EVOLVING HIGH ECCENTRICITY INSPIRALS | FLUXES FROM RAPIDLY ROTATING BLACK HOLES

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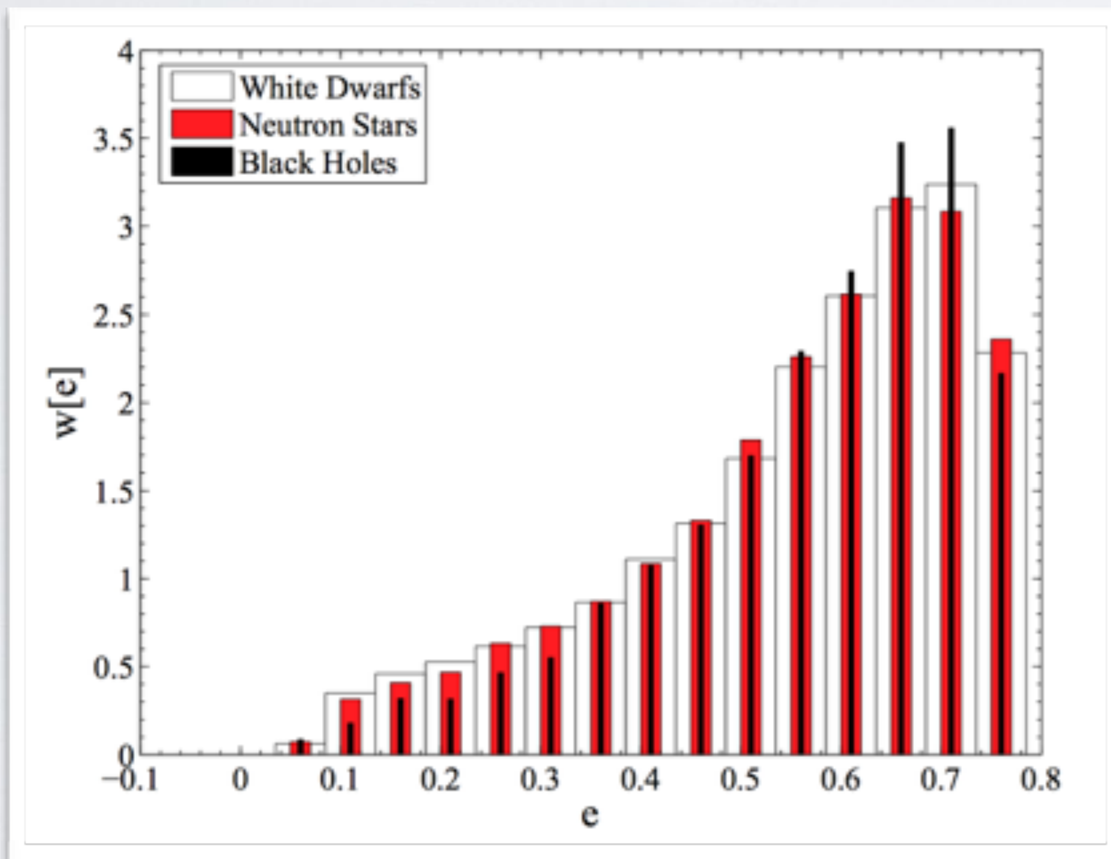
Capra 18, Kyoto

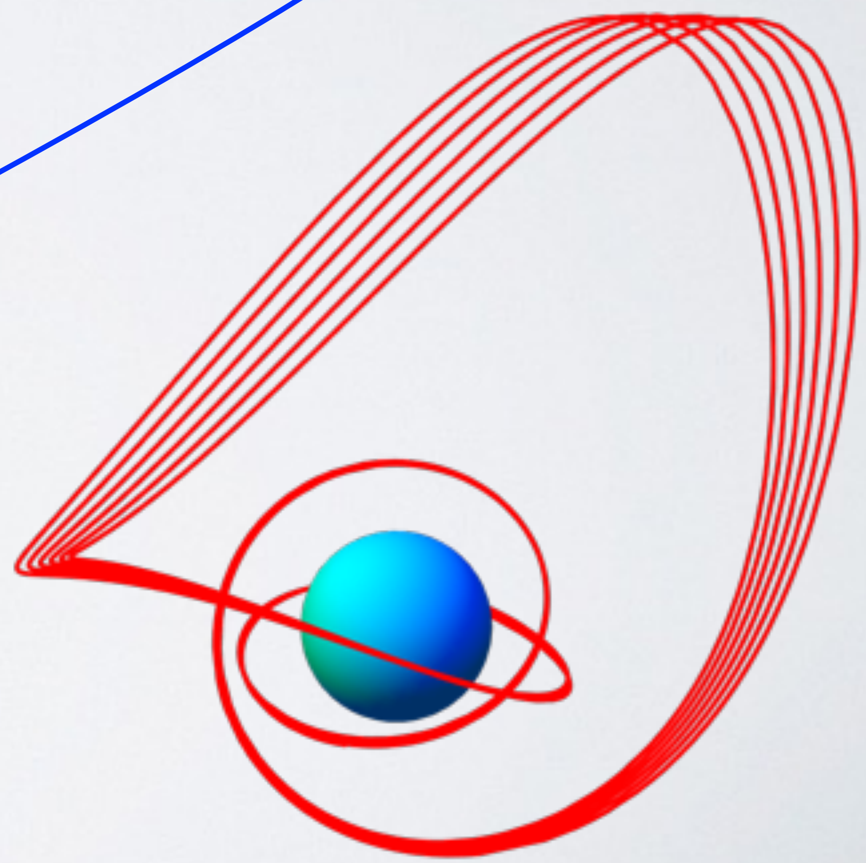
30th June 2015

Part One: Evolving high-eccentricity inspirals

- Effects of the self-force
- Interpolation model
- Computing high eccentricity inspirals

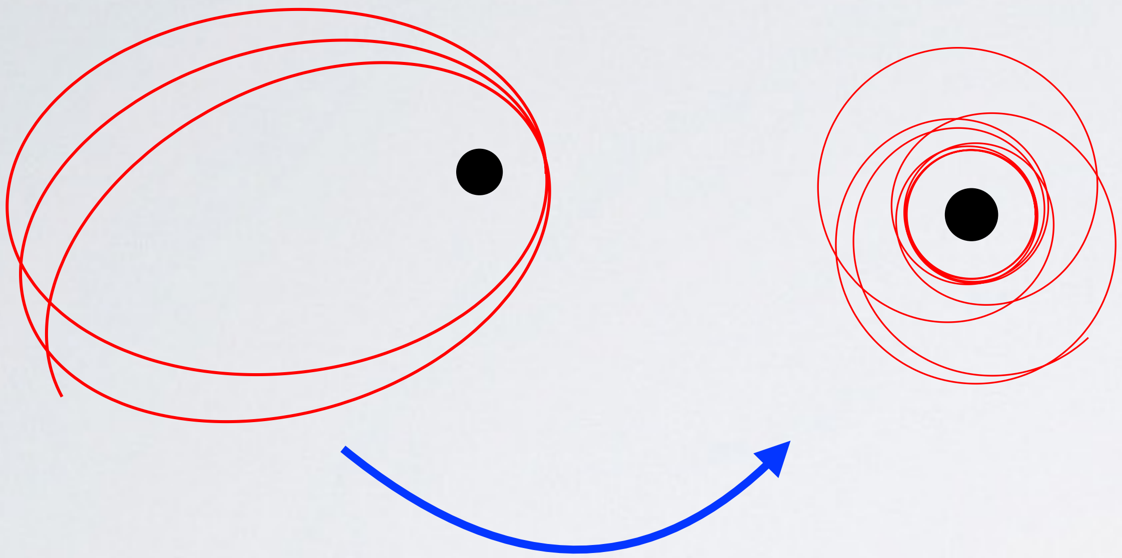


Hopman & Alexander ApJ 629 (2005) 362-372



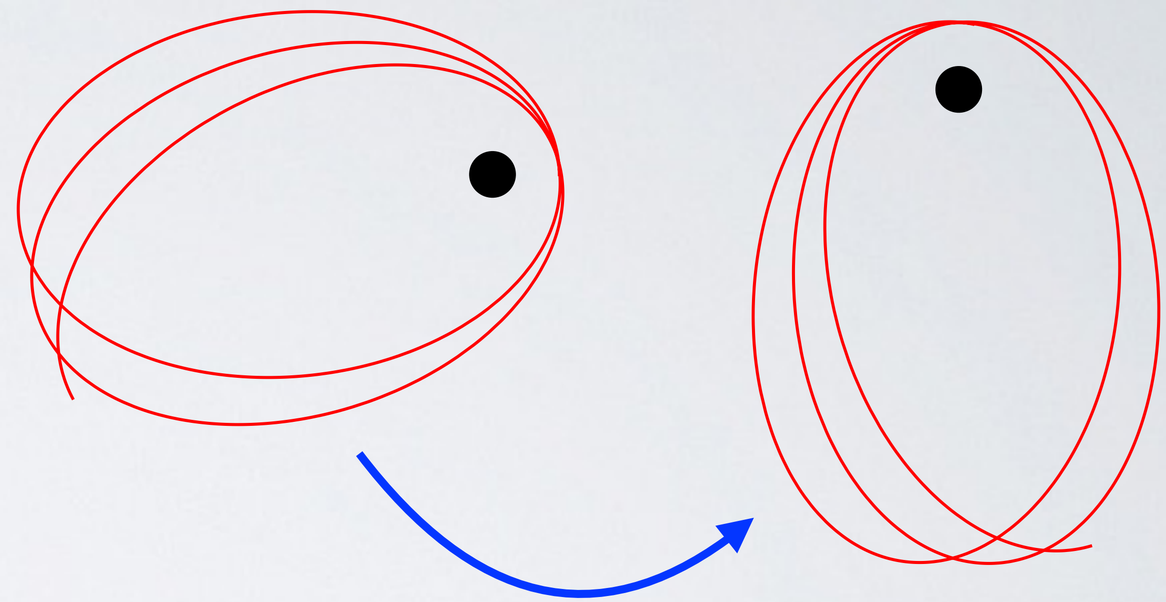
Effects of the self-force $F^\alpha = F^\alpha_{\rm diss} + F^\alpha_{\rm cons}$

