Consiglio Nazionale delle Ricerche

Istituto per le Applicazioni del Calcolo "Mauro Picone"

High-order analytical self-force calculations

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(work(s) in collaboration with T. Damour and A. Geralico)

19th Capra Meeting on Radiation Reaction in General Relativity, June 27- July 1, 2016 Meudon (FR)

Plan of the talk

(short)

→**Review** of high-order analytic self-force formalism in a Schwarzschild spacetime: Bini-Damour approach and list of main results

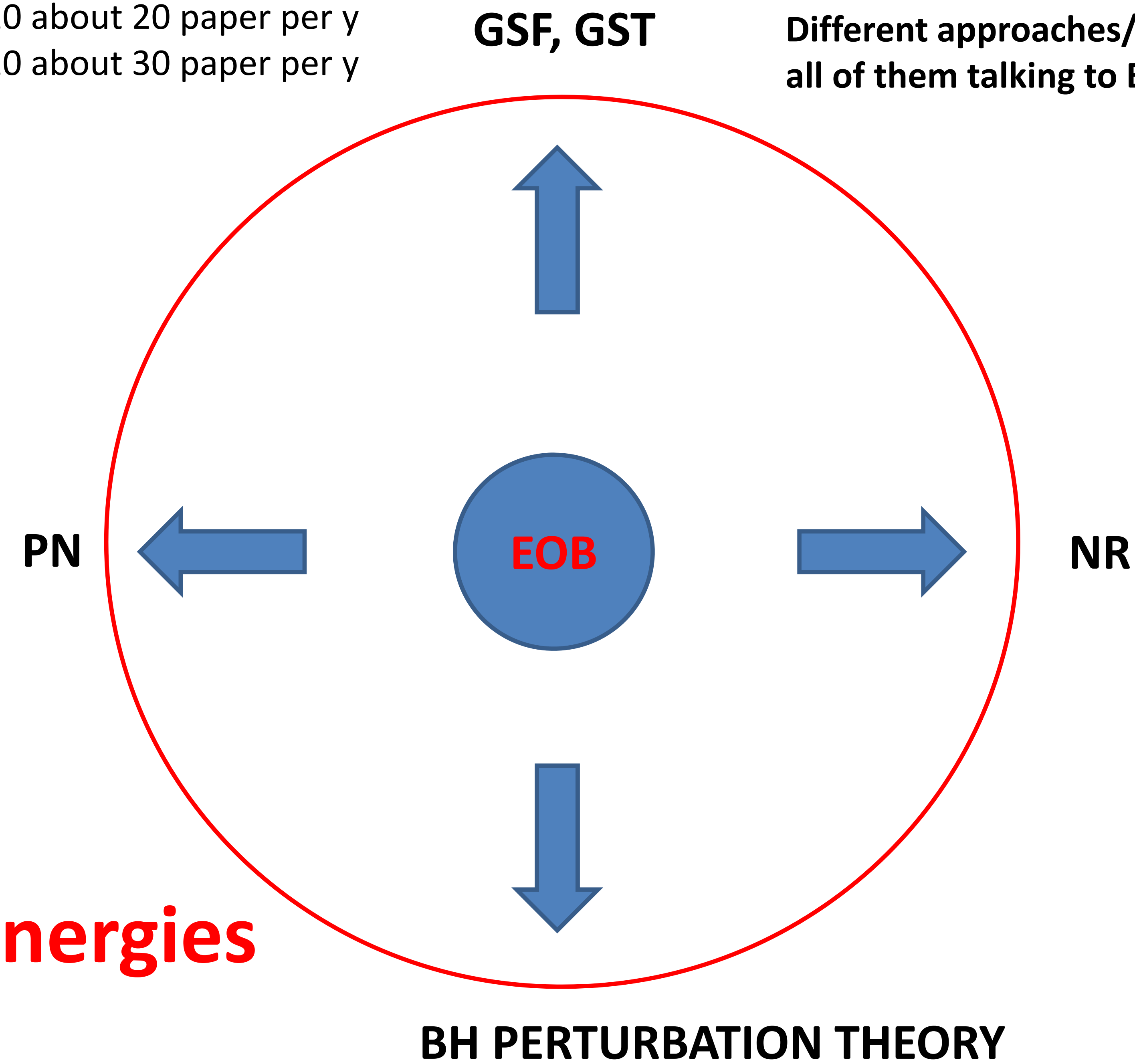
→**Review** of high-order analytic self-force formalism in a Kerr spacetime: Bini-Damour-Geralico list of main results

→**Review** of main applications to PN and EOB formalisms

→**Works in progress**

< 2010 about 20 paper per y
> 2010 about 30 paper per y

Different approaches/formalisms all of them talking to EOB!



Synergies

Minimal details…

Introduction

A test particle of (small) mass m_1 is orbiting a Schwarzschild or a (slowly rotating) Kerr bh of (large) mass m_2, moving on a circular/eccentric equatorial path.

The background metric is perturbed, and one can analytically reconstruct it by solving the associated metric or curvature perturbation equations.

Imagine that one aims at starting analytical SF calculation…what is needed?

