Self-force meets post-Newtonian theory and more...

Abhay Shah
(University of Southampton)

with J Friedman, B Whiting, N Johnson McDaniel,
A Nagar, A Pound, A Le Tiec

*Special thanks to Cesar, Adam and Marta

Overlap between self-force, post-Newtonian and EOB theories

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Part I

- PN coefficients of ΔU in Kerr

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$\uparrow$ Detweiler's redshift invariant

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Part I

- PN coefficients of ΔU in Kerr

  ↑ Detweiler's redshift invariant

Part II

- Resummation of pN flux or waveforms

Part I

Motivation for studying invariants:

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- Binding energy and angular momentum of the binary system are linked with ΔU and its first derivative.

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- $\Delta\lambda_{B/E}$ transcribe to the dynamically significant EOB description of tidal interaction energy.

Part I

Work in the radiation gauge

Brief summary

Part I

Work in the radiation gauge

Perturbed
Weyl scalars,
ψ_0 or ψ_4

Part I

Work in the radiation gauge

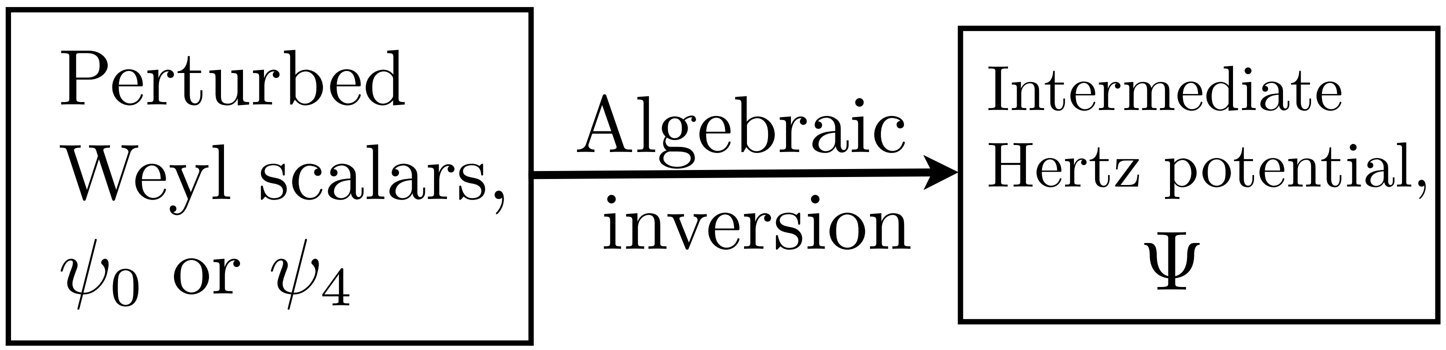
Perturbed Weyl scalars, ψ_0 or ψ_4

Homogenous solutions of Teukolsky equation are written as a sum over known analytical functions

Mano-Suzuki-Takasugi

Part I

Work in the radiation gauge



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Teukolsky-Starobinsky identities

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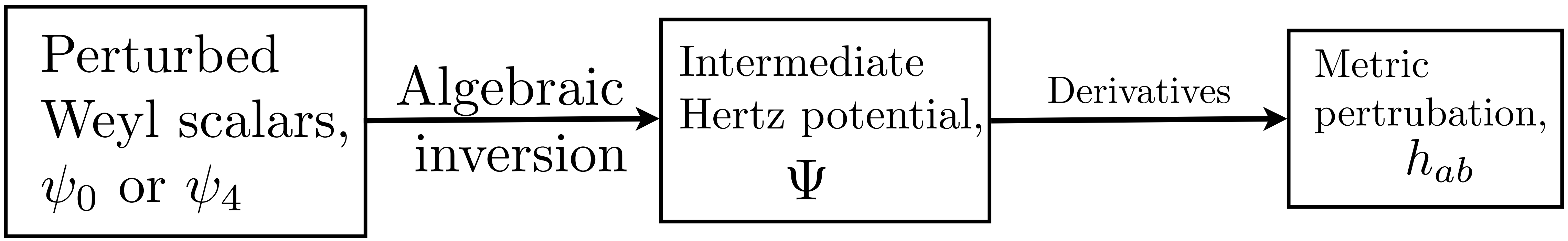

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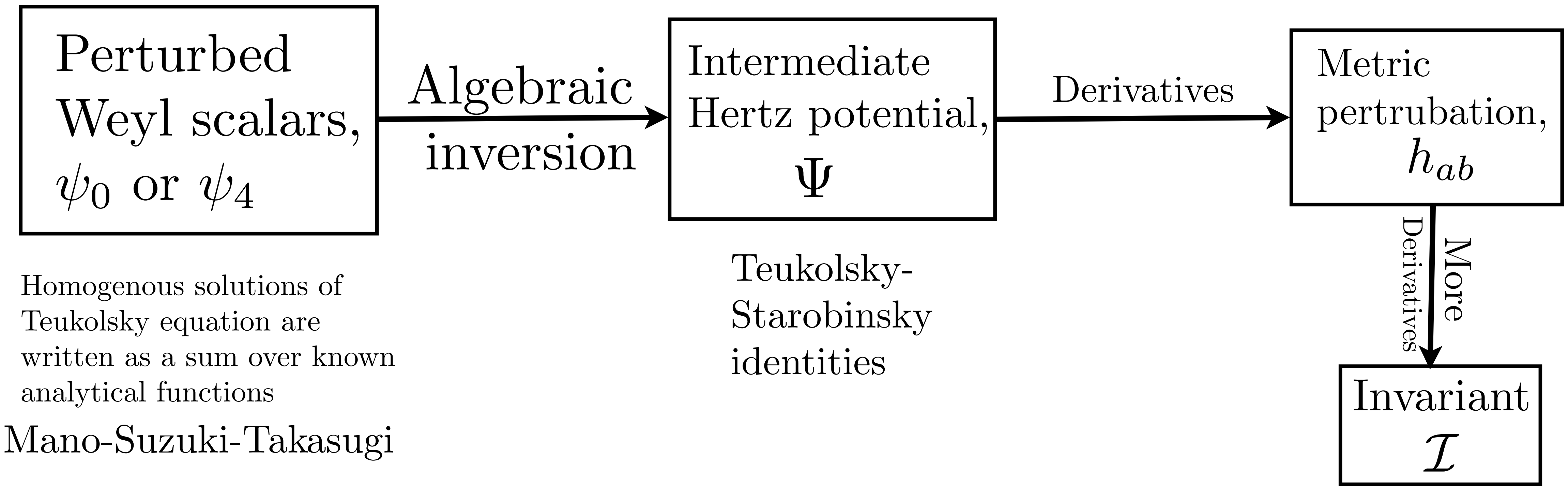

