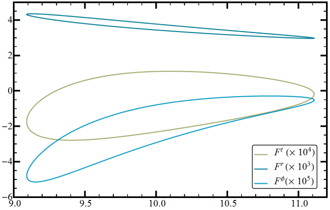
Self-force corrections to eccentric orbits in Kerr

Maarten van de Meent

University of Southampton



Capra 18, Kyoto, 30 June 2015

arXiv:1506.04755

Outline

Goal

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Numerically obtain (regular part of) perturbed metric generate by a particle on an eccentric equatorial geodesic in Kerr spacetime.

The trouble with Kerr

- Linearized Einstein equation in Lorenz gauge on a Kerr background is not separable.
- (Nor in any other known gauge. No Regge-Wheeler.)
- Can't solve linearized Einstein equation (directly) in frequency domain.
- Time domain needs to be solved in 2+1 dimensions. (or solve coupled equations in 1+1D).

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Metric reconstruction in radiation gauge

Key facts

- Teukolsky equation for ψ_4 is separable.
- Wald's theorem: ψ_4 contains (almost) all information about a vacuum perturbation of the Kerr metric.

Chrzanowski-Cohen-Kegeles (CCK) reconstruction

ψ_4 → Ψ_{ORG} → $h_{\mu\nu}^{\mathrm{ORG}}$

$$n^\mu h_{\mu\nu}^{\mathrm{ORG}} = 0$$
$$g^{\mu\nu} h_{\mu\nu}^{\mathrm{ORG}} = 0$$

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Self-force in radiation gauge

- Self force formalism originally formulated in Lorenz gauge.
- Later extended to gauges related to Lorenz by a continuous [Barack&Ori, 2001] or bounded [Gralla&Wald, 2008, 2011] gauge transformation.
- Radiation gauge is not in these classes of gauge.
- [Pound,Merlin&Barack, 2013] derived transformation to a 'locally Lorenz' gauge, deriving a mode-sum formula that takes a radiation gauge metric perturbation as its input.

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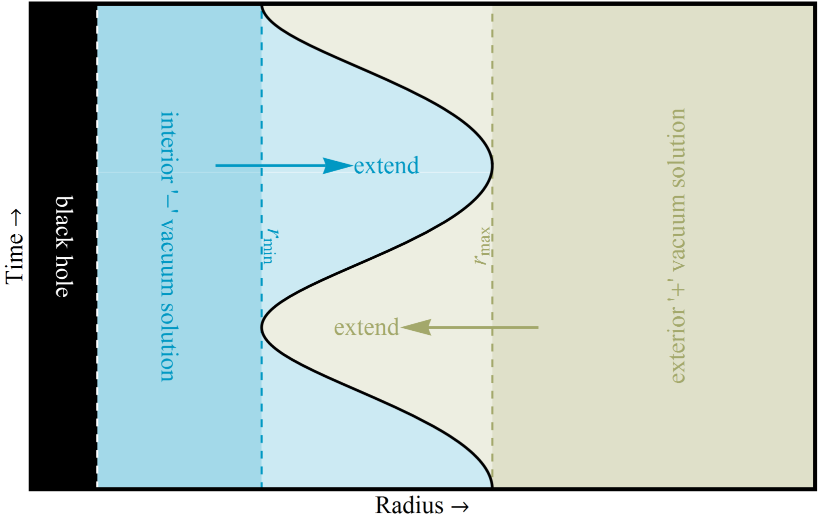
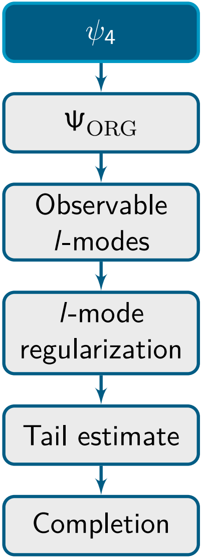
Method of extended homogeneous solutions