General procedure

- Compute the PN and MST solutions of the (radial) RW or Teukolsky equation
- Compute the Green function of the RW or T equation taking into accout the proper source terms

→

- Reconstruct the metric perturbation by using standard procedures

General procedure

- Select the quantity to be computed (in the RW gauge), say δ
- Use the already computed Green function formally in δ (odd and even, left and right)
- Evaluate analytically the jump of δ (if any)
- Identify the series expansion in l of the «B» term i.e. the singular part of δ
- Compare the expanded «B» term with its analytical prediction (if it exists)
- Remove the singular part of δ
- Sum from N to infinity (with $N=l_{max}+1$ if MST solutions used correspond to $l=2,\ldots,l_{max}$)

Metric perturbations in a SCHWARZSCHILD background

$$\left(1-\frac{2M}{r}\right)^2\frac{d^2}{dr^2}R^{(\mathrm{even/odd})}_{lm\omega}+\frac{2M}{r^2}\left(1-\frac{2M}{r}\right)\frac{d}{dr}R^{(\mathrm{even/odd})}_{lm\omega}+[\omega^2-V_l(r)]R^{(\mathrm{even/odd})}_{lm\omega}=S^{(\mathrm{even/odd})}_{lm\omega}(r)$$

$$S^{(\mathrm{even})}_{lm\omega}=s^{(\mathrm{e})}_0\delta(r-r_0)+s^{(\mathrm{e})}_1\delta'(r-r_0)+s^{(\mathrm{e})}_2\delta''(r-r_0)$$

$$S^{(\mathrm{odd})}_{lm\omega}(r)=s^{(\mathrm{o})}_0\delta(r-r_0)+s^{(\mathrm{o})}_1\delta'(r-r_0)$$

$$V_{(\mathrm{RW})}(r)=\left(1-\frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2}-\frac{6M}{r^3}\right)$$

Fundamental equation to be solved with the **Green function** method

A. A single equation to be solved perturbatively

The single equation to be solved perturbatively is then

$$\mathcal{L}^{(r)}_{(\mathrm{RW})}[R^{(\mathrm{even/odd})}_{lm\omega}] = S^{(\mathrm{even/odd})}_{lm\omega}(r)\,.$$

Let us define a quasi-Green function for the RW equation as

$$G(r,r') = \frac{1}{W}\left[R_{(\mathrm{in})}(r)R_{(\mathrm{up})}(r')H(r'-r) + R_{(\mathrm{in})}(r')R_{(\mathrm{up})}(r)H(r-r')\right]$$
$$\equiv G_{(\mathrm{in})}(r,r')H(r'-r) + G_{(\mathrm{up})}(r,r')H(r-r')\,,$$

where

$$W = \left(1-\frac{2M}{r}\right)\left[R_{(\mathrm{in})}(r)\frac{d}{dr}R_{(\mathrm{up})}(r) - \frac{d}{dr}R_{(\mathrm{in})}(r)R_{(\mathrm{up})}(r)\right] = \mathrm{constant}\,.$$

It turns out that

$$\boxed{\mathcal{L}^{(r)}_{(\mathrm{RW})}G(r,r') = f(r')\delta(r-r')\,.}$$

$$f(r) = 1-\frac{2M}{r}$$

$$\mathcal{L}^{(r)}_{(\mathrm{RW})} = \left(1-\frac{2M}{r}\right)^2\frac{d^2}{dr^2} + \frac{2M}{r^2}\left(1-\frac{2M}{r}\right)\frac{d}{dr} + \left[\omega^2 - \left(1-\frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right)\right]$$

R in and R up are two independent solutions of the homogeneous RW equation which are purely ingoing at the bh horizon and purely outgoing at spatial infinity.

The in and up solutions of the homogeneous RWZ equation: I PN solutions

It is enough to compute the PN in solution: a lucky circumstance!

Restore the gravitational constant and the speed of light in the units

$$M \to GM/c^2 \sim M \to M\eta^2 \qquad \omega \to \omega\eta$$

Expand both the RWZ equation and its solution in powers of

$$\eta = \frac{1}{c}$$

$$X^{\mathrm{in(PN)}}_{l\omega}(r) = r^{l+1}[1 + \eta^2 A_2^{(\mathrm{PN},l)} + \eta^4 A_4^{(\mathrm{PN},l)} + \eta^6 A_6^{(\mathrm{PN},l)} + \cdots + \eta^{12} A_{12}^{(\mathrm{PN},l)} + \cdots]$$

$$X^{\mathrm{up(PN)}}_{l\omega}(r) = r^{-l}[1 + \eta^2 A_2^{(\mathrm{PN},-l-1)} + \eta^4 A_4^{(\mathrm{PN},-l-1)} + \eta^6 A_6^{(\mathrm{PN},-l-1)} + \cdots + \eta^{12} A_{12}^{(\mathrm{PN},-l-1)} + \cdots]$$

One does not need to compute the up solution since this is obtained from the in solution by replacing l → -l-1

$$X_1 = GM/r,\ X_2 = (\omega r)^2$$

$$A_0^{\mathrm{in}\,(PN,l)} = 1$$

$$A_2^{\mathrm{in}\,(PN,l)} = -\frac{(l-2)(l+2)}{l}X_1 - \frac{1}{2(2l+3)}X_2$$

$$A_4^{\mathrm{in}\,(PN,l)} = \frac{(l-2)(l-3)(l+2)(l+1)}{(-1+2l)l}X_1^2 + \frac{(l^3-5l^2-14l-12)}{2(l+1)(2l+3)l}X_2X_1 + \frac{1}{8(2l+5)(2l+}$$

$$A_6^{\mathrm{in}\,(PN,l)} = -\frac{(l-2)(l-3)(l-4)(l+2)(l+1)}{3(-1+2l)(l-1)}X_1^3 - \frac{2(15l^4+30l^3+28l^2+13l+24)}{(-1+2l)(2l+1)(l+1)l(2l+3)}X_1^2.$$
$$-\frac{(3l^4-27l^3-154l^2-220l-120)}{24(l+1)l(2l+5)(l+2)(2l+3)}X_1X_2^2 - \frac{}{48(2l+5)(2l+7)(2l+3)}X_2^3$$

