Introduction to gravitational self-force

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Outline

1. Motivation
2. Motion of a point particle
3. Motion of a small extended object
4. Equation of motion at first and second order
    - First-order equation of motion
    - Second-order equation of motion
5. Computing the field
6. Modeling binaries (i.e., concrete results)
    - EMRIs
    - Other binaries

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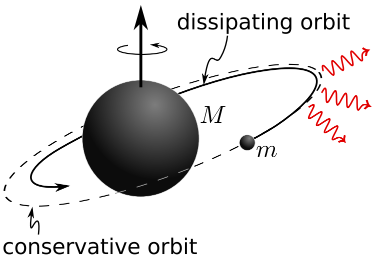
Original motivation: extreme-mass-ratio inspirals (EMRIs)

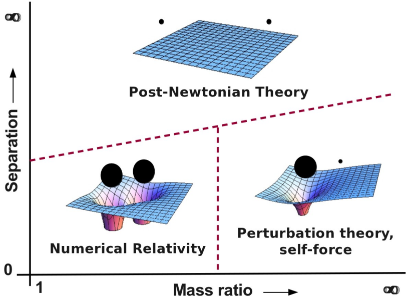


- stellar-mass neutron star or black hole orbits supermassive black hole
- m emits gravitational radiation, loses energy, spirals into M

- waveforms carry very precise information about strong-field dynamics and geometry of spacetime near black hole
- need very accurate model of motion for $\sim M/m \sim 10^5$ orbits—*self-force theory*

Self-force's place in the two-body problem

Binary parameter space



[Leor Barack]

Besides modeling EMRIs, self-force can improve models of ground-based-detector sources

- determine high-order post-Newtonian parameters
- calibrate Effective One Body theory

Also, self-force has surprisingly large domain of validity [Le Tiec et al]
$\Rightarrow$ can *directly* model IMRIs (and even similar-mass binaries?)

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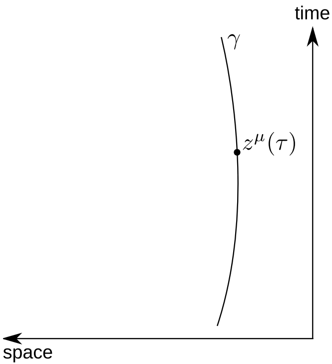
Point particle picture

Linearized theory

- treat m as point particle, with stress-energy $T^{\mu\nu} \sim m\delta^3(x^\rho - z^\rho)$
- write total metric as $g_{\mu\nu} + h_{\mu\nu}$ (e.g., $g_{\mu\nu}$ is metric of M, $h_{\mu\nu}$ is created by m)
- approximate Einstein equation $G_{\mu\nu}[g+h] = 8\pi T_{\mu\nu}$ with *linearized EFE* $\delta G^{\mu\nu}[h] = 8\pi T^{\mu\nu}$

Tail

- part of perturbation propagates slower than light
- light "cone" bends $\therefore$ $h_{\mu\nu}$ depends on past history



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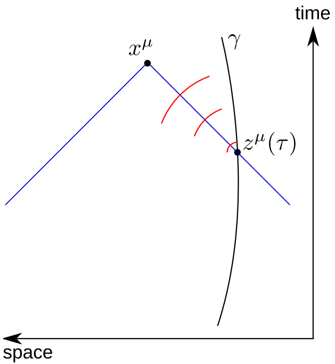

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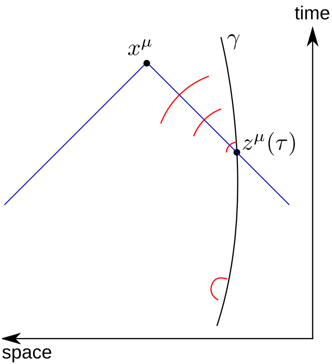

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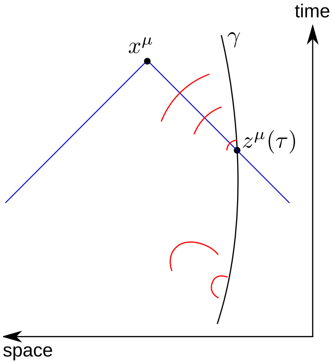

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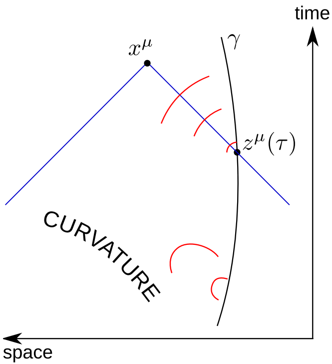