Teukolsky equation



- In the vacuum regions the solution to the Teukolsky equation can be written:

$$\psi_4^{\pm} = \frac{\rho^4}{\sqrt{2\pi}} \sum_{lmn} Z^{\pm}_{lmn} \, {}_{-2}R^{\pm}_{lmn}(r) \, {}_{-2}S_{lmn}(z) e^{i(m\phi - \omega_{mn} t)}$$

- Homogeneous radial solutions, ${}_{-2}R^{\pm}_{lmn}$, are obtained as series of hypergeometric units using the Mano-Suzuki-Tagasugi (MST) formalism.
- The $Z^{\pm}_{lmn}$ are calculated using variation of parameters

$$Z^{\pm}_{lmn} = \int_{r_{\min}}^{r_{\max}} \frac{{}_{-2}R^{\mp}_{lmn}(r) \, {}_{-2}T_{lmn}(r)}{W[{}_{-2}R^{+}_{lmn}, {}_{-2}R^{-}_{lmn}](r)} \, \mathrm{d}r$$

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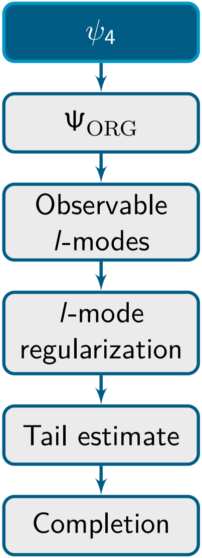

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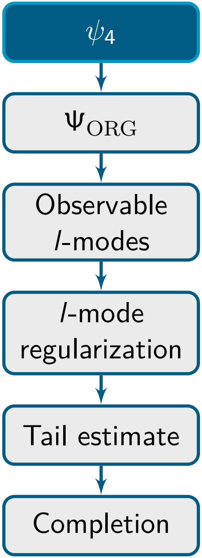

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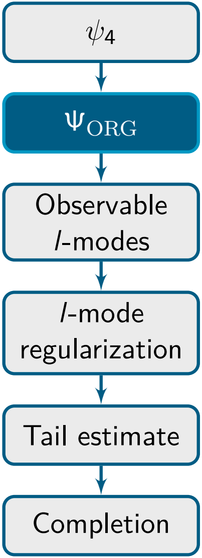
Hertz potential



- Ψ_{ORG} is vacuum solution to Teukolksy equation

$$\Psi^{\pm}_{ORG} = \frac{1}{\sqrt{2\pi}} \sum_{lmn} \Psi^{\pm}_{lmn}\, {}_2R^{\pm}_{lmn}(r)\, {}_2S_{lmn}(z) e^{i(m\phi - \omega_{mn} t)}$$

- In addition

$$\mathcal{D}^4 \bar{\Psi}_{ORG} = \rho^{-4} \psi_4$$

- $\mathcal{D}^4$ separates over Teukolsky modes, so inversion reduces to inverting 2-by-2 matrix for each mode.
- Can be done analytically.[Ori, 2001]

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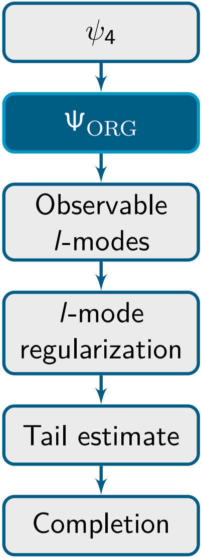

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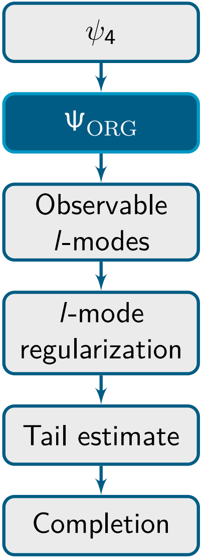

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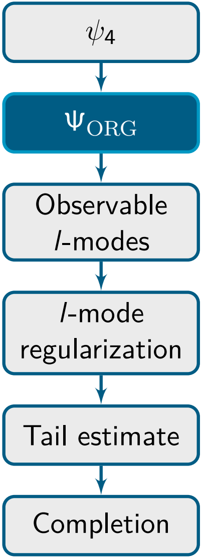