$$A_8^{\mathrm{in}\,(PN,l)} = \frac{(l-2)(l-3)(l-4)(-5+l)(l+2)(l+1)}{6(-1+2l)(2l-3)}X_1^4$$
$$-\frac{\bar{B}_8^{\mathrm{ln}}\ln(r/R)+\bar{B}_8}{6(-1+2l)(2l+1)l^3(2l+3)(l+1)(l-1)}X_2X_1^3$$
$$-\frac{\bar{C}_8^{\mathrm{ln}}\ln(r/R)+\bar{C}_8}{24l(-1+2l)(2l+1)(2l+3)^3(l+1)(2l+5)(l+2)}X_2^2X_1^2$$
$$+\frac{(5l^5-60l^4-645l^3-1788l^2-1928l-840)}{240l(l+3)(2l+3)(l+1)(2l+5)(2l+7)(l+2)}X_1X_2^3$$
$$+\frac{1}{384(2l+9)(2l+3)(2l+5)(2l+7)}X_2^4$$

The magnetic number m is inside X_2

In the up solution this term would have l-2 in the denominator.
Sum over l of cannot include this term: divergencies!

Sum over m, sum over l

$$\sum_{m=-l}^{l} |Y_{lm}(\pi/2,0)|^2 = \frac{2l+1}{4\pi}$$

$$\sum_{m=-l}^{l} m^2 |Y_{lm}(\pi/2,0)|^2 = \frac{2l+1}{8\pi} l(l+1)$$

$$\sum_{m=-l}^{l} m^4 |Y_{lm}(\pi/2,0)|^2 = \frac{2l+1}{32\pi} l(l+1)(3l^2+3l-2)$$

$$\sum_{m=-l}^{l} m^6 |Y_{lm}(\pi/2,0)|^2 = \frac{2l+1}{64\pi} l(l+1)(5l^4+10l^3-5l^2-10l+8)$$

Apart from the l=0,1 (gauge modes) which are computed separately, **one cannot include** A_8 **in the PN solution since this would diverge for l=2.** If one need high-accuracy PN solutions then the contributions due to l=2 should be computed differently!

→ **MST technology**

The in and up solutions of the homogeneous RWZ equation: II MST solutions

$$X^{(\mathrm{in})}_{l\omega}(r) = C^{\nu}_{(\mathrm{in})}(x) \sum_{n=-\infty}^{+\infty} a^{\nu}_{n} \times \bar{F}(n+\nu-1-i\epsilon, -n-\nu-2-i\epsilon, 1-2i\epsilon; x]$$

$$X^{(\mathrm{up})}_{l\omega}(r) = C^{\nu}_{(\mathrm{up})}(z) \sum_{n=-\infty}^{+\infty} a^{\nu}_{n}(-2iz)^{n} \times \bar{\Psi}(n+\nu+1-i\epsilon, 2n+2\nu+2; -2iz)$$

Details in the talk of C. Kavanagh

Here, $x = 1 - c^2 r/2GM$, $z = \omega r/c$, $\epsilon = 2GM\omega/c^3 = 2mGM\Omega/c^3$,

$$\nu = l + \frac{1}{2l+1}\left[-2 - \frac{s^2}{l(l+1)} + \frac{[(l+1)^2 - s^2]^2}{(2l+1)(2l+2)(2l+3)} - \frac{(l^2 - s^2)^2}{(2l-1)2l(2l+1)}\right]\epsilon^2 + O(\epsilon^4)$$

is an ϵ-modified avatar of l

Details in the talk of C. Kavanagh

$$C^{\nu}_{(\mathrm{in})}(x) = c_{(\mathrm{in})} e^{i\epsilon[(x-1)-\ln(-x)]}(1-x)^{-1},$$

$$C^{\nu}_{(\mathrm{up})}(z) = c_{(\mathrm{up})} e^{iz} z^{\nu+1}\left(1 - \frac{\epsilon}{z}\right)^{-i\epsilon} 2^{\nu} e^{-\pi\epsilon} e^{-i\pi(\nu+1)},$$

$$\bar{F}(a,b,c;x) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(c)} \quad F(a,b,c;x),$$

$$\bar{\Psi}(a,b;\zeta) = \frac{\Gamma(a-2)\Gamma(a)}{\Gamma(a^*)\Gamma(a^*+2)}\Psi(a,b;\zeta)$$

[second kummer function, re-expressable in terms of hypergeom]

The in and up solutions of the homogeneous RWZ equation: III HeunC functions

$$\left(1-\frac{2M}{r}\right)^2 R''_{lm\omega} + \frac{2M}{r^2}\left(1-\frac{2M}{r}\right) R'_{lm\omega} + \left[\omega^2 - \left(1-\frac{2M}{r}\right)\left(\frac{l(l+1)}{r^2} - \frac{6M}{r^3}\right)\right] R_{lm\omega} = 0$$

$$\begin{aligned} R_{lm\omega} &= r^3 e^{i\omega r}\Big\{C_1 (r-2M)^{2i\omega M}\mathrm{HeunC}[-4iM\omega, 4i\omega M, 4, -8\omega^2M^2, 4-l-l^2+8\omega^2M^2, 1-\frac{r}{2M}] \\ &+ C_2(r-2M)^{-2i\omega M}\mathrm{HeunC}[-4iM\omega, -4iM\omega, 4, -8\omega^2M^2, 4-l-l^2+8\omega^2M^2, 1-\frac{r}{2M}]\Big\} \end{aligned}$$

One need to transform from the "large variable" (at infinity)

$$x = 1 - \frac{r}{2M}$$

to the "small" variable

$$y = \frac{2M}{r} = \frac{1}{1-x}$$

This variable is not PN small and cannot be used to expand the HeunC functions in power series