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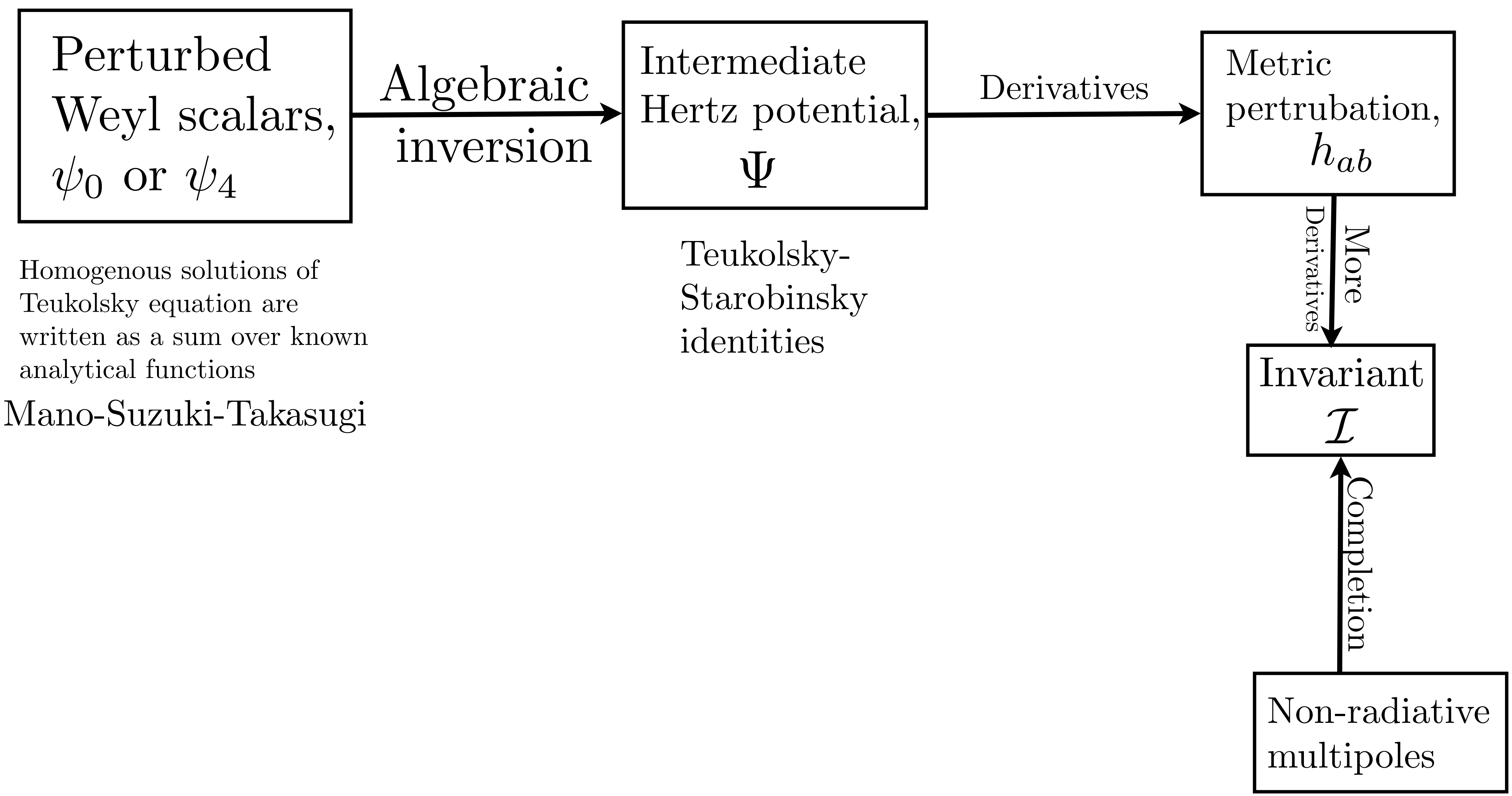