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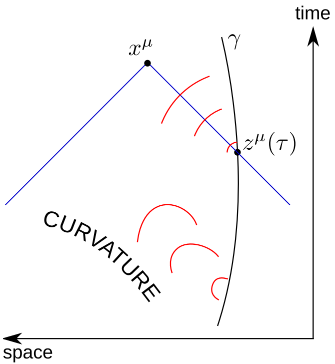

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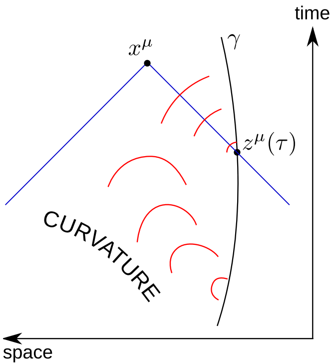

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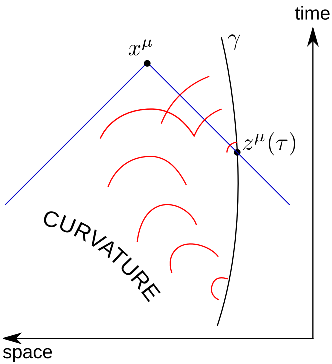

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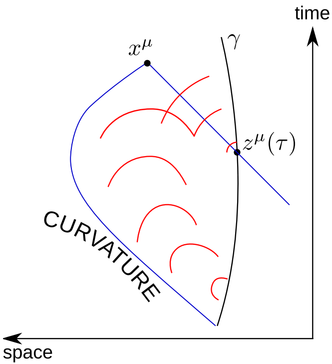


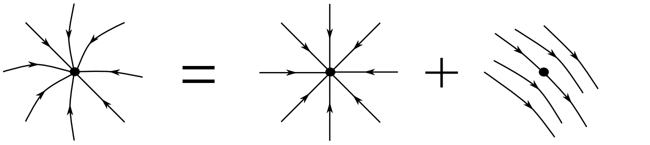
Self-force: geodesic motion in an effective metric

MiSaTaQuWa equation [Mino,Sasaki,Tanaka; Quinn & Wald]

- nonlocal tail acts as potential, exerts force $F^\mu \sim m\nabla^\mu\text{tail}$
- tail isn't nice: non-differentiable, not a solution to a field equation

Generalized equivalence principle [Detweiler &Whiting]

- local field near particle split into two: $h^{(1)}_{\mu\nu} = h^{\mathrm{S}(1)}_{\mu\nu} + h^{\mathrm{R}(1)}_{\mu\nu}$
- $h^{\mathrm{S}(1)}_{\mu\nu} \sim \frac{m}{r} + O(r^0)$; local bound field of particle
- $h^{\mathrm{R}(1)}_{\mu\nu} \sim \text{tail} + \text{local terms}$; smooth solution to source-free EFE
- motion is geodesic in effective metric $g_{\mu\nu} + h^{\mathrm{R}(1)}_{\mu\nu}$



body's field $h_{\alpha\beta}$ = singular field $h^S_{\alpha\beta}$ + regular field $h^R_{\alpha\beta}$

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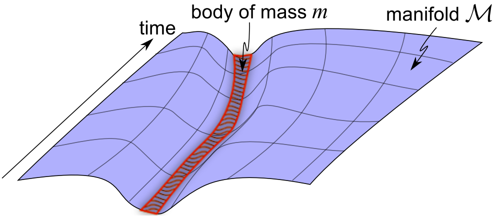
A small extended object moving through spacetime

Fundamental question

how does an object's own gravitational field affect its motion?

Regime: small object

- examine spacetime $(\mathcal{M}, g_{\mu\nu})$ containing object of mass m and external lengthscales $\mathcal{R}$
- seek representation of object's motion when its mass and size are $\ll \mathcal{R}$

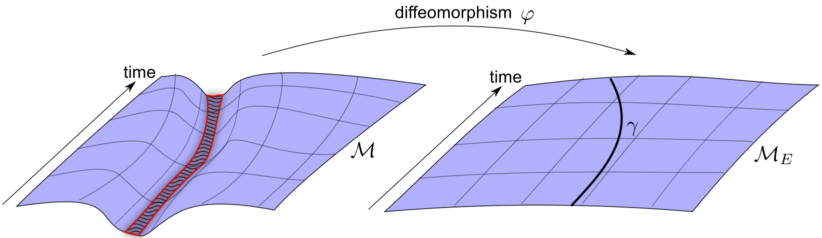


Perturbative description [Mino, Sasaki, Tanaka; Quinn & Wald; Detweiler & Whiting; Gralla & Wald; Pound; Harte]