To overtake this difficulty one expands the HeunC functions of the large variable x in an infinite series of hypergeometric functions (in the same variable x)

$$X^{\nu}_{(\mathrm{in})}(x) = C^{\nu}_{(\mathrm{in})} \sum_{n=-\infty}^{\infty} a^{\nu}_{n} \phi^{(\mathrm{in})}_{n+\nu}(x)$$

$$\epsilon = 2M\omega\eta^3 \equiv \epsilon_0 \eta^3$$

$$\phi^{(\mathrm{in})}_{n+\nu} = \frac{\Gamma(a)\Gamma(b)}{\Gamma(c)} \mathrm{hypergeom}([a,b],[c];x)$$

$$a = n+\nu-1-i\epsilon\,, \qquad b = -n-\nu-2-i\epsilon\,, \qquad c = 1-2i\epsilon$$

$$C^{\nu}_{(\mathrm{in})} = \frac{e^{i\epsilon[(x-1)-\ln(-x)]}}{1-x}\,.$$

Details in the talk of C. Kavanagh

ν is determined by the solution of the recurrence relation

The coefficients of this expansion satisfy a three-term recurrence relation (the same in both the in and up cases) to be solved for each fixed value of l)

$$\alpha_n^\nu a_{n+1} + \beta_n^\nu a_n + \gamma_n^\nu a_{n-1} = 0 \,,$$

$$\begin{aligned}
\alpha_n^\nu &= \frac{-i\epsilon(n+\nu-1+i\epsilon)(n+\nu-1-i\epsilon)(n+\nu+1-i\epsilon)}{(n+\nu+1)(2n+2\nu+3)} \\
\beta_n^\nu &= -l(l+1) + (n+\nu)(n+\nu+1) + 2\epsilon^2 + \frac{\epsilon^2(4+\epsilon^2)}{(n+\nu)(n+\nu+1)} \\
\gamma_n^\nu &= \frac{i\epsilon(n+\nu+2+i\epsilon)(n+\nu+2-i\epsilon)(n+\nu+i\epsilon)}{(n+\nu)(2n+2\nu-1)} \,.
\end{aligned}$$

Details in the talk of C. Kavanagh

$$\begin{aligned}
\nu \;=\; & 2 - \frac{107}{210}\epsilon^2 - \frac{1695233}{9261000}\epsilon^4 - \frac{76720109901233}{480698687700000}\epsilon^6 \\
& - \frac{71638806585865707261481}{389235629236738284000000}\epsilon^8 - \frac{27036066493983382155489949365364 3}{11256262345978018393784760000000000}\epsilon^{10}
\end{aligned}$$

Use the properties of the hypergeometric functions in the map x= 1-1/y

$$\text{hypergeom}([a,b],[c];x) = y^a \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)} \text{hypergeom}([a,c-b],[a-b+1];y)$$
$$+y^b \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)} \text{hypergeom}([b,c-a],[b-a+1];y)$$

and then truncate the expansion at the desired accuracy level!

NO similar property available directly for Heun functions?

Analogous treatment for the up solutions.

One need to transform from the "large variable" (at infinity)

$$x = 1 - \frac{r}{2M}$$

to the "small" variable

$$y = \frac{2M}{r} = \frac{1}{1-x}$$

Low multipoles for h_kk in Schw: l=0,1

Let us introduce the notation

$$\langle \mathcal{F}(r) \rangle = \mathcal{F}(r_0) H(r_0 - r) + \mathcal{F}(r) H(r - r_0)$$

- **Metric perturbation** $l = 0$

$$h_{tt}^{l=0} = 2\delta M f(r) \left[\frac{1}{r_0 f(r_0)} H(r_0 - r) + \frac{1}{r f(r)} H(r - r_0) \right] = 2\delta M f(r) \langle \frac{1}{r f(r)} \rangle$$

$$h_{rr}^{l=0} = \frac{2\delta M}{r [f(r)]^2} H(r - r_0)$$

- **Metric perturbation** $l = 1$ **(odd)**

$$h_{t\phi}^{l=1^-} = -2\delta J r^2 \sin^2\theta \left[\frac{1}{r_0^3} H(r_0 - r) + \frac{1}{r^3} H(r - r_0) \right] = -2\delta J r^2 \sin^2\theta \langle \frac{1}{r^3} \rangle$$

- **Metric perturbation $l = 1$ (even)**

$$
\begin{aligned}
h_{tt}^{l=1^+} &= -2\frac{\delta M}{r}\left(1-\frac{r^3}{r_0^3}\right)\frac{r_0 f(r_0)}{r f(r)}\sin\theta\cos\bar{\phi}H(r_0-r) \\
h_{tr}^{l=1^+} &= 6\Omega\frac{\delta M}{f(r)}\frac{r_0 f(r_0)}{r f(r)}\sin\theta\sin\bar{\phi}H(r_0-r) \\
h_{rr}^{l=1^+} &= -6\frac{\delta M}{r[f(r)]^2}\frac{r_0 f(r_0)}{r[f(r)]}\sin\theta\cos\bar{\phi}H(r_0-r)\,.
\end{aligned}
$$

where we recall

$$
\delta M = \frac{f(r_0)}{\sqrt{1-3\frac{m_2}{r_0}}}\,, \qquad \delta J = \frac{1}{\sqrt{\frac{m_2}{r_0}\left(1-3\frac{m_2}{r_0}\right)}}\,, \qquad f(r) = 1-2\frac{m_2}{r}\,.
$$

These are in general very delicate… the corresponding expressions in Kerr in this form do not exist yet! (to my knowledge)

[last update by Leor Barack at this conference]

