Overspinning a Kerr black hole: the effect of self-force

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Overview

1. Testing the cosmic censorship with binary systems
2. Overspinning a Kerr BH in the geodesic approximation
3. The self-force effect
   - Critical orbits including the SF
   - Censorship conditions including the SF
   - Numerical results
4. Conclusions

What enforces cosmic censorship?

Is CC enforced at the level of the background geometry?

- Test particle in extremal Kerr-Newman space-time (Wald, 1972): **no violations** (only need to look at the kinematics on the background to reach the conclusion)
- Is the censorship encoded in the **geodesic** equations of motions then?

What enforces cosmic censorship?

- The geodesic approximation is not enough for nearly-extremal spacetimes
- Violations were found for Reissner-Nordström (Hubeny, 1999) and Kerr (Jacobson and Sotiriou, 2010)
- These occur when the initial configuration is such that $Q - M \sim m^2$, or $J - M^2 \sim m^2$

A delicate balance

Two competing small parameters in these scenarios:

- deviation from extremality
- mass of the body orbiting the black hole

In nearly-extremal spacetimes the conjecture must be enforced in a more complex way (radiative effects, self-force...)!

Previous works incorporating back-reaction: charged BH

- Isoyama, Sago and Tanaka: reformulate the problem in term of a static configuration, which admits analytical treatment
- Zimmeram, Poisson, Vega and Haas: numerical computation of the EM self-force –found to exert just the right amount of repulsive effect, but neglect potentially important effects of the gravitational self-force

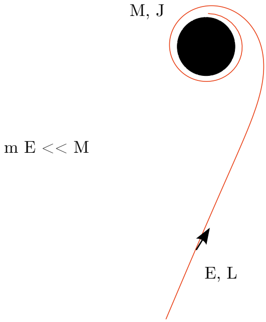
The system under consideration

Binary system composed of

- nearly extremal Kerr BH of mass M and angular momentum J, with spin parameter

$$\tilde{a} := J/M^2 = 1 - \epsilon^2, \quad \epsilon \ll 1$$

- small, non-spinning, non-charged body of mass m, such that $\eta = m/M \ll 1$, on an **equatorial** orbit



Previous works incorporating back-reaction: Kerr BH

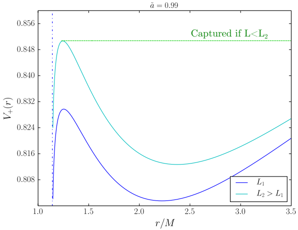
- Previous works (Barausse, Cardoso and Khanna) numerically computed radiative effects for ultra-relativistic orbits
- They showed that radiative effects cannot always prevent the BH from being overspun

In this work we

- Relax ultra-relativistic assumption and allow for fine-tuning
- Account for the full back-reaction (radiative + conservative), working consistently at first order

Finding the overspinning domain in the test particle approximation

- The particle has to overcome a potential barrier $\eta L < \eta L_c(E)$ (exclude deeply bound orbits)



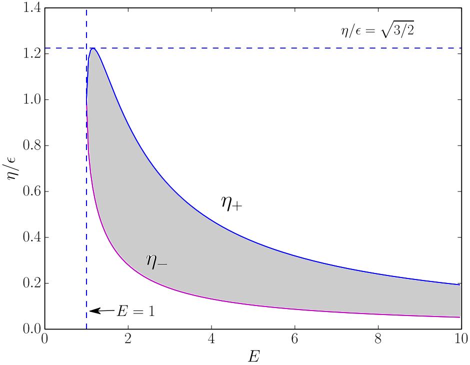
- but also needs to have the right proportion of energy and angular momentum to overspin:

$$(M + \eta E)^2 < aM + \eta L \tag{1}$$

- the maximum width of the L range satisfying both conditions is

$$\max_{\eta} \eta \Delta_L = \frac{\epsilon^2 (E^2 - 1)}{2E^2}$$

Test mass approximation: overspinning domain



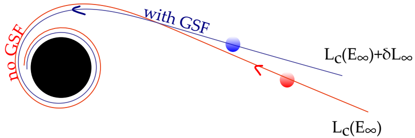
$\forall E > 1$ overspinning achieved ($\Delta_L > 0$) in the range

$$\epsilon\eta_-(E) < \eta < \epsilon\eta_+(E) \tag{2}$$

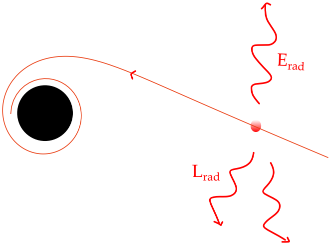
$\Rightarrow$ We will focus on unbound orbits

