Gravitational self-force along marginally bound orbits in Schwarzschild spacetime

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Unbound orbits: why they are interesting

- Give access to new (pseudo) gauge-invariant quantities that can serve as strong-field benchmarks:
  - IBCO frequency shift
  - scattering angles for hyperbolic-like encounters
  - ...
- Relate ADM properties of the binary to SF quantities, already at first order, by exploiting the fact that the two bodies are infinitely separated
- Important for the overspinning problem, where “dangerous” orbits come from infinity
- High-energy scattering of black holes as a model for ultra-relativistic collisions of point-particles

Numerical framework and system considered

- We specialise to marginally bound orbits ($E = 1$) in *Schwarzschild*, where the linearised Einstein equations are fully separable in the time-domain
- We evolve the metric perturbation in Lorenz gauge, on a double null grid (Barack and Sago 2010)

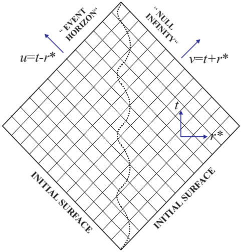


Figure 1: From Barack and Sago, Phys. Rev. D, vol. 81, p. 084021, Apr 2010

Lorenz gauge in time-domain

Lorenz gauge in time-domain

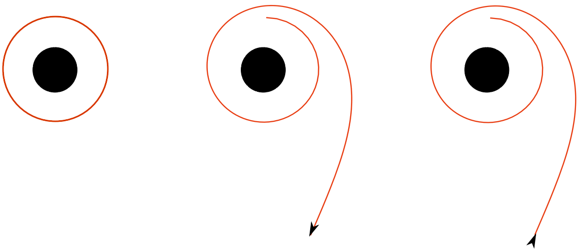
- Can handle any type of orbit
- Framework has been tested thoroughly
- Computationally expensive, slow even at moderate resolutions
- In Kerr must resort to 2+1 evolution: even more expensive!

Ways forward

- Within Lorenz: Parallelisation
- Teukolsky in time-domain?

Marginally bound orbits I

- Three different orbits sharing the same E, L in the geodesic approximation:
  1. IBCO (innermost bound circular orbit) at $r = 4M$
  2. outbound: starting from the IBCO and going out to infinity
  3. inbound: starting from infinity and asymptoting to the IBCO



Marginally bound orbits II

- Impose conditions at infinity and at the whirl $R = R_0 + \delta R$

$$\dot{r}(r \to \infty) = 0,$$
$$\dot{r}(R) = 0,$$
$$\ddot{r}(R) = 0,$$

- Relate energy/angular momentum at the whirl to the ones at infinity through integrals of the SF

$$\delta E(R_0) - \delta E(\infty) = -\int_{\infty}^{R_0} \frac{F_t}{\mu}\frac{dr}{\dot{r}} := \Delta E,$$
$$\delta L(R_0) - \delta L(\infty) = \int_{\infty}^{R_0} \frac{F_\phi}{\mu}\frac{dr}{\dot{r}} := \Delta L.$$

Closed system of equations

Constraint equations

By imposing the circularity conditions and requiring that the small mass is at rest at infinity one gets

$$\delta E(\infty) = 0,$$
$$\delta R = -8M\Delta E - 32M^2 \frac{F^r(R_0)}{\mu},$$
$$\delta L(\infty) = 8M\Delta E - \Delta L.$$

Conservative shift to the IBCO frequency

- The frequency of the IBCO (at fixed energy at infinity) in an asymptotically flat gauge is shifted by the conservative self force

$$\Omega_F^2 = \Omega_0^2 \left(1 - \eta + 6\Delta E + 16\frac{M}{\mu}F^r(4M)\right),$$

where the η term "flattens out" the Lorenz gauge monopole.

- Inbound and outbound orbits are time-reversed versions of each other → use both to compute the conservative self-force along one of the two:

$$F^t_{\text{cons}}(r) = \frac{\left(F^t_{ret,\text{in}}(r) - F^t_{ret,\text{out}}(r)\right)}{2}.$$

Evolution of low modes

In Lorenz gauge the modes $\ell = 0$ and $\ell = 1, m = 1$ do not evolve stably: linear-in-time gauge modes (homogeneous, regular solutions of the field equations) contaminate the data.

Possible strategies

1. Correct initial conditions
2. Generalised Lorenz gauge: $\nabla^\alpha \bar{h}_{\alpha\beta} = H_\beta$, with $H_\beta \to 0$ when $t \gg M$
3. Numerical filtering
4. ...

