Self-force for Cosmic Strings

Barry Wardell
Department of Astronomy, Cornell University
with David Chernoff and Éanna Flanagan

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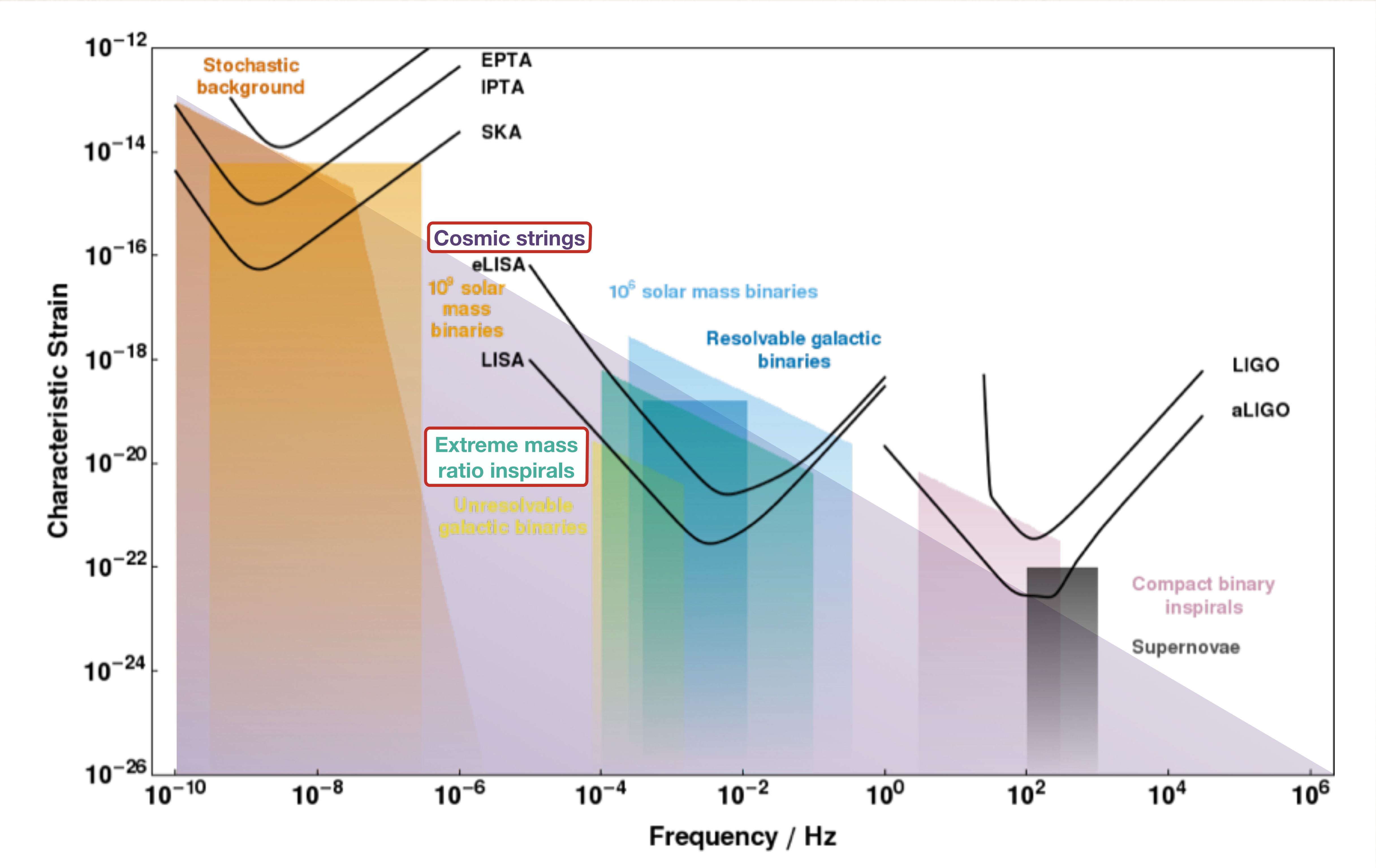
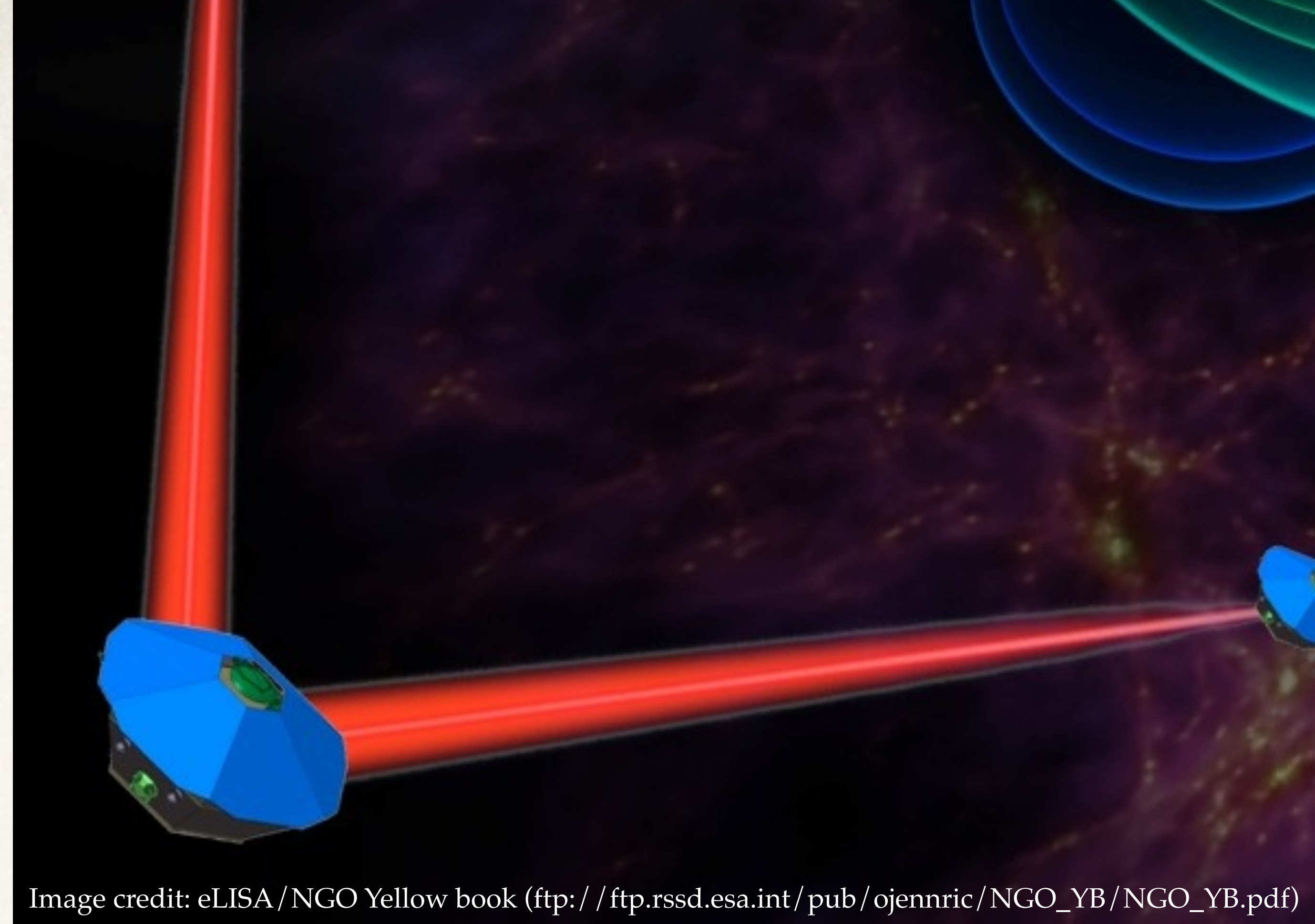
Sources of Gravitational Waves



