# エキセントリックコンパクトバイナリの軌道力学

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# Outline

① Gravitational wave source modelling

② Averaged redshift for eccentric orbits

③ First law of mechanics and applications

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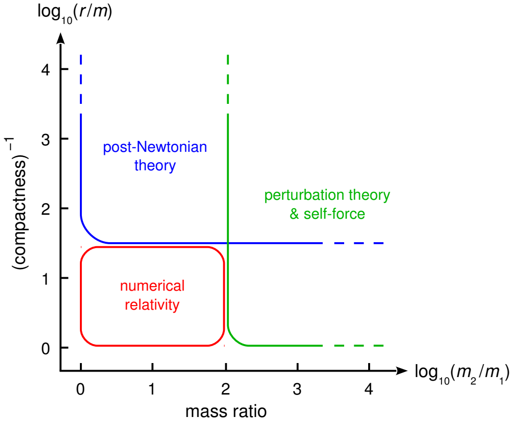
1. Gravitational wave source modelling
2. Averaged redshift for eccentric orbits
3. First law of mechanics and applications

# Source modelling of compact binaries



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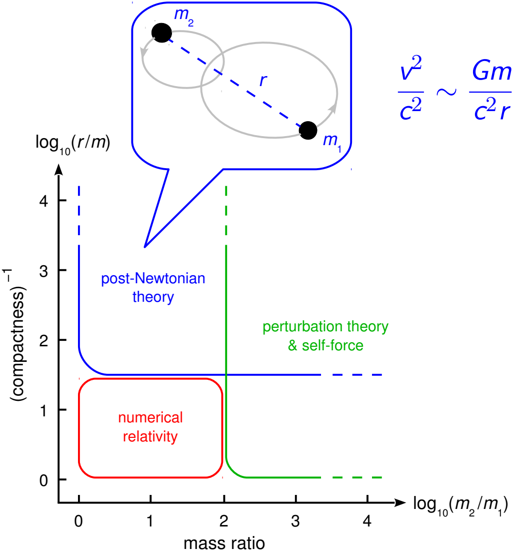
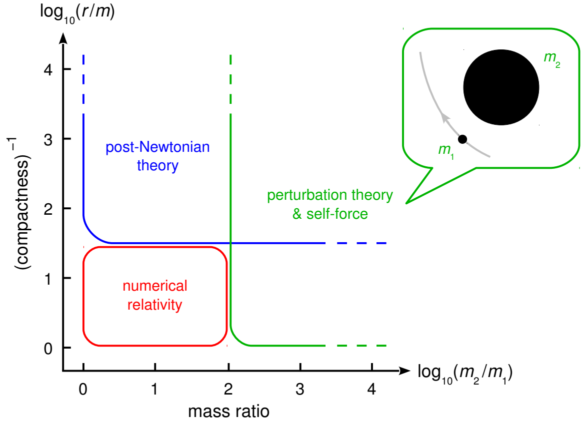



$$\frac{v^2}{c^2} \sim \frac{Gm}{c^2 r} \ll 1$$

# Source modelling of compact binaries

$$q \equiv \frac{m_1}{m_2} \ll 1$$



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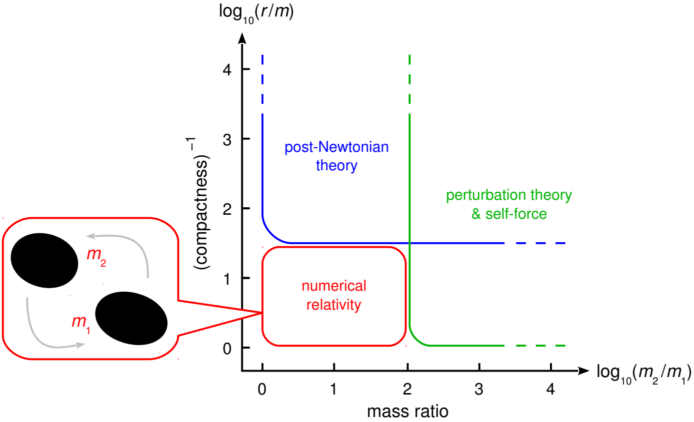