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Hertz potential

ψ_4 → Ψ_{ORG} → Observable l-modes → l-mode regularization → Tail estimate → Completion

Result:

$$\Psi^{+}_{lmn} = (-1)^{l+m} 2 \frac{Z^{+}_{lmn}}{\omega^4_{mn}}$$

$$\Psi^{-}_{lmn} = (-1)^{l+m} \frac{32 Z^{-}_{lmn}}{p_{lm\omega}} (2\kappa)^4 (i(\sigma + 2\omega_{mn}) - 2)_4$$

In addition one can easily obtain ${}_2R^{\pm}_{lmn}$ from ${}_{-2}R^{\pm}_{lmn}$ using that ${}_{-2}\bar{R}^{\pm}_{lmn}/\Delta^2$ is a solution to the spin-2 Teukolsky equation.

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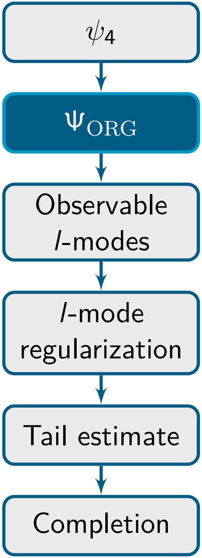

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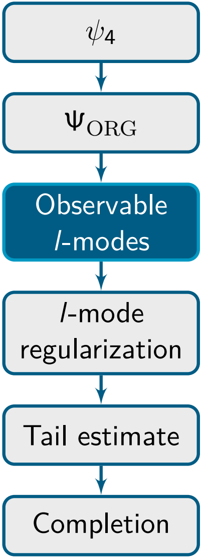
Metric reconstruction



The CCK formalism gives the metric perturbation as

$$h_{\mu\nu}^{ORG} = \hat{\mathcal{H}}_{\mu\nu}^{ORG} \Psi^{ORG} + c.c.$$

Construct expression for *l*-modes observable with following steps:

1. Expand ${}_2S_{lmn} = \sum_{l_2} (b_{mn})_{ll_2}\, {}_2Y_{l_2 mn}$.
2. Apply differential operators replacing $\partial_{\cos\theta}$ with spin lowering operator.
3. Expand ${}_sY_{l_2 mn} = \frac{1}{\sin^{|s|/2}\theta} \sum_{l_2} (\mathcal{A}_{sm})_{ll_2}\, Y_{lm}$.
4. Taylor expand coefficient in $z = \cos\theta$.
5. Drop z^2 and higher terms.

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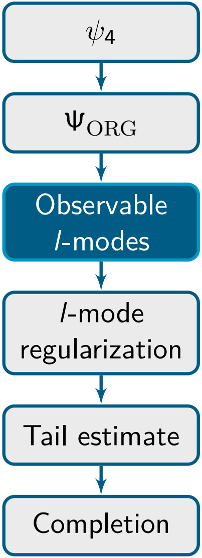

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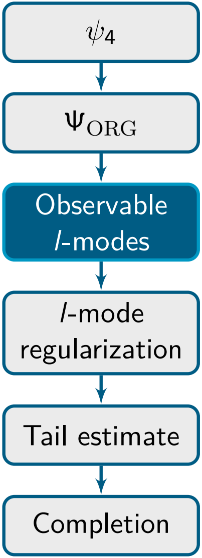

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Mode sum regularization

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$\langle h_{uu} \rangle$

Use outer 'half-string' gauge.

$$\delta_\xi \langle h_{uu} \rangle = 2\langle u^\mu \nabla_\mu (u^\nu \xi_\nu) \rangle \quad - \quad \langle 2\xi_\nu u^\mu \nabla_\mu u^\nu \rangle$$

$$0 \qquad \mathcal{O}(\log s)\ \mathcal{O}(s)$$

GSF

Use discontinuous 'no string' gauge. Can use Lorenz gauge regularization parameters.[Pound,Merlin&Barack, 2013]

