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- *m*-mode decomposition
- separate 2+1D time-domain evolution for each mode
- worldtube scheme
  - finite differencing across the worldtube boundary
  - moving the worldtube
- computing the effective source and puncture function
- finite differencing near the particle (where fields are only C^2)
- mesh refinement

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Sample results

Conclusions, Plans, Lessons Learned

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Work in progress: some goals accomplished, some not yet!

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- worldtube moves in (r, θ) to follow the particle around the orbit
- Cauchy evolution
- fixed mesh refinement; some (finer) grids follow the worldtube/particle
- [now] (almost) causally-disconnected spatial boundaries
  (with mesh refinement this isn't very expensive)
- [future] hyperboloidal outer boundary
  (Zenginoğlu, arXiv:1008.3809 = J. Comp. Phys. 230, 2286)

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If φ_p "closely-enough" approximates $\varphi_{\text{singular}}$ near the particle, then the self-force is given (**exactly!**) by $F_a = q\,(\nabla_a\varphi_r)\big|_{\text{particle}}$

I.e., $S_{\text{effective}}$ compensates for the fact that $\varphi_p \neq \varphi_{\text{singular}}$

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- define "numerical field" $\varphi_{\text{num}} = \begin{cases} \varphi_r & \text{inside the worldtube} \\ \varphi & \text{outside the worldtube} \end{cases}$
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- we actually do m-mode decomposition before introducing worldtube
  $\Rightarrow$ worldtube "lives" in (t, r, θ) space, not full spacetime

Initial data, boundary conditions

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Boundary Conditions:

- [now] use almost causally-disconnected spatial boundaries
  (with mesh refinement this isn't very expensive)
- [future] hyperboloidal outer boundary
  (Zenginoğlu, arXiv:1008.3809 = J. Comp. Phys. 230, 2286)

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Current Status

Equatorial eccentric orbits:

- elliptic-integral puncture fn & effective src
- worldtube moves in (r, θ) to follow the particle around the orbit
- fixed mesh refinement: typically 5 levels, $\Delta r_* = M/4$ to $M/64$;
  finest 3 refinement levels move to follow worldtube
- typical worldtube size particle $\pm 5M$ in r_*, particle $\pm \pi/8$ (22.5°) in θ
- 4th order finite differencing in space & time

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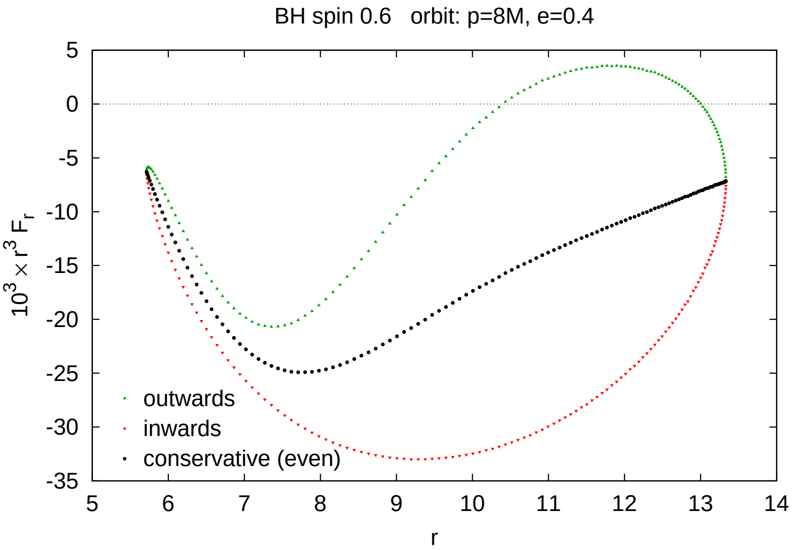
Generic (inclined eccentric) orbits:

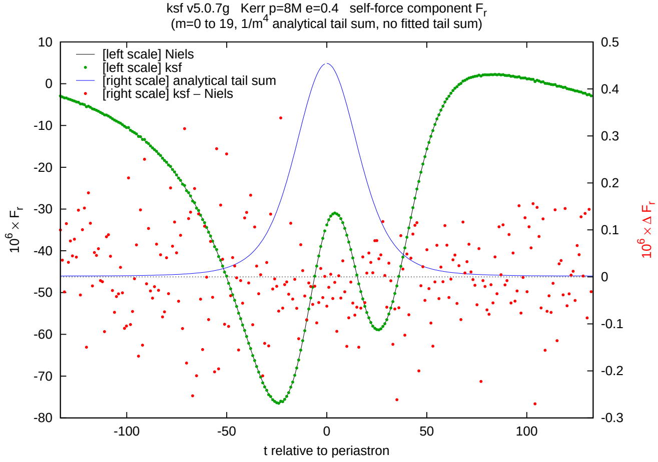
- our first attempt at an effective source had ~ 20 million terms
  $\Rightarrow$ impractical to compile machine-generated C code
- we are starting to explore various ideas to reduce the complexity, and are optimistic we can solve this