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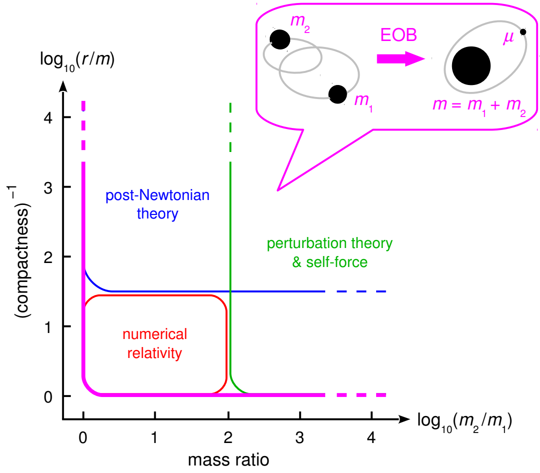

# Comparing the predictions from these methods

## Why?

- Independent checks of long and complicated calculations
- Identify domains of validity of approximation schemes
- Extract information inaccessible to other methods
- Develop a universal model for compact binaries

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✔ Using coordinate-invariant relationships

## What?

- Gravitational waveforms at future null infinity
- Conservative effects on the **orbital dynamics**

# Comparing the predictions from these methods

| Paper | Year | Methods | Observable | Orbit | Spin |
|---|---|---|---|---|---|
| Detweiler | 2008 | SF/PN | redshift observable | | |
| Blanchet et al. | 2010 | SF/PN | redshift observable | | |
| Damour | 2010 | SF/EOB | ISCO frequency | | |
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| Le Tiec et al. | 2013 | NR/SF/PN | periastron advance | | ✓ |
| Damour et al. | 2014 | NR/PN/EOB | scattering angle | hyperbolic | |
| Bini, Damour; Shah et al.; Blanchet et al | 2014 | SF/PN | redshift observable | | |
| Dolan et al.; Bini, Damour | 2014 | SF/PN | precession angle | | ✓ |
| Isoyama et al. | 2014 | SF/PN/EOB | ISCO frequency | | ✓ |
| Akcay et al. | 2015 | SF/PN | averaged redshift | eccentric | |

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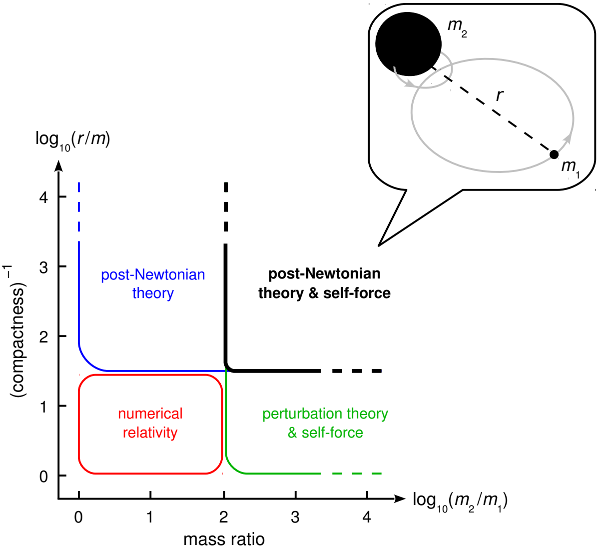
# Post-Newtonian expansions and black hole perturbations



# Redshift invariant for circular orbits

- It measures the redshift of light emitted from the point particle [Detweiler 2008]

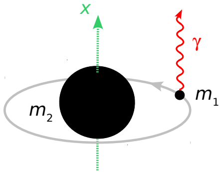
$$\frac{\mathcal{E}_{\text{obs}}}{\mathcal{E}_{\text{em}}} = \frac{(p^a u_a)_{\text{obs}}}{(p^a u_a)_{\text{em}}} = z$$

