NUMERICAL APPROACHES TO CALCULATING THE SELF-FORCE AND RELATED QUANTITIES

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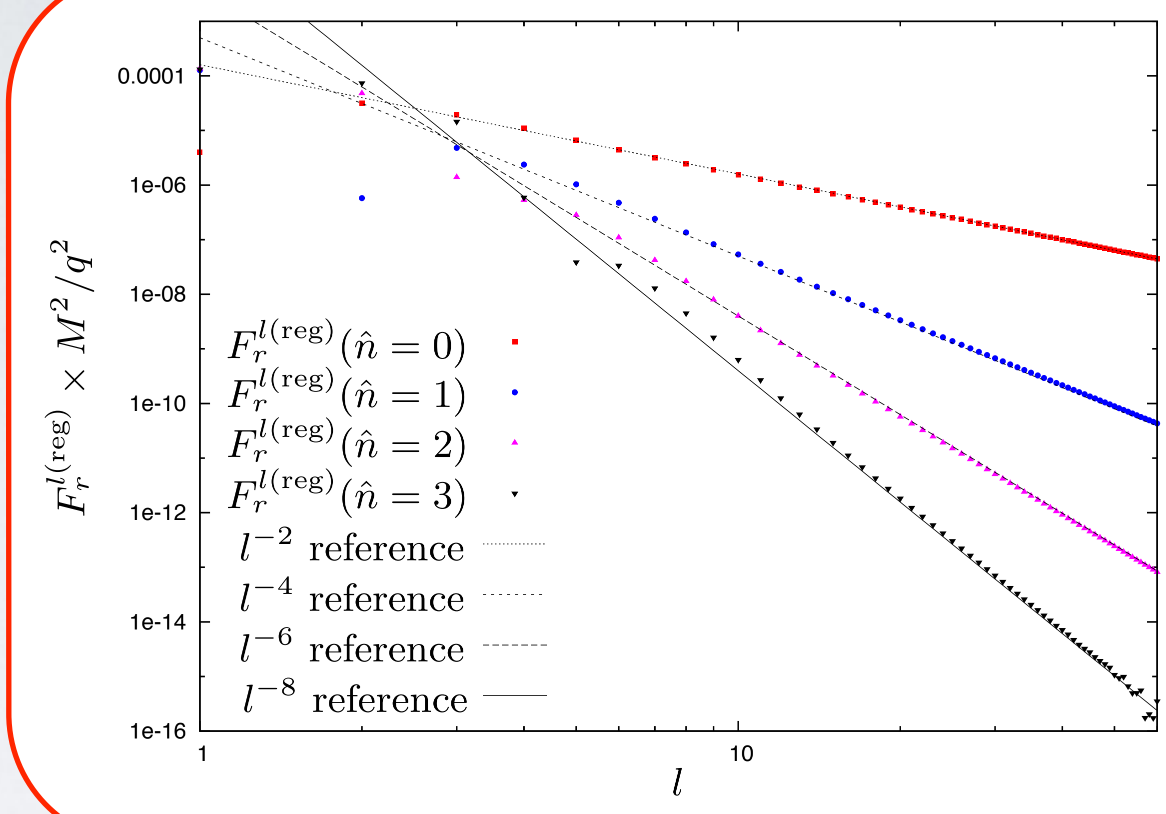
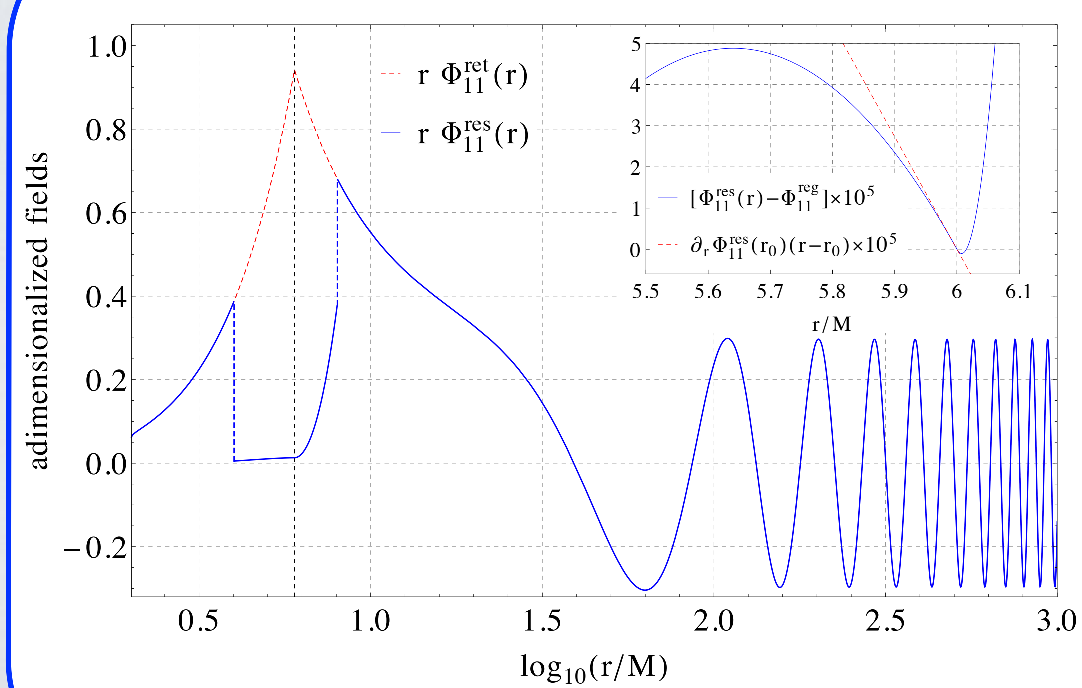
Three main approaches

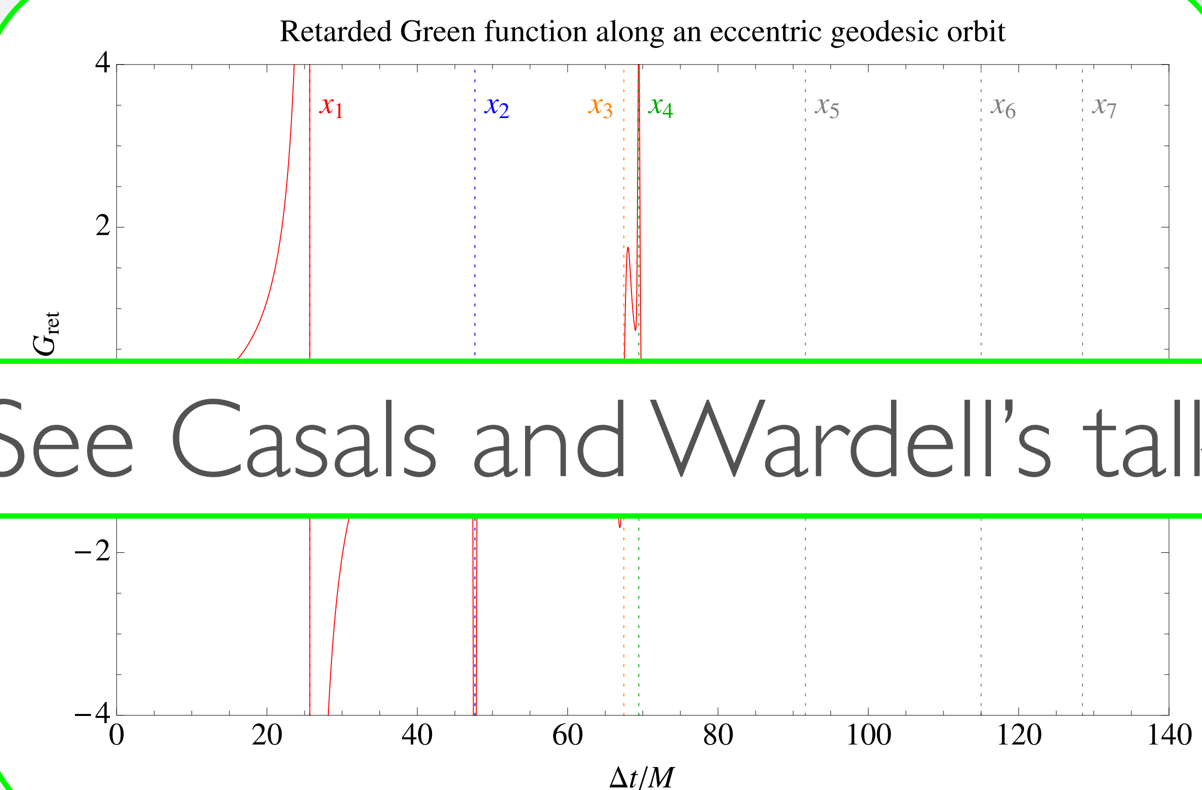
Mode-sum

Effective-source

Green function







See Casals and Wardell's talk

Mode-sum

$$F_\alpha = \nabla_\alpha \Phi^R \qquad \Box\Phi^{\mathrm{ret}/S} = -4\pi\rho \qquad \Box\Phi^R = 0$$

Decompose into spherical-harmonics

$$\Phi^R(x_p) = \lim_{x\to x_p}[\Phi^{\mathrm{ret}}(x) - \Phi^S(x)] \qquad \Phi_l^{R/\mathrm{ret}/S} = \sum_{m=-l}^{l} Y_{lm}\oint \Phi^{R/\mathrm{ret}/S}Y^*_{lm}(\theta,\varphi)d\Omega$$

$$\Phi^R(x_p) = \sum_{l=0}^{\infty}[\Phi_l^{\mathrm{ret}}(x_p) - \Phi_l^S(x_p)]$$

See Heffernan's talk

$$\boxed{\Phi^R = \sum_{l=0}^{\infty}[\Phi^{\mathrm{ret}} - ZL^2 - AL - B - CL^{-1}] - \frac{D_1}{(2l-1)(2l+3)} + \ldots}$$

Naturally suited to scalar-fields in Schwarzschild spacetime.