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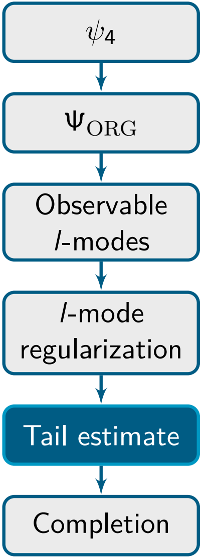

- After subtracting regularization parameters, the sum over l-modes still only converges as $1/l_{max}$.
- The 'tail' of the sum can be estimated by fitting a power series in $1/l$ to the partial sums of the known l-modes.
- Number of terms in power series that can be fitted accurately grows with l_{max}. (If l-modes are accurate enough.)
- Effective convergence is faster than any polynomial in l_{max}.

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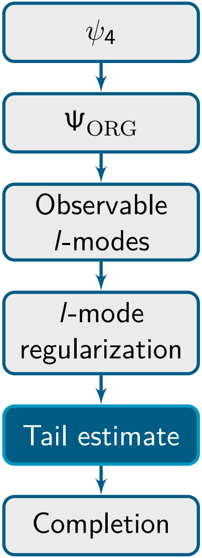

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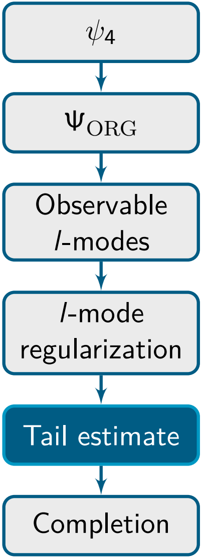

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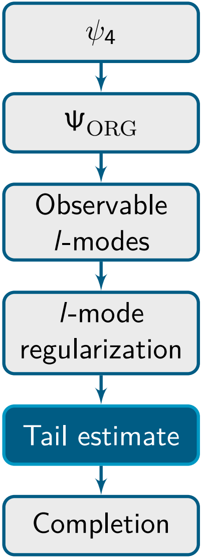

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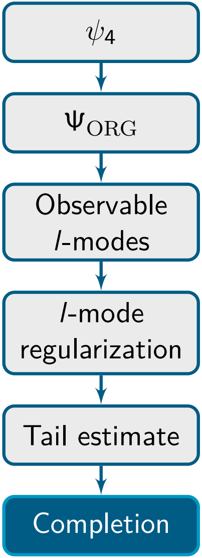

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- see Cesar's talk.

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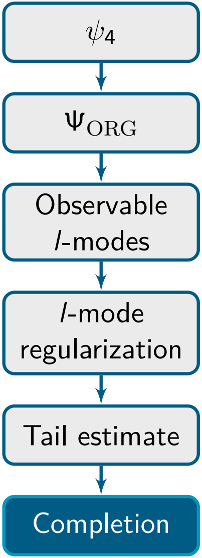

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Generalized redshift

Effective conservative metric:

$$g_1 \equiv g_0 + h^R_{cons,\mu\nu}$$

Generalized redshift

$$U \equiv \langle \frac{\mathrm{d}t}{\mathrm{d}\tau} \rangle = \frac{T_r}{\mathcal{T}_r}$$

Compare at fixed orbital frequencies: [Akcay et al.,2015]

$$\Delta U(\Omega_r, \Omega_\phi) = U_1(\Omega_r, \Omega_\phi) - U_0(\Omega_r, \Omega_\phi) = \frac{T_r}{2\mu\mathcal{T}_r} \langle h^R_{uu} \rangle$$