- treat object as source of perturbation of external background spacetime $(\mathcal{M}_E, g_{\mu\nu})$:

$$\mathsf{g}_{\mu\nu} = g_{\mu\nu} + \epsilon h^{(1)}_{\mu\nu} + \epsilon^2 h^{(2)}_{\mu\nu} + \dots$$

- ϵ counts powers of m
- assume object is compact, so as $m \to 0$, linear size $\to 0$ at same rate
- seek representation of motion in $(\mathcal{M}_E, g_{\mu\nu})$



Body in exact spacetime

Representation of motion in external spacetime

Challenges in applying perturbation theory

Multiple time scales in binary

- orbital period ($\sim M$)
- time over which inspiral occurs ($\sim M^2/m$)

Multiple length scales

- near small object: scale of object's size ($\sim m$)
- everywhere else: scale of external universe ($\sim M$)

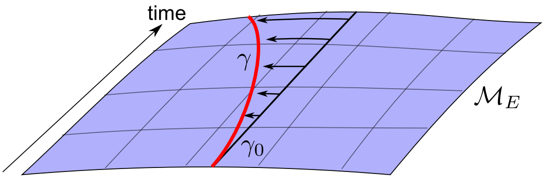
Identifying object's position, spin, higher moments

- point particle not valid in nonlinear field theory such as GR
- how do we capture bulk parameters without worrying about details of object's composition?
- how do we best represent the small object's bulk motion (e.g., identify its "center")?

Long time scales: self-consistent approximation

Worldline

rather than find small corrections to zeroth-order motion, seek worldline $\gamma(\tau, \epsilon)$ that tracks object's bulk motion

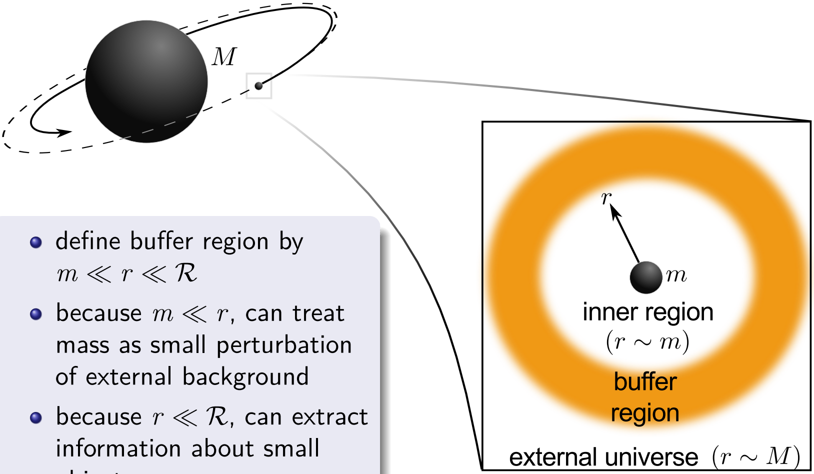


Self-consistent expansion

- since $h_{\mu\nu}$ depends on γ, can't expand $h_{\mu\nu}$ in regular power series without also expanding γ
- allow γ to depend on ϵ and assume expansion of form

$$\begin{aligned} \mathrm{g}_{\mu\nu}(x, \epsilon) &= g_{\mu\nu}(x) + h_{\mu\nu}(x, \epsilon; \gamma) \\ &= g_{\mu\nu}(x) + \epsilon h^{(1)}_{\mu\nu}(x; \gamma) + \epsilon^2 h^{(2)}_{\mu\nu}(x; \gamma) + \ldots \end{aligned}$$

Small length scales: matched asymptotic expansions

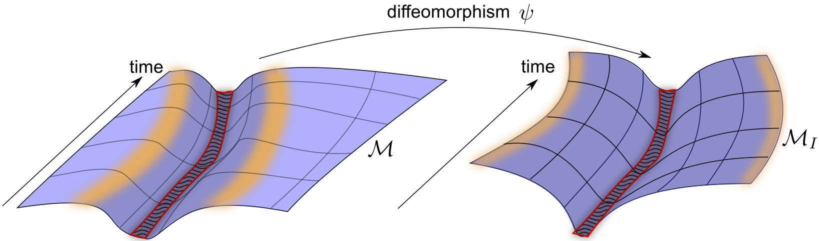


- define buffer region by $m \ll r \ll \mathcal{R}$
- because $m \ll r$, can treat mass as small perturbation of external background
- because $r \ll \mathcal{R}$, can extract information about small object