The GI quantity we are interested in

$$h_{kk}(t,r,\theta,\phi) = h_{tt}^{(\mathrm{RWZ})} + 2\Omega h_{t\phi}^{(\mathrm{RWZ})} + \Omega^2 h_{\phi\phi}^{(\mathrm{RWZ})}$$
$$= \sum_{l,m} e^{-i\omega_m t} h_{kk\,lm\omega}^{(\mathrm{RWZ})}$$

$$h_{kk}^{(l,m)} = h_{kk,lm}^{(\mathrm{even})} + h_{kk,lm}^{(\mathrm{odd})}.$$

$$h_{kk,lm}^{(\mathrm{odd})} = -|\partial_\theta Y_{lm}(\pi/2,0)|^2 \frac{8\pi\Gamma}{r_0^3 W\Lambda} M f_0^2$$
$$\times \left[r_0 \frac{dX_{l\omega}^{(\mathrm{in})}}{dr_0} + X_{l\omega}^{(\mathrm{in})} \right] \left[r_0 \frac{dX_{l\omega}^{(\mathrm{up})}}{dr_0} + X_{l\omega}^{(\mathrm{up})} \right]$$

Final result

$$h^R_{kk} = -2u + 5u^2 + \frac{5}{4}u^3 + \left(-\frac{1261}{24} + \frac{41}{16}\pi^2\right)u^4 + \left(\frac{157859}{960} - \frac{256}{5}\gamma - \frac{128}{5}\ln(u) - \frac{512}{5}\ln(2) - \frac{2275}{256}\pi^2\right)u^5$$
$$+ \left(\frac{284664301}{201600} + \frac{28016}{105}\gamma + \frac{14008}{105}\ln(u) + \frac{63472}{105}\ln(2) - \frac{246367}{1536}\pi^2 - \frac{486}{7}\ln(3)\right)u^6$$
$$- \frac{27392}{525}\pi u^{13/2} + \left(-\frac{413480}{567}\ln(2) + \frac{5044}{405}\ln(u) + \frac{10088}{405}\gamma + \frac{22848244687}{7257600} + \frac{4617}{7}\ln(3)\right.$$
$$\left. + \frac{2800873}{131072}\pi^4 - \frac{608698367}{884736}\pi^2\right)u^7 + o(u^7).$$

From our papers: 9.5 PN
Kavanagh et al: 22.5 PN

Physical relevance of the present result: computation of the EOB potential a(u) at high PN order

$$a(u) = -\frac{1}{2} h_{kk}^{\rm subtracted} - \frac{u(1-4u)}{\sqrt{1-3u}} \, .$$

We find

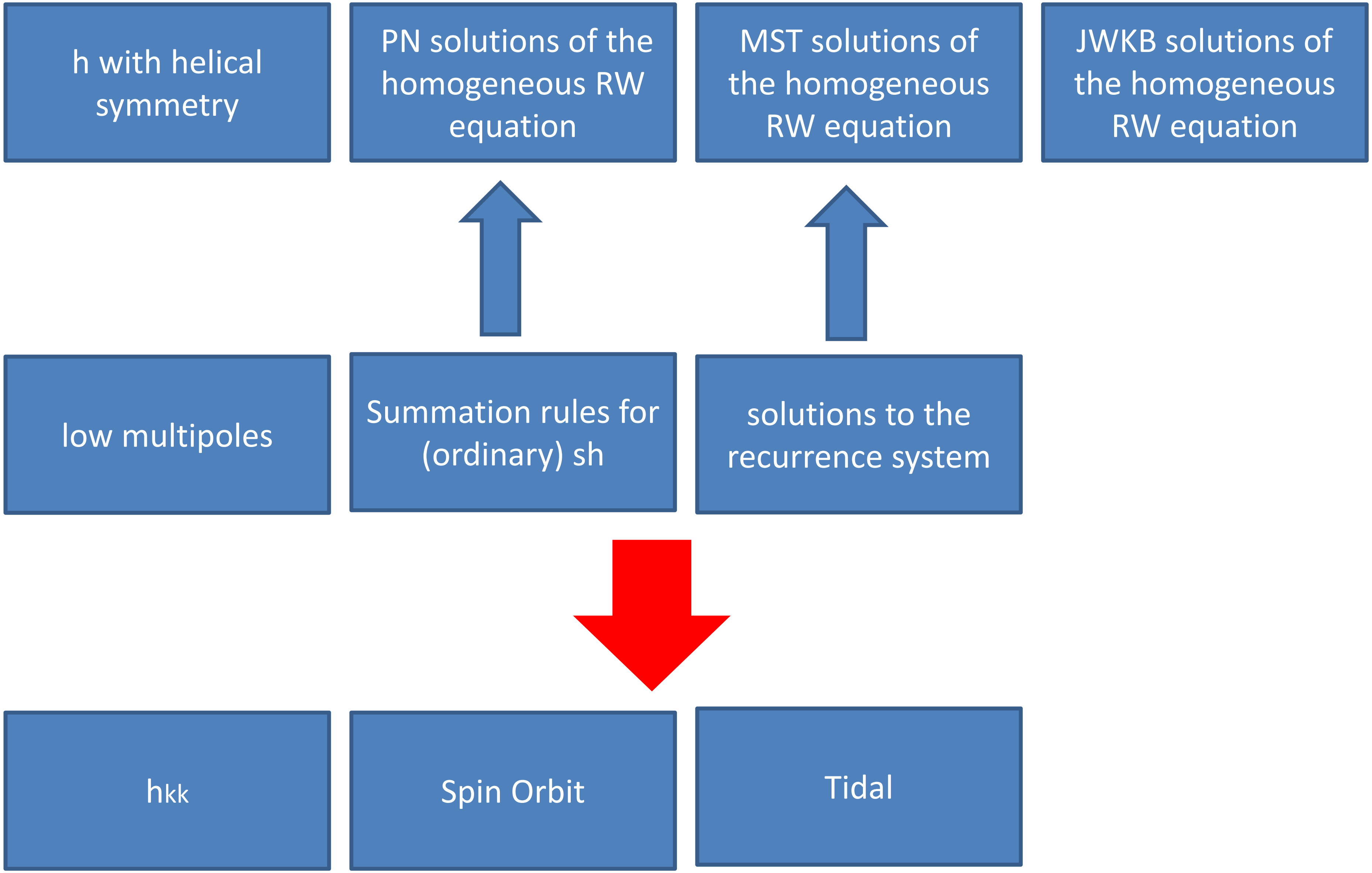
$$a(u) = 2u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right) u^4 + \left(-\frac{4237}{60} + \frac{128}{5}\gamma + \frac{64}{5}\ln(u) + \frac{256}{5}\ln(2) + \frac{2275}{512}\pi^2\right) u^5$$