Schwarzschild: Comparison with Lorenz gauge results

p	e	Here	Akcay et al. 2015	Barack and Sago 2011
10	0.10	−0.1277540232(10)	−0.1277540(3)	−0.1277554(7)
15	0.10	−0.07687063237(5)	−0.0768706(2)	−0.0768709(1)
20	0.10	−0.055221659739(6)	−0.05522166(7)	−0.05522177(4)
100	0.10	−0.010101234326660(2)	−0.0101012344(10)	—
10	0.20	−0.123647888(2)	−0.123648(3)	−0.1236493(7)
15	0.20	−0.07431375582(4)	−0.07431376(9)	−0.0743140(1)
20	0.20	−0.0534085449572(6)	−0.05340854(9)	−0.05340866(4)
100	0.20	−0.0097893279221005(14)	−0.0097893274(4)	—
10	0.30	−0.1168019818(2)	−0.1168020(6)	−0.1168034(6)
15	0.30	−0.07007684538(10)	−0.0700768(5)	−0.0700771(1)
20	0.30	−0.050403774160(6)	−0.05040377(4)	−0.05040388(4)
100	0.30	−0.009270280959(2)	−0.009270281(4)	—
10	0.40	−0.107220(6)	−0.107221(2)	−0.1072221(5)
15	0.40	−0.0641988(4)	−0.064199(1)	−0.0641991(1)
20	0.40	−0.0462337(4)	−0.0462337(9)	−0.04623383(4)
100	0.40	−0.0085452(4)	−0.0085453(2)	—

Schwarzschild: Comparison with PN

- Range of orbits with $100 < p < 1000$ and $0 < e < 0.4$
- Get all known PN coefficients (3PN) with 5 digits of accuracy.

4PN

$\times$	p^{-5}	$p^{-5} \log p$
e^2	$-345.37(5)$	$19.733(5)$
e^4	$737(4)$	$-48.6(4)$
e^6	$-185(12)$	$7(2)$

5PN

$\times$	p^{-6}	$p^{-6} \log p$
e^2	-2000 ± 400	-40 ± 20

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e^4	$737(4)$	$-48.6(4)$
e^6	$-185(12)$	$7(2)$

5PN

$\times$	p^{-6}	$p^{-6} \log p$
e^2	-2000 ± 400	-40 ± 20

Schwarzschild: Comparison with PN

- Range of orbits with $100 < p < 1000$ and $0 < e < 0.4$
- Get all known PN coefficients (3PN) with 5 digits of accuracy.

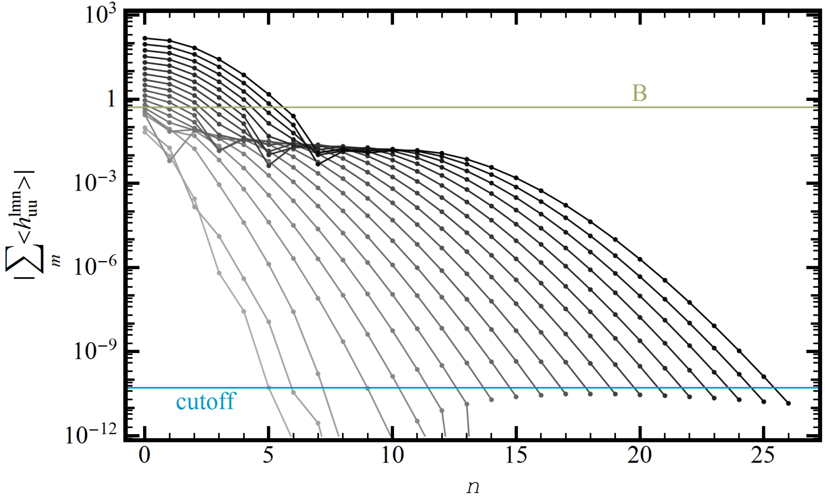
4PN

$\times$	p^{-5}	$p^{-5}\log p$
e^2	$-345.37(5)$	$19.733(5)$
e^4	$737(4)$	$-48.6(4)$
e^6	$-185(12)$	$7(2)$