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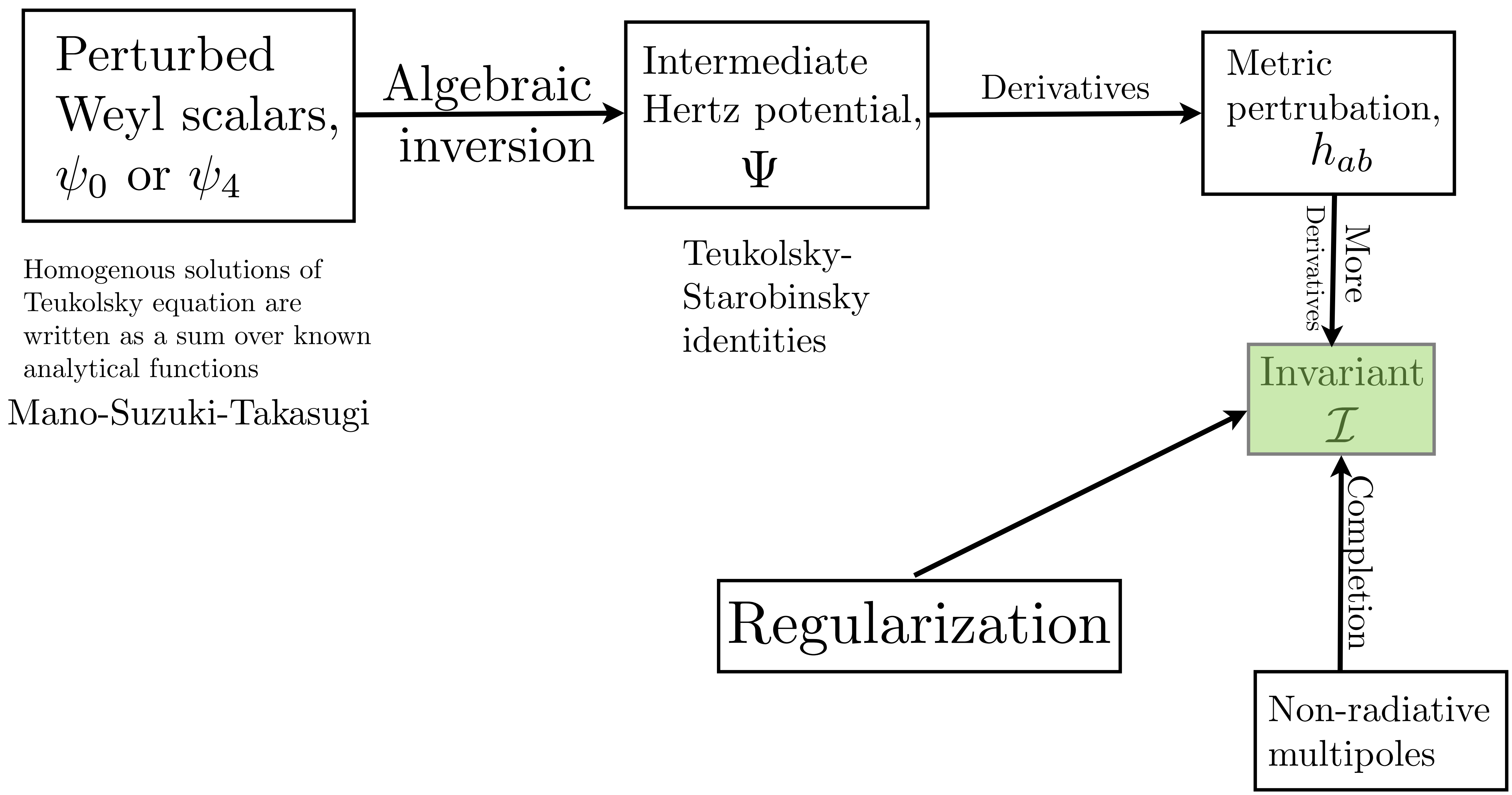

Part I

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- PN coefficients of $\Delta\lambda$ (only quad.) for circular orbits in Schwarzschild.
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(upto 21.5 pN)

- PN coefficients of $\Delta \psi$ for circular orbits in Schwarzschild.
(upto 20 pN)

- PN coefficients of $\Delta \lambda$ (only quad.) for circular orbits in Schwarzschild.
(upto 20 pN)

Circular orbits
in Kerr

Part I

Some time ago...

For circular orbits in Schwarzschild spacetime,
ΔU calculated at $R = 10^{30}$ is

-1.00000000000000000000000000000200000000000000000000000000000
50000000000000000000000000000276879026944375926029516417389721
70966870515028095201627087899062768105743309376603189941880278
16361571144500463025987555823873508773732630498814136599970277
86575686074084067914128508079111306343107928492524235720165174
51597322374056850065539280591675533919250107477196313095263829
27293694642818893511072719809144430520914924437700413544073053
13556805846793462 82 X 10^{-30}

Part I

Some time ago...

For circular orbits in Schwarzschild spacetime,
ΔU calculated at $R = 10^{30}$ is

-1.00000000000000000000000000000200000000000000000000000000000
50000000000000000000000000000276879026944375926029516417389721
70966870515028095201627087899062768105743309376603189941880278
16361571144500463025987555823873508773732630498814136599970277
86575686074084067914128508079111306343107928492524235720165174
51597322374056850065539280591675533919250107477196313095263829
27293694642818893511072719809144430520914924437700413544073053
135568058467934628 2 X 10^{-30}

$$\Delta U = \boxed{\frac{-1}{R}}$$

$$R = (M\Omega)^{-2/3}$$

Part I

Some time ago...

For circular orbits in Schwarzschild spacetime, ΔU calculated at $R = 10^{30}$ is

-1.00000000000000000000000000000200000000000000000000000000000
50000000000000000000000000000027687902694437592602951641738972 1
70966870515028095201627087899062768105743309376603189941880278
16361571144500463025987555823873508773732630498814136599970277
86575686074084067914128508079111306343107928492524235720165174
51597322374056850065539280591675533919250107477196313095263829
27293694642818893511072719809144430520914924437700413544073053
1355680584679346282 X 10^{-30}

$$\Delta U = \boxed{\frac{-1}{R}} + \boxed{\frac{-2}{R^2}}$$

$R = (M\Omega)^{-2/3}$

Part I

Some time ago...

For circular orbits in Schwarzschild spacetime,
ΔU calculated at $R = 10^{30}$ is

-1.0000000000000000000000000000020000000000000000000000000000000
50000000000000000000000000000276879026944375926029516417389721
70966870515028095201627087899062768105743309376603189941880278
16361571144500463025987555823873508773732630498814136599970277
86575686074084067914128508079111306343107928492524235720165174
51597322374056850065539280591675533919250107477196313095263829
27293694642818893511072719809144430520914924437700413544073053
1355680584679346282 X 10^{-30}

$$\Delta U = \boxed{\frac{-1}{R}} + \boxed{\frac{-2}{R^2}} + \boxed{\frac{-5}{R^3}}$$

$$R = (M\Omega)^{-2/3}$$

Part I

Some time ago...