Dissipative



$(\mathcal{E}, \mathcal{L}, \mathcal{Q})$ change

Conservative



$(\mathcal{E}, \mathcal{L}, \mathcal{Q})$ constant

$\mathcal{O}(q^{-1})$: Orbit-averaged dissipative component of the SF

$\mathcal{O}(q^{-1/2})$: Resonances, oscillating SF no longer averages out

$$\mathcal{O}(q^{0}) : \begin{cases} \text{Oscillatory component of the dissipative SF} \\ \text{Conservative component of the SF} \\ \text{Orbit-averaged dissipative piece of the second-order SF} \end{cases}$$

Effects of the self-force

$\mathcal{O}(q^{-1})$: Orbit-averaged dissipative component

$\mathcal{O}(q^{-1/2})$: ~~Resonances, oscillating SF no longer averages out~~

$\mathcal{O}(q^{0})$:
- Oscillatory component of the dissipative SF
- Conservative component of the SF
- ~~Orbit-averaged dissipative piece of the second-order SF~~

$$m_1 = 10^6 M_\odot$$
$$m_2 = 10 M_\odot$$
$$q = 10^{-5}$$

Goal: track phase evolution to within 0.01 radians

$F^\alpha =$	$\langle F_1^\alpha \rangle +$	$F_{1(\mathrm{osc})}^\alpha +$	$\langle F_2^\alpha \rangle$
Required Accuracy	10^{-7}	10^{-3}	10^{-3}
Code	RWZ	Lorenz	-
Code Accuracy	10^{-10}-10^{-9}	10^{-10}-10^{-3}	-

Hopper & Evans: Phys. Rev. D 82.084010

Osburn et al: Phys. Rev. D 90.104031

Orbital parameterization

Geometrically intuitive parameterization

$$p \equiv \frac{2r_{\max}r_{\min}}{M(r_{\max}+r_{\min})} \qquad e \equiv \frac{r_{\max}-r_{\min}}{r_{\max}+r_{\min}}$$

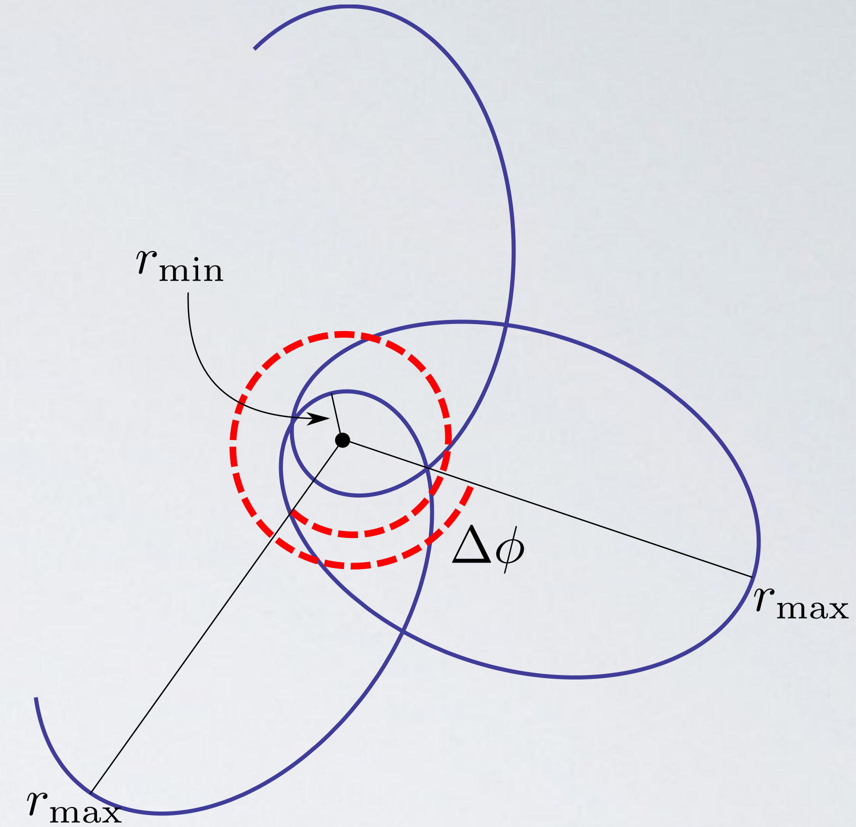
$$r(t) = \frac{pM}{1+e\cos[\chi(t)-\chi_0]}$$

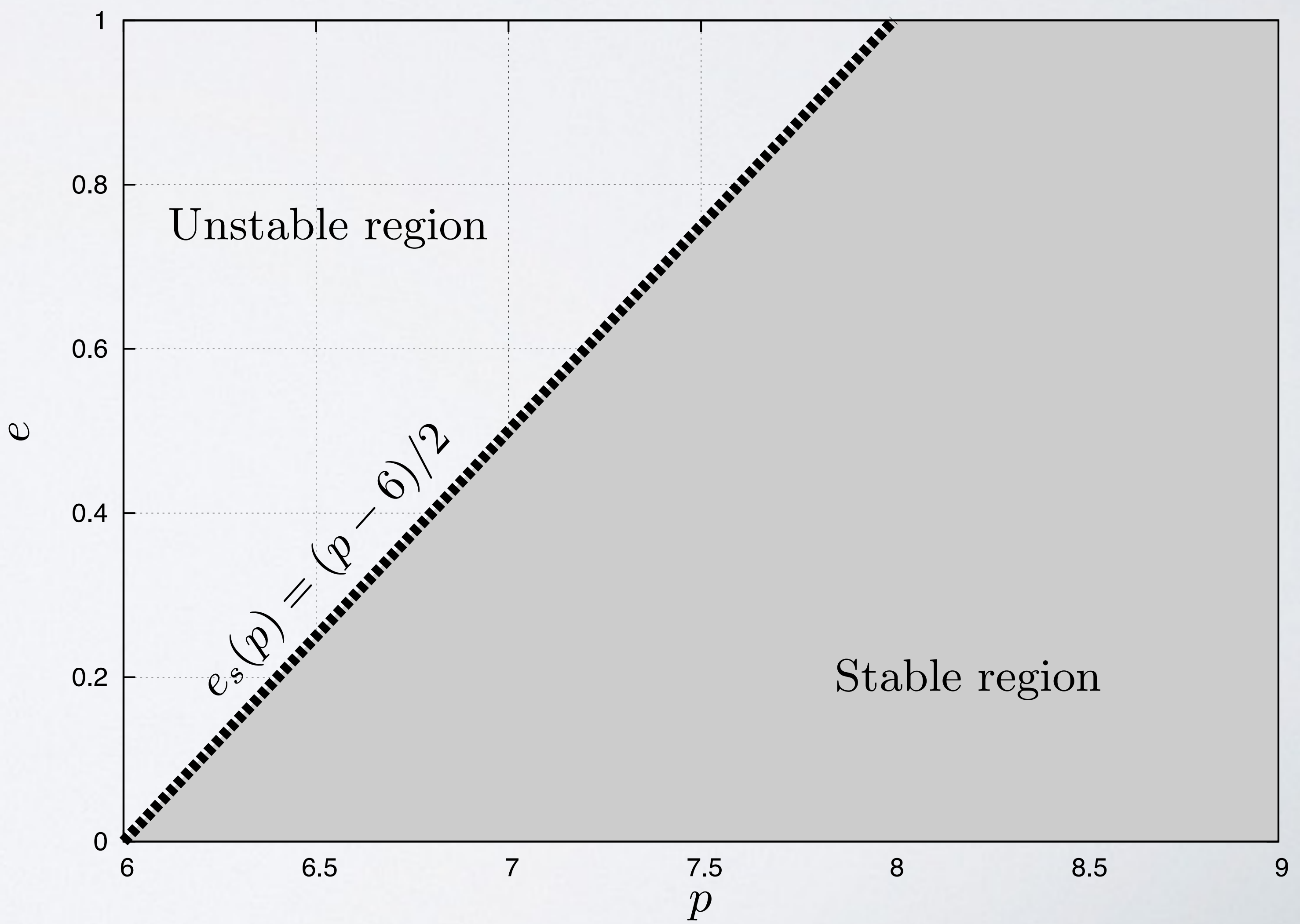
one-to-one mapping:

$$(\mathcal{E},\mathcal{L}) \leftrightarrow (p,e)$$

Note: $(\mathcal{E},\mathcal{L}) \to (\Omega_r,\Omega_\varphi)$

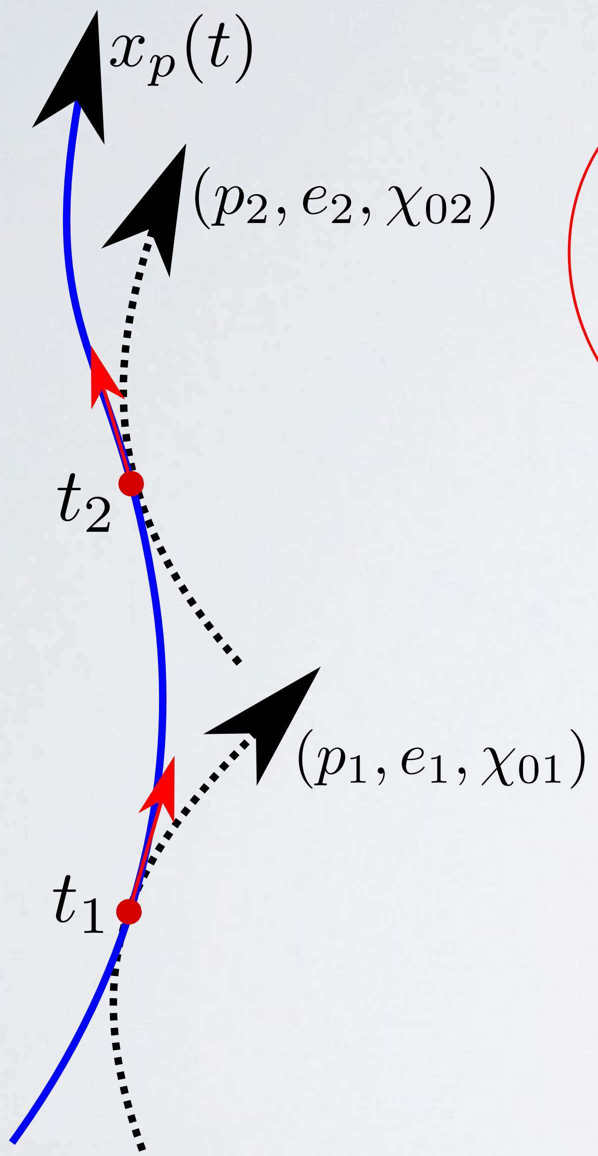
is not one-to-one

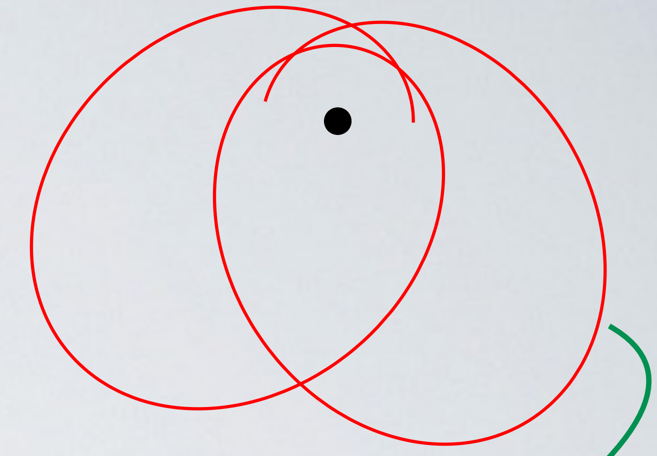


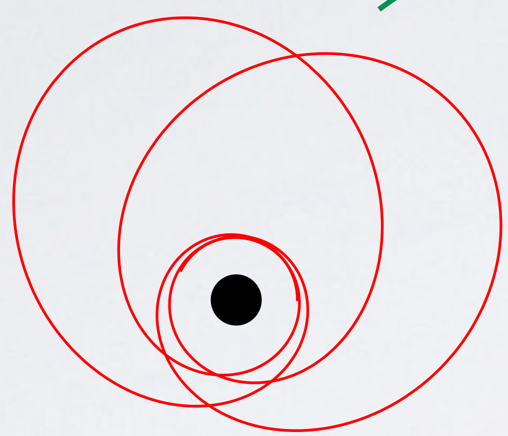


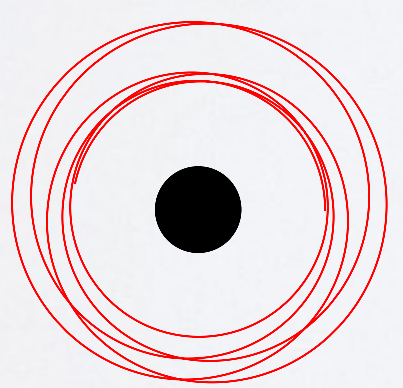
Orbit evolution with the self-force

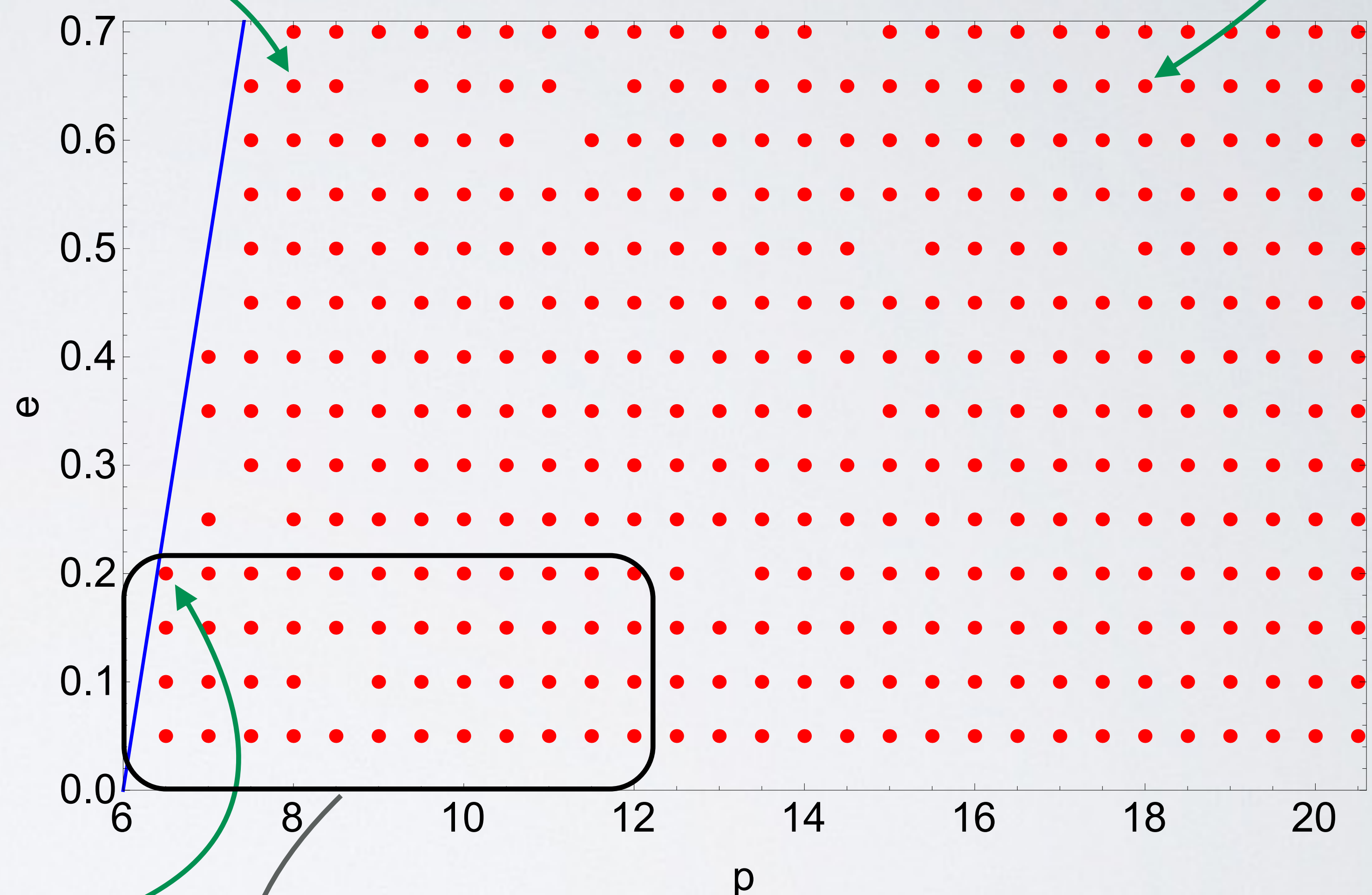
$$u^\beta \nabla_\beta u^\alpha = F^\alpha \implies \begin{cases} r'(\tau) = \dots \\ \varphi'(\tau) = \dots \\ t'(\tau) = \dots \end{cases}$$











Osculating orbits with geodesic self-force

NW, S. Akcay, L. Barack, J. Gair and N. Sago: Phys. Rev. D 85.061501(R)

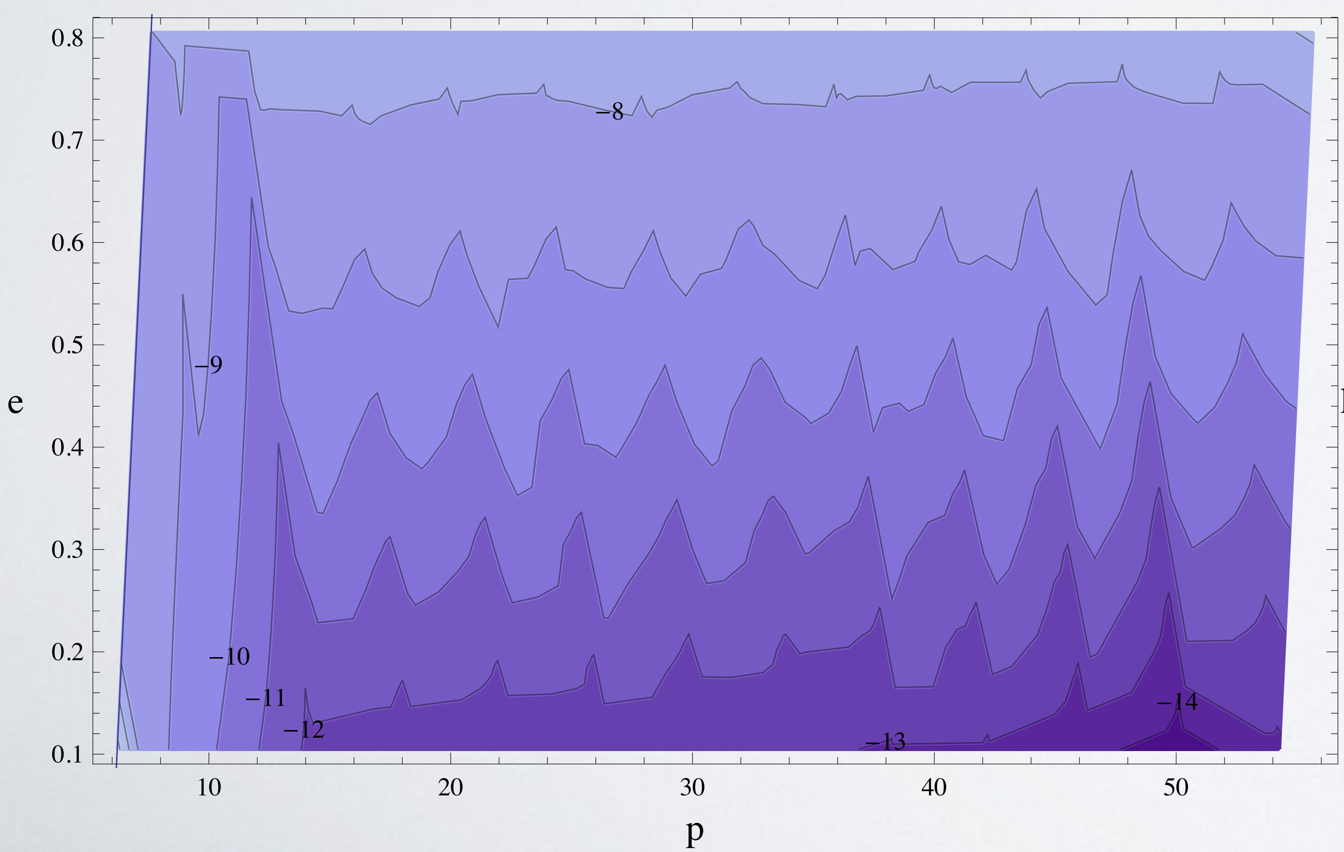
$$u^\beta \nabla_\beta u^\alpha = F^\alpha \implies \begin{cases} p'(v) = f_p(p, e, v, F^r, F^\varphi) \\ e'(v) = f_e(p, e, v, F^r, F^\varphi) \\ \chi_0'(v) = f_{\chi_0}(p, e, v, F^r, F^\varphi) \end{cases}$$