Matched asymptotic expansions: the *inner expansion*

Zoom in on object

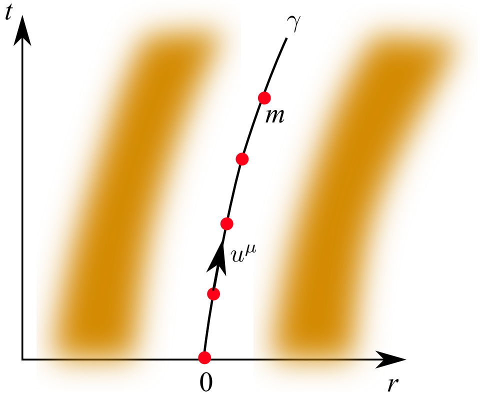
- use scaled coords $\tilde{r} \sim r/\epsilon$ to keep size of object fixed, send other distances to infinity as $\epsilon \to 0$
- unperturbed object defines background spacetime $g_{I\mu\nu}$ in inner expansion
- buffer region at asymptotic infinity $r \gg m$
  $\Rightarrow$ can define multipole moments without integrals over object



Position at first order: self-consistent definition

Mass dipole about γ

We want to find worldline γ for which $M^\mu = 0$

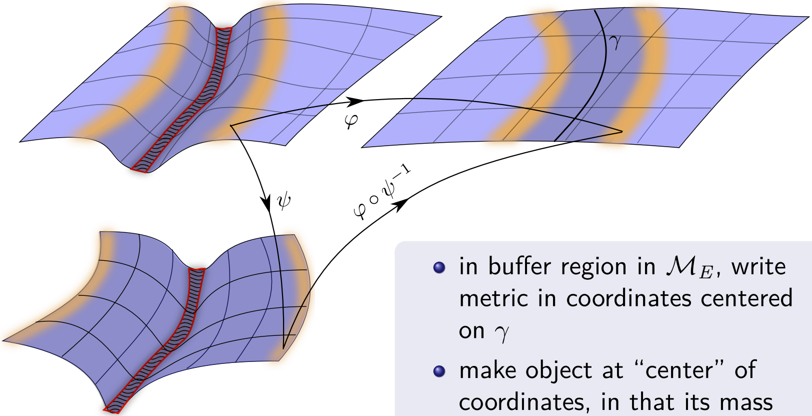


- work in coordinates centered on unspecified γ
- calculate mass dipole M^μ of inner background $g_{I\mu\nu}$
- first-order acceleration of γ: whatever ensures $M^\mu \equiv 0$

Position in self-consistent expansion (continued)

Enforce a relationship between the expansions

...to define a worldline for all time, even for black hole



- in buffer region in $\mathcal{M}_E$, write metric in coordinates centered on γ
- make object at "center" of coordinates, in that its mass dipole vanishes in $\mathcal{M}_I$

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Solving the EFE in buffer region

Expansion for small r

- in coordinates centered on γ, allow all negative powers of r in $h^{(n)}_{\mu\nu}$
- but inner expansion must not have negative powers of ϵ
  $\Rightarrow$ most negative power of r in $\epsilon^n h^{(n)}_{\mu\nu}$ is $\frac{\epsilon^n}{r^n} = \frac{\epsilon^n}{\epsilon^n \tilde{r}^n} = \frac{1}{\tilde{r}^n}$

Therefore

$$h^{(n)}_{\mu\nu} = \frac{1}{r^n} h^{(n,-n)}_{\mu\nu} + r^{-n+1} h^{(n,-n+1)}_{\mu\nu} + r^{-n+2} h^{(n,-n+2)}_{\mu\nu} + \dots$$

Information from inner expansion

- $1/\tilde{r}^n$ terms arise from asymptotic expansion of zeroth-order background in inner expansion
  $\Rightarrow$ $h^{(n,-n)}_{\mu\nu}$ is determined by multipole moments of isolated object

Form of solution in buffer region

What appears in the solution?

- put expansion into nth-order wave equation, solve order by order in r
- expand each $h^{(n,p)}_{\mu\nu}$ in spherical harmonics (wrt angles on sphere around $r = 0$)
- given a worldline γ, the solution at all orders is fully characterized by
  1. object's multipole moments (and corrections thereto): $\sim \frac{Y^{\ell m}}{r^{\ell+1}}$
  2. smooth solutions to vacuum wave equation: $\sim r^\ell Y^{\ell m}$
- everything else made of (linear or nonlinear) combinations of the above

Self field and regular field

- multipole moments define $h^{\mathrm{S}(n)}_{\mu\nu}$; interpret as bound field of object
- smooth homogeneous solutions define $h^{\mathrm{R}(n)}_{\mu\nu}$; free radiation, determined by global boundary conditions