$$+ \left(-\frac{1066621}{1575} - \frac{14008}{105}\gamma - \frac{7004}{105}\ln(u) - \frac{31736}{105}\ln(2) + \frac{246367}{3072}\pi^2 + \frac{243}{7}\ln(3)\right) u^6$$

$$+ \frac{13696}{525}\pi u^{13/2}$$

$$+ \left(\frac{206740}{567}\ln(2) - \frac{2522}{405}\ln(u) - \frac{5044}{405}\gamma - \frac{1360201207}{907200} - \frac{4617}{14}\ln(3) - \frac{2800873}{262144}\pi^4 + \frac{608698367}{1769472}\pi^2\right) u^7 \, .$$

Schwarzschild, RW gauge: Maple codes



2016 status of the art of the main GSF results in Schwarzschild

Pioneering/inspiring papers: a selection

Detweiler, Barack, Sago, Damour, Blanchet, Whiting, Le Tiec, Barausse, Buonanno, Akcay…

[1] S. L. Detweiler, "A Consequence of the gravitational self-force for circular orbits of the Schwarzschild geometry," Phys. Rev. D **77**, 124026 (2008) [arXiv:0804.3529 [gr-qc]].

[2] L. Barack and N. Sago, "Gravitational self-force correction to the innermost stable circular orbit of a Schwarzschild black hole," Phys. Rev. Lett. **102**, 191101 (2009) [arXiv:0902.0573 [gr-qc]].

[3] L. Blanchet, S. L. Detweiler, A. Le Tiec and B. F. Whiting, "Post-Newtonian and Numerical Calculations of the Gravitational Self-Force for Circular Orbits in the Schwarzschild Geometry," Phys. Rev. D **81**, 064004 (2010) [arXiv:0910.0207 [gr-qc]].

[4] T. Damour, "Gravitational Self Force in a Schwarzschild Background and the Effective One Body Formalism," Phys. Rev. D **81**, 024017 (2010) [arXiv:0910.5533 [gr-qc]].

[5] L. Barack and N. Sago, "Beyond the geodesic approximation: conservative effects of the gravitational self-force in eccentric orbits around a Schwarzschild black hole," Phys. Rev. D **83**, 084023 (2011) [arXiv:1101.3331 [gr-qc]].

[6] L. Blanchet, S. L. Detweiler, A. Le Tiec and B. F. Whiting, "High-Order Post-Newtonian Fit of the Gravitational Self-Force for Circular Orbits in the Schwarzschild Geometry," Phys. Rev. D **81**, 084033 (2010) [arXiv:1002.0726 [gr-qc]].

[7] L. Barack, T. Damour and N. Sago, "Precession effect of the gravitational self-force in a Schwarzschild spacetime and the effective one-body formalism," Phys. Rev. D **82**, 084036 (2010) [arXiv:1008.0935 [gr-qc]].

[8] A. Le Tiec, L. Blanchet and B. F. Whiting, "The First Law of Binary Black Hole Mechanics in General Relativity and Post-Newtonian Theory," Phys. Rev. D **85**, 064039 (2012) [arXiv:1111.5378 [gr-qc]].

[9] A. Le Tiec, E. Barausse and A. Buonanno, "Gravitational Self-Force Correction to the Binding Energy of Compact Binary Systems," Phys. Rev. Lett. **108**, 131103 (2012) [arXiv:1111.5609 [gr-qc]].

[10] E. Barausse, A. Buonanno and A. Le Tiec, "The complete non-spinning effective-one-body metric at linear order in the mass ratio," Phys. Rev. D **85**, 064010 (2012) [arXiv:1111.5610 [gr-qc]].

[11] S. Akcay, L. Barack, T. Damour and N. Sago, "Gravitational self-force and the effective-one-body formalism between the innermost stable circular orbit and the light ring," Phys. Rev. D **86**, 104041 (2012) [arXiv:1209.0964 [gr-qc]].

Most of people here

Circular orbits (including spin precession and tidals)

Most of people here

[1] D. Bini and T. Damour, "Gravitational radiation reaction along general orbits in the effective one-body formalism," Phys. Rev. D **86**, 124012 (2012) [arXiv:1210.2834 [gr-qc]].

[2] D. Bini and T. Damour, "Analytical determination of the two-body gravitational interaction potential at the fourth post-Newtonian approximation," Phys. Rev. D **87**, no. 12, 121501 (2013) [arXiv:1305.4884 [gr-qc]].

[3] D. Bini and T. Damour, "High-order post-Newtonian contributions to the two-body gravitational interaction potential from analytical gravitational self-force calculations," Phys. Rev. D **89**, no. 6, 064063 (2014) [arXiv:1312.2503 [gr-qc]].

[4] D. Bini and T. Damour, "Analytic determination of the eight-and-a-half post-Newtonian self-force contributions to the two-body gravitational interaction potential," Phys. Rev. D **89**, no. 10, 104047 (2014) [arXiv:1403.2366 [gr-qc]].