Generated using Gravitational Wave Sensitivity Curve Plotter by Christopher Moore, Robert Cole and Christopher Berry

Cosmic Strings

- Can be treated as approximately one-dimensional objects (Nambu-Goto).
- Predicted by cosmological models of early-universe phase transitions, field theory models, string theory.
- Radiate gravitational waves at all frequencies (power law) $\Rightarrow$ LIGO, LISA and PTA
- Gravitational-wave detections $\Rightarrow$ upper limits on string tension $\Rightarrow$ constrain physical models.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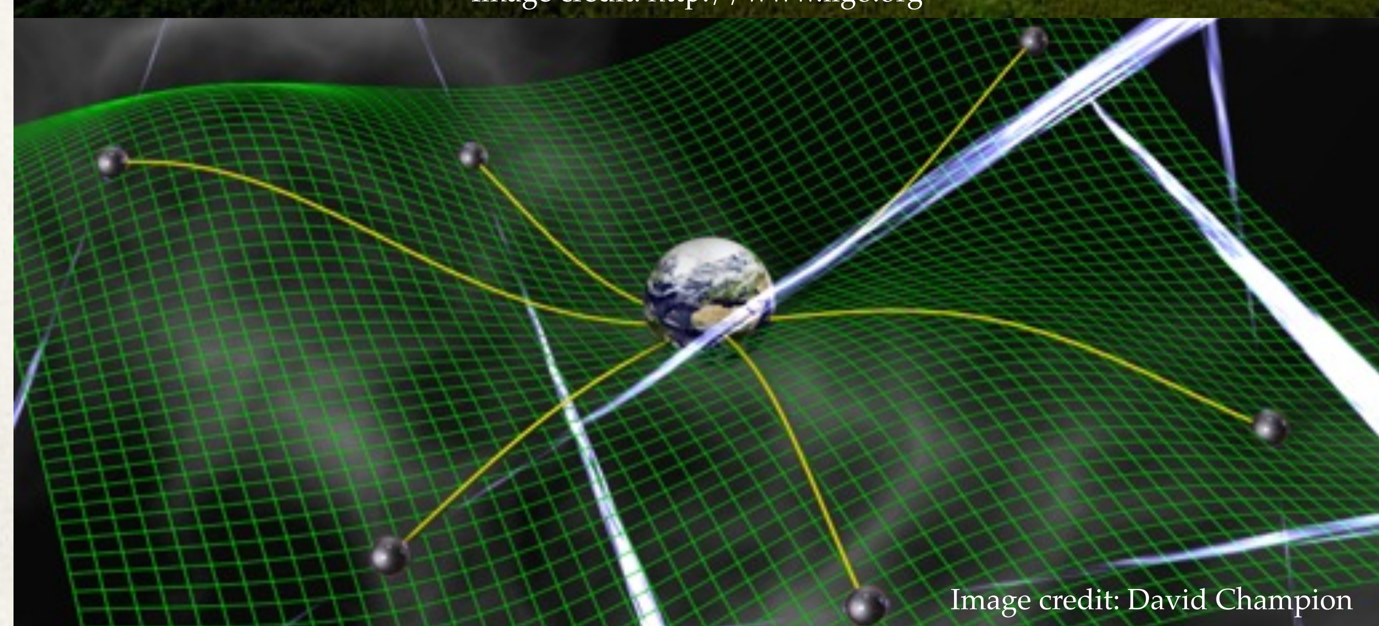
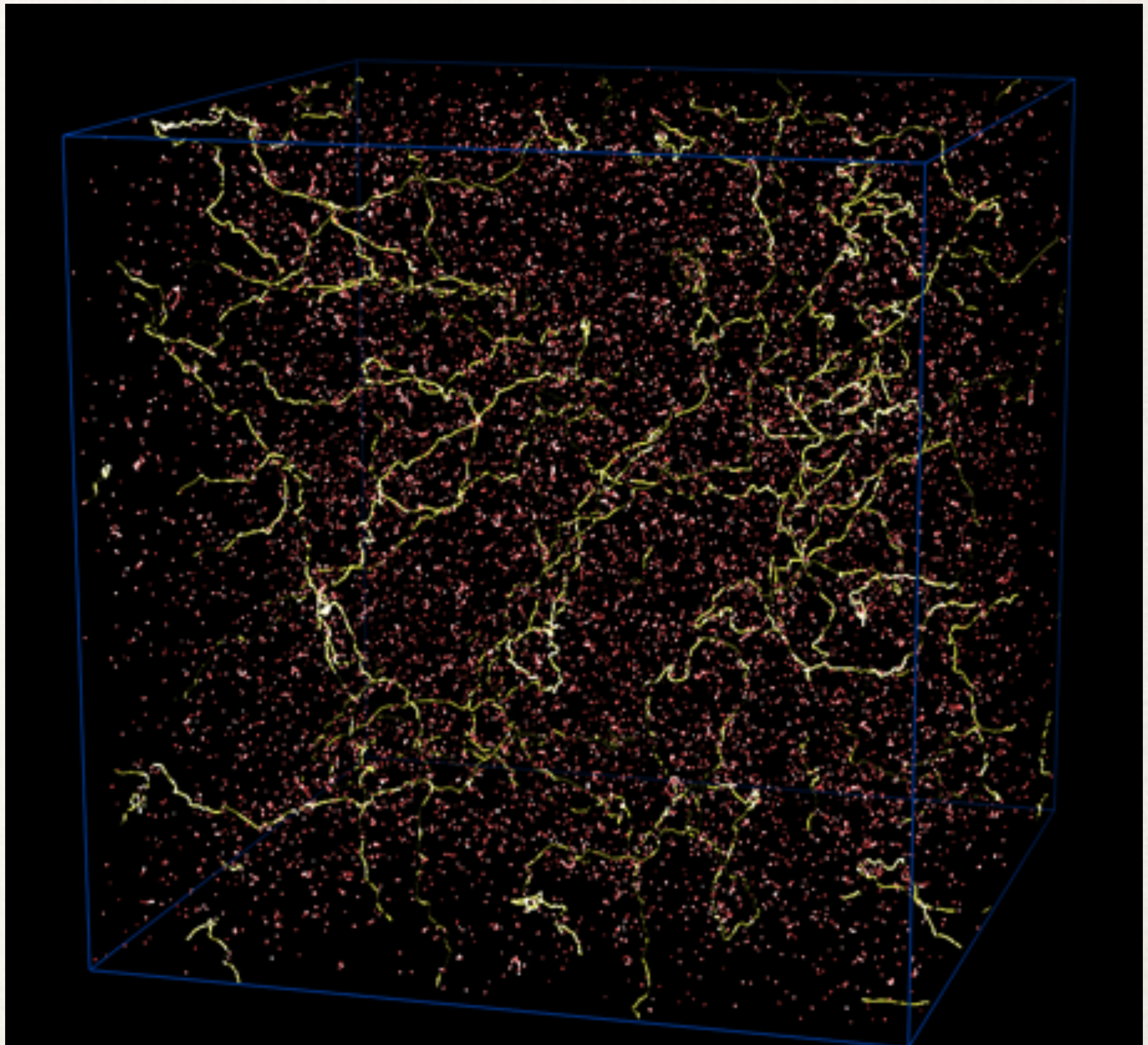