The GSF effect

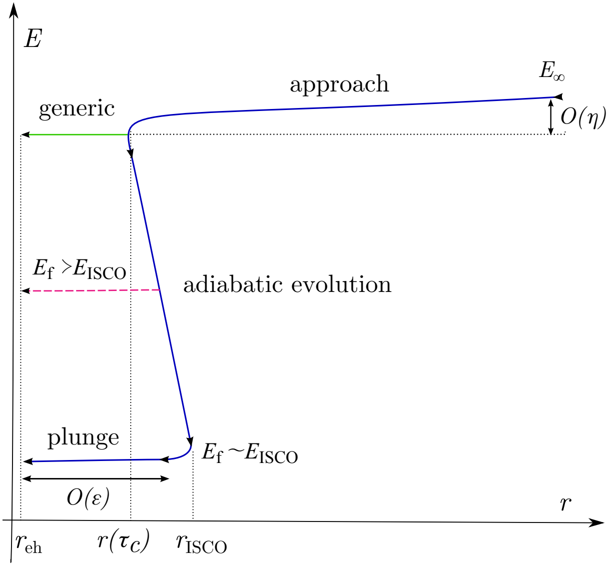
1. Shift in the parameters of the "critical" orbits (defining the separatrix between scatter and plunge)



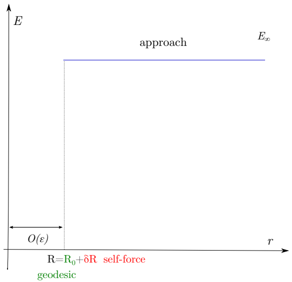
2. The small body radiates energy and angular momentum



Evolution of critical, unbound orbits including the self-force



Conservative shift in the critical angular momentum



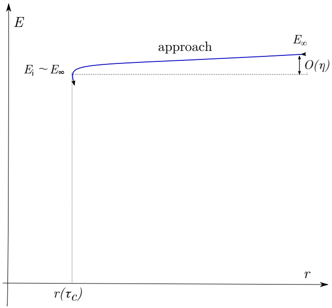
- Contribution to $\delta L_\infty(E_\infty)$ from the quasi-circular motion is shown to be negligible at leading order in η, ϵ
-
$$\delta L_\infty^{cons}(E_\infty) = -\frac{1}{2\eta}\int_{-\infty}^{\tau_c}(2F_t^{cons} + F_\phi^{cons})d\tau := W_{cons},$$

  where τ_c is some arbitrary time at the end of approach (such that $R - r(\tau_c) \ll 1$).

The effect of the dissipative self-force

1. The total $\delta L_\infty(E_\infty)$ contains a dissipative contribution
2. The final Bondi energy and angular momentum of the system do not correspond to the ADM ones (some radiation is emitted to null infinity)

Shift in the critical angular momentum with the full self-force



$$\delta L_\infty(E_\infty) = \delta L_\infty^{cons}(E_\infty) - \mathcal{W}_{appr}^{+}/\eta$$

where $W_{appr}^{+} = -\frac{1}{2\eta}\int_{-\infty}^{\tau_c}(2F_t^{diss} + F_\phi^{diss})d\tau$ (at leading order, **only** the **approach** contributes).

Censorship condition with the self-force

The final state is a black hole iff

$$(E_{ADM}(E_\infty) - \mathcal{E}^+(E_\infty, L_\infty))^2 - [L_{ADM}(L_\infty, E_\infty) - \mathcal{L}^+(E_\infty, L_\infty)] \geq 0$$