[5] D. Bini and T. Damour, "Two-body gravitational spin-orbit interaction at linear order in the mass ratio," Phys. Rev. D **90**, no. 2, 024039 (2014) [arXiv:1404.2747 [gr-qc]].

[6] D. Bini and T. Damour, "Gravitational self-force corrections to two-body tidal interactions and the effective one-body formalism," Phys. Rev. D **90**, no. 12, 124037 (2014) [arXiv:1409.6933 [gr-qc]].

[7] D. Bini and T. Damour, "Detweiler's gauge-invariant redshift variable: Analytic determination of the nine and nine-and-a-half post-Newtonian self-force contributions," Phys. Rev. D **91**, 064050 (2015) [arXiv:1502.02450 [gr-qc]].

[8] D. Bini and T. Damour, "Analytic determination of high-order post-Newtonian self-force contributions to gravitational spin precession," Phys. Rev. D **91**, no. 6, 064064 (2015) [arXiv:1503.01272 [gr-qc]].

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[10] P. Nolan, C. Kavanagh, S. R. Dolan, A. C. Ottewill, N. Warburton and B. Wardell, "Octupolar invariants for compact binaries on quasicircular orbits," Phys. Rev. D **92**, no. 12, 123008 (2015) [arXiv:1505.04447 [gr-qc]].

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Eccentric orbits (no spin, no tidals)

Most of people here

[1] D. Bini, T. Damour and A. Geralico, "Confirming and improving post-Newtonian and effective-one-body results from self-force computations along eccentric orbits around a Schwarzschild black hole," Phys. Rev. D **93**, no. 6, 064023 (2016) [arXiv:1511.04533 [gr-qc]].

[2] D. Bini, T. Damour and a. Geralico, "New gravitational self-force analytical results for eccentric orbits around a Schwarzschild black hole," Phys. Rev. D **93**, no. 10, 104017 (2016) [arXiv:1601.02988 [gr-qc]].

[3] S. Hopper, C. Kavanagh and A. C. Ottewill, "Analytic self-force calculations in the post-Newtonian regime: eccentric orbits on a Schwarzschild background," Phys. Rev. D **93**, no. 4, 044010 (2016) [arXiv:1512.01556 [gr-qc]].

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PN accuracy level

In **h_kk** in Schwarzschild we have performed our computation up to The 9.5 PN accuracy level, using the MST solution up to l=8 included. We have converted these results into the main EOB radial potential, a(u).

We have not identified theoretical difficulties or subtleties in going beyond this level. The latter seem purely technical and related to the memory resources of the PCs or to the used software. **(?)**

Kavanagh et al. were able to go much beyond our results, i.e., up to 22.5 PN.

Maple could not even convert (on my PC…) Kavanagh's very long, mathematica expressions into its own format. At a certain order I had to work with PN term-by-term conversion. Better to know this!

Kerr, Teukolsky equation: Maple codes

h with helical symmetry

PN

MST

JWKB

low multipoles
(interior metric not fully available yet!)

Summation rules for (ordinary) spheroidal→sh

solutions to the recurrence system

hkk

Spin Orbit

tidal

Curvature perturbations in a Kerr spacetime

Teukolsky formalism to metric reconstruction in Kerr replaces RWZ formalism in Schwarzschild

A.G.Shah, J.L.Friedman and T.S.Keidl,

EMRI corrections to the angular velocity and redshift factor of a mass in circular orbit about a Kerr black hole,

Phys. Rev. D 86, 084059 (2012)

2016 status of the art of the main GSF results in Kerr

Pioneering/inspiring papers: Shah, Friedman, Keidl, Blanchet, Buonanno, Le Tiec

[1] A. G. Shah, J. L. Friedman and T. S. Keidl, "EMRI corrections to the angular velocity and redshift factor of a mass in circular orbit about a Kerr black hole," Phys. Rev. D **86**, 084059 (2012) [arXiv:1207.5595 [gr-qc]].

[2] L. Blanchet, A. Buonanno and A. Le Tiec, "First law of mechanics for black hole binaries with spins," Phys. Rev. D **87**, 024030 (2013) [arXiv:1211.1060 [gr-qc]].

Circular orbits

Most of people here

[1] D. Bini, T. Damour and A. Geralico, "Spin-dependent two-body interactions from gravitational self-force computations," Phys. Rev. D **92**, no. 12, 124058 (2015) Erratum: [Phys. Rev. D **93**, no. 10, 109902 (2016)] [arXiv:1510.06230 [gr-qc]].

[2] A. G. Shah, Self-force meets post-Newtonian theory and more..., talk presented at 18th Capra Meeting on Radiation Reaction in General Relativity, Kyoto University, Kyoto, Japan, June 29 - July 2, 2015

[3] A. G. Shah, Overlap between black hole perturbation theory and post-Newtonian formalism, talk delivered at XIV Marcel Grossmann Meeting on General Relativity, University of Rome "La Sapienza," Rome, Italy, July 12-18, 2015

[4] C. Kavanagh, A. C. Ottewill and B. Wardell, "Analytical high-order post-Newtonian expansions for spinning extreme mass ratio binaries," Phys. Rev. D **93**, no. 12, 124038 (2016) [arXiv:1601.03394 [gr-qc]].

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Eccentric equatorial orbits

[1] D. Bini, T. Damour and A. Geralico, "High post-Newtonian order gravitational self-force analytical results for eccentric orbits around a Kerr black hole," Phys. Rev. D **9**, to appear (2016) arXiv:1602.08282 [gr-qc].

[2] M. van de Meent and A. G. Shah, "Metric perturbations produced by eccentric equatorial orbits around a Kerr black hole," Phys. Rev. D **92**, 064025 (2015) [arXiv:1506.04755 [gr-qc]].