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First and second order solutions

First order

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- $h^{S(1)}_{\mu\nu} \sim 1/r + O(r^0)$ defined by mass monopole m
- $h^{R(1)}_{\mu\nu}$ is undetermined homogenous solution regular at $r = 0$
- evolution equations: $\dot{m} = 0$ and $a^\mu_{(0)} = 0$
  (where $\frac{D^2 z^\mu}{d\tau^2} = a^\mu_{(0)} + \epsilon a^\mu_{(1)} + \ldots$)

Second order

- $h^{(2)}_{\mu\nu} = h^{S(2)}_{\mu\nu} + h^{R(2)}_{\mu\nu}$
- $h^{S(2)}_{\mu\nu} \sim 1/r^2 + O(1/r)$ defined by
  1. monopole correction δm
  2. mass dipole M^μ (set to zero)
  3. spin dipole S^μ
- evolution equations: $\dot{S}^\mu = 0$, $\delta\dot{m} = \ldots$, and $a^\mu_{(1)} = \ldots$

Equation of motion

MiSaTaQuWa-Mathisson-Papapetrou equation

$$a^\alpha_{(1)} = -\tfrac{1}{2}\left(g^{\alpha\delta} + u^\alpha u^\delta\right)\left(2h^{R(1)}_{\delta\beta;\gamma} - h^{R(1)}_{\beta\gamma;\delta}\right)u^\beta u^\gamma + \tfrac{1}{2m}R^\alpha{}_{\beta\gamma\delta}u^\beta S^{\gamma\delta}$$

- self-force (due to regular field) + Papapetrou spin-force
- together with $\dot{m} = 0$ and $\dot{S}^{\mu\nu} = 0$
- through order ϵ, small object moves as a test body in $g_{\mu\nu} + h^R_{\mu\nu}$

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Why second order?

Modeling EMRIs

- inspiral occurs very slowly, on time scale $t \sim 1/m$
- neglecting second-order self-force leads to error in acceleration $\delta a^\mu \sim m^2$
  $\Rightarrow$ error in position $\delta z^\mu \sim m^2 t^2$
  $\Rightarrow$ after time $t \sim 1/m$, error $\delta z^\mu \sim 1$

$\therefore$ accurately describing orbital evolution requires second-order force

Modeling IMRIs and similar-mass binaries

- second-order self-force should yield highly accurate model for IMRIs
- will fix terms quadratic in mass in post-Newtonian and Effective One Body theory

Position at second order: mass-centered gauges

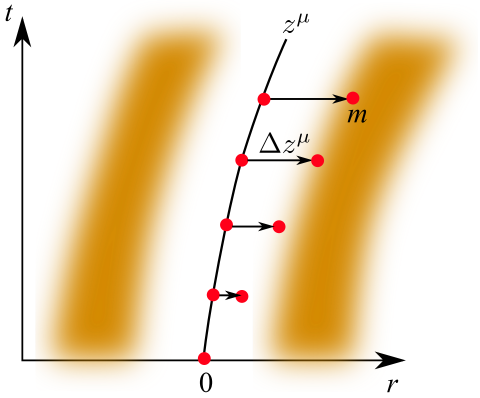
Problem

- mass dipole moment defined for asymptotically flat spacetimes
- beyond zeroth order, inner expansion is not asymptotically flat

Solution

- find gauge in which field is manifestly mass-centered on γ
- define position in other gauges by referring to transformation to that mass-centered gauge

Position at second order [Pound]



- start in gauge mass-centered on z^μ
- demand that transformation to practical (e.g., Lorenz) gauge does not move z^μ
- i.e., insist $\Delta z^\mu = 0$
- ensures worldline in the two gauges is the same

Self-consistent equation of motion [Pound]

Neglecting object's spin and quadrupole moment,

$$\frac{D^2 z^\mu}{d\tau^2} = \frac{1}{2}\left(g^{\mu\nu} + u^\mu u^\nu\right)\left(g_\nu{}^\rho - h^{\mathrm{R}\rho}_\nu\right)\left(h^{\mathrm{R}}_{\sigma\lambda;\rho} - 2h^{\mathrm{R}}_{\rho\sigma;\lambda}\right) u^\sigma u^\lambda + O(\epsilon^3)$$

- here $h^{\mathrm{R}}_{\mu\nu} = \epsilon h^{\mathrm{R}(1)}_{\mu\nu} + \epsilon^2 h^{\mathrm{R}(2)}_{\mu\nu}$