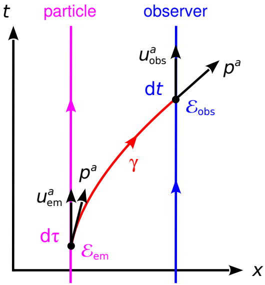
- It is a constant of the motion associated with the helical Killing field $k^a$:

$$z = -k^a u_a$$

- In coordinates adapted to the symmetry:

$$z = \frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{1}{u^t}$$



# Averaged redshift for eccentric orbits

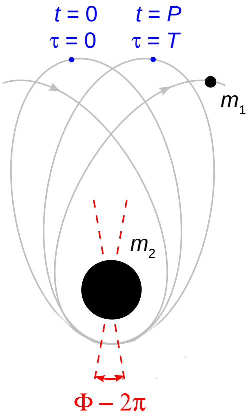
- Generic eccentric orbit parameterized by the two invariant frequencies

$$n = \frac{2\pi}{P}\,, \quad \omega = \frac{\Phi}{P}$$

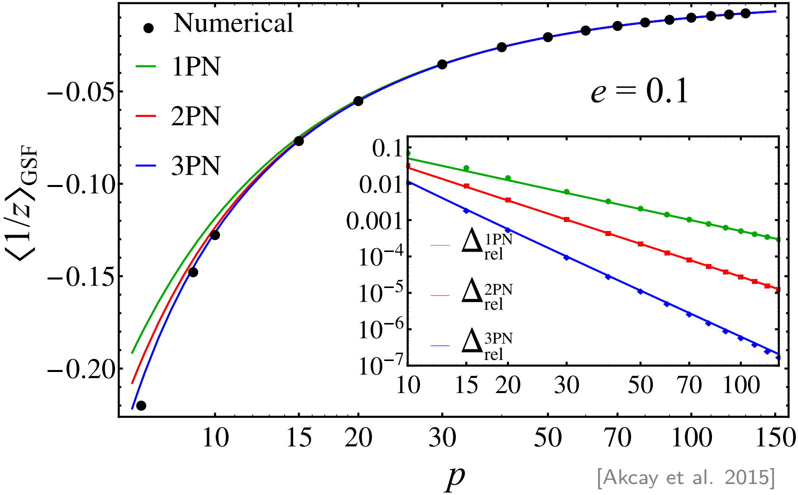
- Time average of $z = \mathrm{d}\tau/\mathrm{d}t$ over one radial period [Barack & Sago 2010]

$$\langle z \rangle \equiv \frac{1}{P}\int_0^P z(t)\,\mathrm{d}t = \frac{T}{P}$$

- Coordinate-invariant relation $\langle z \rangle(n,\omega)$ is well defined in GSF and PN frameworks



# Averaged redshift *vs* semi-latus rectum




[Akcay et al. 2015]

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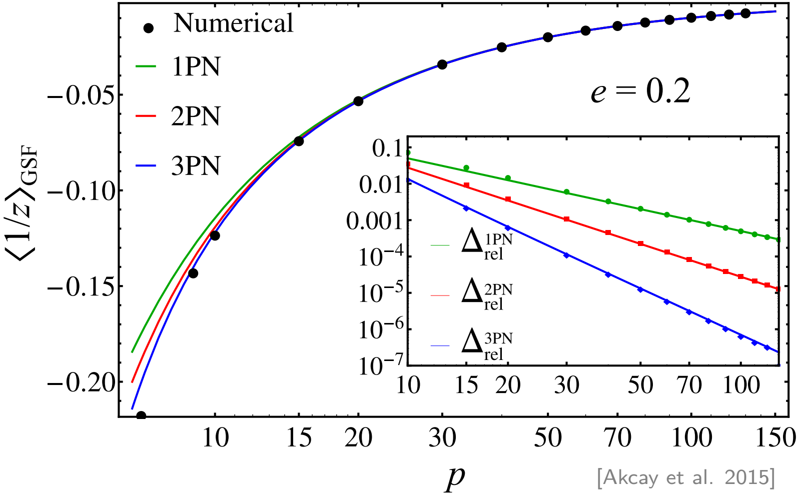