Orbit evolution with the self-force

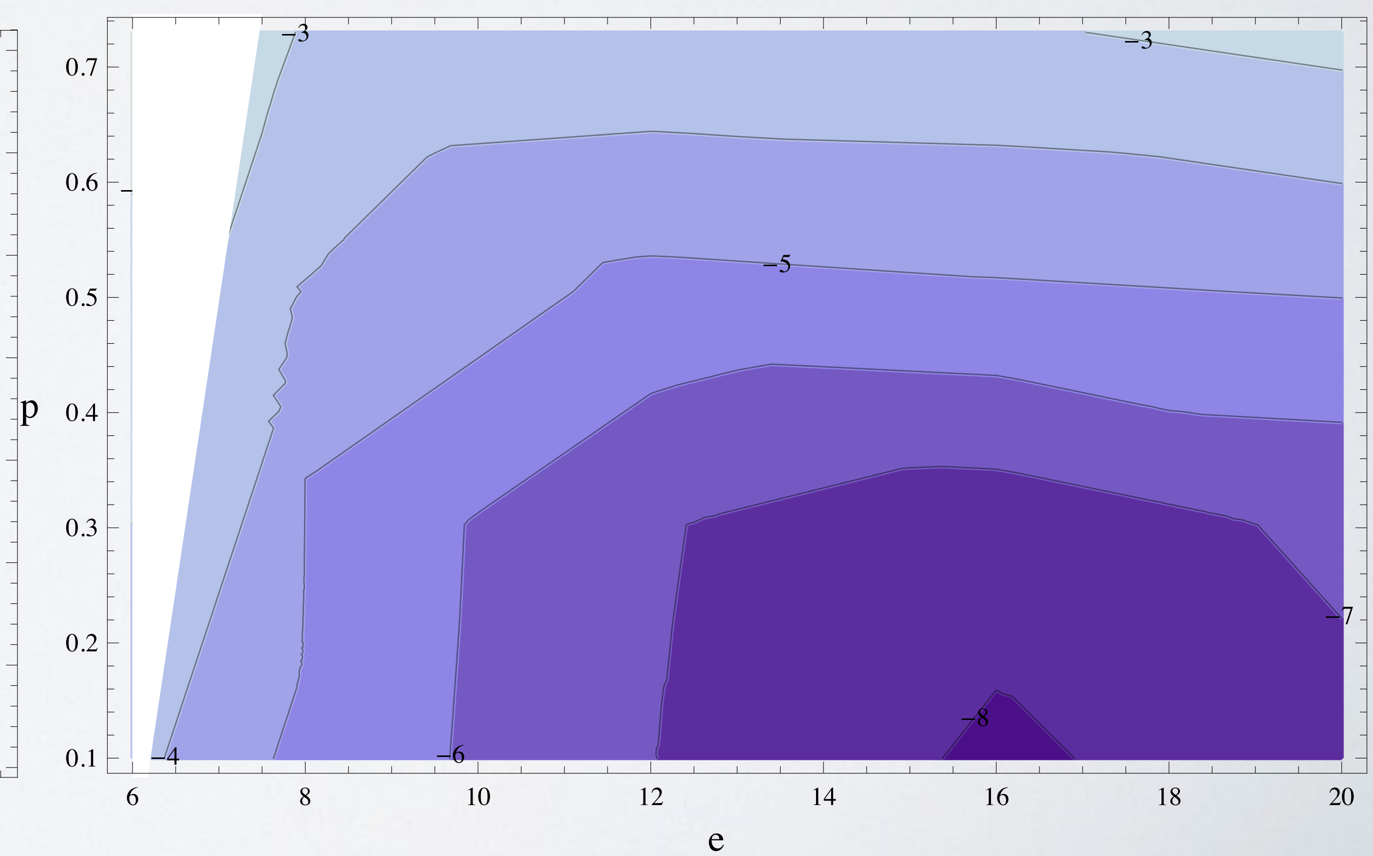
How good is our fit?

$F^{\alpha} =$	$\langle F_1^{\alpha} \rangle +$	$F_{1(\mathrm{osc})}^{\alpha} +$	$\langle F_2^{\alpha} \rangle$
Required Accuracy	10^{-7}	10^{-3}	10^{-3}
Code	RWZ	Lorenz	-
Code Accuracy	10^{-10}-10^{-9}	10^{-10}-10^{-3}	-

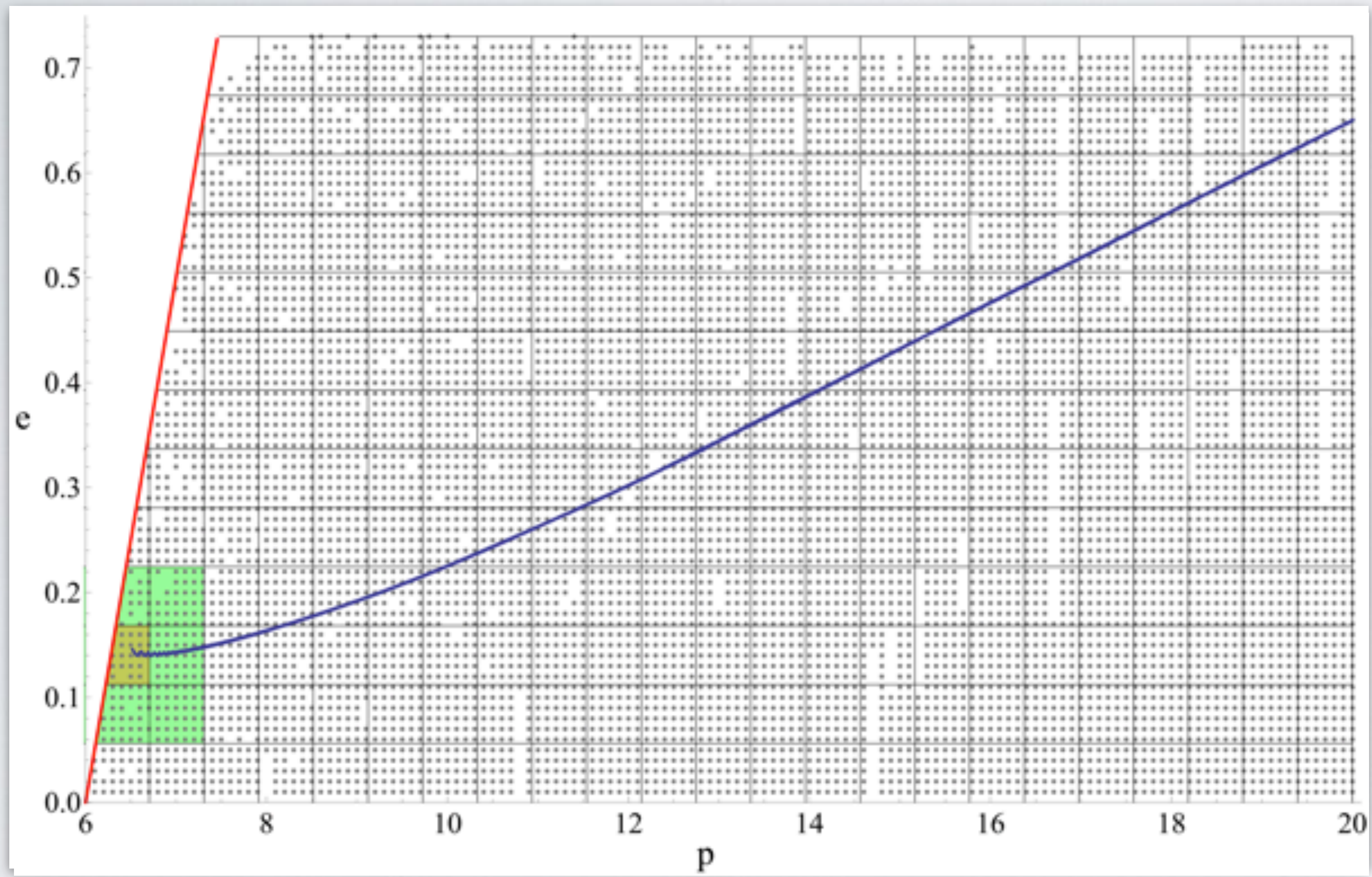
Adiabatic interpolation: $\mathrm{Log}_{10}(F^t$ relative error)