Image credit: David Champion

Cosmic String Loop Formation

- Cosmic string loops form from large string networks.
- Whenever a string self-intersects it breaks apart, leaving two smaller strings with kinks behind.
- Kinks eventually evolve into cusps.
- Kinks, cusps are generically expected to be present in cosmic strings.
- Galactic-scale strings essentially moving in flat space.


B. Allen & E. P. Shellard

Perturbative sources of GWs

- Disparity of length-scales ⇒ not well suited to Numerical Relativity.
- Well approximated as perturbation of some background spacetime.
- Ignore any internal structure.
- Point particle/string approximation ⇒ divergent perturbations

	EMRI	Cosmic String
Small parameter	Mass ratio,	String tension, G
Modelled as	Zero-dimensional point particle	One-dimensional string
Length scale	Mass of larger black hole (typically ~10	Sub-galactic scale (also higher harmonics)
Background spacetime	Rotating black hole (Kerr)	Flat space (Minkowski)
Divergence of self-field	1/	log(

Perturbative sources of GWs

	EMRI	Cosmic String
Parametrisation	Worldline,	Worldsheet,
Stress-energy	$m\int_\gamma \frac{g^\alpha{}_\mu(x,z)g^\beta{}_\nu(x,z)\dot z^\mu \dot z^\nu}{\sqrt{-g_\mu\nu\dot z^\mu\dot z^\nu}}\delta_4(x,z)d\lambda$	$-\frac{G\mu}{\sqrt{-g}}\int\sqrt{-\gamma}\gamma^{AB}\partial_A z^\alpha\partial_B z^\beta\delta_4(x,z)d\zeta d\tau$
Unperturbed Equation of Motion	$\frac{d^2z^\mu}{d\tau^2}+\Gamma^\mu_{\alpha\beta}\frac{d^2z^\mu}{d\tau^2}\frac{d^2z^\mu}{d\tau^2}=0$	$\frac{1}{\sqrt{\gamma}}\partial_A(\sqrt{\gamma}\gamma^{AB}\partial_B z^\rho)+\gamma^{AB}z^\lambda_{,A}z^\mu_{,B}\Gamma^\rho_{\lambda\mu}=0$ $\frac{\partial z^\rho}{\partial\zeta^2}-\frac{\partial z^\rho}{\partial\tau^2}=0$
First order equation of motion	$\frac{d^2\xi^\mu}{d\tau^2}+\Gamma^\mu_{\alpha\beta}\frac{d^2\xi^\mu}{d\tau^2}\frac{d^2\xi^\mu}{d\tau^2}$ $=\frac{1}{2}\perp^{\mu\nu}(2\nabla_\lambda h_{\mu\rho}-\nabla_\mu h_{\lambda\rho})u^\lambda u^\rho$	$\perp^\rho_\chi\bar\nabla_\mu\bar\nabla^\mu\xi^\chi-2\bar\nabla_\mu\xi^\alpha K^\mu{}_\alpha{}^\rho+\perp^{\beta\rho}P^{\mu\nu}R_{\mu\varepsilon\nu\beta}\xi^\varepsilon=K^{\alpha\beta\rho}h_{\alpha\beta}-\frac{1}{2}\perp^\rho_\beta P^{\lambda\tau}\left(\nabla_\lambda h^\beta_\tau-\frac{1}{2}\nabla^\beta h_{\lambda\tau}\right)$ $\frac{\partial^2\xi^\rho}{\partial\zeta^2}-\frac{\partial^2\xi^\rho}{\partial\tau^2}=K^{\alpha\beta\rho}h_{\alpha\beta}-\frac{1}{2}\perp^{\rho\beta}P^{\lambda\tau}\left(\nabla_\lambda h_{\beta\tau}-\frac{1}{2}\nabla_\beta h_{\lambda\tau}\right)$