For circular orbits in Schwarzschild spacetime,
ΔU calculated at $R = 10^{30}$ is

```
-1.00000000000000000000000000000200000000000000000000000000000
50000000000000000000000000000276879026944375926029516417389721
70966870515028095201627087899062768105743309376603189941880278
16361571144500463025987555823873508773732630498814136599970277
86575686074084067914128508079111306343107928492524235720165174
51597322374056850065539280591675533919250107477196313095263829
27293694642818893511072719809144430520914924437700413544073053
135568058467934628 2 X 10^-30
```

$$\Delta U = \frac{-1}{R} + \frac{-2}{R^2} + \frac{-5}{R^3} + \frac{\left(\frac{-121}{3} + \frac{41}{32}\pi^2\right)}{R^4} + \cdots$$

$$R = (M\Omega)^{-2/3}$$

Part I

If we subtracted the *known* PN-series from it

$$\Delta U - \left(\frac{-1}{R} + \frac{-2}{R^2} + \frac{-5}{R^3} + \frac{\left(\frac{-121}{3} + \frac{41}{32}\pi^2\right)}{R^4} + \frac{\frac{64}{5}\log(R)}{R^5} + \frac{\frac{956}{105}\log(R)}{R^6}\right)$$

= -114.348951367572602952040002444836538764412
8652844070388692348480929255963692827665976343
7619372125523054160542189937045693826002042714
825538690979057075189... X 10^{-150}

Part I

If we subtracted the *known* PN-series from it

$$\Delta U - \left(\frac{-1}{R} + \frac{-2}{R^2} + \frac{-5}{R^3} + \frac{\left(\frac{-121}{3} + \frac{41}{32}\pi^2\right)}{R^4} + \frac{\frac{64}{5}\log(R)}{R^5} + \frac{\frac{956}{105}\log(R)}{R^6}\right)$$

= -114.34895136757260295204000244483653876441286528440703886923484809292559636928276659763437619372125523054160542189937045693826002042714825538690979057075189... X 10^{-150}

$$\Delta U = \frac{-1}{R} + \frac{-2}{R^2} + \frac{-5}{R^3} + \frac{\left(\frac{-121}{3} + \frac{41}{32}\pi^2\right)}{R^4} + \frac{\frac{-592384 - 196608\gamma + 10155\pi^2 - 393216\log(2)}{7680}}{R^5} + \frac{\frac{64}{5}\log(R)}{R^5} + \frac{\frac{956}{105}\log(R)}{R^6} + \cdots$$

Part I

$$\gamma_7 = 52.1752380952380952380952380952380952380952380952380952380952380952375380434891643 31\cdots$$

Part I

$$\gamma_7 = 52.17\boxed{523809}523809523809523809523809$$
$$523809523809523809523809523809$$
$$523809523753804348916433 1\cdots$$

Part I

$$\gamma_7 = 52.17523809523809523809523809523809523809523809523809523809523809523809523753804348916433 1\cdots$$

More than 11 repetition cycles

$$\gamma_7 = \frac{27392}{525}$$

Part I

Extracting spin-dependent pN parameters by just staring at ΔU calculated for a given R and a.

Part I

For a circular, equatorial orbit in Kerr,
ΔU for $R = 10^{20}$ & $a = 10^{-3}$ is

-1.00000000000000000000199999999999976666671666667666665133333610212455833187814869894029453526501586987499048011280649959622939855508421935344551422768537510337622732112348892363161717032837228070495809740777820065101791518318969626899096371783942651929072216356874825589293038885925936231530270966005434715385446834849809423556296080091790221654604861949284553553586807490001071 65 X 10-20

Part I

If we subtract the known a-independent PN-terms from it, we get

2.3333333333332333334866666666666657111118811111211110470000333
61101449462298052019375307453520567434360997918586330437827422
85790789258060941125270718078251256276684354908208083322490189
27613456043301239146966332282381109537868453512458593272282243
42962251368467464109905674383013754100571040568288348387074195
38774336951906007694268879889268637496375852038699098746116349
07608445730228968534690145005883609409871989370801221523747813
79214778427338448055324558181161062657272652196860626477197489
998371250766911017733058814377904976565274243148 X 10^{-53}

Part I

$$\Delta U - \Delta U_{\text{Schw PN}}$$

2.333333333333233334866666666666665711111881111121111047000033 3
6110144946229805201937530745352056743436099791858633043782742 2
857907892580609411252707180782512562766843549082080833224901 89
276134560433012391469663322823811095378684535124585932722822 43
429622513684674641099056743830137541005710405682883483870741 95
387743369519060076942688798892686374963758520386990987461163 49
076084457302289685346901450058836094098719893708012215237478 13
792147784273384480553245581811610626572726521968606264771974 89
99837125076691101773305881437790497656527424314 8 X 10^{-53}

-53 = -50 + -3

$R = 10^{20}$

$a = 10^{-3}$

$\frac{1}{R^{5/2}}$ → -50

a → -3

⇒ $\frac{a}{R^{5/2}}$

Part I

$$\Delta U - \Delta U_{\mathrm{Schw\ PN}}$$

2.3333333333332333334866666666666571111188111112111104700003336110144946229805201937530745352056743436099791858633043782742285790789258060941125270718078251256276684354908208083322490189276134560433012391469663322823811095378684535124585932722822434296225136846746410990567438301375410057104056828834838707419538774336951906007694268879889268637496375852038699098746116349076084457302289685346901450058836094098719893708012215237478137921477842733844805532455818116106265727265219686062647719748999837125076691101773305881437790497656527424314 8 X 10^{-53}

$$\frac{\frac{7}{3}a}{R^{5/2}}$$

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}}$$

-0.99999846666666666676222214522222122222863332999722318838710
352813139580258798127658989723354147470028955059104754254407 52
723922080626152550820770566489784251252500108431440571987729 00
320941863670010509522237954648798208747400610510899037108196 48
658692234276589503195792327622927650449849462591379455899638 14
273256390644534440646958369574812946342345872169842572488760 31
043647986431883274497239234613439625321118095855195411855490 59
948852780087751521722706760606811364727068561358433349620825 66
4223156002745189554283567680590901851069 91 X 10^{-66}

$R = 10^{20}$

$a = 10^{-3}$

-66 = -60 + -6

$\frac{1}{R^3}$ → -60

a^2 → -6

⇒ $\frac{a^2}{R^3}$

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}}$$

-0.999998466666666666762222145222221222228633329997223188387103528131395802587981276589897233541474700289550591047542544075272392208062615255082077056648978425125250010843144057198772900320941863670010509522237954648798208747400610510899037108196486586922342765895031957923276229276504498494625913794558996381427325639064453444064695836957481294634234587216984257248876031043647986431883274497239234613439625321118095855195411855490599488527800877515217227067606068113647270685613584333496208256642231560027451895542835676805909018510699 1 X 10^{-66}

$$\frac{-a^2}{R^3}$$

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}} - \frac{-a^2}{R^3}$$

15.33333333333237777854777778777771366670002776811612896471868
60419741201872341010276645852529971044940895245745592472760779
19373847449179229433510215748747499891568559428012270996790581
36329989490477762045351201791252599389489100962891803513413077
65723410496804207672377072349550150537408620544100361857267436
09355465559353041630425187053657654127830157427511239689563520
13568116725502760765386560374678881904144804588144509400511472
19912248478277293239393188635272931438641566650379174335776843
99725481044571643231940909814893008 X 10^{-73}

