Ballistic orbits for Gravitational Waves

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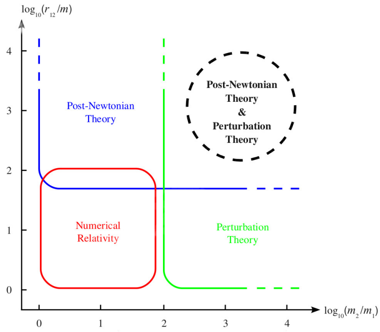
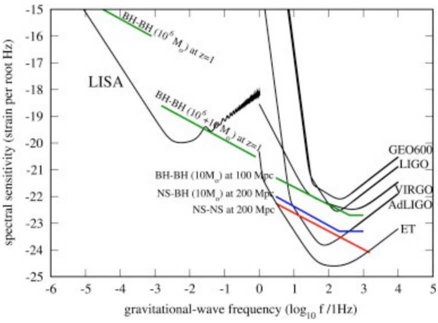
[arXiv:1406.4282]

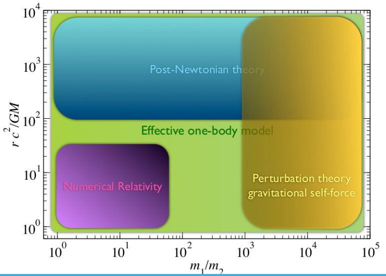
Kyoto 02-07-2015
18th Capra meeting on Radiation Reaction in GR

EMR binaries: $\frac{\mu}{M} \ll 1 \Rightarrow$

- PostNewtonian methods do not help much
- neglect effects of μ on metric
- use full GR (strong gravity)







Different stages of an EMR binary

- inspiral: eccentric orbit, adiabatic approximation
- plunge: inspiralling orbit
- merger and ringdown

μ and M get closer to each other and the orbit circularizes (but [Cutler et al 1994], [Tanaka et al 1993])

The transition between inspiral and plunge takes place in the region of the last stable orbit (ISCO)

Schwarzschild Black Holes

HORIZON: $r = 2M$ LIGHT RING: $r = 3M$ ISCO: $r = 6M$

Schwarzschild's solution

$$d\tau^2 = e^{2\nu}(dt)^2 - e^{2\psi}(d\phi - \omega dt - q_2 d\theta - q_3 dr)^2 - e^{\mu_2}(d\theta)^2 - e^{\mu_3}(dr)^2$$

↓

Spherical symmetry: expand in spherical harmonics EM tensor and metric

the **Zerilli-Moncrief** (even) and **Regge-Wheeler** (odd) equations respectively polar $(\nu, \psi, \mu_2, \mu_3)$ and axial (ω, q_2, q_3) perturbations

Transform to RW gauge

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^{*2}} - V^{lm}_{e/o}\right] \Psi^{lm}_{e/o}(t, r^*) = F^{lm}_{e/o}\frac{\partial}{\partial r}\delta(r - r_p(t)) + G^{lm}_{e/o}\delta(r - r_p(t))$$

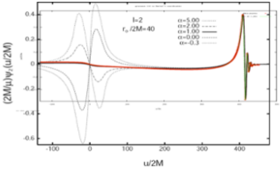
From RW-ZM to the gravitational wave

Two polarizations for the gravitational wave in TT gauge

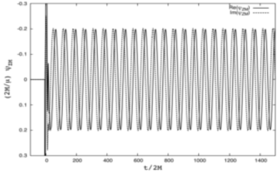
$$h_+ - ih_\times = \frac{1}{r}\sum_{lm}\left(\Psi_e^{lm} - 2i\int_{-\infty}^{t}\Psi_o^{lm}dt'\right)V_{AB}^{l}\bar{m}^A\bar{m}^B$$

- $T_{\mu\nu}^{GW} = \frac{1}{64\pi} < D_\mu h^{\alpha\beta} D_\nu h_{\alpha\beta} >$
- $P = \frac{1}{32\pi}\sum_{lm}\frac{(l+2)!}{(l-2)!}\left(|\dot{\Psi}_e^{lm}|^2 + 4|\Psi_o^{lm}|^2\right)$
- $\frac{dL}{dt} = \mathcal{R}e\left[\frac{i}{64\pi}\sum_{lm} m\frac{(l+2)!}{(l-2)!}\left(\dot{\Psi}_e^{lm}\Psi_e^{*lm} + 4\Psi_o^{*lm}\int_{-\infty}^{t}\Psi_o^{*}(t')dt'\right)\right]$

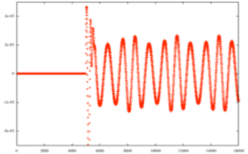
Some waveforms in time domain



Radial infall quadrupole wave for infall from $r_0 = 30M$ [Lousto,Price 1997]



Circular orbit $r = 12M$, $(l, m) = (2, 2)$ wave [Martel 2005]



Eccentric orbit from geodesic deviations, $(l, m) = (2, 2)$ wave [Koekoek,van Holten 2011]

So..