Less so for:

- Kerr spacetime
- Gravitational perturbations

See NWs talk

$$\bar{h}_{\alpha\beta} = \sum_{lm}\sum_{i=1}^{10}\bar{h}^{(i)lm}Y^{(i)lm}_{\alpha\beta}$$

Effective-source $F_\alpha = \nabla_\alpha \Phi^R$ $\quad \Box\Phi^{\mathrm{ret}/S} = -4\pi\rho$ $\quad \Box\Phi^R = 0$

Basic idea is to move the singular field into the source

$$\Box\Phi^{\mathrm{ret}} = \Box(\Phi^R + \Phi^S) \quad \longrightarrow \quad \Box\Phi^{\mathrm{res}} = -4\pi\rho - \Box(\mathcal{W}\Phi^P)$$

Mode-sum relies on lm-modes being finite at the particle
Use effective-source otherwise, e.g.,

- 2+1 calculations
- 3+1 calculations
- 2nd perturbative order

See Thornburg's talk

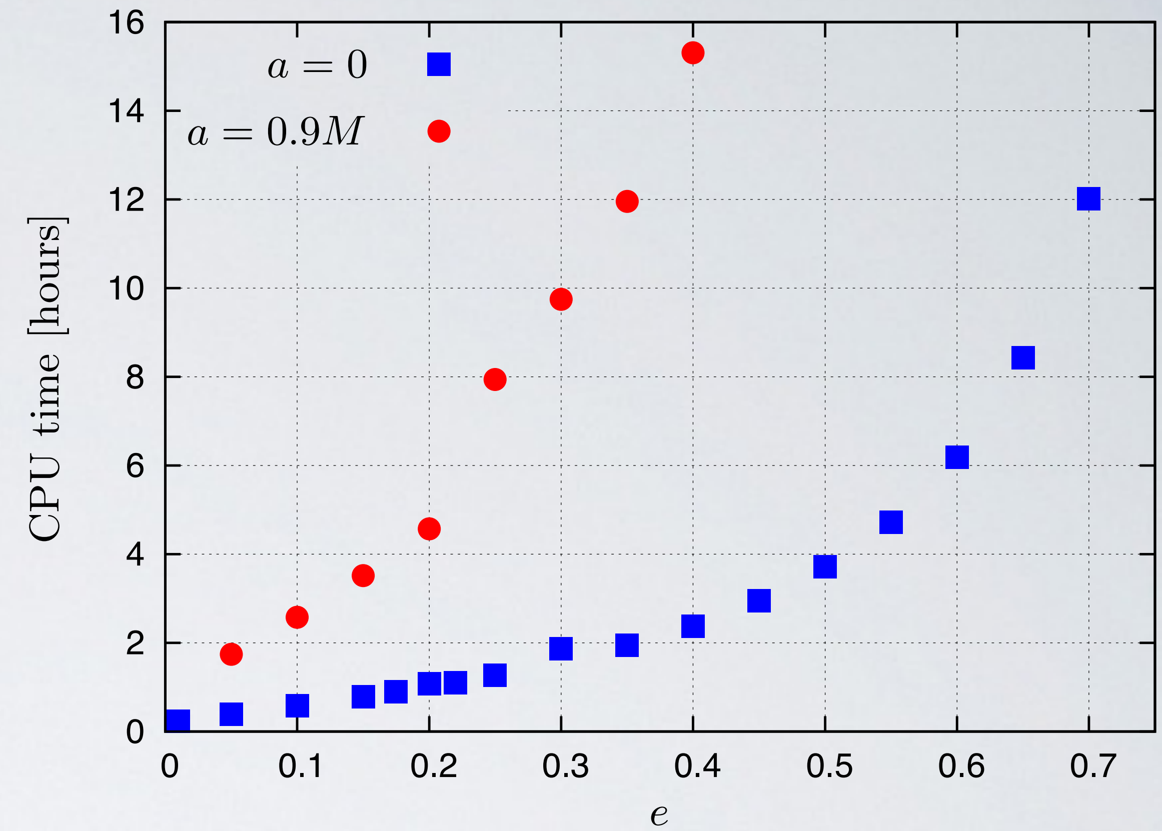
See NWs talk

As with mode-sum, a higher-order puncture improves the numerical calculation

Frequency-domain

$$\Phi^{\text{ret}} = \int \sum_{lm} R(r) Y(\theta, \varphi) e^{-i\omega t} \, d\omega$$

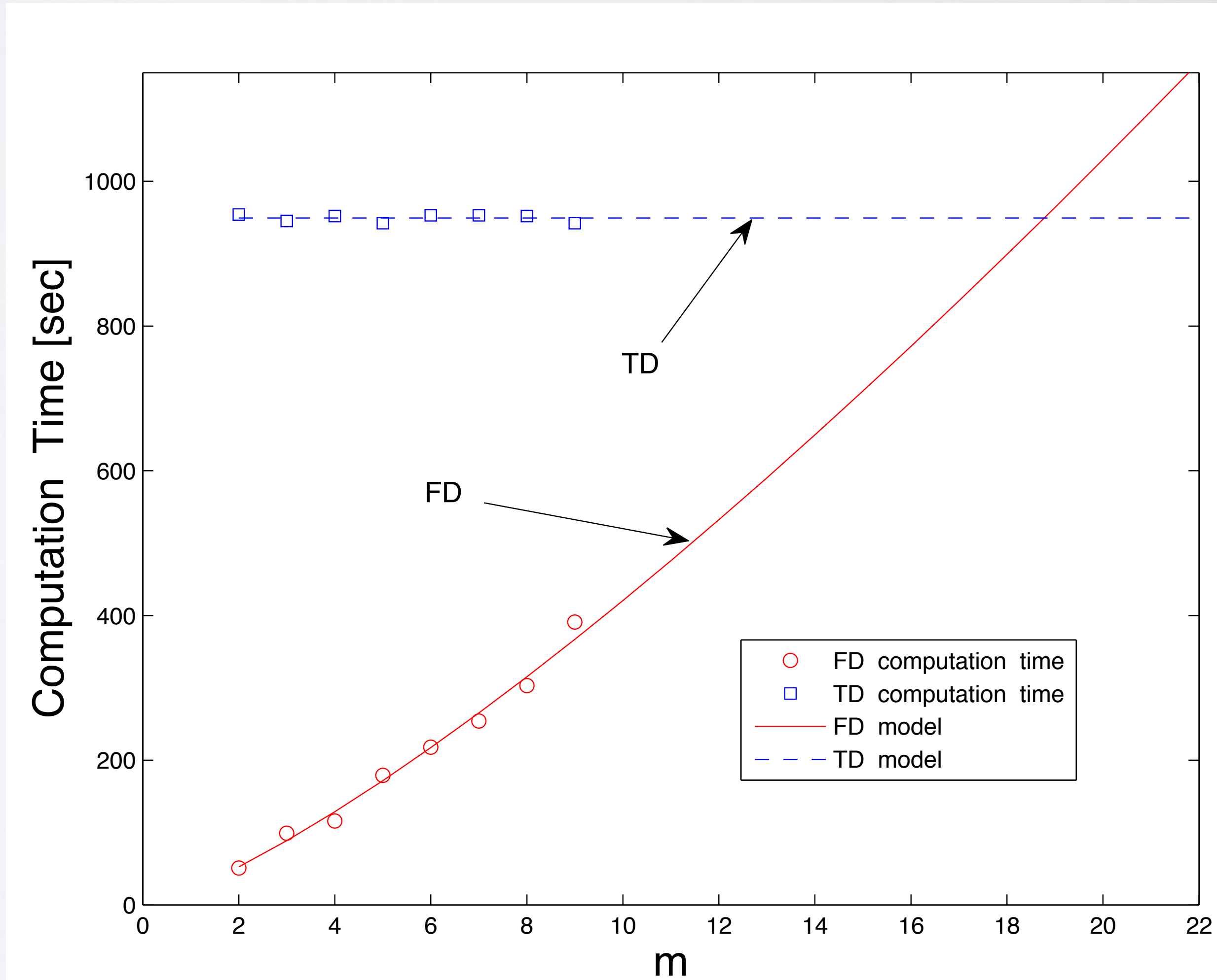
- For geodesic motion the spectrum is discrete
- Solve ODEs