$R = 10^{20}$

$a = 10^{-3}$

-73 = -70 + -3

$\frac{1}{R^{7/2}} \rightarrow -70$, $a \rightarrow -3$

$$\Rightarrow \frac{a}{R^{7/2}}$$

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}} - \frac{-a^2}{R^3}$$

15.33333333333237777854777778777771366670002776811612896471868
60419741201872341010276645852529971044940895245745592472760779
19373847449179229433510215748747499891568559428012270996790581
36329989490477762045351201791252599389489100962891803513413077
65723410496804207672377072349550150537408620544100361857267436
09355465559353041630425187053657654127830157427511239689563520
13568116725502760765386560374678881904144804588144509400511472
19912248478277293239393188635272931438641566650379174335776843
99725481044571643231940909814893008 X 10^{-73}

$$\frac{\frac{46}{3}a}{R^{7/2}}$$

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}} - \frac{-a^2}{R^3} - \frac{\frac{46}{3}a}{R^{7/2}}$$

-9.55554785555545555619666633305565217204368614647291359213146
099232305668748080336228839243808758774086057255413959485884154
103899823117584585833441764773905321062336542751970033438428555
712879821315420807339438442323704415298199202556760992283652
912566095626098378318279592471278923297147606589723977867773980
29170290814627967567920550317590582209364376981319765216607830
572567946772958654451429188528745188823932821861134210848550560
400939401446980604018946917666829541589975564893360785228876
169010139242351844032438954958070 X 10^{-86}

$R = 10^{20}$

$a = 10^{-3}$

-86 = -80 + -6

$\frac{1}{R^4}$ → -80 a^2 → -6

a^2 ⇒ $\frac{a^2}{R^4}$

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}} - \frac{-a^2}{R^3} - \frac{\frac{46}{3}a}{R^{7/2}}$$

-9.5555478555554555619666633305565217204368614647291359213146
09923230566874808033622883924380875877408605725541395948588415
41038998231175845858334417647739053210623365427519700334384285
55712879821315420807339438442323704415298199202556760992283652
91256609562609837831827959247127892329714760658972397786777398
02917029081462796756792055031759058220936437698131976521660783
05725679467729586544514291885287451888239328218611342108485505
60400939401446980604018946917666829541589975564893360785228876
169010139242351844032438954958070 X 10^{-86}

$$\frac{\frac{-86}{9}a^2}{R^4}$$

Part I

Careful:

• Coefficient of $\frac{a^i}{R^j}$ has to be of order unity or "so", and grow consistently with each pN order for a given choice of i.

• The i in $\frac{a^i}{R^j}$ has to be a positive integer.

• After subtracting $\frac{c_{i,j}\, a^i}{R^j}$, one should see a "consistent" reduction in magnitude that would help you in predicting the next higher-order pN coefficient.

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}} - \frac{-a^2}{R^3} - \frac{\frac{46}{3}a}{R^{7/2}} - \frac{\frac{-86}{9}a^2}{R^4}$$

77.00000099999358889222499903383511869409082641963424094563232
4988680747521932671631174679678146949830014159606967140145165 5
7324379709697221137907816502344932190128035855221171269998426 7
5734240134748216117113231851140257356352998794563271902642989 4
5992945717723727596308427663225840794896583157768778157526385 2
6474092758798763500523796497334619117857423579033894772498298 7
6087825969011041263670268103667316227336944213447070049951546 1
6154108574951536608637888726013965579990662194770326679386545 4
16313203711523116600597485532717 6 X 10^{-93}

$R = 10^{20}$

$a = 10^{-3}$

-93 = -90 + -3

$\frac{1}{R^{9/2}}$ → -90

a → -3

a ⇒ $\frac{a}{R^{9/2}}$

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}} - \frac{-a^2}{R^3} - \frac{\frac{46}{3}a}{R^{7/2}} - \frac{\frac{-86}{9}a^2}{R^4}$$

77.000000999993588892224999033835118694090826419634240945632324988680747521932671631174679678146949830014159606967140145165573243797096972211379078165023449321901280358552211712699984267573424013474821611711323185114025735635299879456327190264298945992945717723727596308427663225840794896583157768778157526385264740927587987635005237964973346191178574235790338947724982987608782596901104126367026810366731622733694421344707004995154616154108574951536608637888726013965579990662194770326679386545416313203711523116600597485532717 6 X 10^{-93}

$$\frac{77\,a}{R^{9/2}}$$

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}} - \frac{-a^2}{R^3} - \frac{\frac{46}{3}a}{R^{7/2}} - \frac{\frac{-86}{9}a^2}{R^4} - \frac{77\,a}{R^{9/2}}$$

0.99999358889222499903383511869409082641963424094563232498868O
747521932671631174679678146949830014159606967140145165573243 79
709697221137907816502344932190128035855221171269998426757342 40
134748216117113231851140257356352998794563271902642989459929 45
717723727596308427663225840794896583157768778157526385264740 92
758798763500523796497334619117857423579033894772498298760878 25
969011041263670268103667316227336944213447070049951546161541 08
574951536608637888726013965579990662194770326679386545416313 20
37115231166005974855327176 3421 X 10^{-99}

$R = 10^{20}$

$a = 10^{-3}$

-99 = -90 + -9

$\frac{1}{R^{9/2}}$ → -90

a^3 → -9

⇒ $\frac{a^3}{R^{9/2}}$

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}} - \frac{-a^2}{R^3} - \frac{\frac{46}{3}a}{R^{7/2}} - \frac{\frac{-86}{9}a^2}{R^4} - \frac{77\,a}{R^{9/2}}$$