The only orbits that can potentially overspin are the ones for which

$$L_\infty = 2E_\infty + O(\eta, \epsilon)$$

Reduction to near critical orbits

$$(E_{ADM}(E_\infty) - \mathcal{E}^+(E_\infty, L_\infty))^2 - [L_{ADM}(L_\infty, E_\infty) - \mathcal{L}^+(E_\infty, L_\infty)] \geq 0$$

$$\Downarrow$$

$$(\eta E_\infty - \mathcal{E}^+)^2 - \mathcal{W}^+ + \eta\left[2\,\delta E_{ADM} - \left(\delta L_{ADM}^{cons} - \mathcal{W}^+_{appr}/\eta\right)\right] + \eta W_\infty + \epsilon^2 \geq 0$$

$$\Downarrow$$

$$(\eta E_\infty - \mathcal{E}^+)^2 - \mathcal{W}^+_{quasicirc} - \mathcal{W}^+_{plunge} + \eta(2\,\delta E_{ADM} - \delta L_{ADM}^{cons}) + \eta W_\infty + \epsilon^2 \geq 0$$

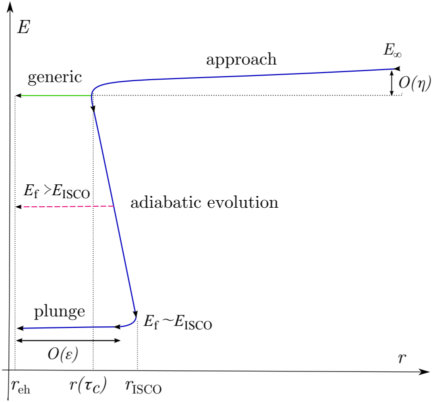
In the above we set $E_{ADM} = M\,[1 + \eta(E_\infty + \delta E_{ADM}(E_\infty))]$ (and similarly for L) and $W_\infty = 2E_\infty - L_\infty$.

Classification of near-critical orbits

We divide near-critical orbits into two families

1. **fine-tuned**, for which $\mathcal{E}^+, \mathcal{L}^+ \sim \eta$
2. **generic**, for which $\mathcal{E}^+, \mathcal{L}^+ \sim \eta^2 \log(\eta) \to$ radiative effects drop out at first order

$$(\eta E_\infty - \cancel{\mathcal{E}^+})^2 - \cancel{\mathcal{W}^+_{qc}} + \eta(2\,\delta E_{ADM} - \delta L^{cons}_{ADM}) + \eta W_\infty + \epsilon^2 \geq 0$$



Overspinning with near-critical orbits

- If one factors out the η and ϵ dependence from every term , the censorship condition can be rewritten in the compact form

$$\Phi := \epsilon^2 + \epsilon\eta F + \eta^2 H \geq 0,$$

- One can show that Φ is minimized by exactly critical orbits $\rightarrow$ generic orbits can be reduced to a **one-parameter** family of orbits

Censorship condition for generic orbits

For generic orbits

$$\Phi := \epsilon^2 + \epsilon\eta F + \eta^2 H \geq 0,$$

where

- $F := -\sqrt{6E_\infty^2 - 2}$
- $H := E_\infty^2 + 2\delta\hat{E}_{ADM} - \delta\hat{L}_{ADM}^{cons}$

Overspinning is averted provided that

$$\delta\hat{L}_{ADM} \leq \frac{1}{2}(1 - E_\infty^2) \qquad E_{ADM} = \text{fixed}$$

The shift in L_c must be negative enough to close the window where overspinning was possible in the test particle approx.

Overspinning with fine-tuned orbits

If one factors out the η and ϵ dependence from every term, the censorship condition can be rewritten as

$$\Phi := \epsilon^2 + \epsilon\eta F + \eta^2 H \geq 0,$$

where

- $F = -\hat{\mathcal{W}}^{+}_{quasicirc} + W_{\infty}$
- $H = (E_{\infty} - \hat{\mathcal{E}}^{+})^2 + 2\delta\hat{E}_{ADM} - \delta\hat{L}^{cons}_{ADM}$

A necessary and sufficient censorship condition for fine-tuned orbits is

$$H \geq \min\left(F/2, 0\right)^2$$

The effect of fine-tuning

Evaluation of the condition

The radiative contribution needs to be numerically computed. For this purpose it is convenient to introduce

$$\mathcal{R}(E) := \dot{\mathcal{E}}^-(E)/\dot{\mathcal{E}}^+(E)$$

The radiative terms featuring in the censorship condition for fine-tuned orbits can be conveniently re-expressed in terms of $\mathcal{R}(E)$:

$$\hat{\mathcal{E}}^+ = -\int_{E_\infty}^{E_f} \frac{dE}{1+\mathcal{R}(E)}$$

$$\hat{\mathcal{W}}_{qc}^+ = \int_{E_\infty}^{E_f} \frac{b(E)}{1+\mathcal{R}(\mathcal{E})} dE,$$

where $b(E)$ is defined through $\Omega = 1/2 - 1/4b(E)\epsilon + O(\epsilon^2)$.