Time-domain

- Finite difference methods
- Pseudo-spectral methods
- Hyperboloidal layers

High eccentricity orbits
Self-consistent evolution



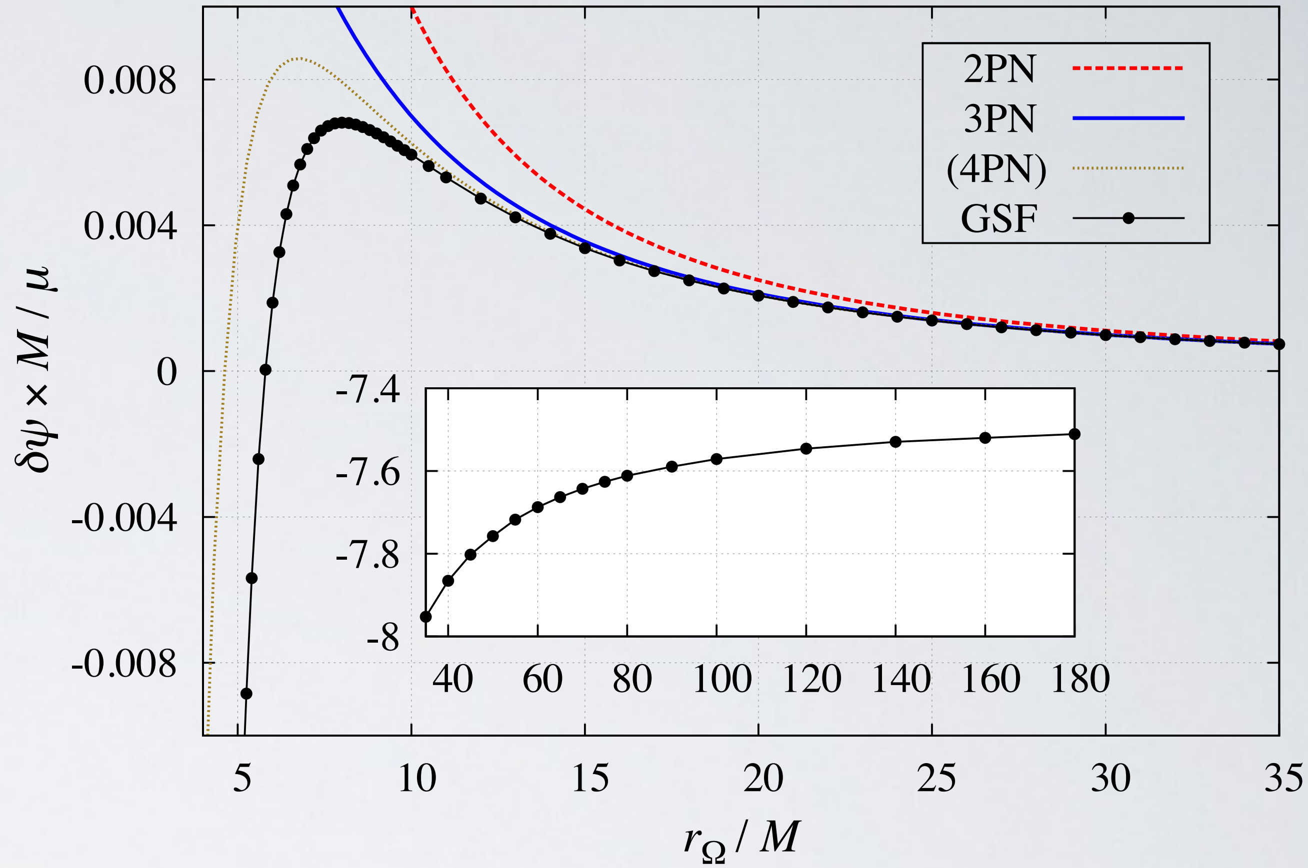
From Barton etal. (2008)