0.999993588892224999033835118694090826419634240945632324988680
747521932671631174679678146949830014159606967140145165573243797
0969722113790781650234493219012803585522117126999842675734240
1347482161171132318511402573563529987945632719026429894599294 5
717723727596308427663225840794896583157768778157526385264740927
5879876350052379649733461911785742357903389477249829876087825
9690110412636702681036673162273369442134470700499515461615410 8
574951536608637888726013965579990662194770326679386545416313203
71152311660059748553271763421 X 10^{-99}

$$\frac{a^3}{R^{9/2}}$$

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}} - \frac{-a^2}{R^3} - \frac{\frac{46}{3}a}{R^{7/2}} - \frac{\frac{-86}{9}a^2}{R^4} - \frac{77\,a}{R^{9/2}} - \frac{a^3}{R^{9/2}}$$

-64.111077750009661648813059091735803657590543676750113192524
780673283688253203218530501699858403930328598548344267562029030
277886209218349765506780987196414477882873000157324265759865251
783882886768148859742643647001205436728097357010540070542822762
724036915723367741592051034168422312218424736147352590724120
123649947620350266538088214257642096610522750170123912174030988
958736329731896332683772663055786552929950048453838458914250484
633913621112739860344200093378052296733206134545836867962884
768833994025144672823657821 4 X 10-106

$R = 10^{20}$

$a = 10^{-3}$

-106 = -100 + -6

$\frac{1}{R^5}$ → -100

a^2 → -6

a^2 ⇒ $\frac{a^2}{R^5}$

Part I

$$\Delta U - \Delta U_{\text{Schw PN}} - \frac{\frac{7}{3}a}{R^{5/2}} - \frac{-a^2}{R^3} - \frac{\frac{46}{3}a}{R^{7/2}} - \frac{\frac{-86}{9}a^2}{R^4} - \frac{77\,a}{R^{9/2}} - \frac{a^3}{R^{9/2}}$$

-64.1110777500096616488130590917358036575905436767501131925247
8067328368825320321853050169985840393032859854834426756202903 0
2778862092183497655067809871964144778828730001573242657598652 5
1783882886768148859742643647001205436728097357010540070542822 7
6272403691572336774159205103416842231221842473614735259072412 0
1236499476203502665380882142576420966105227501701239121740309 8
8958736329731896332683772663055786552929950048453838458914250 4
8463391362111273986034420009337805229673320613454583686796288 4
7688339940251446728236578214 X 10^{-106}

$$\frac{\frac{-577}{9}a^2}{R^5}$$

Part I

$$\Delta U_{\text{Kerr}} = \Delta U_{\text{Schw-PN}} + \frac{\frac{7}{3}a}{R^{5/2}} + \frac{-a^2}{R^3} + \frac{\frac{46}{3}a}{R^{7/2}} + \frac{\frac{-86}{9}a^2}{R^4} + \frac{77\,a + a^3}{R^{9/2}}$$
$$+ \frac{\frac{-577}{9}a^2}{R^5} + \frac{\left(\frac{31168}{96} + \frac{29\pi^2}{32}\right)a + \frac{1526}{81}a^3}{R^{11/2}} + \frac{\left(\frac{-1147}{3} + \frac{593\pi^2}{512}\right)a^2 - 2a^4}{R^6}$$
$$+ \frac{\left(\frac{348047}{150} + \frac{352\gamma}{5} - \frac{6349\pi^2}{64} + \frac{416\ln(2)}{3}\right)a + \frac{13625}{81}a^3}{R^{13/2}} + \frac{\frac{-176}{5}a\ln(R)}{R^{13/2}}$$
$$+ \cdots$$

Polygamma functions enter at 6.5 PN order, i.e., at $O\left(\frac{1}{R^{15/2}}\right)$.

Part II

Resummation of fluxes or waveforms

Motivation:

Find an analytical form for linear-in-mass-ratio part of the flux or waveform valid in all region.

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Find an analytical form for linear-in-mass-ratio part of the flux or waveform valid in all region.

Resummation introduced by Damour-Iyer-Nagar:

Factorized multipolar resummation where each $h_{l,m}$ is factored into five parts and each of these parts is resummed individually.

Part II

$$h_{\ell,m} = h^{\mathrm{N}}_{\ell,m} \times S_{\ell,m} \times T_{\ell,m} \times e^{i\delta_{\ell,m}} \times {\rho_{\ell,m}}^{\ell}$$

↑

Newtonian
contribution

$$= \frac{1}{R} n^{(p)}_{\ell,m} \, (-1)^{\ell+p} \, v^{\ell+p} \, Y_{\ell+p,m}$$

Part II

$$h_{\ell,m} = h^{\mathrm{N}}_{\ell,m} \times S_{\ell,m} \times T_{\ell,m} \times e^{i\delta_{\ell,m}} \times {\rho_{\ell,m}}^{\ell}$$

Source,
$\mathcal{E}$ or $\mathcal{J}$

Part II

$$h_{\ell,m} = h^{\mathrm{N}}_{\ell,m} \times S_{\ell,m} \times T_{\ell,m} \times e^{i\delta_{\ell,m}} \times {\rho_{\ell,m}}^{\ell}$$

↑

"Tail term"

$$\frac{\Gamma(\ell+1-2i\hat{k})}{\Gamma(\ell+1)}\, e^{\pi\hat{k}}\, e^{2i\hat{k}\ln(2\hat{k}c)}$$

$$(\ \hat{k} = m\Omega\ )$$

Part II

$$h_{\ell,m} = h^{\mathrm{N}}_{\ell,m} \times S_{\ell,m} \times T_{\ell,m} \times e^{i\delta_{\ell,m}} \times {\rho_{\ell,m}}^{\ell}$$

Dephasing term

Part II

$$h_{\ell,m} = h^{\mathrm{N}}_{\ell,m} \times S_{\ell,m} \times T_{\ell,m} \times e^{i\delta_{\ell,m}} \times {\rho_{\ell,m}}^{\ell}$$