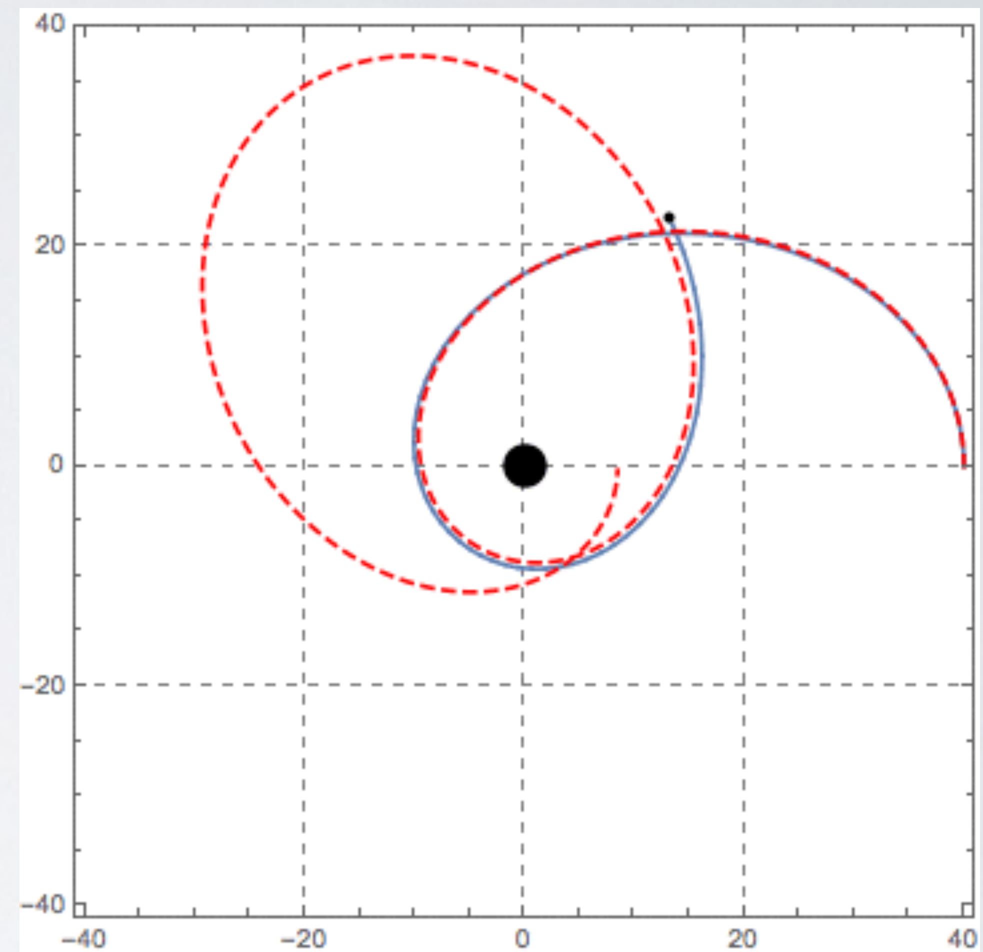


Oscillatory interpolation: $\mathrm{Log}_{10}(F^t$ relative error)



Orbit evolution with the self-force





Previous work used global fits. In this work we use local fitting to achieve high accuracy

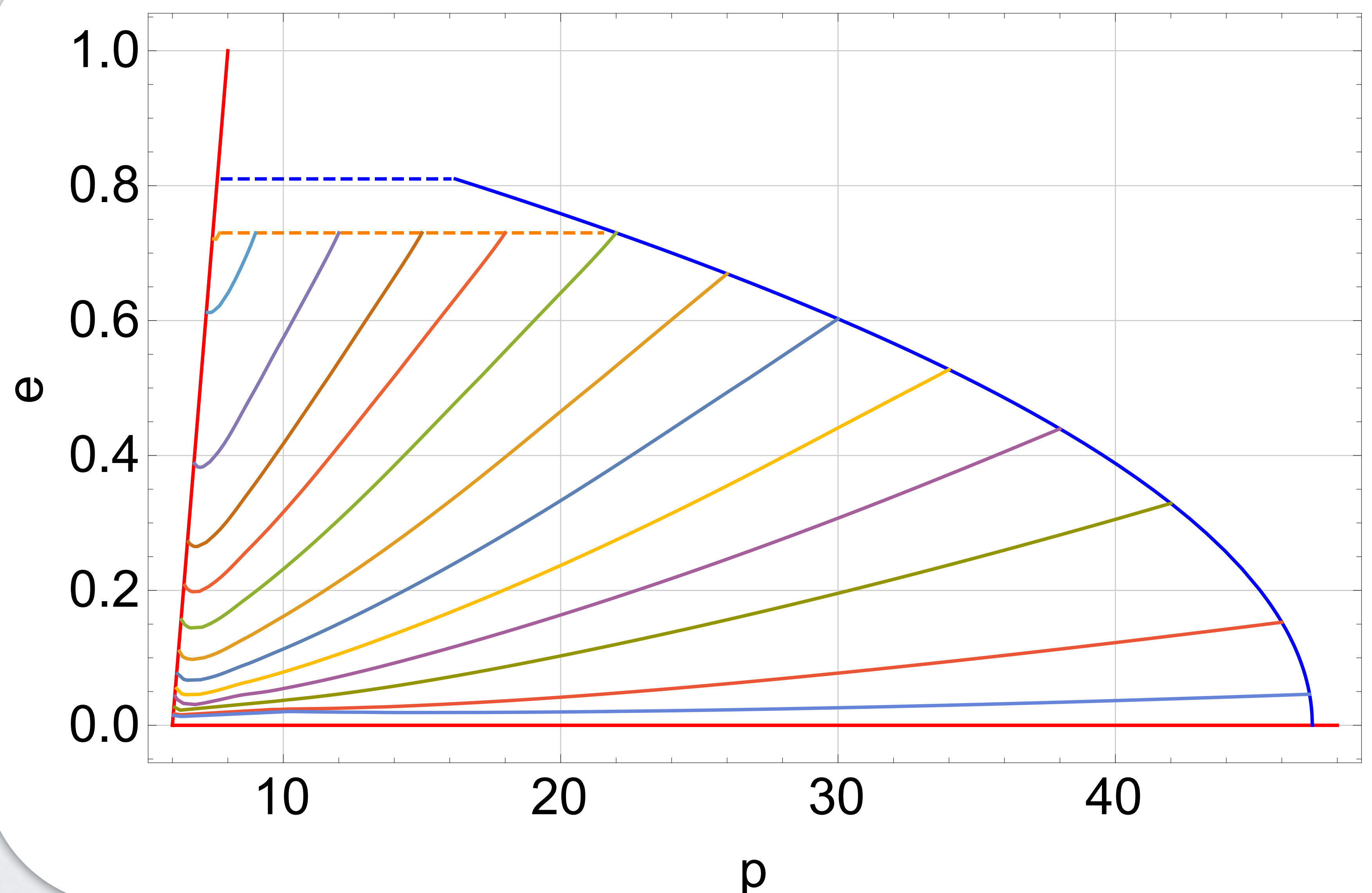
As the particle inspirals it moves from one region to another. Interpolate over containing box and 8 surrounding boxes. All interpolation coefficients pre-computed.

Importance of conservative effects

Evolution: $(p(v), e(v), \chi_0(v))$

Conservative only

$$r = \frac{p}{1 + e\cos(\chi - \chi_0)}$$



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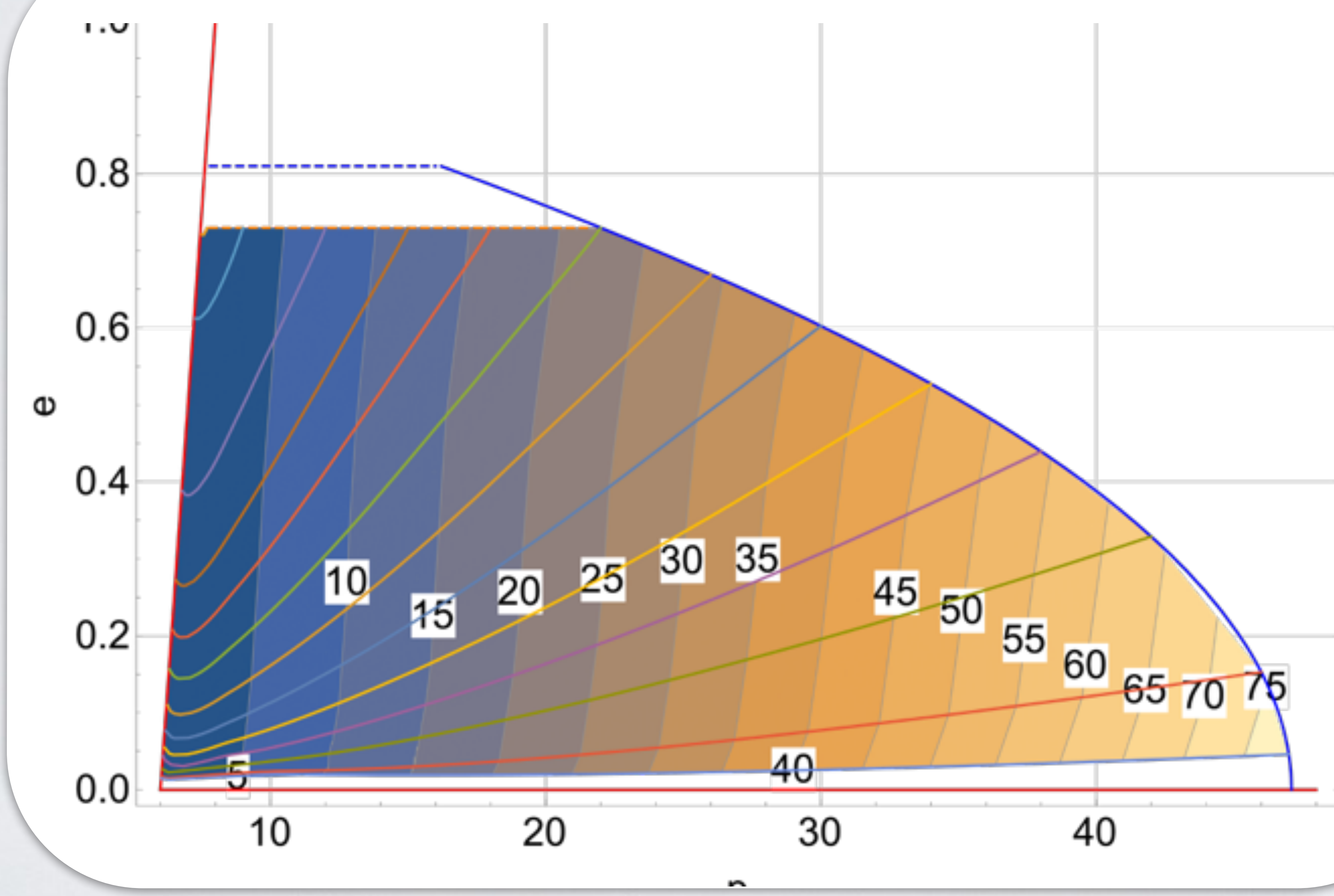