Recent results: gauge-invariant quantities

Before 2013: ΔU

After 2013: $\Delta\psi, \Delta\lambda_i^{E/B}, \Delta\chi$

See Dolan's talk



Comparisons with PN, NR and calibration of EOB

Desire for highly accurate numerics

$$\Delta U \propto r_0^{-1}$$

$$\Delta\psi \propto r_0^{-2}$$

$$\Delta\lambda_i^{E/B} \propto r_0^{-3}$$

$$\boxed{\Delta^R = \sum_{l=0}^{\infty}(\Delta^{\rm ret} - ZL^2 - AL - B)}$$

e.g., At $r_0 = 5000M$ $\quad \Delta U^{\rm err} = 10^{-18}$

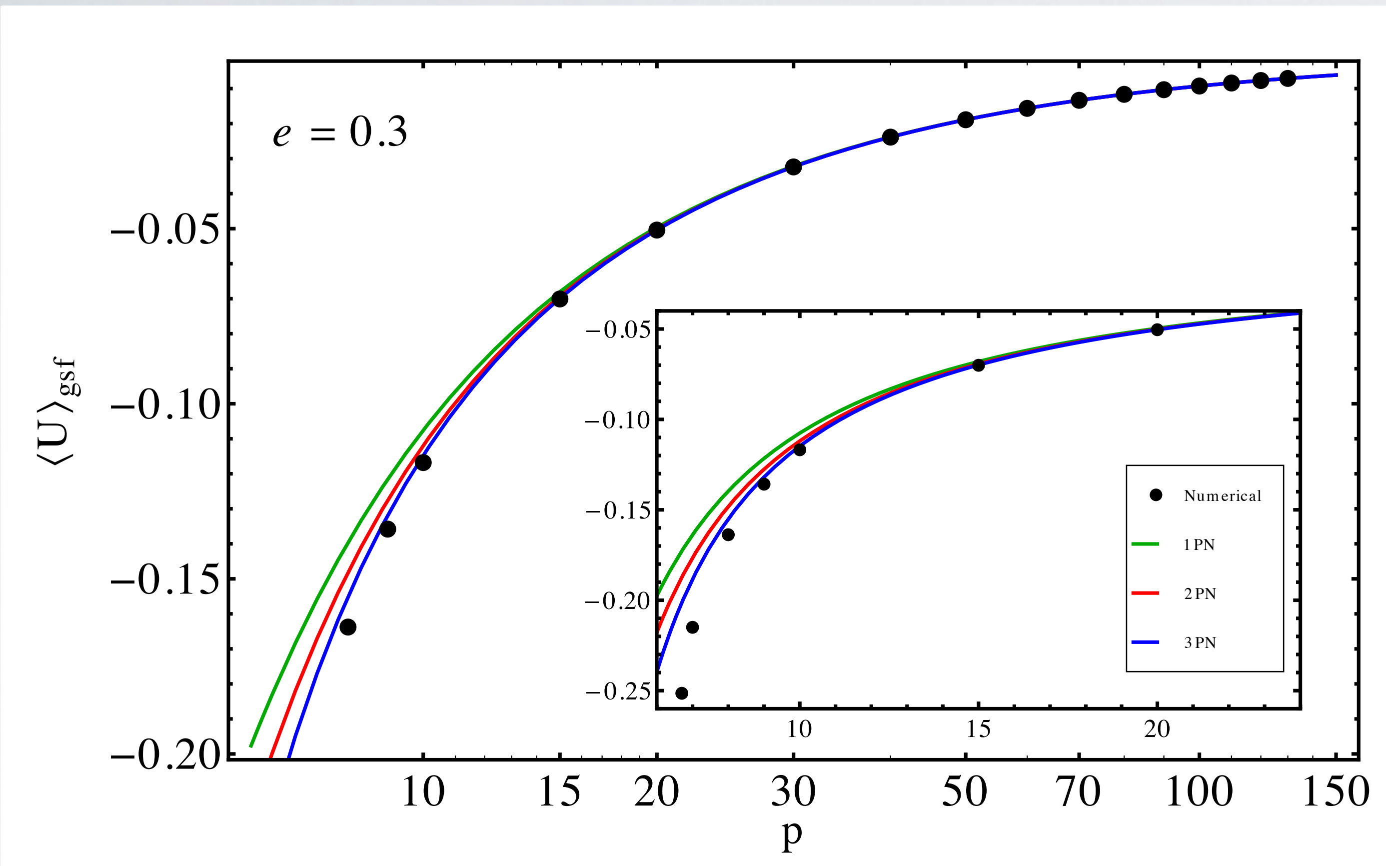
$$\Delta\psi^{\rm err} = 10^{-15}$$

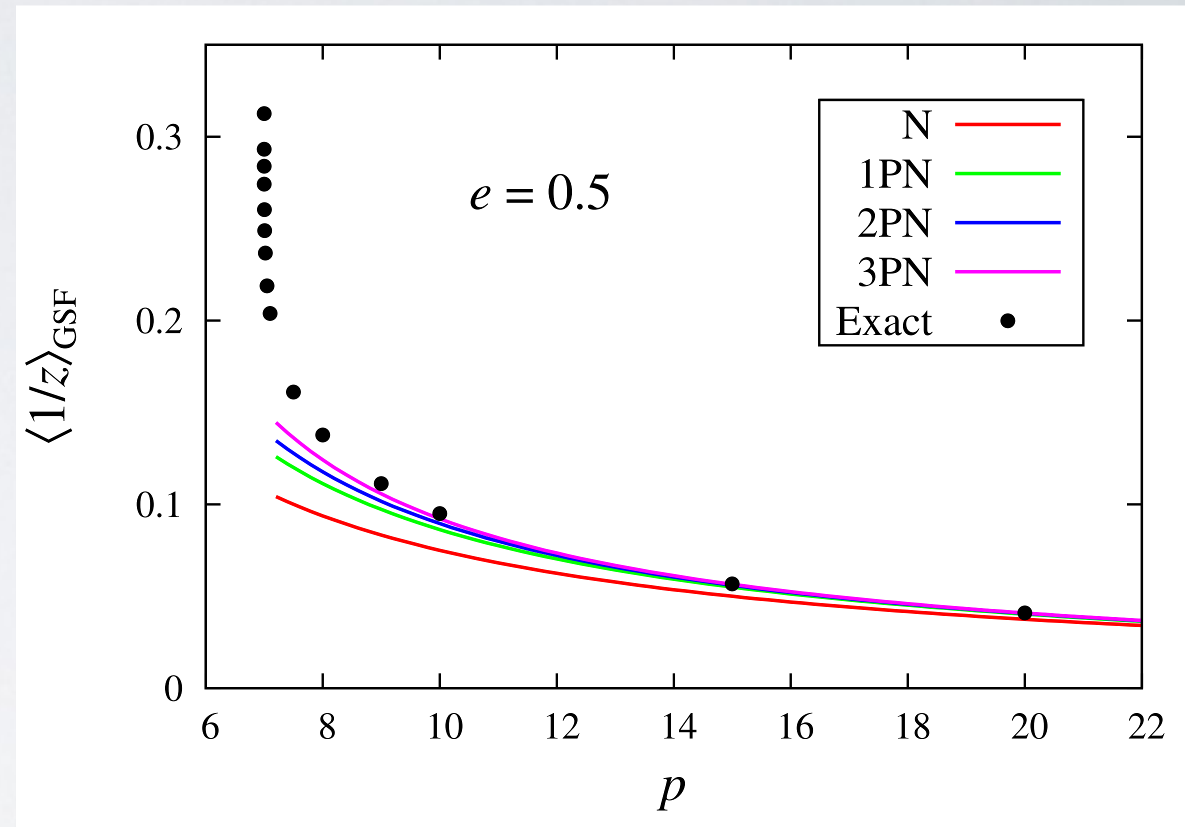
$$\Delta\lambda^{\rm err} = 10^{-12}$$

See Nolan's talk

Recent results: gauge-invariant quantities

Comparison between GSF and PN for eccentric orbits $\langle \Delta U \rangle$





$$\langle \Delta U \rangle = \cancel{\langle \Delta U_F \rangle}^{0} + \langle \Delta U_h \rangle$$

$$= \frac{1}{2} \langle h^{R,\mathrm{cons}}_{uu} \rangle$$

From Akcay etal (in prep)