Unperturbed motion

Motion of a Cosmic String

- A one-dimensional string traces out a two-dimensional *worldsheet* in spacetime.
- Concentrate on the case of closed cosmic string loops.
- Parametrise worldsheet $z(\tau,\zeta)$ by time coordinate, τ, and spatial coordinate, ζ, which picks out a location on the string.
- Periodic in τ and ζ, period L.
- "Geodesic" equation of motion

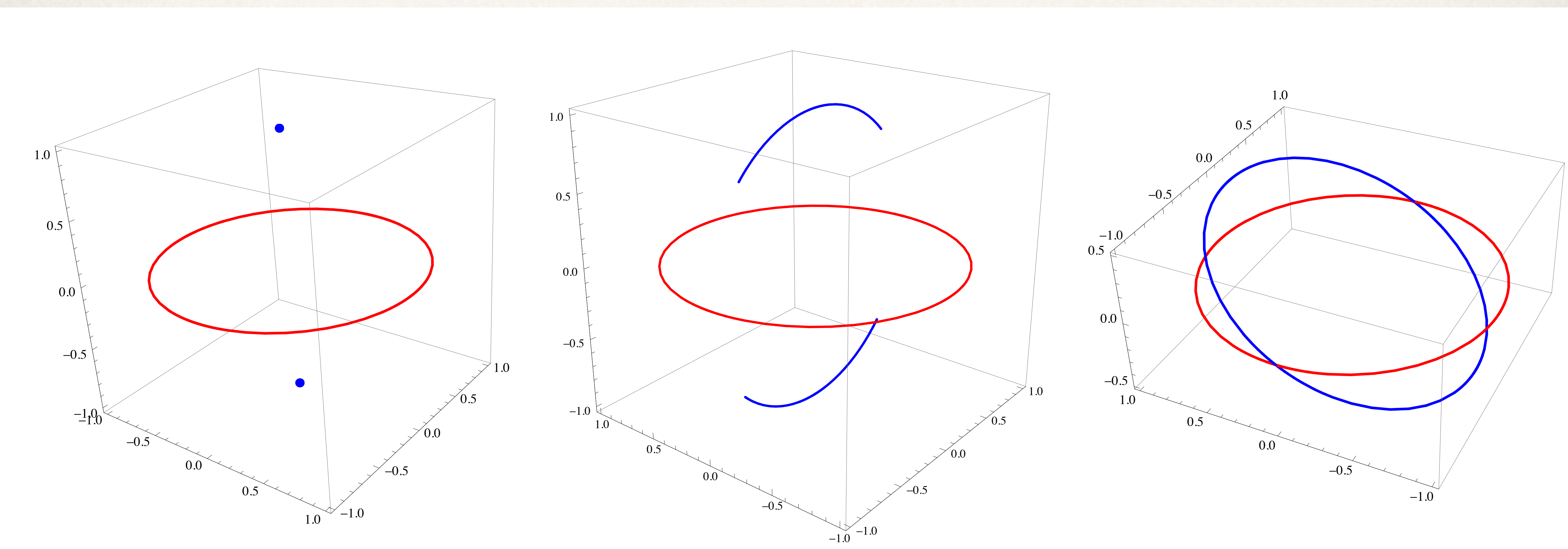
$$\frac{\partial^2 z^a}{\partial \tau^2} - \frac{\partial^2 z^a}{\partial \zeta^2} = 0$$

Kinks and Cusps

- A generic string will have either a kink or a cusp.
- Tangent-sphere representation

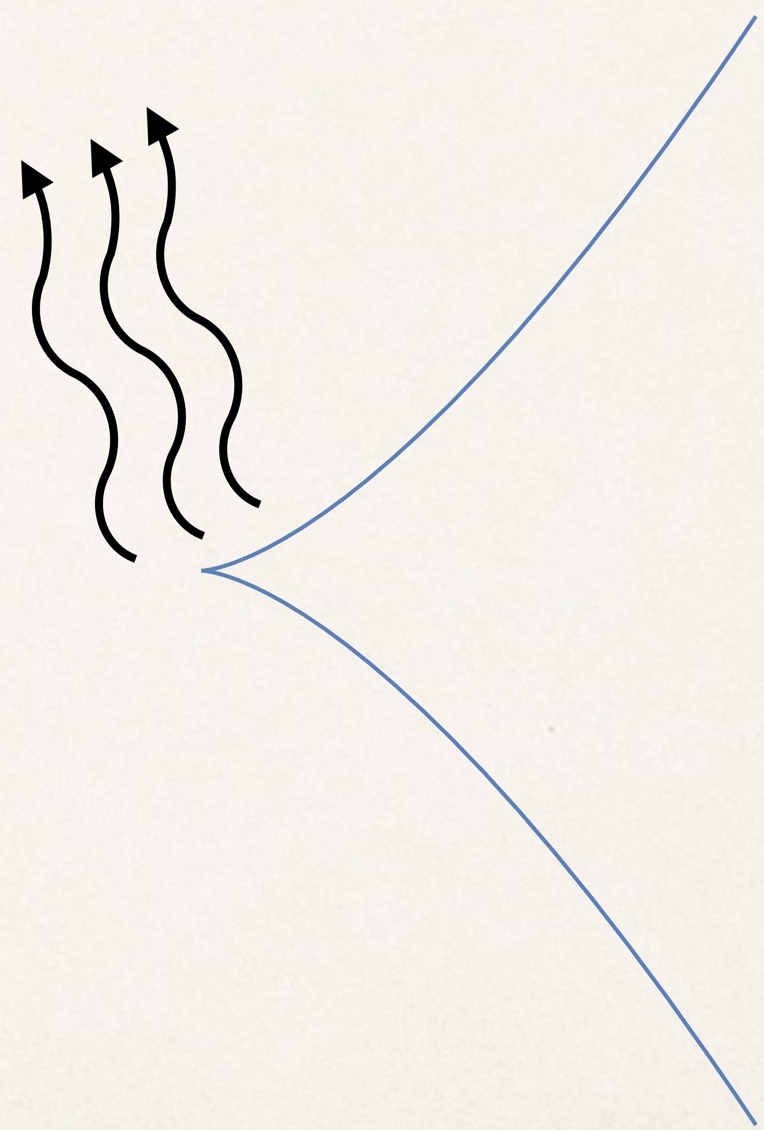
$$\vec{z}\,'(\tau + \zeta) \qquad \vec{z}\,'(\tau - \zeta)$$