↑ Whatever left

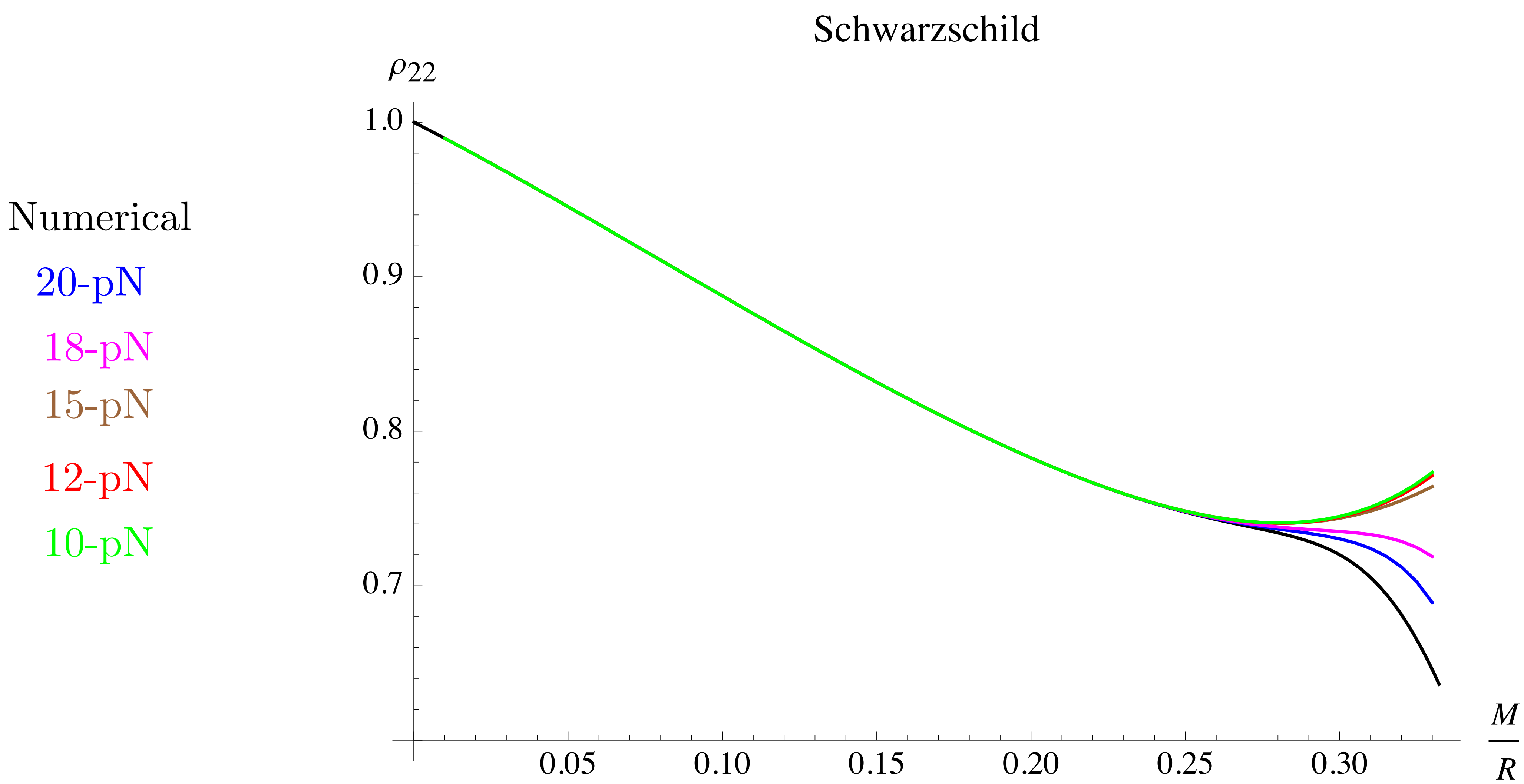
Part II

Motive $\rightarrow$ Analytical $\rho_{\ell,m}$ is valid everywhere.

Part II

Motive $\rightarrow$ Analytical $\rho_{\ell,m}$ is valid everywhere.

Problem $\rightarrow$ Standard pN series doesn't converge well in strong field.



Part II

Proposal $\rightarrow$ Hybridize it and impose agreement with pN.

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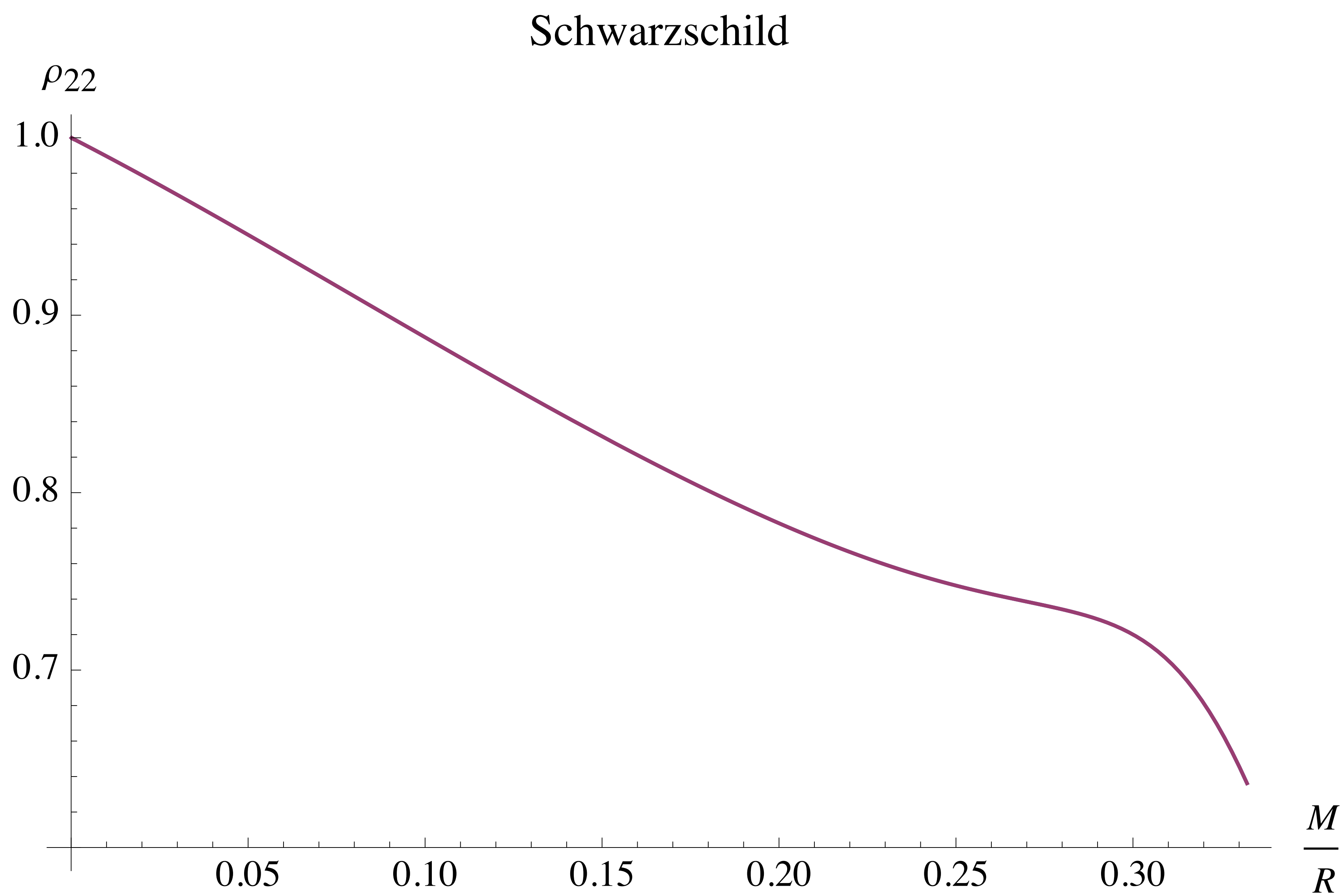
Proposal → Hybridize it and impose agreement with pN.