Also see Evans, Forseth and Osburn's talks

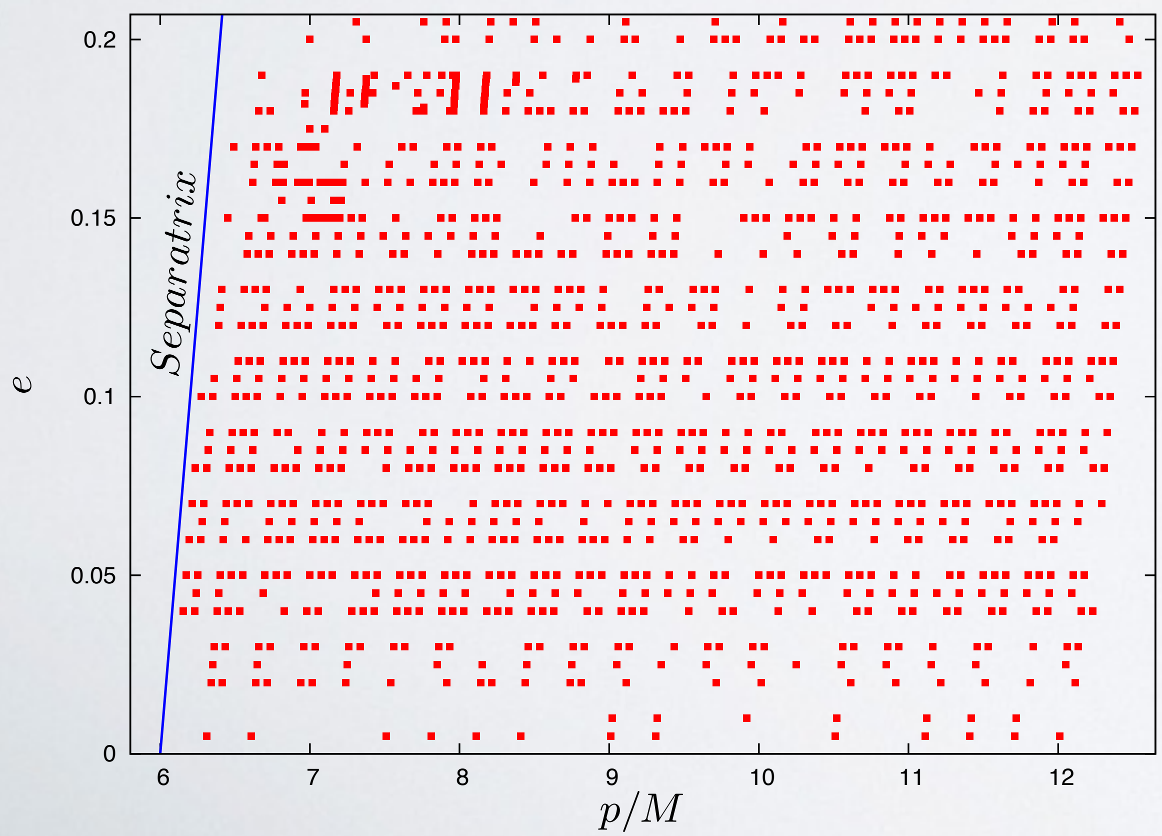
Orbit evolution: Schwarzschild inspiral

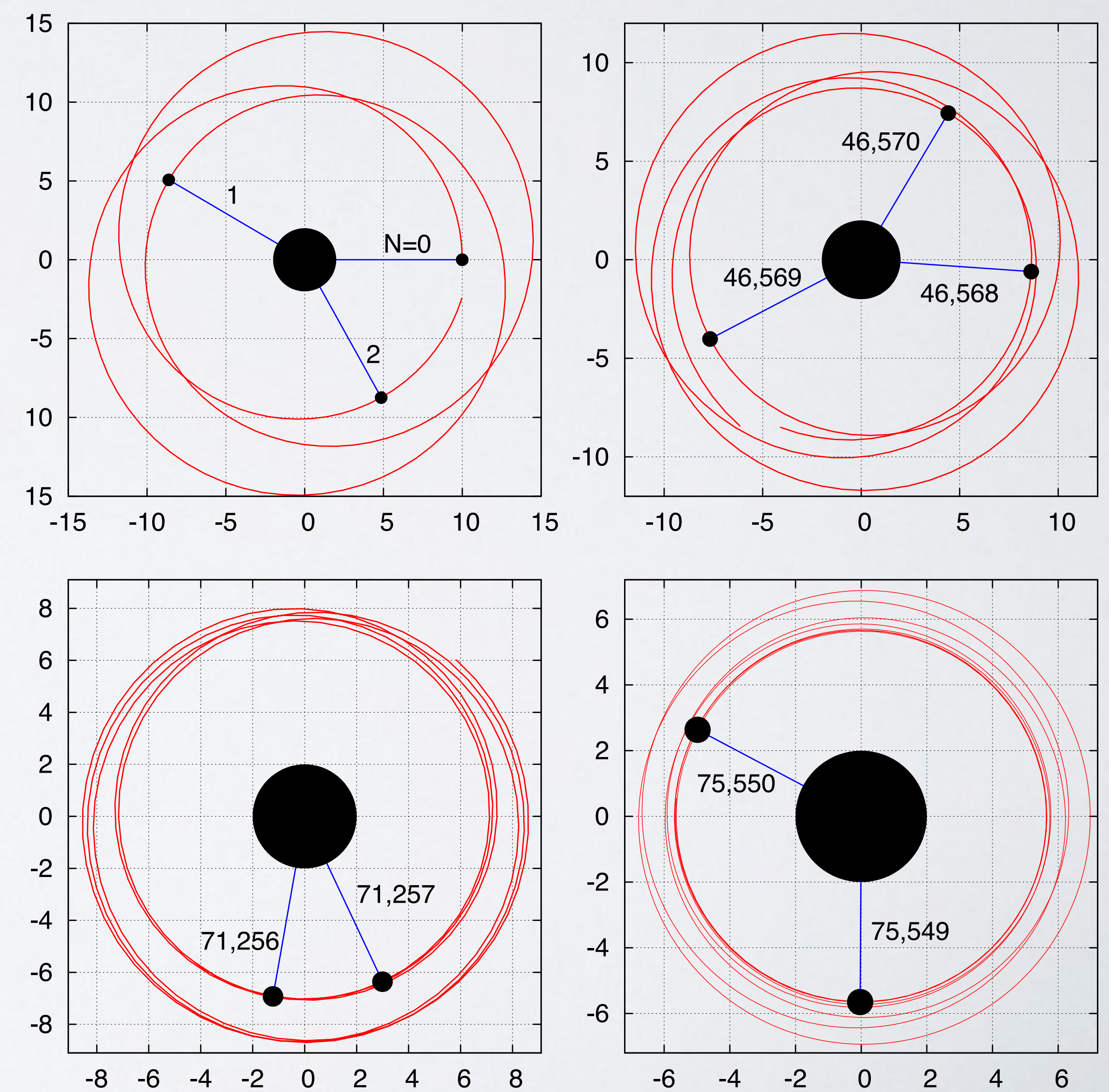
$\mathcal{O}(\epsilon^{-1})$: Orbit-averaged dissipative component of

$\mathcal{O}(\epsilon^{-1/2})$: ~~Resonances, oscillating SF no longer averages out~~

$\mathcal{O}(\epsilon^{0})$: Oscillatory component of the dissipative SF
Conservative component of the SF
~~Orbit averaged dissipative piece of the second order SF~~

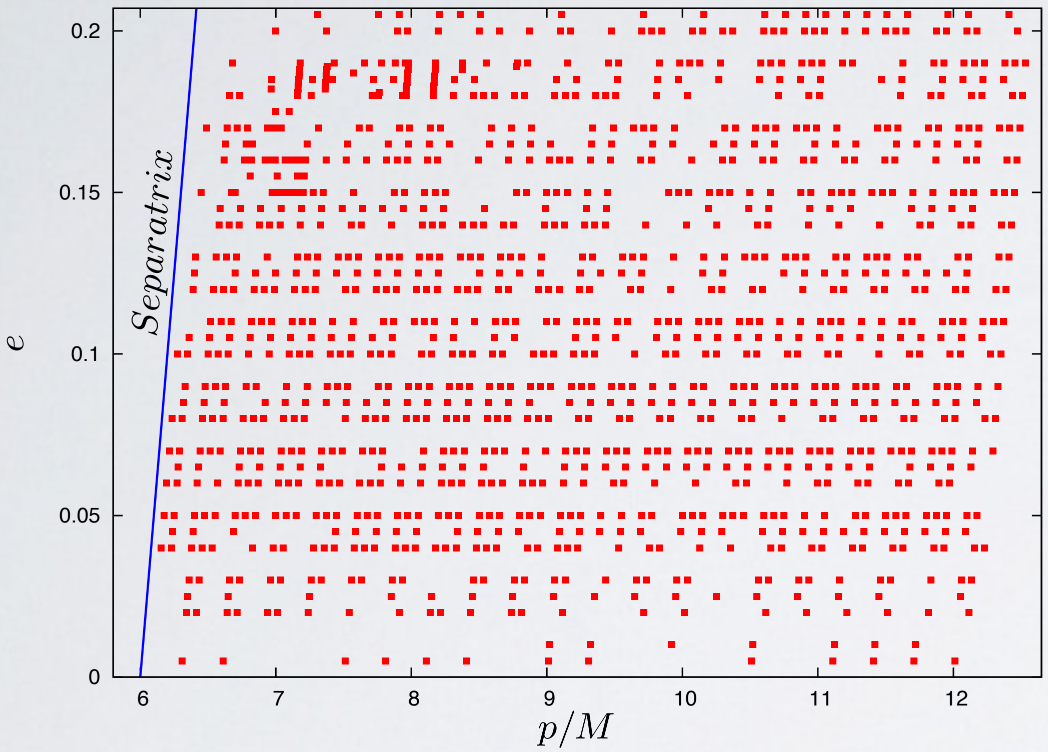
Geodesic self-force evolution: assume at each time that the GSF is that of the tangent geodesic





Inspiral comparison: scalar case

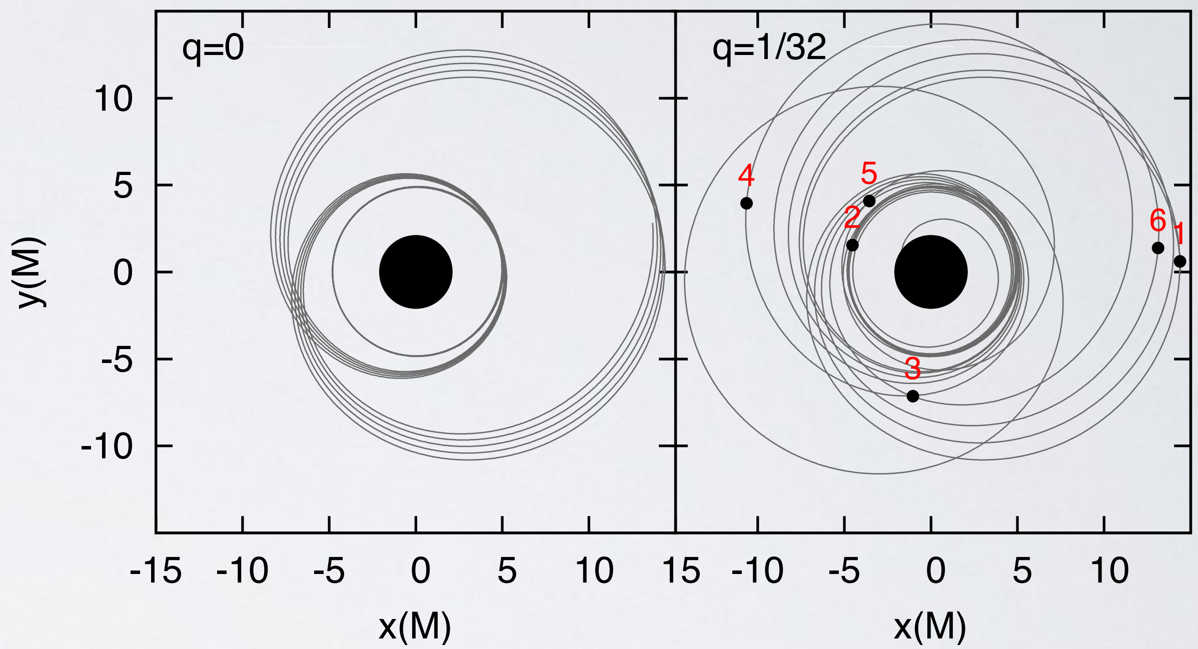
Osculating orbits with geodesic SSF



Fit model with 1000 geodesics worth of SSF data computed using FD code using mode-sum regularization