- Difficult (impossible?) to find string configurations which don't contain either kinks, cusps or self-intersections.



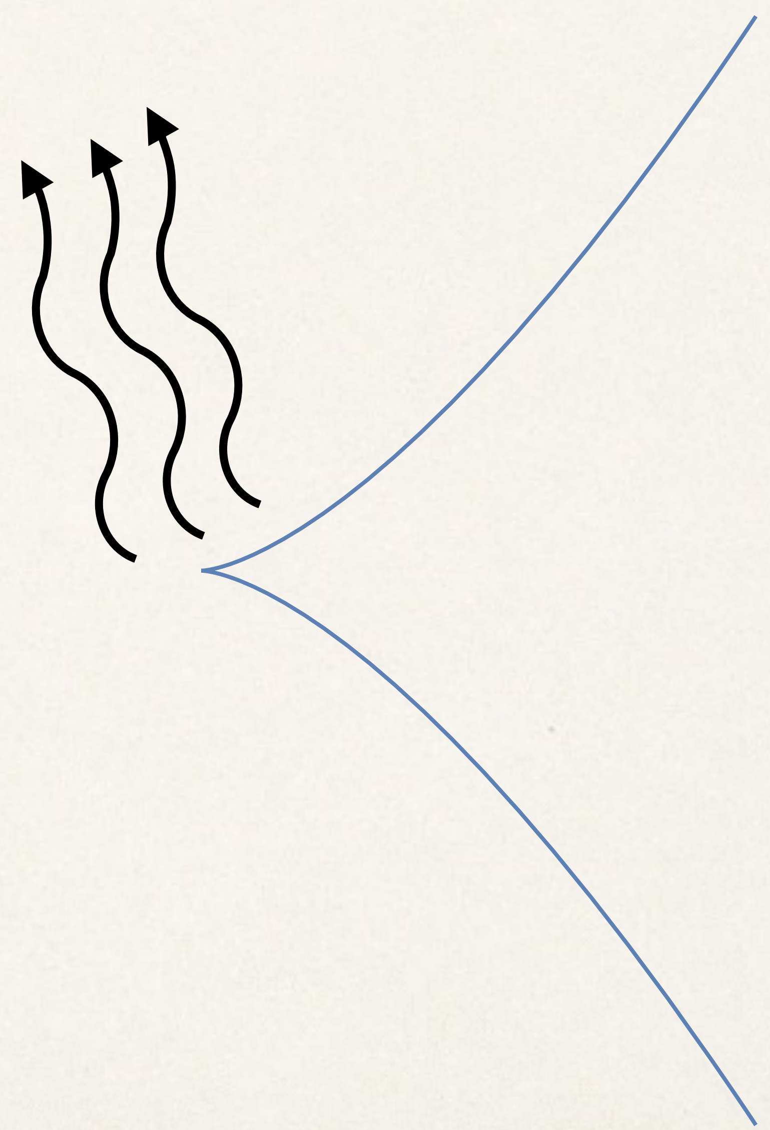
Kinks and Cusps

- Kinks and cusps are highly relativistic, smooth parts of the string much less so.
- Radiation from kinks, and cusps in particular, is strongly amplified relative to smooth string.
- Gravitational radiation dominated by cusp.
- Cusp models used for bounds on string tension from gravitational wave detectors.



Gravitational Radiation

- But, strings can couple to gravity.
- Gravitational radiation damping causes string to evaporate.
- Which happens first, evaporation or cusp formation?
- If evaporation happens first, then existing gravitational wave bounds may be missing a key component in their models.



Self-force for Cosmic Strings

Self-force for Cosmic Strings

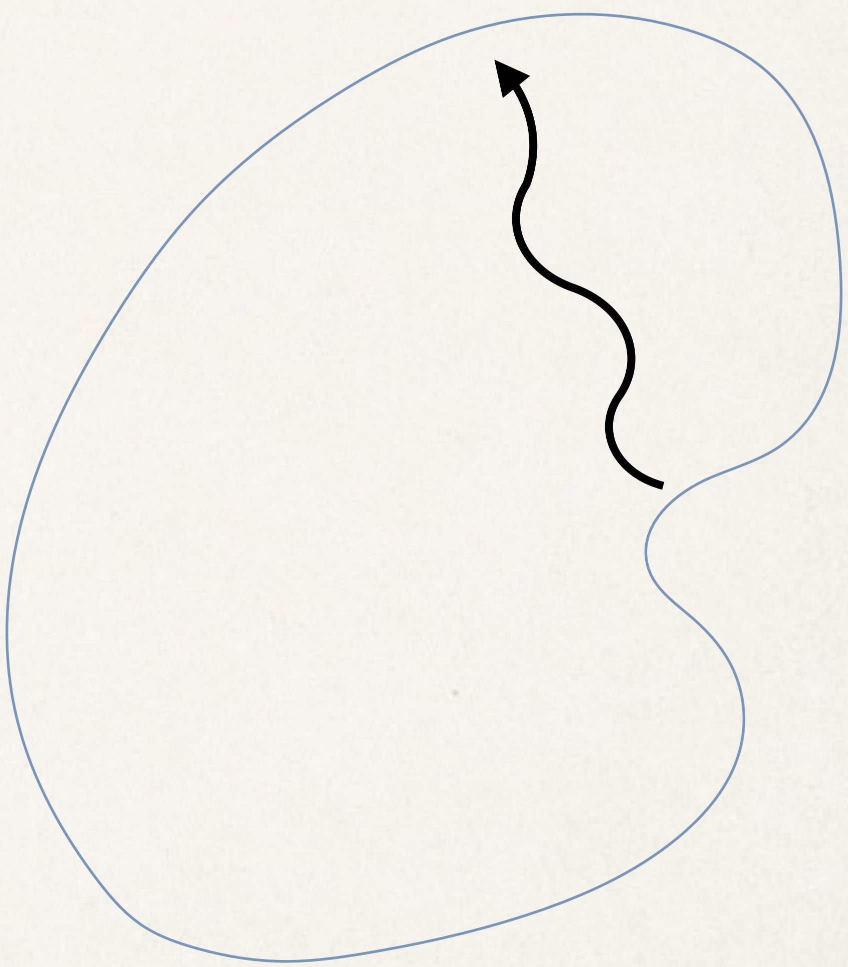
- When gravitational backreaction is taken into account, the string feels a self-force at linear order in string tension, μ

