# Aspects of Steven Detweiler's approach to second-order

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# Steve and Gauge

Steve (with emphasis): “Anything physical must be gauge invariant.”

- 2nd order paper makes no mention of gauge
- Detweiler Redshift variable
- Initially did S-R (Detweiler-Whiting) split without gauge (2000)

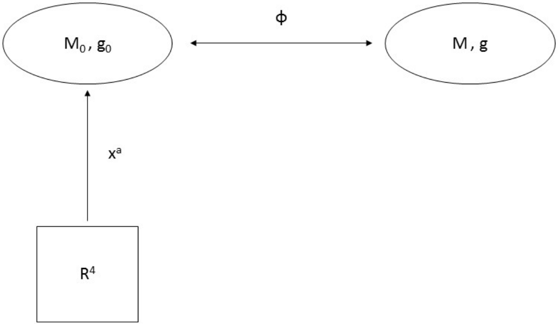
# Gauge

- Gauge Invariance
- Gauge Freedom
- Gauge Transformation
- Gauge
- Gauge in Perturbation Theory
- Gauge in General Relativity
- Gauge in QFT

# Gauge Confusion

- Gauge Invariance
- Gauge Freedom
- Gauge Transformation
- Gauge
- Gauge in Perturbation Theory
- Gauge in General Relativity
- Gauge in QFT

Description of a gauge transformation from Bardeen: "In discussing perturbations one is dealing with two spacetimes–the physical, perturbed spacetime and a fictitious background spacetime. A one-to-one correspondence between points in the background and points in the physical spacetime carries a set of coordinates from the background to the physical spacetime and defines a choice of "gauge." A change in the correspondence, keeping the background coordinates fixed is called a gauge transformation, to be distinguished from a coordinate transformation which changes the labeling of points in the background and physical spacetime together."



# Gauge Invariance

Background $M_0$ and Physical $M$

Gauge

- Correspondence (diffeomorphism) $\phi : M_0 \to M$

Perturbation

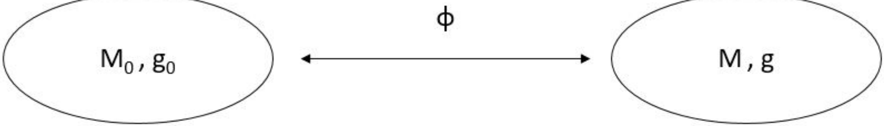
- Difference between pullback and background: $h = \phi^* g - g_0$

Gauge Transformation

- Switch from $\phi$ to $\psi$: $h_\phi - h_\psi = \pounds_\xi g_0$

Gauge Invariant

- $T$ on $M$ such that $\phi^* T$ is the same for all $\phi$ (Sachs 1963)
- But this is not quite right...




$T$ on $M$ such that $\phi^* T$ is the same for all $\phi$ (Sachs 1963)

- Too restrictive on T (only vanishing tensor fields, constant or zero scalar fields)
- $\phi^* T$ is a tensor field on *background* $M_0$
  - we care about value on $M$
  - we make measurements on $M$
- Consider instead $\phi_* T_0$

# Practical Understanding

- Gauge as arbitrary diffeomorphism
  - Very Abstract
  - Admittedly useless for calculations
- More familiar notions of gauge set conditions on $h_{ab}$
  - RW gauge - some components zero
  - Detweiler's Easy Gauge - some other components zero
- How to set a gauge
  - Start in some arbitrary gauge (no components zero)
  - Transform to another arbitrary gauge: $h_\phi - h_\psi = \pounds_\xi g$
  - Choose $\xi$ "carefully"

# A-K Notation

- Choose Schwarzschild coordinates

$$v_a = (-1, 0, 0, 0), \qquad n_a = (0, 1, 0, 0),$$

- Metric Perturbation Decomposition

$$h_{ab}^{\ell m} = \mathrm{A}\, v_a v_b Y^{\ell m} + 2\,\mathrm{B}\, v_{(a} Y_{b)}^{E,\ell m} + 2\,\mathrm{C}\, v_{(a} Y_{b)}^{B,\ell m} + 2\,\mathrm{D}\, v_{(a} Y_{b)}^{R,\ell m}$$
$$+ \mathrm{E}\, T_{ab}^{T0,\ell m} + \mathrm{F}\, T_{ab}^{E2,\ell m} + \mathrm{G}\, T_{ab}^{B2,\ell m}$$
$$+ 2\,\mathrm{H}\, T_{ab}^{E1,\ell m} + 2\,\mathrm{J}\, T_{ab}^{B1,\ell m} + \mathrm{K}\, T_{ab}^{L0,\ell m}.$$