NW

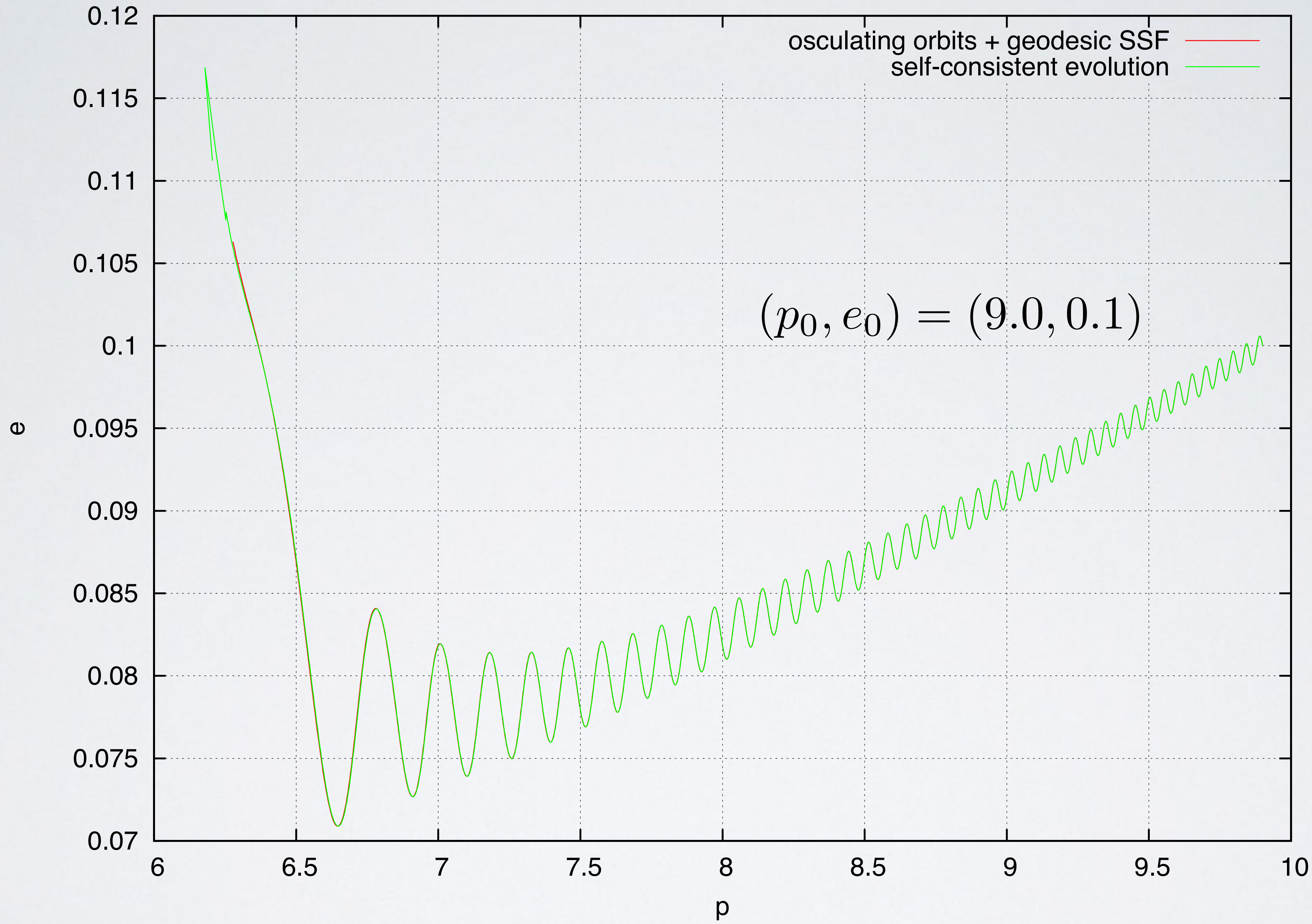
Self-consistent evolution



3+1 time-domain code using the effective source approach

Diener, Vega and Wardell

Inspiral comparison: scalar case

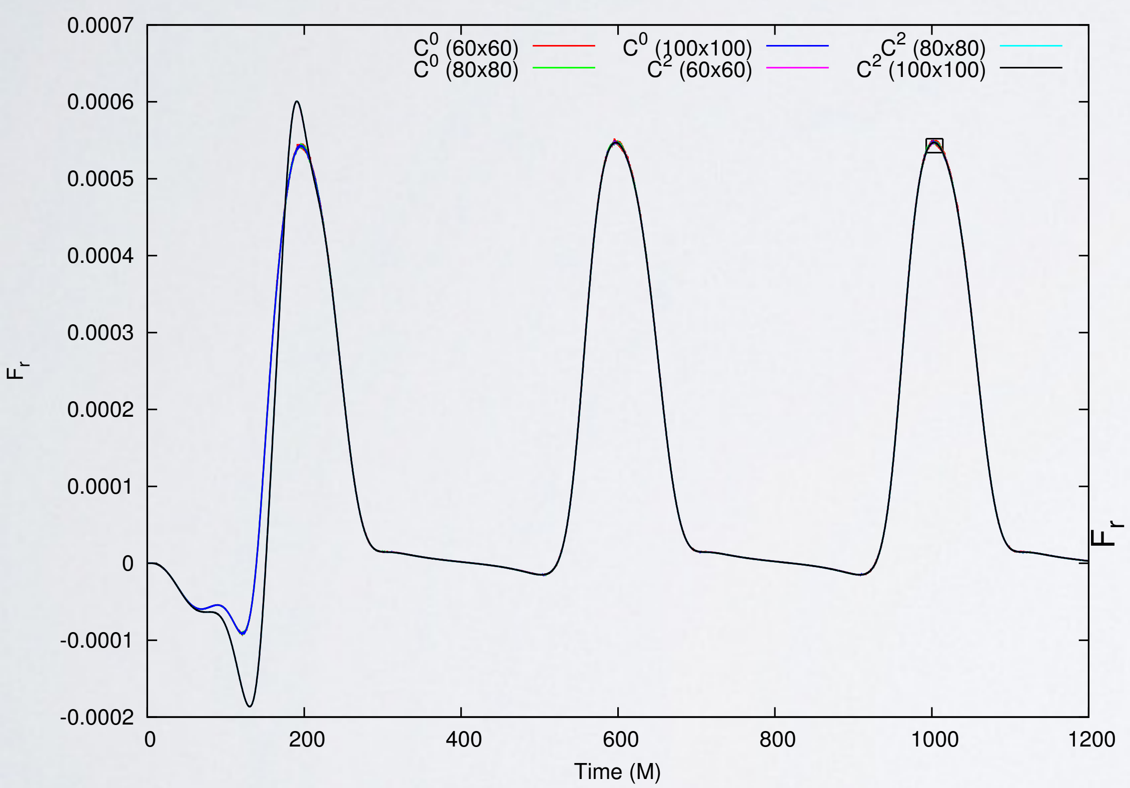


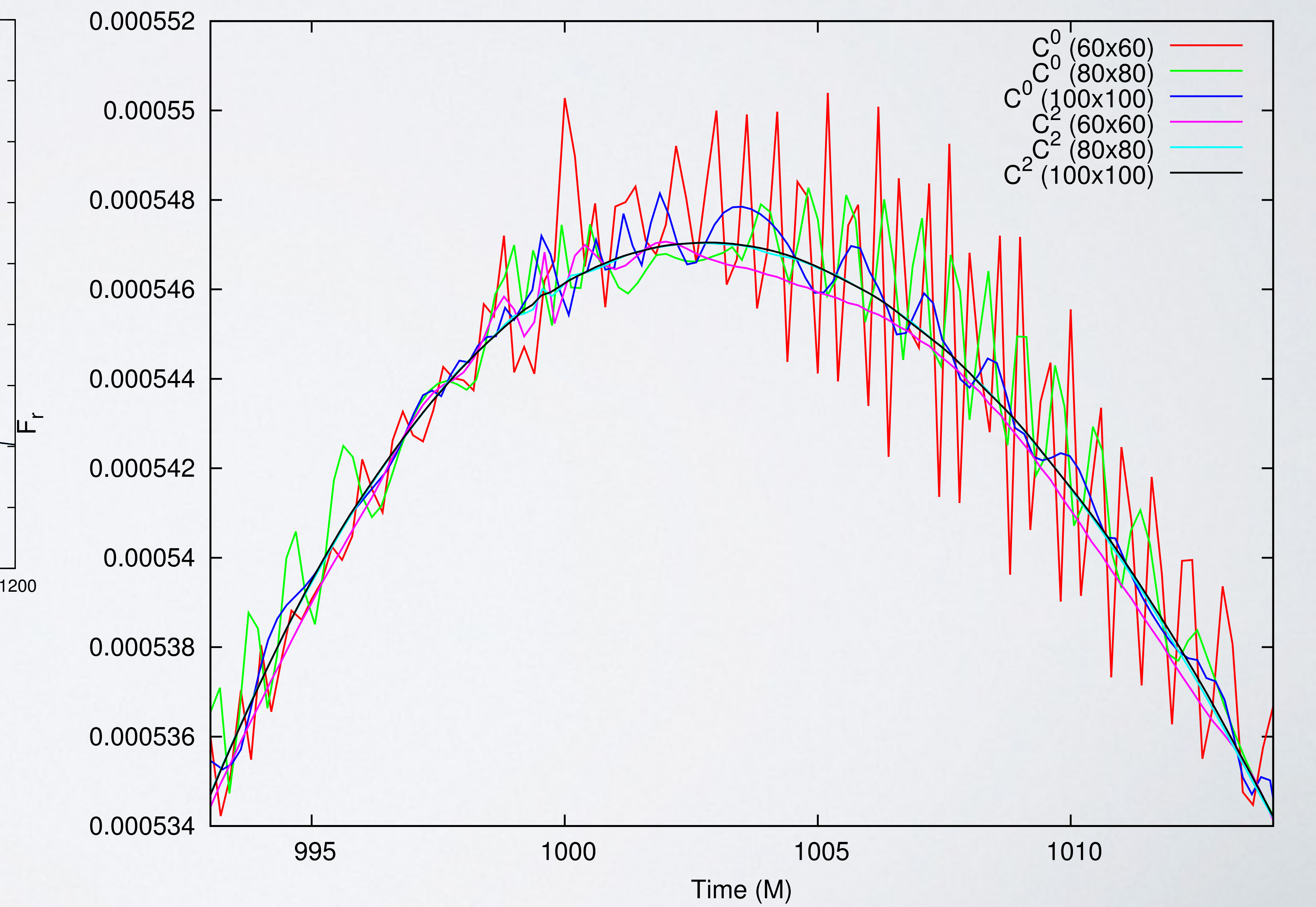
Evolutions are indistinguishable to within the numerical error from the 3+1 code