Methods to calculate δL_{ADM}

1. The hard way: numerically compute the force along unbound orbits
2. The easier way: compute the shift on circular orbits, using the Hamiltonian formalism of Isoyama et al. or the 1st law of binary black-hole mechanics (which give $\delta L_{ADM}(\Omega)$)

We can already tell what the outcome of applying 2) is...

Conservative δL_{ADM}: how to compute it

The shift in the critical angular momentum can be related to the SF correction to the redshift $z := 1/u^t$

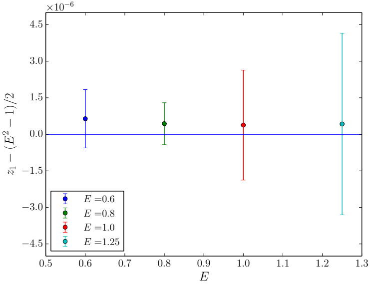
$$\delta L_{ADM}(E) = -\eta Z_1(E),$$

where $Z_1(E) := \lim_{\epsilon \to 0} z_1(\Omega(E;\epsilon), \epsilon)$

- Evaluate the correction to the redshift for a sequence of **nearly-extremal** spacetimes with $\epsilon \ll 1$
- Take the limit $\epsilon \to 0$, at fixed energy.
- Evaluate the censorship condition

$$Z_1(E) \geq \frac{1}{2}(E^2 - 1)$$

Conservative δL_{ADM}: numerical results



Remarks

- **Non-critical** orbits **cannot** overspin
- For critical, non-exponentially fine-tuned orbits, the BH appears to be **saturated** within the first order self-force approximation

Conservative δL_{ADM}: analytical derivation?

- The RHS of the censorship condition

$$Z_1(E) \geq \frac{1}{2}(E^2 - 1)$$

turns out to be equal to contribution to z_1 coming from the low multipoles $\ell = 0, \ell = 1$ in the limit $\epsilon \to 0$

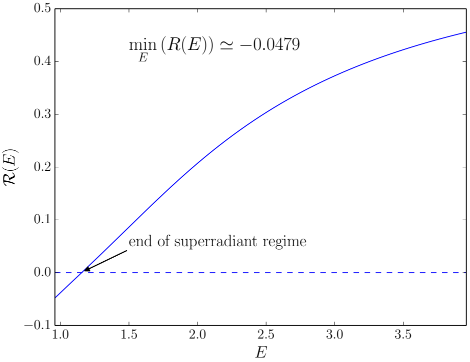
$$\lim_{\epsilon \to 0} z_1^{\ell=0} + z_1^{\ell=1} = \lim_{\epsilon \to 0} \frac{1}{2} z_0 (\delta h_{uu}^{\ell=0} + \delta h_{uu}^{\ell=1}) = \frac{1}{2}(E^2 - 1)$$

- Assuming $h_{uu}^{\ell \geq 2}$ is finite in the limit $\epsilon \to 0$, we have $\lim_{\epsilon \to 0} z_1^{l>=2} = \lim_{\epsilon \to 0} \frac{1}{2} z_0 (\delta h_{uu}^{\ell \geq 2}) = 0$

Fine-tuned orbits cannot overspin

Assuming $Z_1(E) = \frac{1}{2}(E_\infty^2 - 1)$, then one can show that the censorship condition is satisfied as long as

$$\mathcal{R}(E) := \dot{\mathcal{E}}^- / \dot{\mathcal{E}}^+ \geq -1/3$$



Conclusions

- Working at first order, overspinning is ruled out for non-critical and critical, fine-tuned orbits $\Rightarrow$ as expected, the inclusion of self-force works in favour of cosmic censorship
- Critical, non (exponentially) fine-tuned orbits represent a special case, where the second-order SF seems to be needed
- It would be interesting to compare the result of a numerical SF computation on unbound orbits with the one obtained using the 1st law framework