- Can be translated to RW notation
  - New notation introduced “with trepidation”
- Gauge vector

$$\xi_a = \mathsf{P}\, v_a Y_{\ell m} + \mathsf{R}\, n_a Y_{\ell m} + \mathsf{S}\, Y_a^{E,\ell m} + \mathsf{Q}\, Y_a^{B,\ell m},$$

# Gauge Transformation

$$\Delta \mathrm{A} = -2\left(\frac{\partial}{\partial t}\mathrm{P}\right) - \left(\frac{2(r-2M)M}{r^3}\right)\mathrm{R}$$

$$\Delta \mathrm{B} = \frac{1}{r}\mathrm{P} - \frac{\partial}{\partial t}\mathrm{S}$$

$$\Delta \mathrm{D} = \left(\frac{\partial}{\partial r} - \frac{2M}{r(r-2M)}\right)\mathrm{P} - \frac{\partial}{\partial t}\mathrm{R}$$

$$\Delta \mathrm{E} = \frac{2(r-2M)}{r^2}\mathrm{R} - \frac{\ell(\ell+1)}{r}\mathrm{S}$$

$$\Delta \mathrm{F} = \frac{2}{r}\mathrm{S}$$

$$\Delta \mathrm{H} = \frac{1}{r}\mathrm{R} + \left(\frac{\partial}{\partial r} - \frac{1}{r}\right)\mathrm{S}$$

$$\Delta \mathrm{K} = \left(2\frac{\partial}{\partial r} + \frac{2M}{r(r-2M)}\right)\mathrm{R}$$

$$\Delta \mathrm{C} = -\frac{\partial}{\partial t}\mathrm{Q}$$

$$\Delta \mathrm{G} = \frac{2}{r}\mathrm{Q}$$

$$\Delta \mathrm{J} = \left(\frac{\partial}{\partial r} - \frac{1}{r}\right)\mathrm{Q}$$

# Gauge Invariants

Solve Algebraically

$$Q = \frac{1}{2} r \Delta G$$
$$\frac{dQ}{dr} = \Delta J + \frac{1}{2} \Delta G$$

$$\Delta J - \frac{r}{2} \left( \frac{\partial}{\partial r} \Delta\ G \right) = 0.$$

Thus

$$\alpha = J - \frac{r}{2} \left( \frac{\partial}{\partial r} G \right)$$

Steve's method gives 8 of these

# Easy Gauge

- Haven't chosen components of gauge vector
- Easy Gauge: choose gauge vector so that $B = E = F = G = 0$
- Reduces complexity of gauge invariants

$$
\begin{aligned}
o = {} & \frac{1}{2}\left(\frac{\partial}{\partial r}\mathrm{A}\right) + \frac{2M}{r-2M}\left(\frac{\partial}{\partial t}\mathrm{B}\right) + \frac{\partial}{\partial t}\mathrm{D} + \frac{r^2}{2(r-2M)}\left(\frac{\partial^2}{\partial t^2}\mathrm{E}\right) \\
& + \frac{r[4M + r\ell(\ell+1)]}{4(r-2M)}\left(\frac{\partial^2}{\partial t^2}\mathrm{F}\right) + \frac{M}{2r}\left(\frac{\partial}{\partial r} - \frac{1}{r}\right)\mathrm{E} + \frac{M\ell(\ell+1)}{4r}\left(\frac{\partial}{\partial r} - \frac{1}{r}\right)\mathrm{F}.
\end{aligned}
$$

- Can also write Einstein tensor in terms of gauge-invariants

# Second-Order

Steve: "When I write papers I like to keep myself grounded. I don't like to introduce a lot of formalism."

- Use point particle
- Expand Einstein Tensor
- Sub in Deweiler-Whiting decomposition
- Rearrange and cancel

# Point Particles in GR

- Geroch and Traschen - No point particles in GR
- Worst behaved metrics still can't produce point particle
- Example: Schwarzschild solution
  - "On the gravitational field of a mass point according to Einstein's theory" (Schwarzschild 1916)
  - Only mixed stress-energy is defined $T^a{}_b = -m\delta^3(x)\delta^a{}_0\delta^0{}_b$
  - Anything else has products of distributions
  - Colombeau algebra not fully developed (GR makes it worse)
- Shows up similarly in $G^{(2)}_{ab}(g^0, h^1)$ on worldline
- Detweiler gives special attention to the worldline