$$\frac{\partial^2 z^a}{\partial \tau^2} - \frac{\partial^2 z^a}{\partial \zeta^2} = F^a \propto \nabla_b h^1_{cd}$$

- As with point particles, Lorenz gauge metric perturbation h^1_{cd} can be computed by convolving with the Green function.
- $\nabla_a h^1_{cd}$ diverges like r^{-1} near the worldsheet, but it turns out that the combination as it appears in the self-force is finite (except at discontinuities such as kinks and cusps).

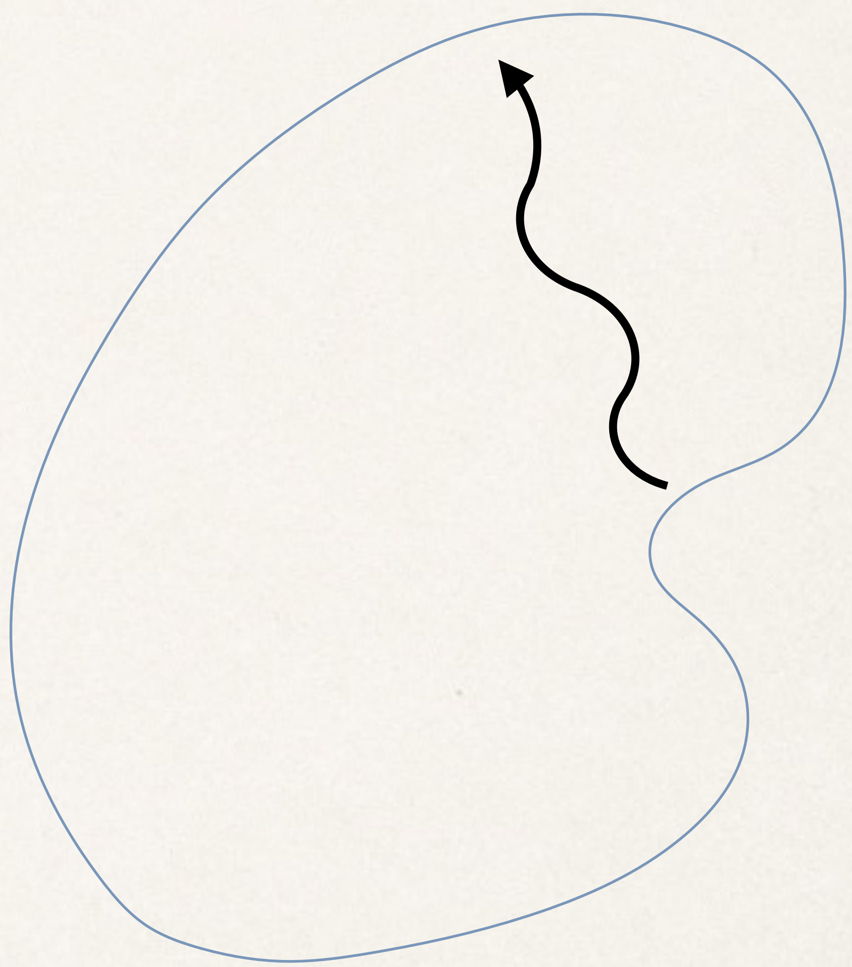
Self-force via World*sheet* Integration

- History dependence now comes about in two ways.
- There is a backscattering tail term as with point particles.
- There is also a direct-propagation contribution arising from radiation from one part of the string directly propagating and hitting another part of the string.
- Even in flat space the self-force is non-trivial and has a history dependence.



Evolving a Cosmic String with backreaction

- In principal straightforward.
- Compute self-force at a point on the string by integrating retarded Green function over all points in the past.
- Repeat for each point on the string.
- Update the orbit using the self-force as the right-hand side in the equation of motion.
- Repeat for an evaporation timescale $\sim 1/\mu$.



Evolving a Cosmic String with backreaction

$$\bar{h}_{\alpha\beta}(x) = \int G^{\text{ret}}_{\alpha\alpha'\beta\beta')(x,x')} T^{\alpha'\beta'} \sqrt{-g} d^4x'$$
$$= G\mu \int \sqrt{-\gamma'} P_{\alpha'\beta'} \delta[\sigma(x^\gamma, z^\gamma(\tau', \zeta'))] d\tau' d\zeta'$$