Part One: Evolving high-eccentricity inspirals
Recap and future directions

This work

- High eccentricity inspirals into a Schwarzschild black hole

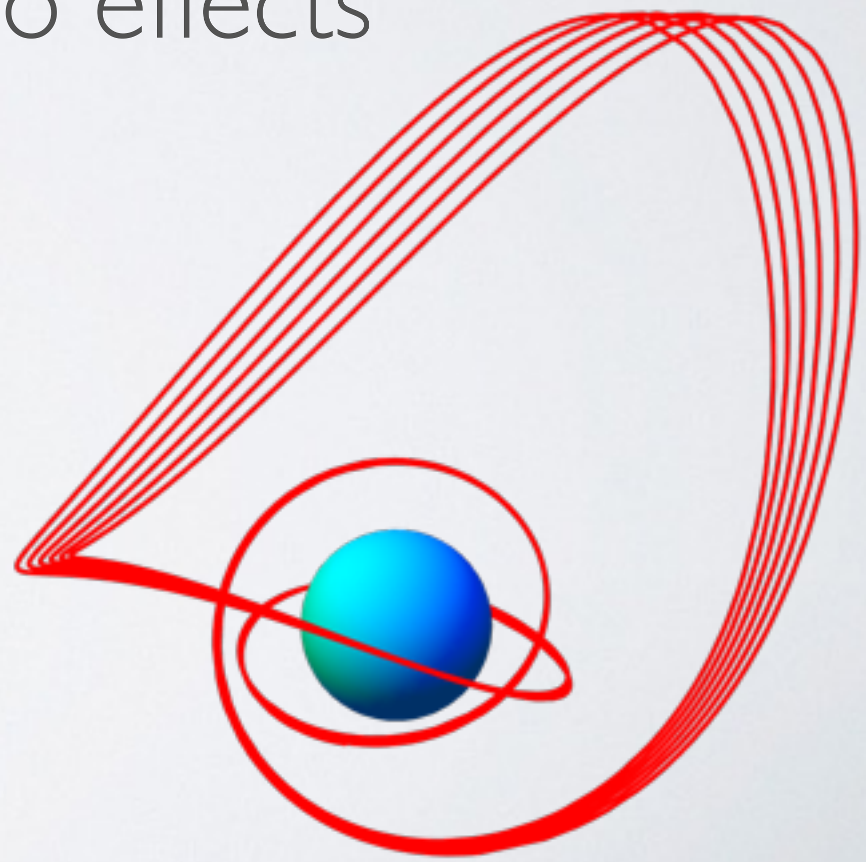
See: Pound, Wardell, Detweiler and Tanaka's talks tomorrow

Open questions:

- Second-order-in-the-mass-ratio effects
- How good is the geodesic self-force approximation?
- Inspirals in Kerr spacetime

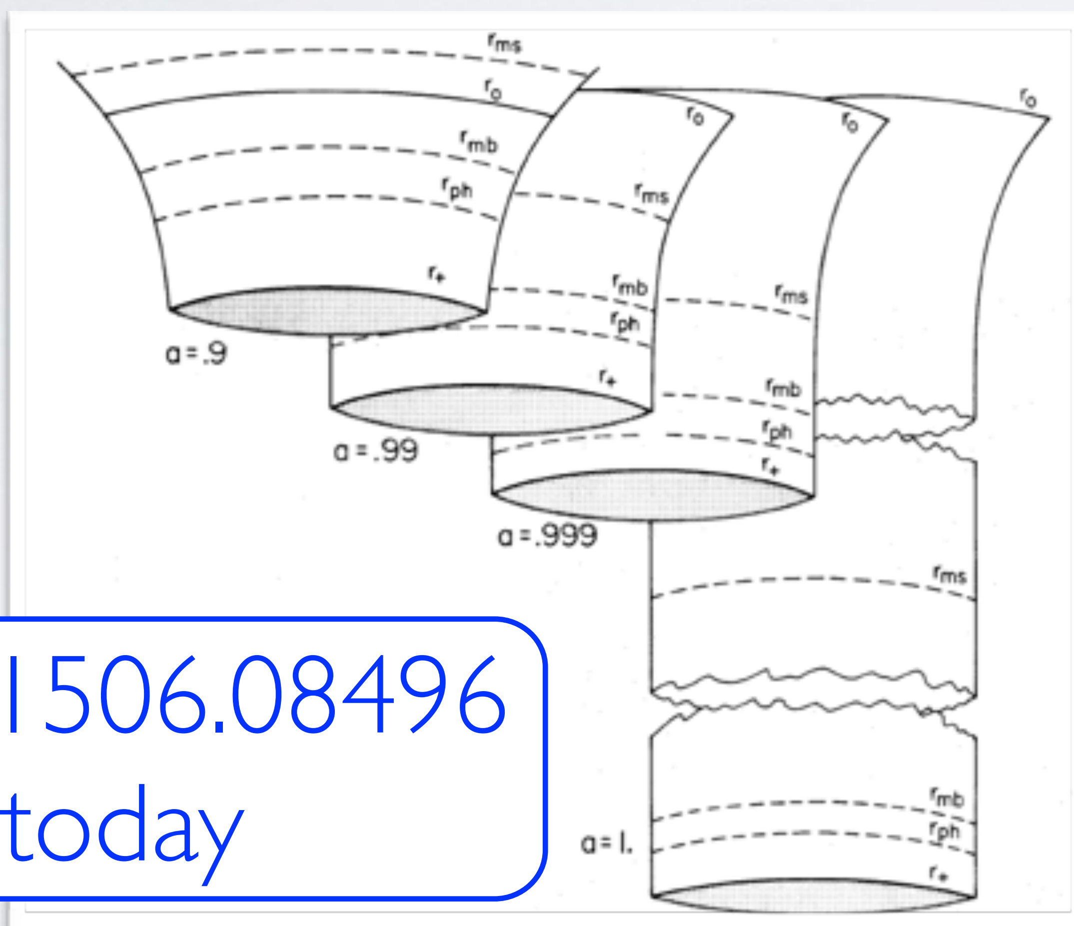
Kerr self-force: van de Meent's talk

See Diener's talk on Thursday

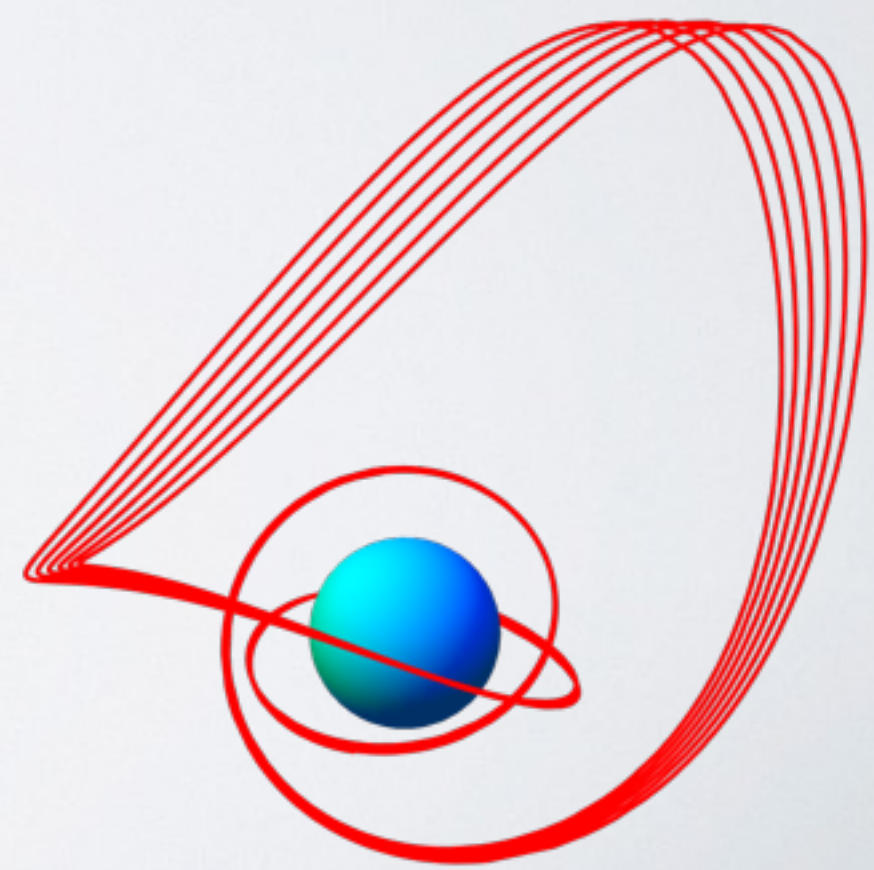


Part Two: Fluxes from rapidly rotating black holes

- Particle on the ISCO
- Analytic and numerical calculations
- Comparison of results



arXiv:1506.08496
today



Particle on the ISCO

Parametrize using **ε** for the spin: $\epsilon = \sqrt{1 - a^2/M^2}$

Every non-extreme (**ε**>0) black hole has an ISCO

$$r_{\mathrm{ISCO}} = M\left(1 + \frac{1}{2^{1/3}}\epsilon^{2/3} + \mathcal{O}(\epsilon^{4/3})\right)$$

We put a particle on the ISCO and compute the power radiated as **ε**→0:

Analytically, to leading-order in **ε**

$$\boxed{\frac{d\mathcal{E}}{dt} = C\epsilon^p}$$

Previous work estimated p=2/3.

We confirm, find interesting mode structure, and get C.