Generalized equivalence principle

- z^μ satisfies geodesic equation in $g_{\mu\nu} + h^{\mathrm{R}}_{\mu\nu}$
- recall: here $g_{\mu\nu} + h^{\mathrm{R}}_{\mu\nu}$ is a "physical" field in the sense of satisfying vacuum EFE
- extends results of Detweiler-Whiting to second order

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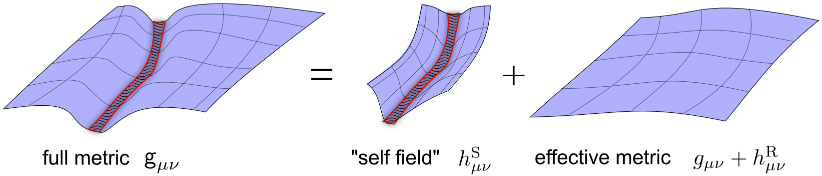
Effective interior metric

From self-field to singular field

- $h^{\rm S}_{\mu\nu}$ and $h^{\rm R}_{\mu\nu}$ derived only in buffer region
- simply extend them to all $r > 0$ (and $r = 0$, for $h^{\rm R}_{\mu\nu}$)
- does not change field in buffer region or beyond



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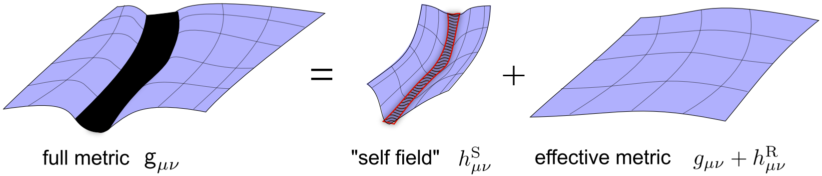

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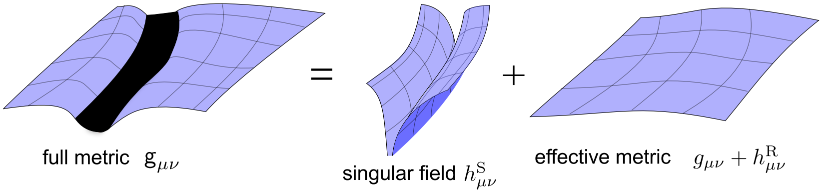
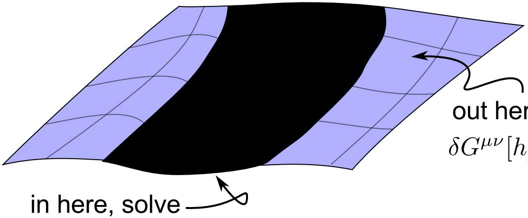


Obtaining global solution

Puncture/effective source scheme

- define $h^{\mathcal{P}}_{\mu\nu}$ as small-r expansion of $h^{\rm S}_{\mu\nu}$ truncated at finite order in r
- define $h^{\mathcal{R}}_{\mu\nu} = h_{\mu\nu} - h^{\mathcal{P}}_{\mu\nu} \simeq h^{\rm R}_{\mu\nu}$



$$\delta G^{\mu\nu}[h^{(2)}_{\rho\sigma}] = -\delta^2 G^{\mu\nu}[h^{(1)}_{\rho\sigma}]$$

$$\delta G^{\mu\nu}[h^{\mathcal{R}(2)}_{\rho\sigma}] = -\delta^2 G^{\mu\nu}[h^{(1)}_{\rho\sigma}] - \delta G^{\mu\nu}[h^{\mathcal{P}(2)}_{\rho\sigma}]$$

The point...

- to calculate effective metric "inside" object and full metric everywhere else, all you need is $h^{\rm S}_{\mu\nu}$ found in buffer region

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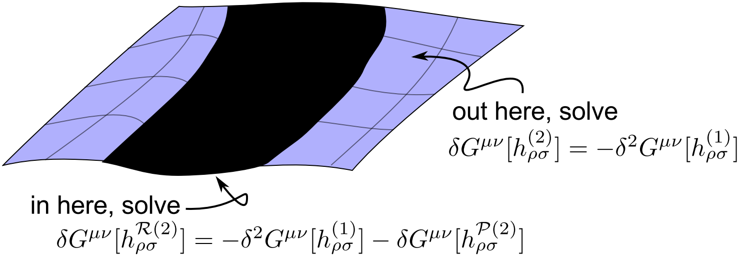

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- to calculate effective metric "inside" object and full metric everywhere else, all you need is $h^{\rm S}_{\mu\nu}$ found in buffer region

Particles and punctures

A particle at linear order

can show that the linear field $h^1_{\mu\nu}$ is identical to one sourced by a point mass m moving on γ