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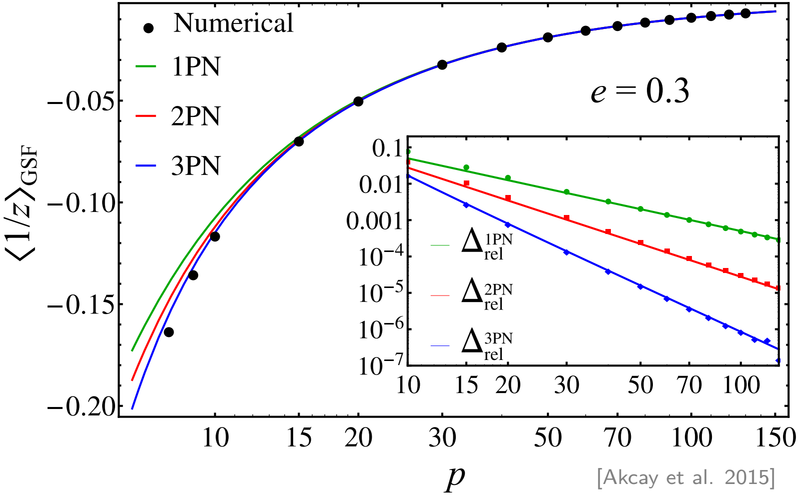


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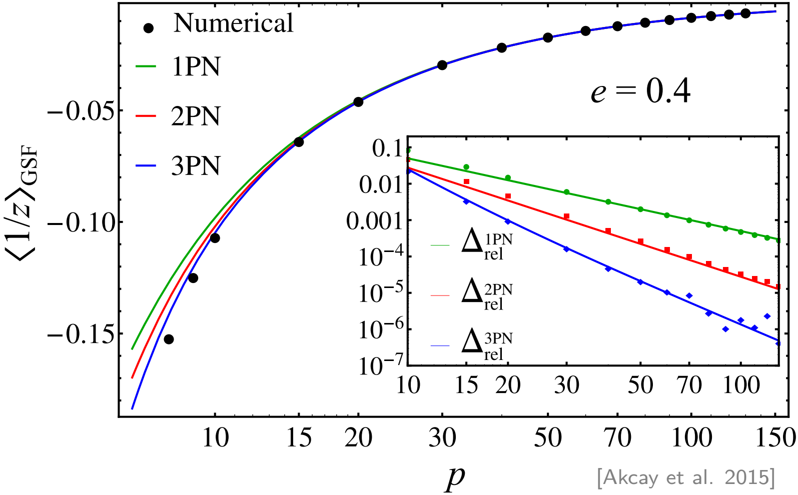


[Akcay et al. 2015]

# Extracting post-Newtonian coefficients

| | Coeff. | Exact value [Akcay et al. 2015] | Fitted value [Akcay et al. 2015] | Fitted value [Meent, Shah 2015] |
|---|---|---|---|---|
| 1PN | $e^2$ | $4$ | $4.0002(8)$ | $4 \pm 6 \times 10^{-12}$ |
| | $e^4$ | $-2$ | $-2.00(1)$ | $-2 \pm 4 \times 10^{-10}$ |
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| 3PN | $e^2$ | $-14.312097\cdots$ | $-14.5(4)$ | $-14.3120980(5)$ |
| | $e^4$ | $83.382963\cdots$ | | $83.38298(7)$ |
| | $e^6$ | $-36.421975\cdots$ | | $-36.421(3)$ |

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New coefficients at 4PN and 5PN orders [van de Meent, Shah 2015]

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# First law of binary mechanics

- Canonical ADM Hamiltonian $H$ of two point masses $m_a$
- Variation $\delta H$ + Hamilton's equation + orbital averaging:

$$\delta M = \omega\, \delta L + n\, \delta R + \sum_a \langle z_a \rangle\, \delta m_a$$