Numerically using a new high-precision Teukolsky code

Solve the Sasaki-Nakamura equation in Mathematica with new, more accurate boundary conditions

Analytic calculation

Different possible limits as $\epsilon \to 0$:

- BL exteme Kerr
- NHEK (near-Horizon-extremal-Kerr)
- near-NHEK

Hallmark of a singular perturbation problem.
Solution: matched asymptotic expansions

Define dimensionless radial coord:

$$\boxed{\begin{aligned} x &= \frac{r - r_+}{r_+} \\ x_0 &= 2^{1/3}\epsilon^{2/3} \end{aligned}}$$

far zone: $x >> \epsilon^{2/3}$

$$x^2 R''(x) + 2xR'(x) + \left[m^2(2 + x + x^2/4) - K\right] R(x) = 0$$

near zone: $x << 1$

$$x(x+2\epsilon)R''(x) + 2(x+\epsilon)R'(x)\hat{V}R(x) = Nx_0\delta(x - x_0)$$

Matching in overlap region ($\epsilon^{2/3} << x << 1$) gives the result

Results for fluxes (but not other quantities) turn out to be the same as a NHEK calculation. Can also derive the same formula from Kerr/CFT. See Porfryiadis and Strominger arXiv:1401.3746

Numerical calculation

Homogeneous Sasaki-Nakamura equation

$$\frac{dX}{dr_*^2} - F(r)\frac{dX}{dr_*} - U(r)X(r) = 0$$

Expand asymptotic behaviour as

$$X^\infty = e^{i\omega r_*} \sum_{k=0}^{k_{\max}^\infty} a_k^\infty (\omega r_{\text{out}})^{-k},$$

$$X^H = e^{-i(\omega - m\Omega_H)r_*} \sum_{k=0}^{k_{\max}^H} a_k^H (r_{\text{in}} - r_+)^k$$

Previous authors have only used: $k_{\max}^\infty = 3, \; k_{\max}^H = 0$
We have the full recursion relations

Algorithm (in Mathematica):

(i) construct BCs
(ii) solve homogeneous SN equation
(iii) Transform to Teukolsky variables
(iv) matching at the particle
(v) compute the fluxes

Analytic Results

Formulae for the infinity and horizon flux for the scalar (s=0) and gravitational (s=-2) case

$$\frac{d\mathcal{E}_\infty}{dt} = C\epsilon^p \qquad p = \frac{4}{3}\mathrm{Re}[h]$$

$$h = \frac{1}{2} + \sqrt{K - 2m^2 + \frac{1}{4}}$$

spheroidal eigenvalue

$$x_0 = 2^{1/3}\epsilon^{2/3}$$

Gravitational case

$$\dot{\mathcal{E}}_H = -\frac{m_0^2}{2^6 3^2 M^2}\frac{1}{|\mathcal{C}|^2}x_0 e^{-|m|\pi}\frac{|\Gamma(h-im-s)|^2}{|\Gamma(2h)|^2}\left|\hat{\mathcal{M}} + \frac{b}{1-\hat{b}}\frac{\Gamma(1-h-im-s)}{\Gamma(1-2h)}\hat{\mathcal{W}}\right|^2$$

$$\dot{\mathcal{E}}_\infty = \frac{m_0^2}{2^5 3^3 M^2}(3x_0 m^2/2)^{2\mathrm{Re}[h]}e^{|m|\pi}\left|\frac{2h-1}{1-\hat{b}}\frac{\Gamma(h-im-s)\Gamma(h-im+s)}{\Gamma(2h)^2}\hat{\mathcal{W}}\right|$$

$$\hat{b} = \frac{\Gamma(1-2h)^2}{\Gamma(2h-1)^2}\frac{\Gamma(h-im-s)}{\Gamma(1-h-im-s)}\frac{\Gamma(h-im+s)}{\Gamma(1-h-im+s)}\left(\frac{3x_0m^2}{2}\right)^{2h-1}$$

$$\hat{\mathcal{M}} = 2\left[(h^2-h+6-im)S + 4(2i+m)S' - 4S''\right]M_{im-2,h-\frac{1}{2}}(3im/2) - (h-2+im)\left[(4+3im)S - 8iS'\right]M_{im-1,h-\frac{1}{2}}(3im/2)$$

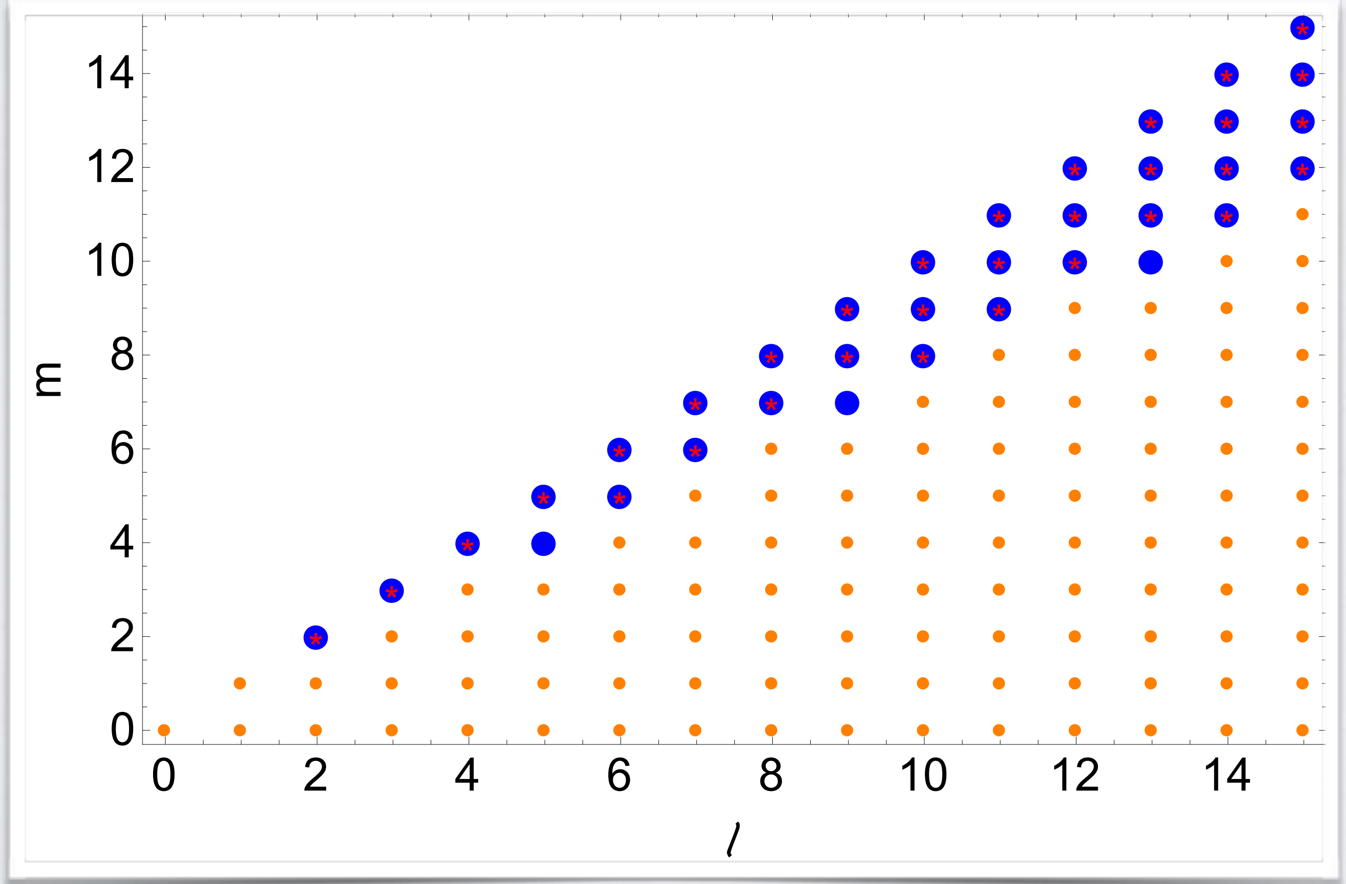
$$\hat{\mathcal{W}} = 2\left[(h^2-h+6-im)S + 4(2i+m)S' - 4S''\right]W_{im-2,h-\frac{1}{2}}(3im/2) + \left[(4+3im)S - 8iS'\right]W_{im-1,h-\frac{1}{2}}(3im/2)$$

$$|\mathcal{C}|^2 = [(-2+h)^2+m^2][(-1+h)^2+m^2][h^2+m^2][(1+h)^2+m^2]$$

Analytic Results: mode structure

$$\frac{d\mathcal{E}_\infty}{dt} = C\epsilon^p \qquad p = \frac{4}{3}\mathrm{Re}[h] \qquad h = \frac{1}{2} + \sqrt{K - 2m^2 + \frac{1}{4}}$$

$$\text{If } K - 2m^2 + 1/4 \leq 0, \quad p = 2/3$$
$$\text{If not}, \quad p > 2/3$$



● = dominant (grav)

* = dominant (scalar)

● = subdominant (both)