Self-force for $e = 0.4$ orbit



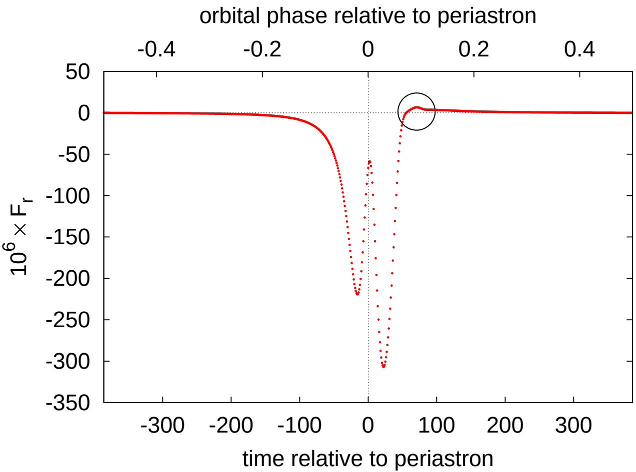
Comparison with Frequency-Domain Results

Big thanks to **Niels Warburton (Dublin)** for frequency-domain results!



Wiggles/Ripples (1)

We think these are caused by the particle (outbound) crossing the future light cone of its inbound trajectory. See Niels Warburton talk.



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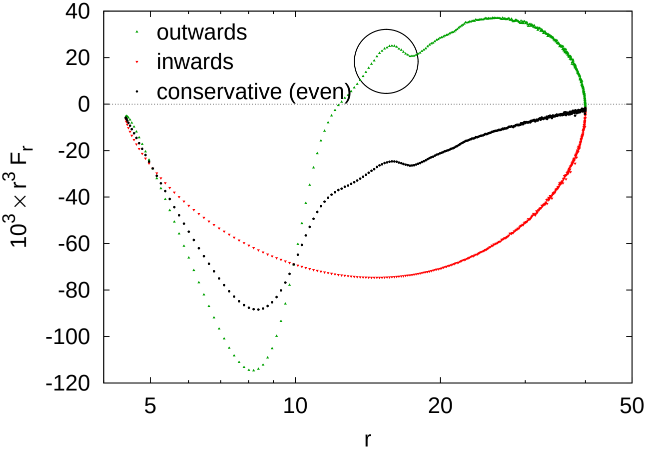
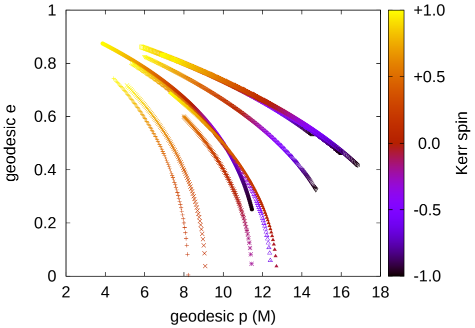


Isofrequency Kerr Geodesics

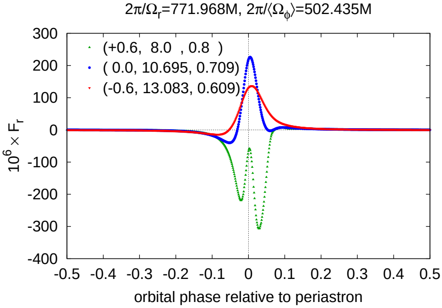
Basic idea:

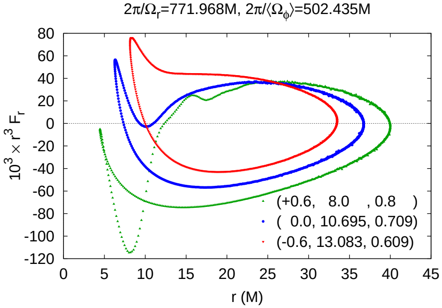
- parameterize bound equatorial geodesics in Kerr spacetime by (a, p, e) where $a =$ Kerr spin and (p, e) characterize the geodesic
- since $\Omega_r = \Omega_r(a, p, e)$ and $\langle \Omega_\phi \rangle = \langle \Omega_\phi \rangle(a, p, e)$, in general we can find a 1-parameter family of (a, p, e) with the **same $\boldsymbol{\Omega_r}$ and $\boldsymbol{\langle \Omega_\phi \rangle}$**
- that is, these are geodesics in **different-spin Kerr spacetimes** which have the **same dominant GW frequencies**



Self-force for Isofrequency Kerr Geodesics

Very different self-forces $\Rightarrow$ very different orbital evolutions $\Rightarrow$ break degeneracies





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Highly eccentric orbits:

- numerical errors & cost per unit of evolution time
  seem to be only weakly dependent on eccentricity
- main limits are cost due to long orbital period ⇒ long evolution time

Directions for further research

Things that don't (yet) work well:

- $S_{\text{effective}}$ is very expensive to calculate ($\gtrsim \frac{2}{3}$ of code's CPU time)
- dynamically adjust mesh refinement and/or worldtube around the orbit
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