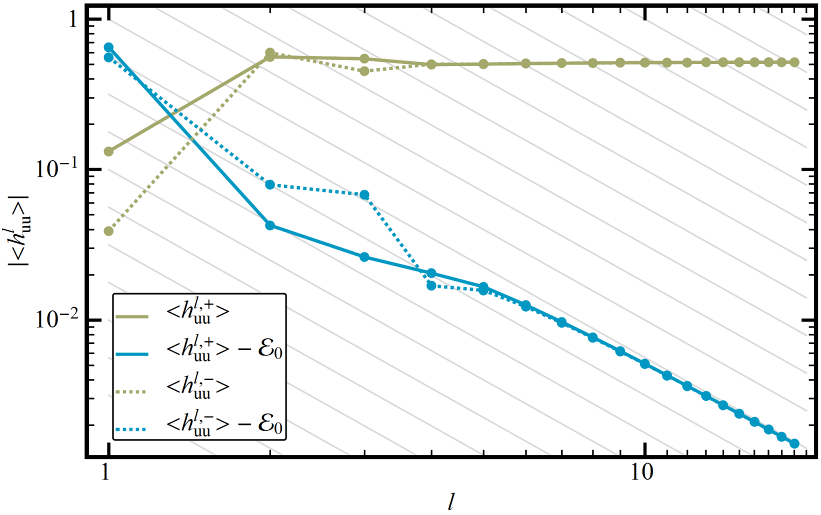
5PN

$\times$	p^{-6}	$p^{-6}\log p$
e^2	-2000 ± 400	-40 ± 20

Eccentric orbit in Kerr: *n*-mode convergence



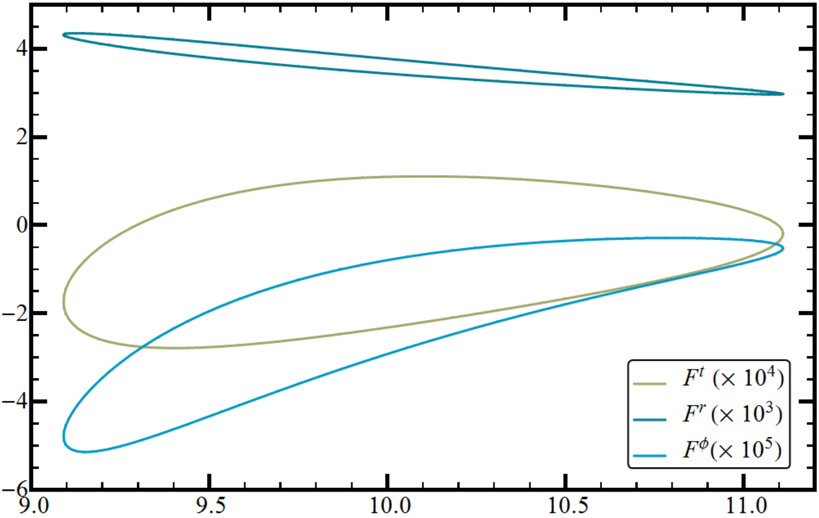
Eccentric orbit in Kerr: l-mode convergence (regularization)