Controlling Maple's results

- Danger in use of symbolic simplifications
- Term-by-term factorization of long expressions
- «Visual» check (for fixed values of parameters) of leading order terms for any series expansion
- Tricks and tools «a la TD»

[Most of the coefficients of the log terms in the various expressions computed by us were already obtained «*by hands*» by TD]

First successful achievements: from GSF to EOB

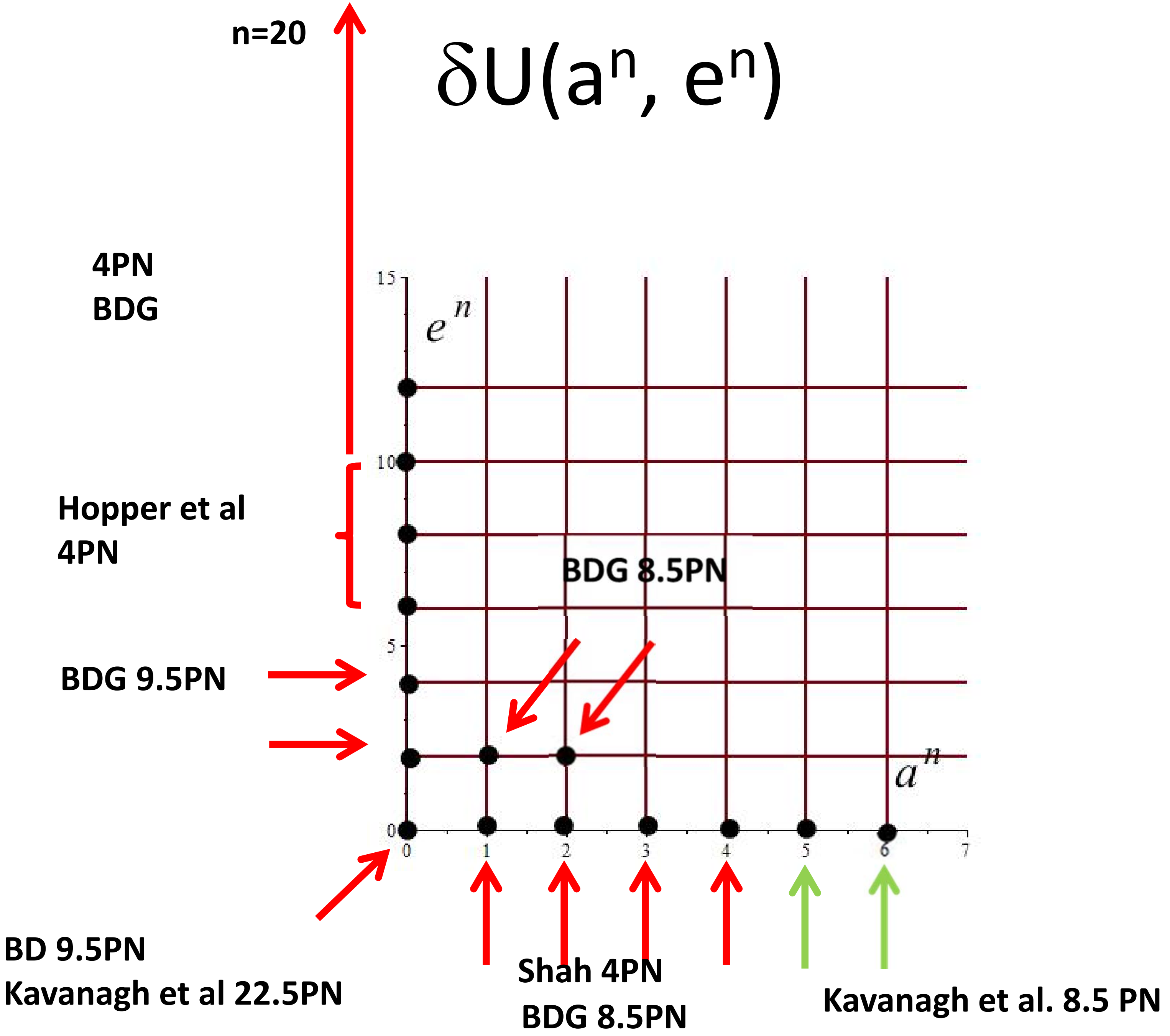
Our work had significant impact on the GSF and EOB description of compact binaries:

1. The first **explicit, analytic** computation of the 4PN contribution to the redshift function along circular orbits in Schwarzschild.
2. The (consequent) computation of the 4PN contribution main radial potential of EOB.

3. Our combination of PN and numerical-GSF computation of spin-orbit effects led to an improved estimate of the TIDAL contribution to the radial EOB potential of NS which was successfully compared to state of the art coalescing NS simulations, see:

S. Bernuzzi, A. Nagar, T. Dietrich, T. Damour
Modeling the Dynamics of Tidally Interacting Binary Neutron Stars up to the Merger
Phys.Rev.Lett. 114 (2015) 16, 161103

What we did up to now…



To do/in progress

- No complete results are available yet for the spin precession frequency, the tidal invariants along circular orbits in Kerr.
- No complete results are available yet for small inclined orbits.

Subtleties for the low multipoles in Kerr?

Apparently ONLY the exterior metric for equatorial geos in Kerr is well known.
Evaluating continuous functions can be done from «right» by using the exterior metric.
What about discontinuous fucntions? What about non-equatorial orbits? What about non-geo equatorial orbits?

Works in progress

(almost done)

1) Scalar self-force in Reissner-Nordstrom

2) Spin precession in Kerr

3) Tidal invariants in Kerr

but…in progress,
see below

4) Small inclined orbits in Kerr

5)…

Thanks for your kind attention!