$$\rho_{22}^{\text{Resum}} = \frac{1 + \sum_{i=1}^{i_{\text{max}}} \frac{n_i}{R^i}}{1 + \sum_{j=1}^{j_{\text{max}}} \frac{d_j}{R^j}}$$

Choose n_i and d_j such that the Taylor-expanded ρ_{22}^{Resum} agree with pN-expansion of ρ_{22}. Left with some undetermined coefficients.

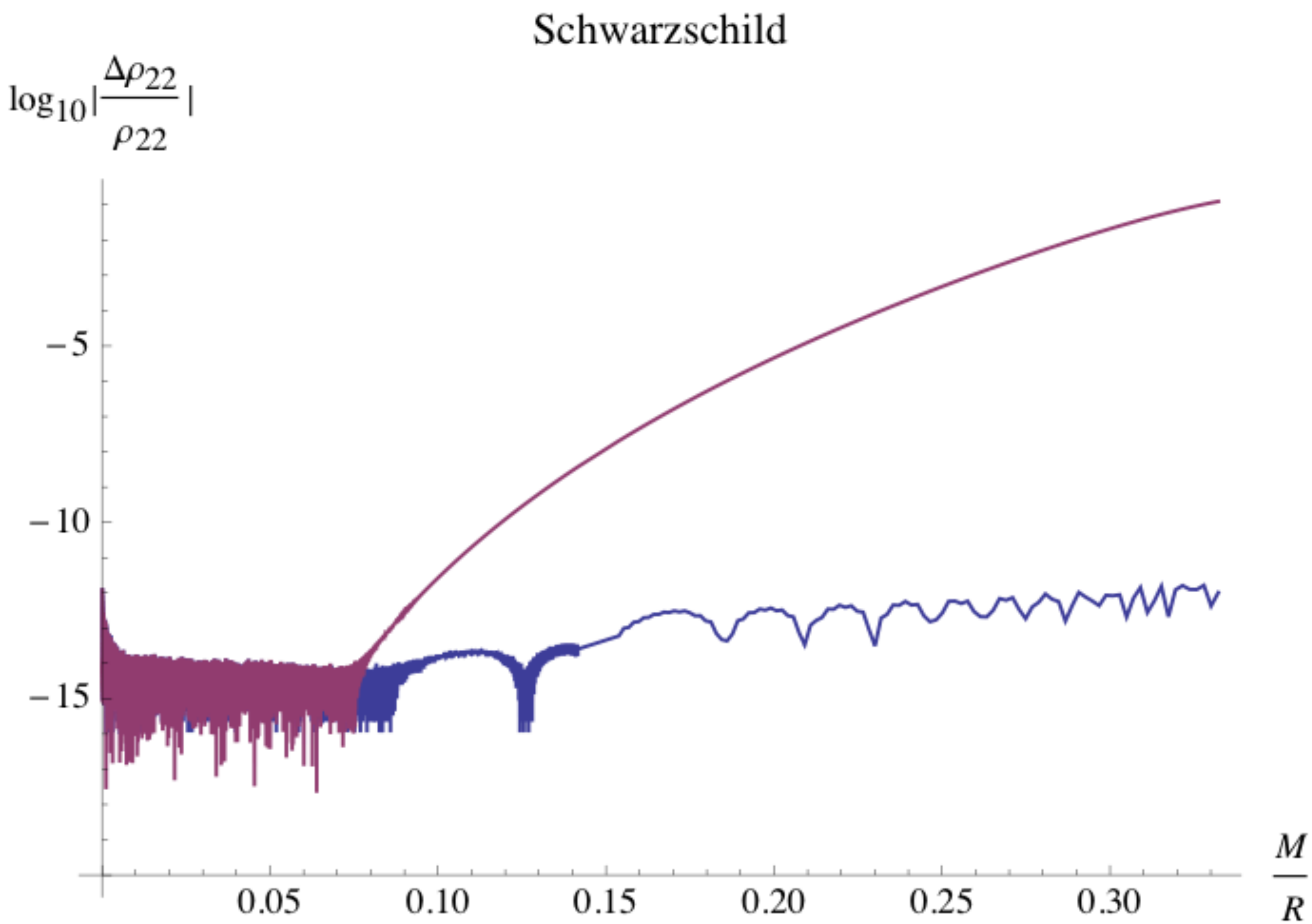
Fix these remaining coefficients, n_i and d_j, by imposing agreement with numerical ρ_{22}.

Part II



Overlap

Part II



Schwarzschild
$\log_{10}|\frac{\Delta\rho_{22}}{\rho_{22}}|$
−5
−10
−15
0.05
0.10
0.15
0.20
0.25
0.30
$\frac{M}{R}$

Doable work in the not-so-distant-future

- Calculate PN parameters of spin and tidal invariants in Kerr.
- Resummation of ingoing and outgoing fluxes in Kerr.