$\Rightarrow$ *exactly recover the point particle picture*

A puncture at any order

the field outside the object, and the object's bulk motion, can be computed by *replacing the object with a puncture equipped with a set of multipole moments*

$\Rightarrow$ generalizes the idea of a point particle

(A note on "regularization": we never introduce divergent quantities that have to be regularized—every quantity is finite at every step of every calculation)

Outline

1. Motivation
2. Motion of a point particle
3. Motion of a small extended object
4. Equation of motion at first and second order
   - First-order equation of motion
   - Second-order equation of motion
5. Computing the field
6. Modeling binaries (i.e., concrete results)
   - EMRIs
   - Other binaries

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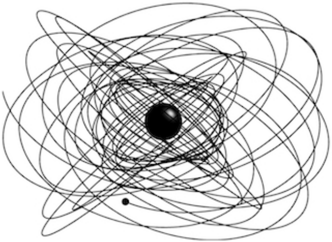
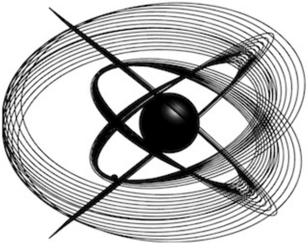
Geodesics in Kerr



[Steve Drasco]

- geodesic characterized by three constants of motion:
  1. energy E
  2. angular momentum L_z
  3. Carter constant Q, related to orbital inclination

- E, L_z, Q related to frequencies of r, ϕ, and θ motion
- *resonances* occur when two frequencies have a rational ratio



[Steve Drasco]

Hierarchy of self-force models [Hinderer & Flanagan]

- when self-force is accounted for, E, L_z, and Q evolve with time
- on an inspiral timescale $t \sim \frac{M^2}{m}$, the phase of the gravitational wave has an expansion

$$\phi = \frac{M}{m}\left[\phi_0 + \frac{m}{M}\phi_1 + O\!\left(\frac{m^2}{M^2}\right)\right]$$

- a model that gets ϕ_0 right is probably enough for signal detection in many cases
- a model that gets both ϕ_0 and ϕ_1 is enough for parameter extraction
- passage through resonances complicates the expansion, but post-adiabatic requirements are unaltered

Hierarchy of self-force models [Hinderer & Flanagan]

Adiabatic order

determined by

- averaged dissipative piece of F_1^{μ}

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Hierarchy of self-force models [Hinderer & Flanagan]

Adiabatic order

determined by
- averaged dissipative piece of F_1^μ

Post-adiabatic order

determined by
- averaged dissipative piece of F_2^μ
- conservative piece of F_1^μ
- oscillatory dissipative piece of F_1^μ

$$\phi = \frac{M}{m}\left[\phi_0 + \frac{m}{M}\phi_1 + O\left(\frac{m^2}{M^2}\right)\right]$$

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- a model that gets both ϕ_0 and ϕ_1 is enough for parameter extraction
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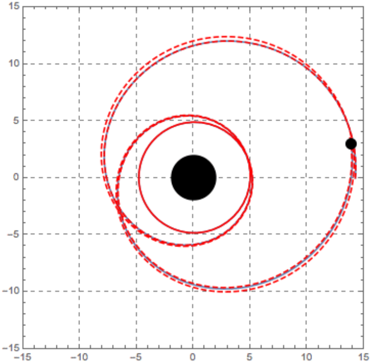
Orbital evolution

- adiabatic evolution schemes in Kerr already devised and implemented (modulo resonances) [Mino, Drasco et al, Sago et al]



[Warburton et al 2011, image courtesy of Warburton]

- also, complete inspirals simulated in Schwarzschild including full F_1^μ
- should soon be possible in Kerr
- but still need F_2^μ for accurate post-adiabatic inspiral

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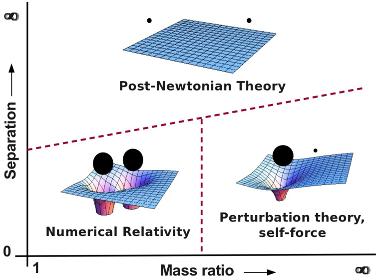
Improving other binary models

PN and EOB models have been improved using data for *conservative* effects of the self-force (computed by "turning off" dissipation)

- orbital precession [Barack et al.]
- ISCO shift [Barack and Sago, Isoyama et al.]
- Detweiler's redshift invariant $\frac{dt}{d\tau^R}$ on circular orbits [Detweiler, Shah et al., Dolan and Barack]
- averaged redshift $\left\langle \frac{dt}{d\tau^R} \right\rangle$ on eccentric orbits [Barack et al., van de Meent & Shah]