Stages of an EMR binary

- inspiral: eccentric orbits, adiabatic
- waveforms, SF corrections, etc..
- what about plunge?
- quasi-circular orbit/inspiralling orbit

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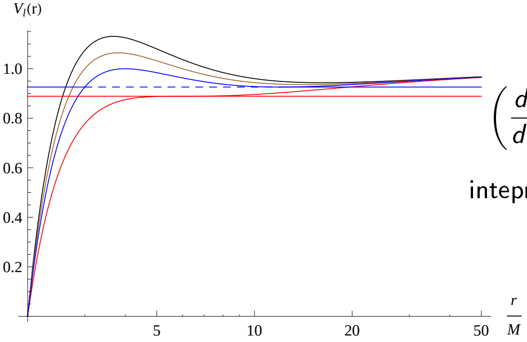
Lagrangian formulation of Schwarzschild metric

- $\mathcal{L} = \frac{1}{2}\left[\left(1 - \frac{2M}{r}\right)\dot{t}^2 - \frac{\dot{r}^2}{1-\frac{2M}{r}} - r^2\dot{\theta}^2 - (r^2\sin^2\theta)\dot{\varphi}^2\right] = const$
- energy and angular momentum first integrals of the motion
- EMR binaries circularize until $\sim 6M$

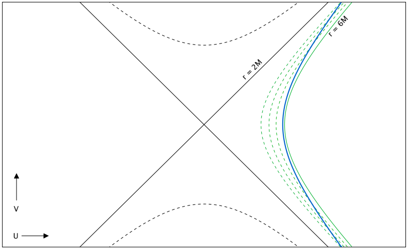


$$\left(\frac{dr}{d\tau}\right)^2 + \left(1 - \frac{2M}{r}\right)\left(1 + \frac{L^2}{r^2}\right) = E^2$$

intepret the terms as kinetic energy
and
potential energy

From the effective potential V_l:

- $V_\ell = \left(1 - \frac{2M}{r}\right)\left(1 + \frac{\ell^2}{r^2}\right)$
- for $\ell^2 > 12M^2$ there are two kinds of circular orbits
- V_ℓ extremized for $R_\pm = \frac{\ell^2}{2M}\left(1 \pm \sqrt{1 - \frac{12M^2}{\ell^2}}\right)$



$$\xi = \sqrt{1 - \frac{12M^2}{\ell^2}}$$

$$\varepsilon_\pm^2 = V_\ell[R_\pm] = \frac{2}{9}\frac{(2 \pm \xi)^2}{1 \pm \xi}$$

Introduce the ballistic orbit

$$r(\varphi) = \frac{6M}{1 + 2\xi + 3\xi \cot^2\left(\frac{A}{2}\varphi\right)}$$

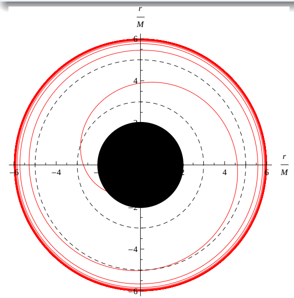
in $1-1$ correspondence with stable circular orbits $(r = R_+)$!