Sample results: ΔU in Kerr

p	e	$a = -0.9$	$a = 0.0$	$a = 0.9$
$p_{\mathrm{ISO}} + 1$	0.00	$-0.1591225238(2)$	$-0.2208475274(2)$	$-0.40871659(2)$
$p_{\mathrm{ISO}} + 1$	0.10	$-0.150998651(3)$	$-0.209375588(9)$	$-0.391205(6)$
$p_{\mathrm{ISO}} + 1$	0.20	$-0.142034175(2)$	$-0.195877797(5)$	$-0.3635027(6)$
$p_{\mathrm{ISO}} + 1$	0.30	$-0.13127905(3)$	$-0.179591180(7)$	$-0.327109(4)$
$p_{\mathrm{ISO}} + 1$	0.40	$-0.1182790(4)$	$-0.160212(3)$	$-0.283764(2)$
$p_{\mathrm{ISO}} + 10$	0.00	$-0.062991300291(4)$	$-0.07205505742909(0)$	$-0.09093033701(3)$
$p_{\mathrm{ISO}} + 10$	0.10	$-0.061183242(1)$	$-0.070241089(2)$	$-0.089218442(2)$
$p_{\mathrm{ISO}} + 10$	0.20	$-0.058180786(3)$	$-0.066951046(3)$	$-0.085406440(1)$
$p_{\mathrm{ISO}} + 10$	0.30	$-0.054043290(2)$	$-0.06226898(1)$	$-0.0796315937(2)$
$p_{\mathrm{ISO}} + 10$	0.40	$-0.048833176(5)$	$-0.0562888674(2)$	$-0.072058378(4)$
$p_{\mathrm{ISO}} + 100$	0.00	$-0.00939536223942(0)$	$-0.00961638326555(0)$	$-0.00994293246401(0)$
$p_{\mathrm{ISO}} + 100$	0.10	$-0.00927472002(8)$	$-0.00950017309(4)$	$-0.0098334067(4)$
$p_{\mathrm{ISO}} + 100$	0.20	$-0.0089653010(1)$	$-0.00918951172(9)$	$-0.0095209535(1)$
$p_{\mathrm{ISO}} + 100$	0.30	$-0.00846904194(5)$	$-0.00868616413(8)$	$-0.00900719351(3)$
$p_{\mathrm{ISO}} + 100$	0.40	$-0.007788207(3)$	$-0.00799229975(7)$	$-0.0082942072(9)$
$p_{\mathrm{ISO}} + 1000$	0.00	$-0.00099341331118(0)$	$-0.00099601693770(0)$	$-0.00099959564940(0)$
$p_{\mathrm{ISO}} + 1000$	0.10	$-0.000983179941(7)$	$-0.00098584074(1)$	$-0.000989496817(4)$
$p_{\mathrm{ISO}} + 1000$	0.20	$-0.000953066056(2)$	$-0.000955719551(2)$	$-0.000959363369(3)$
$p_{\mathrm{ISO}} + 1000$	0.30	$-0.000903093610(8)$	$-0.000905672227(9)$	$-0.000909211662(8)$
$p_{\mathrm{ISO}} + 1000$	0.40	$-0.0008332886(4)$	$-0.000835722608(2)$	$-0.000839062961(5)$

Eccentric orbit in Kerr: GSF (hot of the press)



Orbit: $a = 0.9$, $p = 10$, $e = 0.1$

Conclusions

- MST + CCK forms powerful framework for frequency domain GSF calculations.
- First calculation of $\langle h_{uu} \rangle$ and *GSF* for eccentric orbits is Kerr spacetime.

What's next: generic orbits in Kerr!

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What's next: generic orbits in Kerr!

And now what?

ψ_4	Generic orbits	
Ψ_{ORG}	Generic orbits	
Observable l-modes	h_{uu}/GSF eq. orbs	Generic orbits other quantities
l-mode regularization	GSF: Generic orbit	h_{uu}: Generic orbit
Tail estimate	any orbit/quantity	
Completion	equatorial orbits	Generic orbits

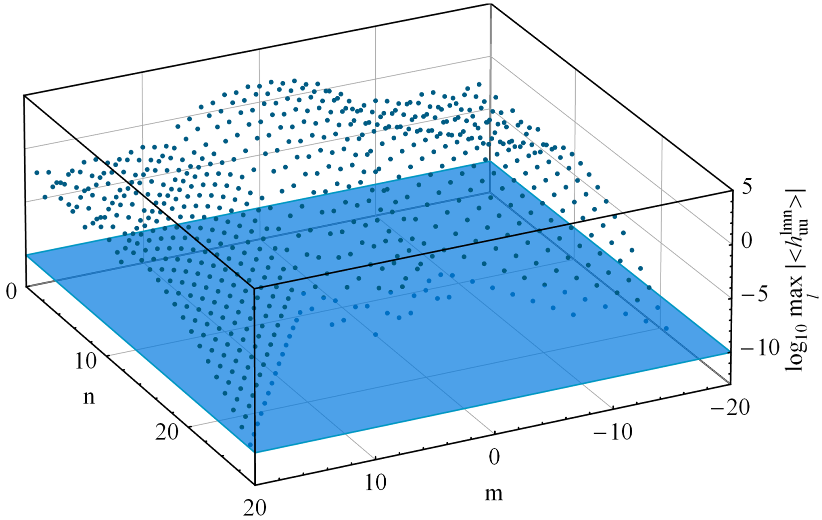
The end

Thank you for listening!

For more details see

MvdM and AG Shah, arXiv:1506.04755

n-mode convergence (*mn*-modes)



Interior vs. Exterior limit