Binary parameter space



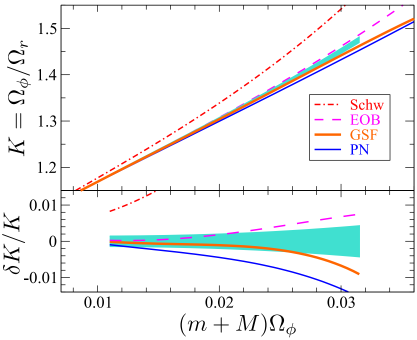
[Leor Barack]

- spin precession [Dolan et al.]
- quadrupolar and octupolar self-tides [Dolan et al, Damour and Bini]

Using SF to *directly* model other binaries

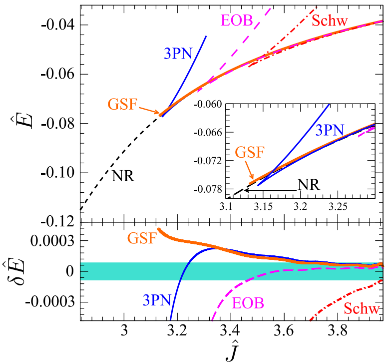
Comparisons for **equal-mass** binaries

Orbital precession



[Le Tiec et al]

Gravitational binding energy



- SF results use "mass symmetrized" model: $\frac{m}{M} \to \frac{mM}{(m+M)^2}$
- with mass-symmetrization, second-order self-force might be able to directly model even similar-mass binaries

Calculations performed as of 2005

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓		
	generic	✓		
Kerr	circular			
	generic (w/o resonances)			
	generic (w/ resonances)			

Calculations performed as of 2006

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓		
	generic	✓		
Kerr	circular	✓		
	generic (w/o resonances)			
	generic (w/ resonances)			

Calculations performed as of 2007

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓	✓	
	generic	✓		
Kerr	circular	✓		
	generic (w/o resonances)			
	generic (w/ resonances)			

Calculations performed as of 2009

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓	✓	
	generic	✓		
Kerr	circular	✓		
	generic (w/o resonances)	✓		
	generic (w/ resonances)			

Calculations performed as of 2010

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓	✓	
	generic	✓	✓	
Kerr	circular	✓		
	generic (w/o resonances)	✓		
	generic (w/ resonances)			

Calculations performed as of 2012

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓	✓	
	generic	✓	✓	
Kerr	circular	✓	✓	
	generic (w/o resonances)	✓		
	generic (w/ resonances)			

Calculations performed as of 2015

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓	✓	*underway*
	generic	✓	✓	
Kerr	circular	✓	✓	
	generic (w/o resonances)	✓	*underway*	
	generic (w/ resonances)	*underway*	*underway*	

Calculations performed as of ????

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓	✓	*underway*
	generic	✓	✓	
Kerr	circular	✓	✓	
	generic (w/o resonances)	✓	*underway*	
	generic (w/ resonances)	*underway*	*underway*	**holy grail**

Conclusion

Formalism: motion of a small object

- in principle, no obstacle to going to arbitrary order
- through second order in its mass, a nonspinning compact object moves on a geodesic in a vacuum geometry $g_{\mu\nu} + h^{\mathrm{R}}_{\mu\nu}$
- $h^{\mathrm{R}}_{\mu\nu}$ can be computed by replacing the object with an analytically known puncture

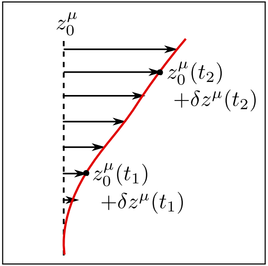
Status of binary modeling

- wealth of numerical results at first order
- calculations at second order are underway [Wednesday talks]
- second-order equations have so far neglected spin and quadrupole moment of object—need to include them through second order for accurate modeling

Using the local self-force in evolution

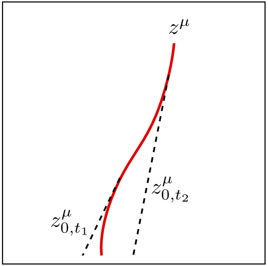
Gralla-Wald

- source of $h^1_{\mu\nu}$ moves on Kerr geodesic z_0^μ
- calculate shift δz^μ relative to z_0^μ
- error grows large with time



Geodesic-source + osculation

- at each t, source moves on instantaneously tangential geodesic $z^\mu_{0,t}$
- error $\sim (m/M)^2$



Self-consistent [Pound]

- source moves on the accelerated orbit z^μ
- only been implemented in scalar field model