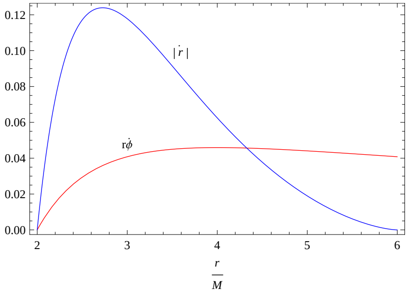
What for? Plunge phase beyond the ISCO, $r < 6M$

μ starts at $r \approx 6M$ towards the horizon

$$r(\varphi) = \frac{6M(1-\delta)}{1 + e\cot^2\left(\frac{A}{2}\varphi\right)}$$

- perturbative parameter $\delta = \frac{2}{3}e = \frac{2\xi}{1+2\xi}$, so $0 < \delta < \frac{2}{3}$
- solve geodesic equations as evolution equations





Analytically solved evolution

$$\frac{1}{(2-e)}\sqrt{\frac{e}{2(1-e)}}\frac{t-t_0}{4M} = \frac{(3-2e)^2 A\varphi}{2(2-e)(1-e)^2} + \frac{e(3-2e)}{2(1-e)}\frac{\cot\frac{A\varphi}{2}}{1+e\cot^2\frac{A\varphi}{2}}$$
$$-\frac{(11-11e+2e^2)\sqrt{e}}{2(1-e)^2}\arctan\left(\frac{1}{\sqrt{e}}\tan\frac{A\varphi}{2}\right)$$
$$-\frac{1}{(2-e)}\sqrt{\frac{e}{2(1-e)}}\,\mathrm{arccotanh}\left(\sqrt{\frac{2(1-e)}{e}}\tan\frac{A\varphi}{2}\right).$$

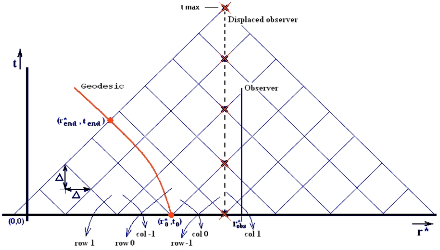
$\nu=\frac{\mu}{M}$	$\delta\times10^3$	$\gamma=\nu^{-2/5}\delta$	r_*/M	total turns	turns after r_*
10^{-7}	1.3	0.82	4.328	18.71	0.49
10^{-6}	3.2	0.80	4.328	11.59	0.50
10^{-5}	7.8	0.78	4.328	7.09	0.50
10^{-4}	18.3	0.73	4.327	4.29	0.50
10^{-3}	41.2	0.67	4.323	2.53	0.50

universal geodesic behavior

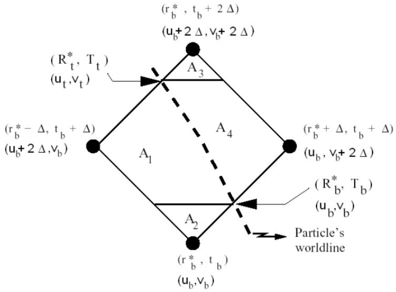
- number of turns $\sim 4\nu^{-1/5}$ [Buonanno 2000, Bernuzzi 2010]
- begin at $\frac{r}{M}=6-\alpha\nu^{2/5}$, [Buonanno 2000, Damour 2007]

Lousto-Price algorithm

$$
\begin{aligned}
\psi^{lm}_{RW/ZM}(t+2\Delta, r^*) \;=\;& -\psi^{lm}_{RW/ZM}(t, r^*)\,\frac{1+\frac{A_2}{4}\,\bar{V}^{lm}_{RW/ZM}(r^*)}{1+\frac{A_3}{4}\,\bar{V}^{lm}_{RW/ZM}(r^*)} - \frac{\int_{cell}\bar{S}^{lm}_{RW/ZM}(t,r)\,dt\,dr^*}{4+\bar{V}^{l}(r^*)A_4}\\
&+ \frac{1-\frac{A_1}{4}\,\bar{V}^{l}_{RW/ZM}(r^*-\Delta)}{1+\frac{A_3}{4}\,\bar{V}^{l}_{RW/ZM}(r^*)}\,\psi^{lm}_{RW/ZM}(t+\Delta, r^*-\Delta)\\
&+ \frac{1-\frac{A_4}{4}\,\bar{V}^{l}_{RW/ZM}(r^*+\Delta)}{1+\frac{A_3}{4}\,\bar{V}^{lm}_{RW/ZM}(r^*)}\,\psi^{lm}_{RW/ZM}(t+\Delta, r^*+\Delta)
\end{aligned}
$$

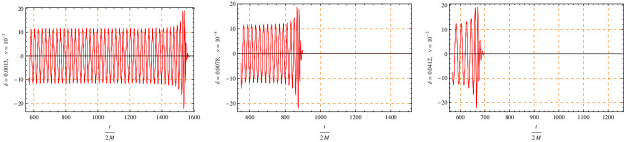


Past light cone where the ZM/RW equations are integrated.
Starting from the values on two rows (times) one retrieves the following ones