Inspiral comparison: scalar case

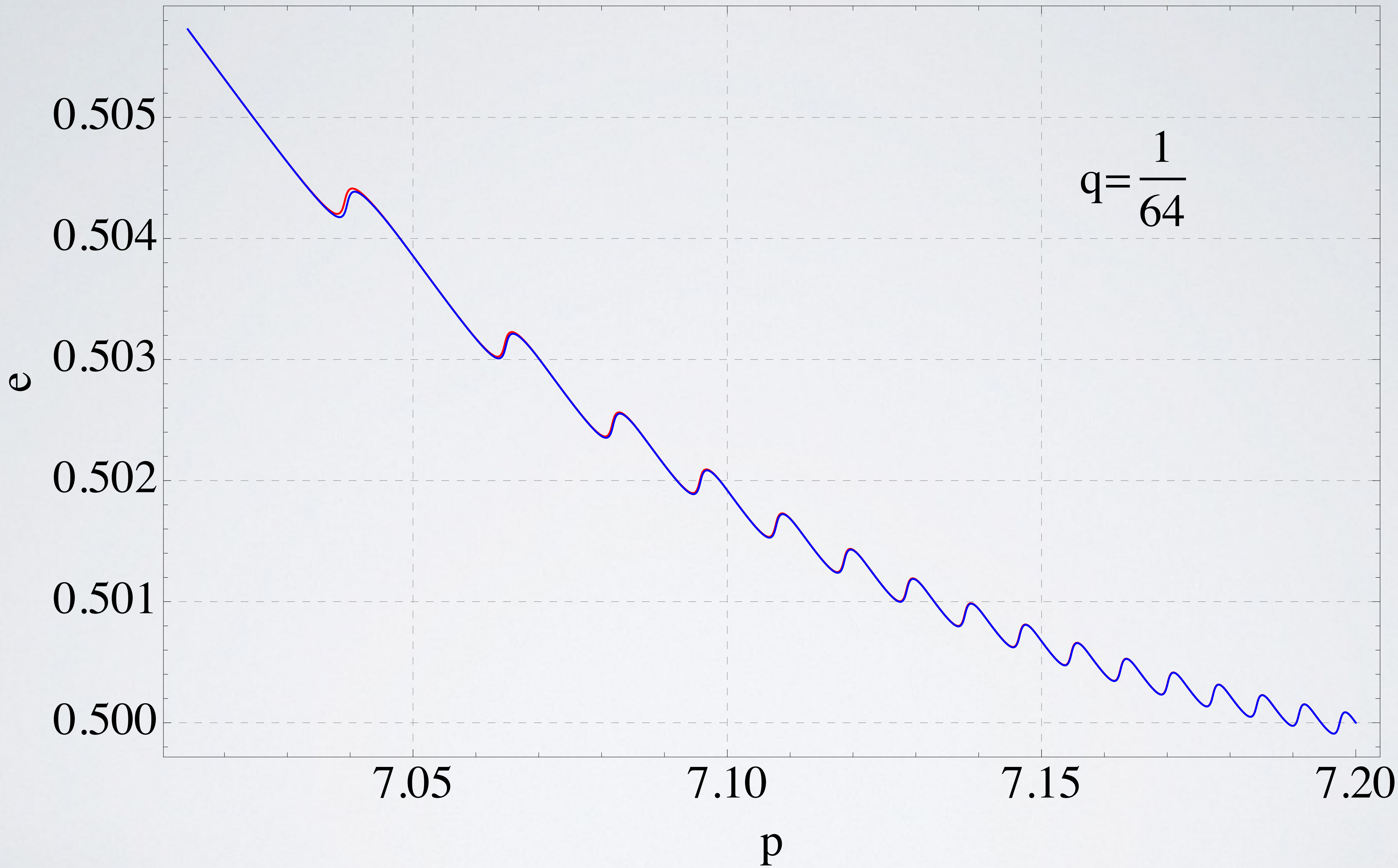
Two ways to proceed:

- Compare higher eccentricity orbits
- Improve accuracy of 3+1 code: increase differentiability of the source