- First integral associated with the variational first law:

$$M = 2\,(\omega L + nR) + \sum_a \langle z_a \rangle\, m_a$$

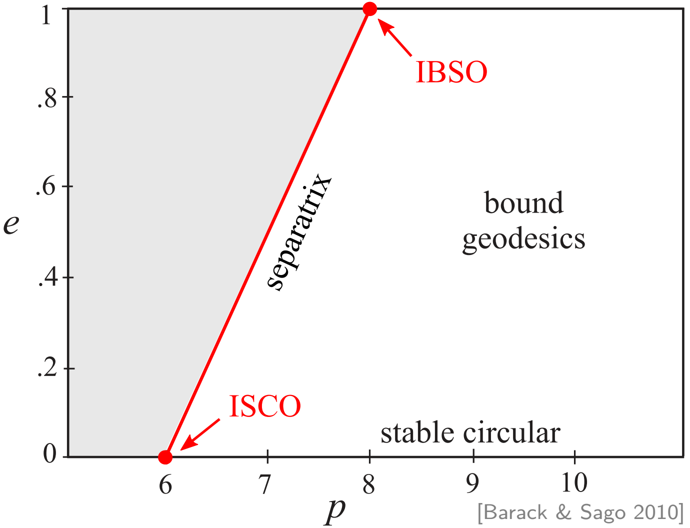
- These relations are satisfied up to *at least* 3PN order

# Applications of the first law

- Conservative dynamics beyond the geodesic approximation
- Shift of the Schwarzschild separatrix and singular curve
- Calibration of EOB potentials for generic bound orbits

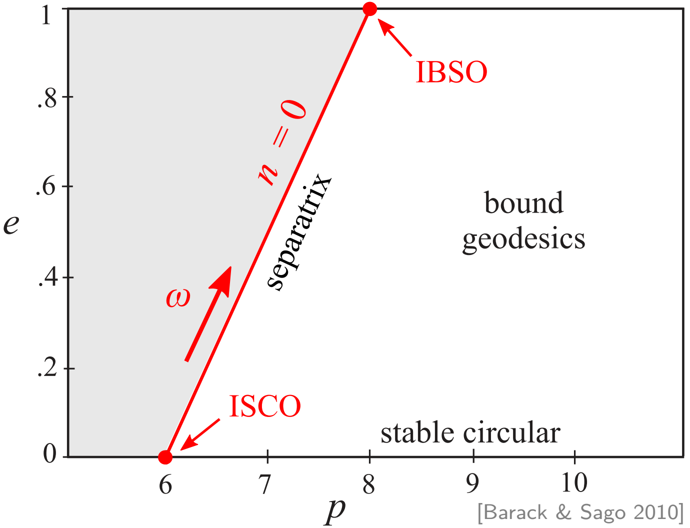
$$\frac{\partial M}{\partial m_1} = \langle z \rangle - \omega \frac{\partial \langle z \rangle}{\partial \omega} - n \frac{\partial \langle z \rangle}{\partial n}$$
$$\frac{\partial L}{\partial m_1} = -\frac{\partial \langle z \rangle}{\partial \omega}$$
$$\frac{\partial R}{\partial m_1} = -\frac{\partial \langle z \rangle}{\partial n}$$

# Schwarzschild separatrix




[Barack & Sago 2010]

# Schwarzschild separatrix




[Barack & Sago 2010]

# Shift of the Schwarzschild separatrix

- Separatrix $\omega = \omega_{\mathrm{sep}}(e)$ characterized by the condition

$$n = 0$$

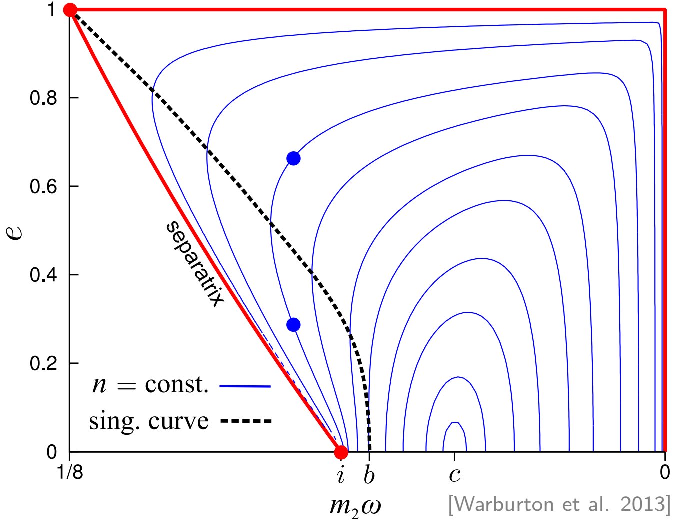
- GSF-induced shift of Schwarzschild ISCO frequency