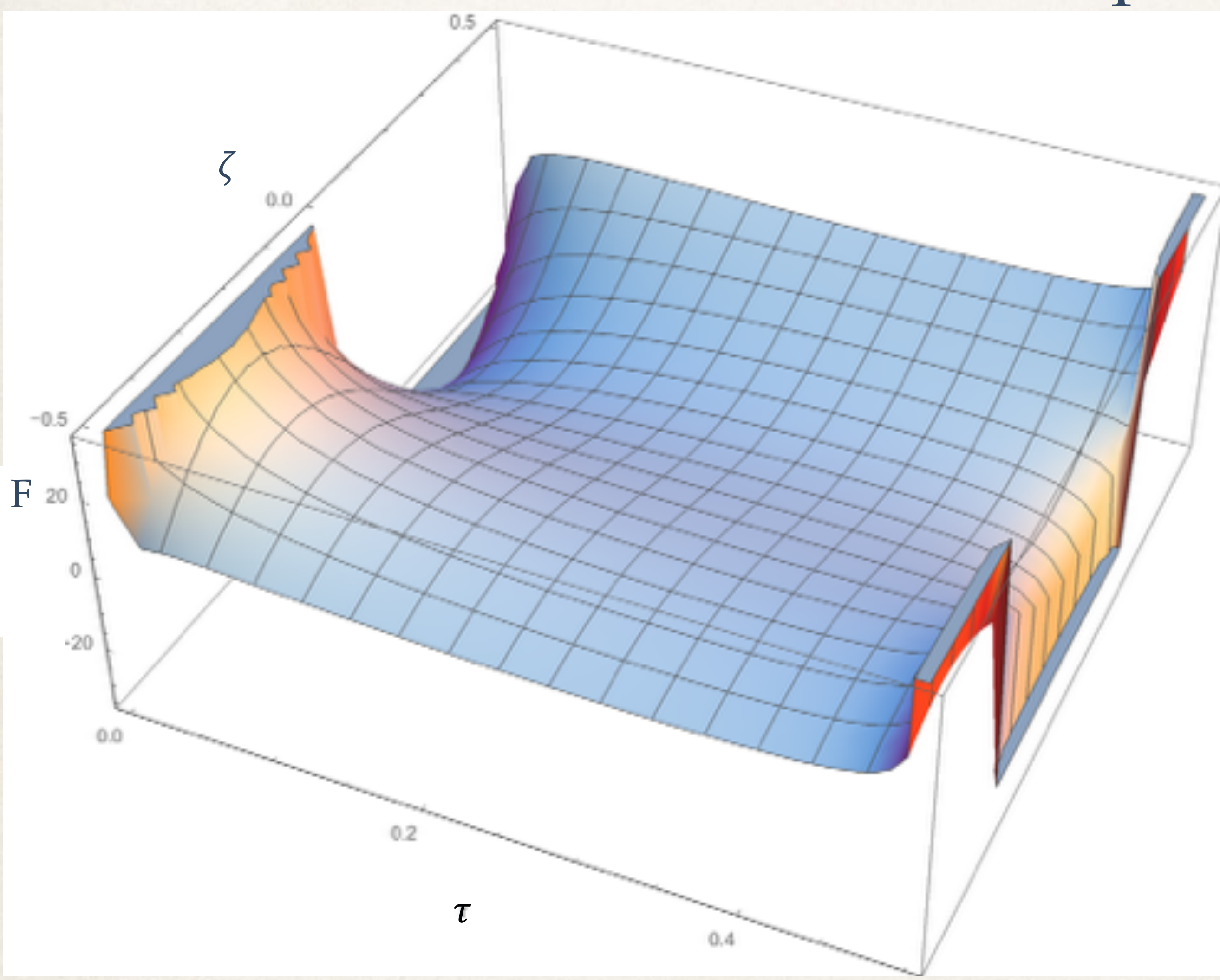
$$h_{\alpha\beta}(x) = G\mu \int \left[\frac{\sqrt{-\gamma'}}{r} \left(P_{\alpha'\beta'} - \frac{1}{2} g_{\alpha'\beta'} P^{\gamma'}{}_{\gamma'} \right) \right]_{\tau'=\tau^{\text{ret}}} d\zeta'$$

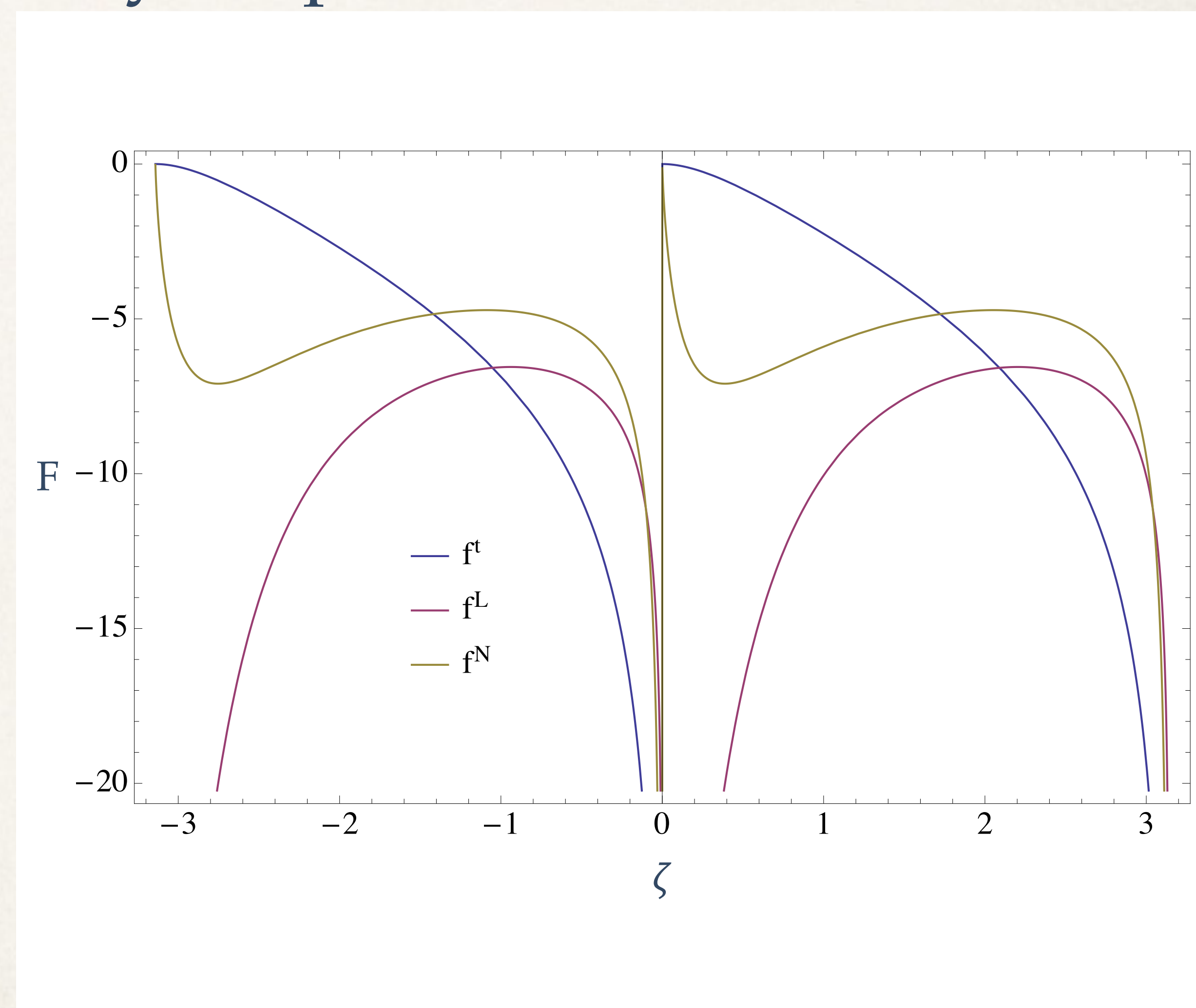
$$F^\rho = G\mu \Big[\perp^\rho{}_\lambda P^{\mu\nu} \left(\nabla_\mu h_\nu{}^\lambda - \frac{1}{2} \nabla^\lambda h_{\mu\nu} \right) - K^{\mu\nu\rho} h_{\mu\nu} \Big]$$