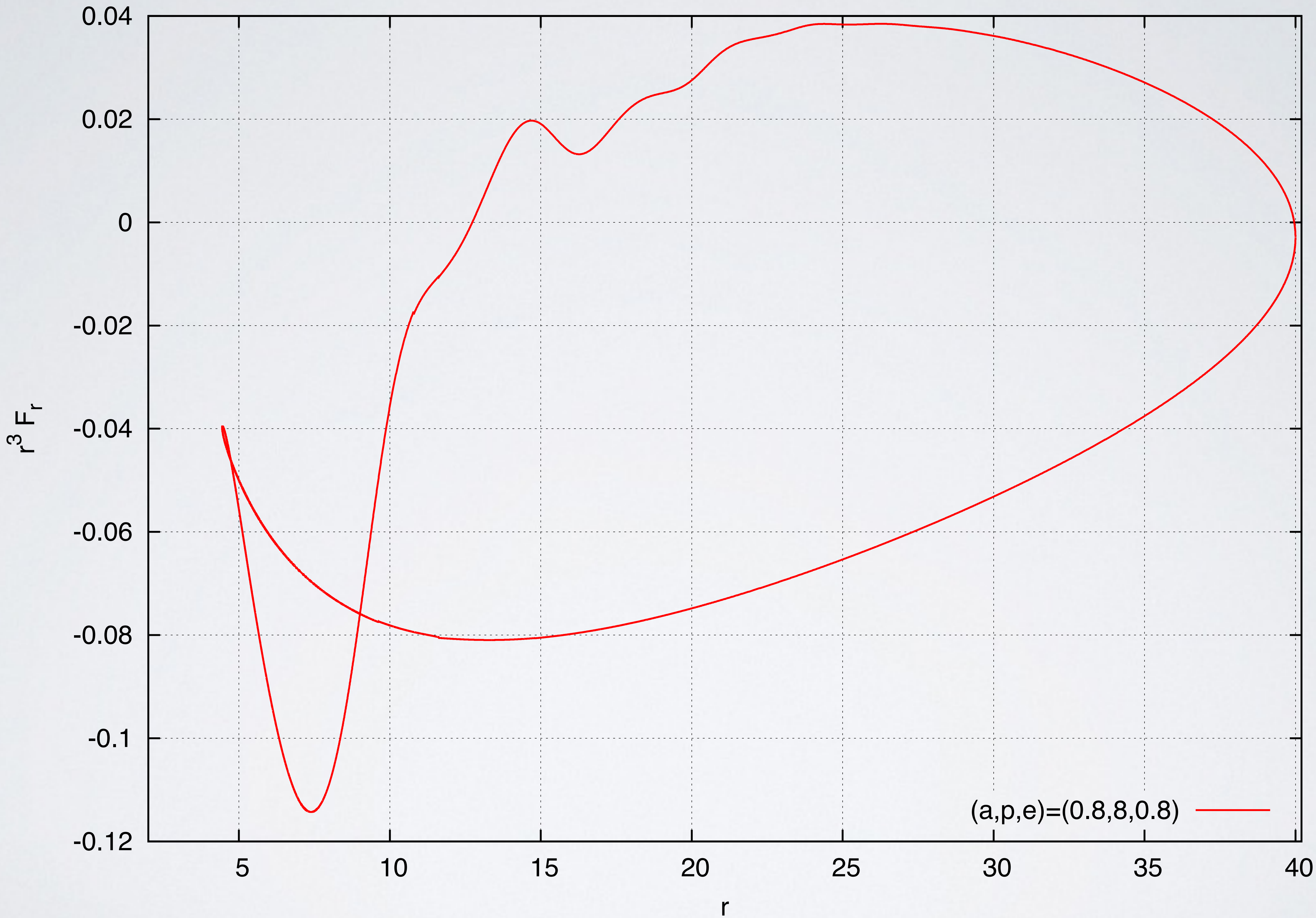


Inspiral comparison: scalar case



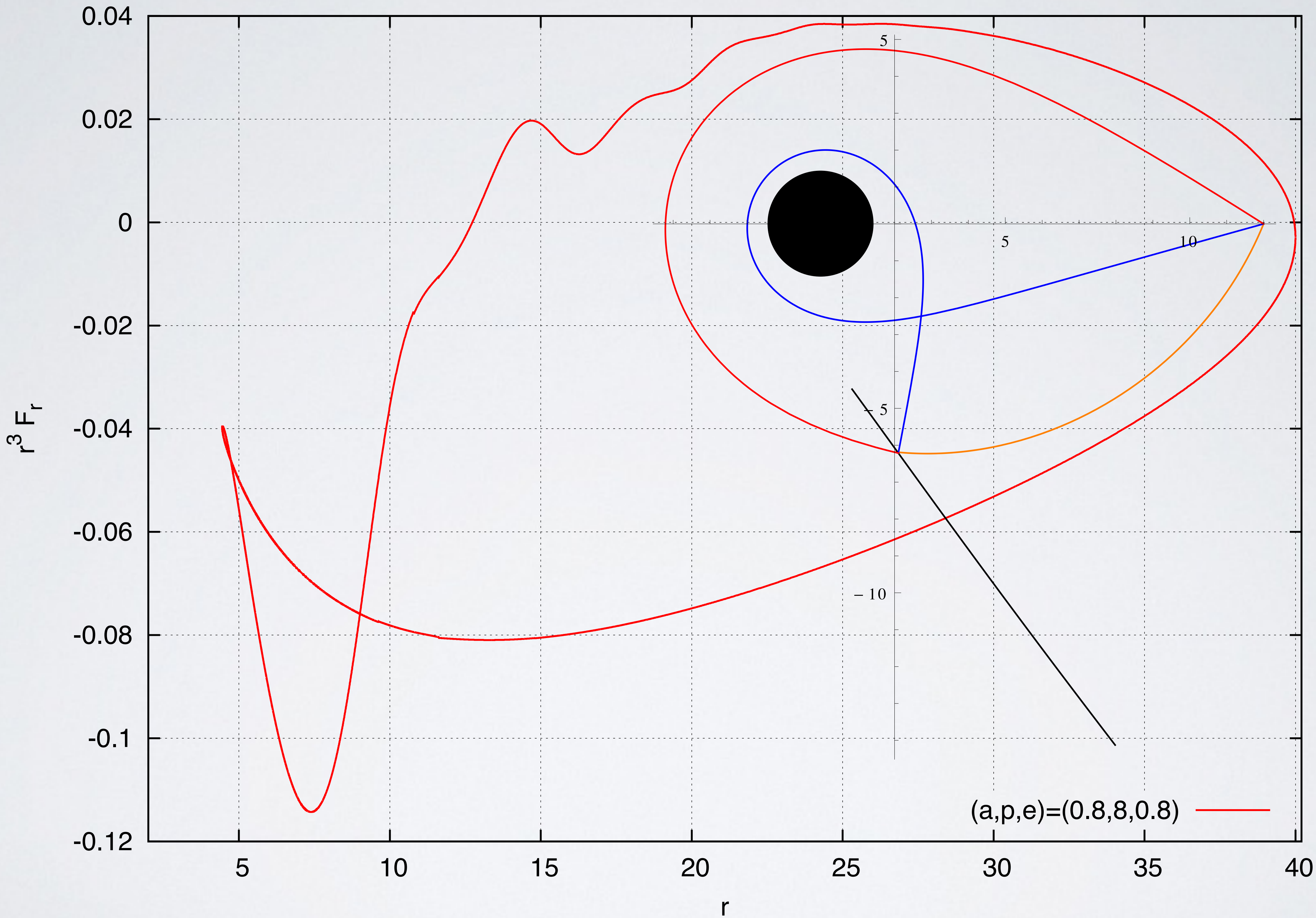
Inspirals are still very similar*

Ripples in Kerr scalar-field self-force



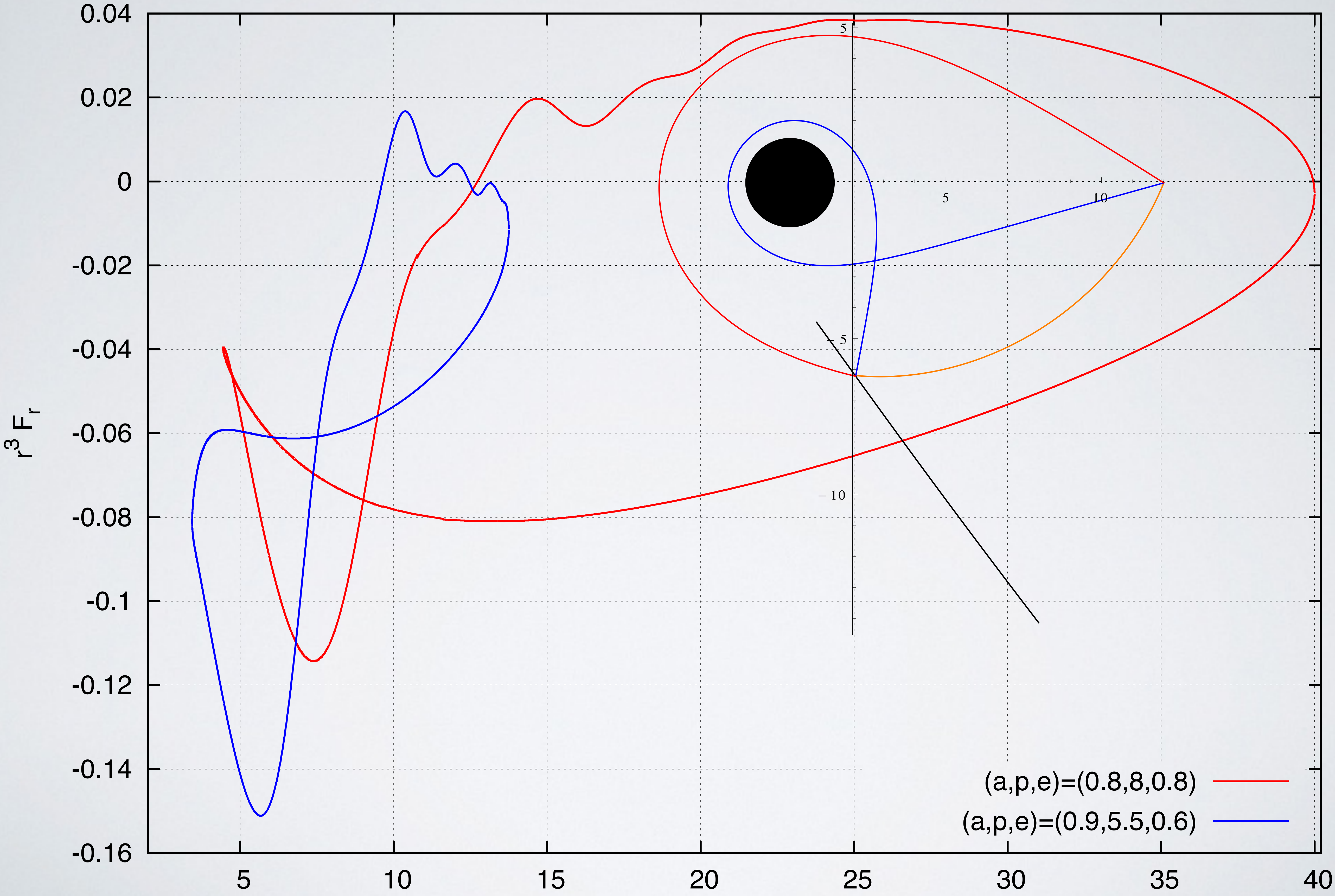
See Thornburg's talk

Ripples in Kerr scalar-field self-force



See Thornburg's talk

Ripples in Kerr scalar-field self-force



The Future…

See Casals and Wardell's talk

Comparison of results in Kerr with Green function approach

See Kavanagh's talk

Inspiral comparison: scalar case. Can we stabilise the Lorenz gauge monopole and dipole in the time-domain?

What can we learn from new gauge-invariant quantities?

Computation of gauge invariant-quantities at 2nd perturbative order

See Pound's talk

… is Abhay