[Barack & Sago 2009; Le Tiec et al. 2012; Akcay et al. 2012]

$$\frac{\Delta\omega_{\mathrm{isco}}}{\omega_{\mathrm{isco}}} = 1.2101539(4)\, q$$

- GSF-induced shift of Schwarzschild IBSO frequency ?
- $\mathcal{O}(q)$ shift in $\omega = \omega_{\mathrm{sep}}(e)$ controlled by $\langle z \rangle_{\mathrm{GSF}}(n, \omega)$

# Schwarzschild singular curve




[Warburton et al. 2013]

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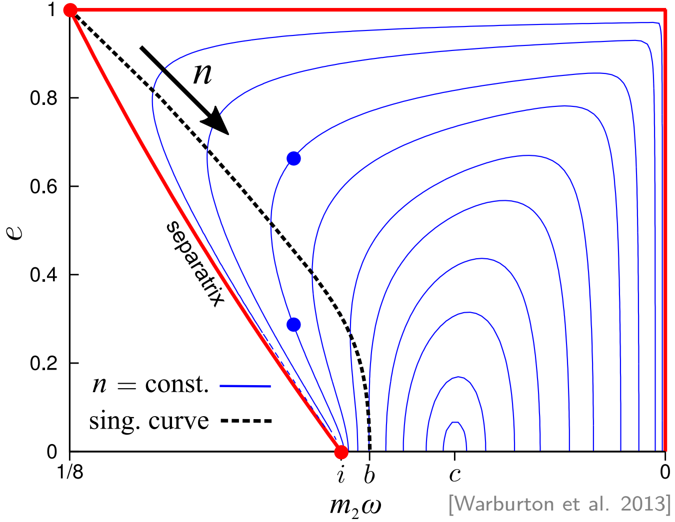



[Warburton et al. 2013]

# Shift of the Schwarzschild singular curve

- Singular curve $\omega = \omega_{\text{sing}}(n)$ characterized by condition

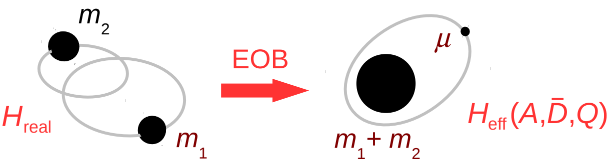
$$\left| \frac{\partial(n, \omega)}{\partial(M, L)} \right| = 0$$

- In the test-particle limit $q \to 0$ this is equivalent to

$$\left[ \left( \partial^2_{n\omega} \langle z \rangle \right)^2 - \partial^2_n \langle z \rangle \, \partial^2_\omega \langle z \rangle \right]^{-1} = 0$$

- $\mathcal{O}(q)$ shift in $\omega = \omega_{\text{sing}}(n)$ controlled by $\langle z \rangle_{\text{GSF}}(n, \omega)$

# EOB dynamics beyond circular motion




- Conservative EOB dynamics determined by "potentials"

$$A = 1 - 2u + \nu\, a(u) + \mathcal{O}(\nu^2)$$
$$\bar{D} = 1 + \nu\, \bar{d}(u) + \mathcal{O}(\nu^2)$$
$$Q = \nu\, q(u)\, p_r^4 + \mathcal{O}(\nu^2)$$

- Functions $a(u)$, $\bar{d}(u)$ and $q(u)$ controlled by $\langle z \rangle_{\text{GSF}}(n,\omega)$

# Summary

- GSF/PN comparison for eccentric orbits relying on $\langle z \rangle(n, \omega)$
- First law of mechanics can be extended to eccentric orbits
- Numerous applications of the first law:
  - Conservative dynamics beyond the geodesic approximation
  - Shift of the Schwarzschild separatrix and singular curve
  - Calibration of EOB potentials for generic bound orbits
  - ...

# Prospects

- GSF/PN comparison for eccentric orbits relying on $\langle \psi \rangle(n, \omega)$
- Extension of the first law to precessing spinning binaries