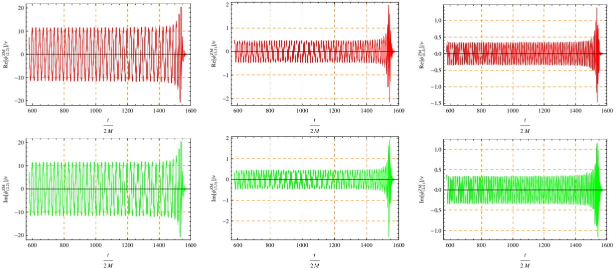
Universal geodesic behavior

- smaller values of δ imply longer quasi-circular phase
- quasi-circular phase gives quasi-periodic emission
- final burst independent of $\delta \Rightarrow \nu$



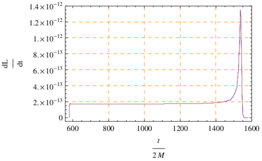
$\psi^{(2,2)}_{ZM}/\nu$ amplitudes for $\delta = (0.0013, 0.0078, 0.0412)$, only real parts shown. The values of δ are chosen to match mass ratios $\nu = (10^{-7}, 10^{-5}, 10^{-3})$.

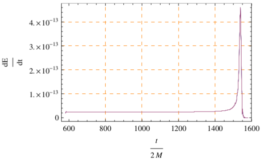
Waveforms obtained with this orbit



(2,2)-(3,3)-(4,4) waves

amplitude decreases fast with increasing *l*-mode





How to help? Geodesic deviations!

Energy and angular momentum of ballistic orbits bigger than ISCO ($\xi = 0$)

$$\varepsilon^2 = \frac{2}{9}\frac{(2+\xi)^2}{1+\xi} = \frac{2(2-e)^2}{9-9e+2e^2} \qquad \ell^2 = \frac{12M^2}{1-\xi^2} = \frac{4M^2(3-2e)^2}{3-4e+e^2}$$

Knowing a simple geodesic in analytical form in time, one can try to turn it into a more general orbit:

$$x^\mu = \bar{x}^\mu + \sigma n^\mu + \frac{1}{2}\sigma^2(k^\mu - \bar{\Gamma}^\mu_{\lambda\nu} n^\lambda n^\nu) + \cdots$$

with $n^\mu = \frac{\partial x^\mu}{\partial \sigma}|_{\sigma=0}$ and $k^\mu = \frac{\partial n^\mu}{\partial \sigma}|_{\sigma=0} + \bar{\Gamma}^\mu_{\lambda\nu} n^\lambda n^\nu$

with the geodesic deviation equation to be solved, to first-order:

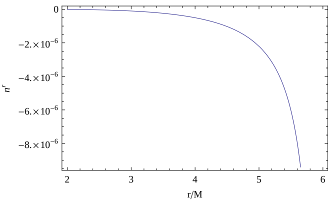
$$\frac{D^2 n^\mu}{D\tau^2} - \bar{R}^\mu{}_{\lambda\nu\kappa}\bar{u}^\mu \bar{u}^\lambda n^\nu = 0$$

Energy and angular momentum get corrections as well

$$\left(1-\frac{2M}{r}\right)\frac{dt}{d\tau}=\varepsilon=\varepsilon_0+\sigma\varepsilon_1+o(\sigma^2)$$

$$r^2\frac{d\varphi}{d\tau}=\ell=\ell_0+\sigma\ell_1+o(\sigma^2)$$

ε_1 and ℓ_1 tunable to any matching orbit: this case $r=6M$



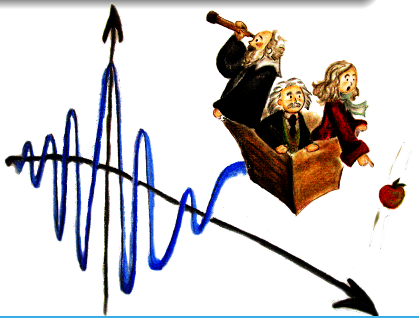
- n^μ obtained analytically
- corrections decrease along plunge
- in preparation. . .

Conclusions..

..and further directions

- explored a sector of geodesic motion
- developed a way to investigate GW during plunge
- use ballistic orbit for geodesic deviations
- obtain a confirmation of the universal behaviour?
- use bal.orbits for GWs

thank you for your attention!



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