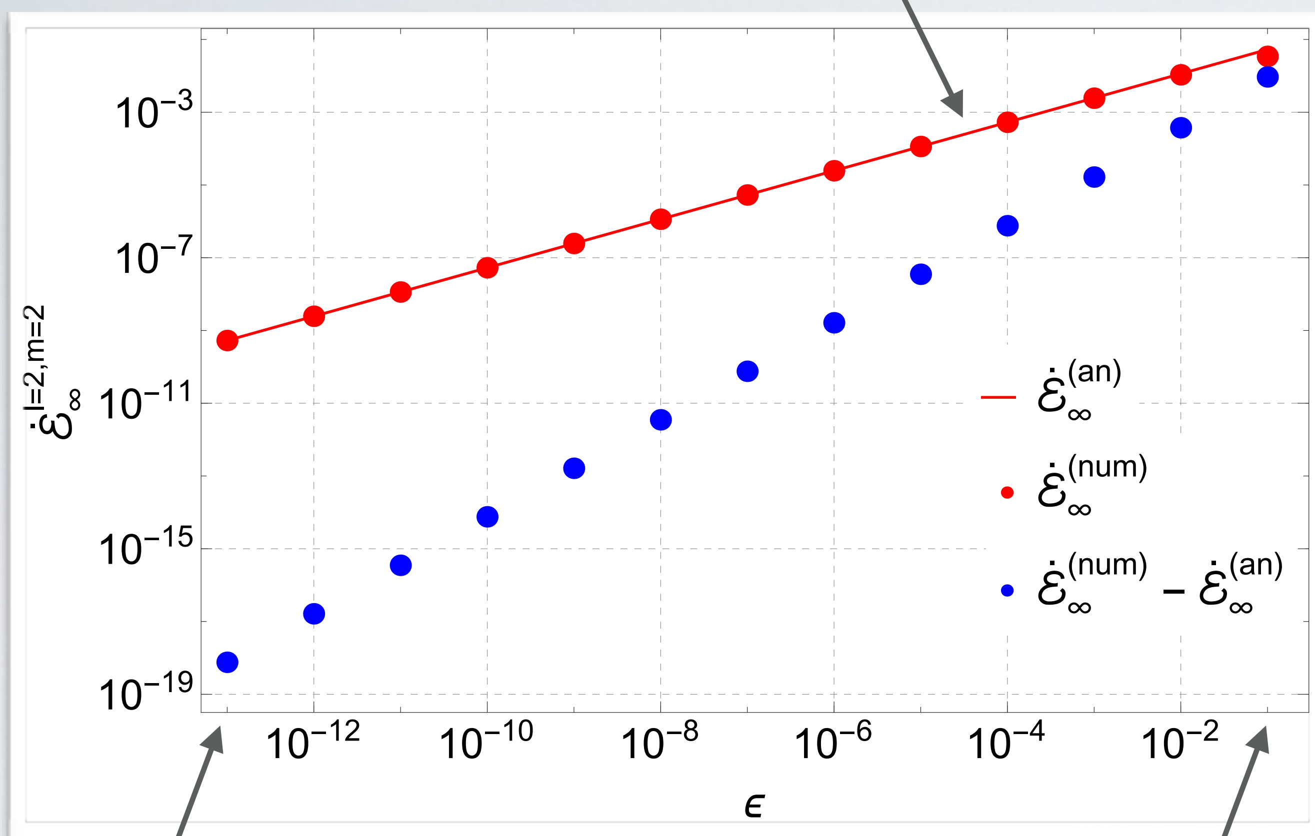
Analogous mode structure observed in study of near extremal quasi-normal modes Yang etal arXiv:1212.3271,1307.8086

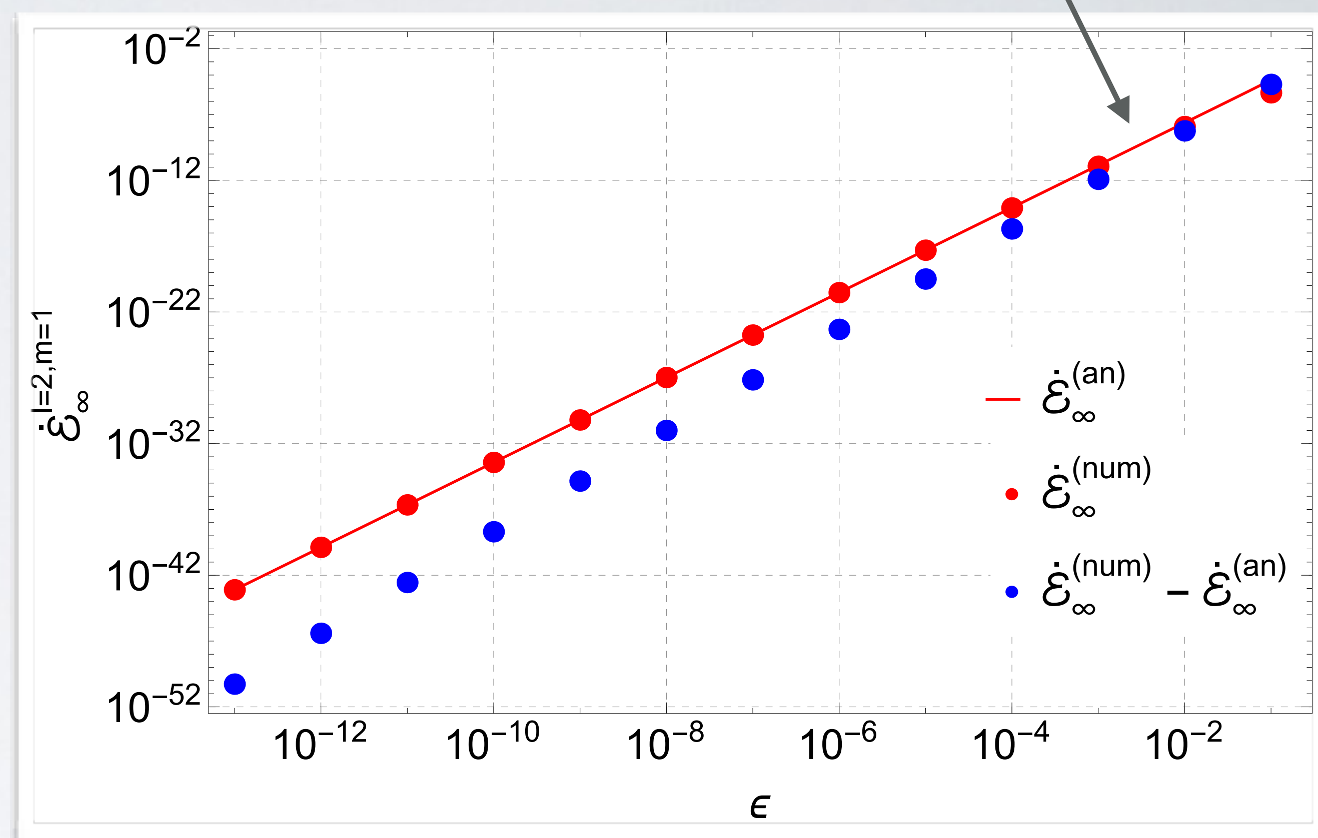
Numerical comparison

$$\frac{d\mathcal{E}_\infty}{dt} = C\epsilon^p$$

p=2/3

p=3.225427

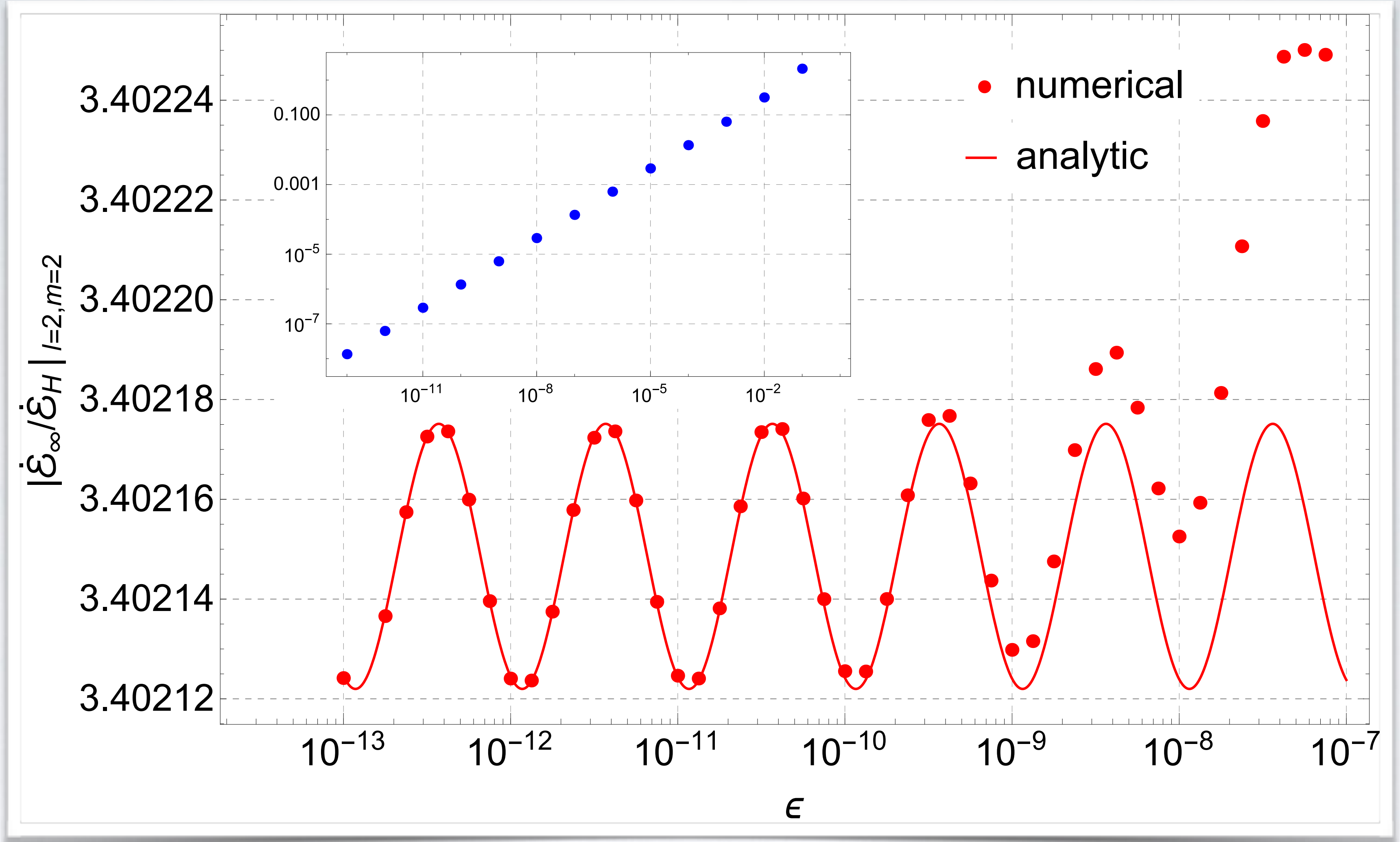




a/M=0.99999999999999999999999995 a/M=0.995

- Dominant scaling (p=2/3) for m~l for infinity flux
- Horizon flux scales as p=2/3 for all modes
- All modes with Re[h]=1/2 also have oscillations

Numerical comparison



- Oscillations clear when you take the ratio of the fluxes
- Inset shows absolute difference between analytic and numerical results for the flux ratio

Part Two: Fluxes from rapidly rotating black holes
Recap and future directions

arXiv:1506.08496
today

This work

- Fluxes for particle on the ISCO (i) analytically, to leading-order in the deviation from extremality and (ii) numerically to high accuracy
- Find excellent agreement between two methods
- Analytic formulae useful to check numerical codes

Future work

- Calculate local quantities both analytically and numerically e.g. ISCO shift
- Useful in study of cosmic censorship?

Colleoni's talk on Thursday