$F^a \sim O(1)$ for smooth strings, $O(\log \varepsilon)$ at discontinuities.

Self-force - divergence near kinks

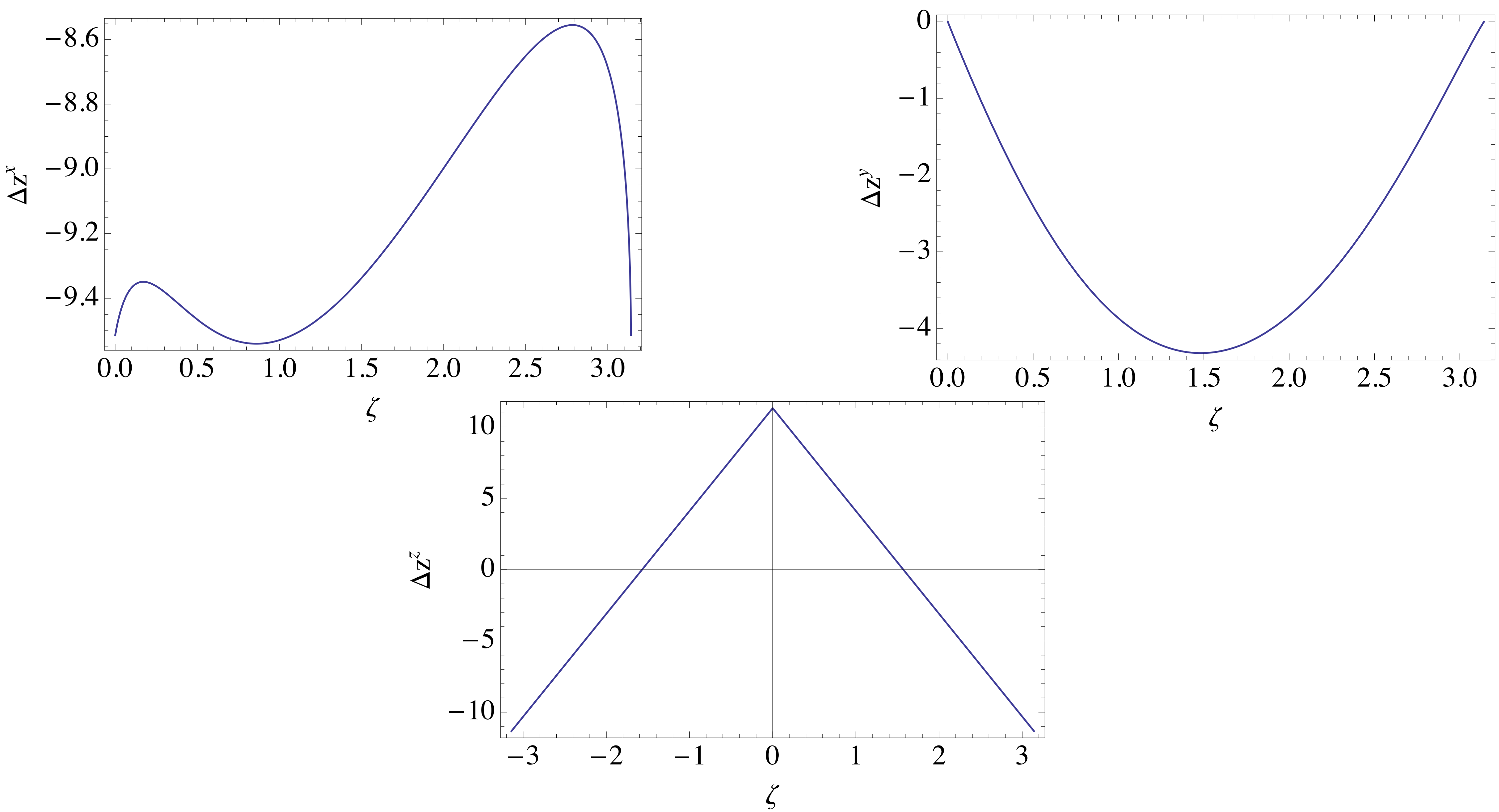
Self-force diverges logarithmically in the vicinity of discontinuities
(kinks and probably cusps)





Self-force - divergence near kinks

Divergence is integrable in the sense that Δz^{μ} is finite.



Self-force - divergence near kinks

But derivative along certain directions in the worldsheet are still divergent