Evolution of low modes: our implementation

$\ell = 0$

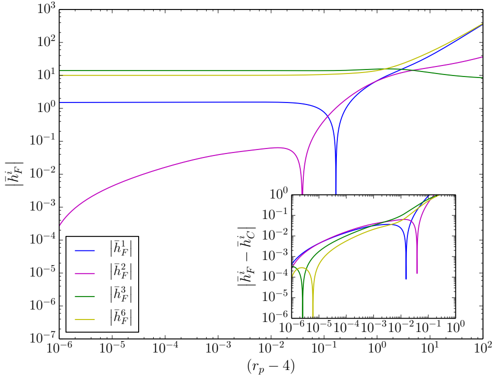
- outbound orbit: use the (analytical) circular solution to construct initial conditions $\rightarrow$ evolution is stable!
- inbound orbit: design a suitable gauge mode with the characteristics observed in the evolution (constant trace, linear-in-t...) and subtract it from the numerical data (Dolan and Barack, 2013)

$\ell = 1, m = 1$

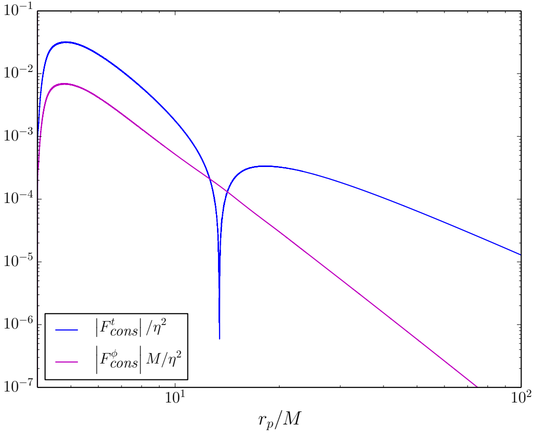
Numerical filtering for both orbits

Numerical filtering: discussion

- Easy to implement, the gauge modes can be subtracted in the post-processing phase
- Ad hoc procedure, needs to be tailored to the specific type of orbit that is being evolved
- Implies loss of accuracy!



How the SF looks like



Computation of ΔE

$$\Delta E = -\int_{-\infty}^{\infty} \frac{F_t}{\mu} d\tau$$

Formally an integral over in infinite domain, in practice

- Neglect the contribution from the region $4M \leq r \leq (4+\epsilon)M$ ($\epsilon \sim 10^{-6}$)
- Fit the data in the far field region $100M \leq r \leq 130M$ to a power-law model and integrate that analytically
- Numerically integrate over the remaining domain.

Current limitations

- Considerable noise coming from $\ell \gtrsim 10$ in the strong field region
- Evolution is expensive! We run over a rather limited domain ($\sim 130M$) and this implies the fit in the far-field region is not extremely accurate

SF and the first law

- The conservative IBCO shift at fixed energy can be computed from the shift in the binding energy at fixed

$$x := ((1+\eta)M\Omega)^{2/3}$$

and it reads

$$\delta\Omega(E) := \Omega - \Omega_0 = -\frac{1}{8M}\left(\eta + 3\delta E|_{x=1/4}\right),$$

- Using the first law of binary black hole mechanics $\delta E|_{x=1/4}$ can be computed from the redshift z and its first derivative
- Following Akcay *et al.* (2012) one can compute $z(x)$ for arbitrary values of x (relying on knowledge of h_{uu} along circular orbits)

Direct comparison of SF-calculation along unbound orbits and first-law!

Results

- Result obtained using the first law

$$\delta\Omega(E) = 0.0692008...\frac{\eta}{M},$$

- Result obtained using our time-domain code

$$\delta\Omega(E) = 0.069(2)\frac{\eta}{M}$$

Results are consistent but at the moment we have limited accuracy

Calibration of EOB: an example

- The shift in the IBCO frequency can be related to the derivative of the function $a(u = 1/r)$, which features in the EOB effective metric

$$ds^2_{eff} = -A(r;\nu)dt^2 + \bar{B}(r;\nu)dr^2 + r^2\left(d\theta^2 + \sin\theta^2\, d\phi^2\right),$$

where $\nu = \mu M/(\mu + M)^2$.

- For the innermost stable circular orbit, $x(u)$ can be related to A

$$x(u) = u\left(\frac{-\partial_u A(u)}{2}\right)^{1/3} \quad \text{Damour 2010}$$

- For EMRIs $\nu \sim \eta \ll 1$

$$A(u;\nu) = 1 - 2u + a(u)\nu + O(\nu^2)$$

and compare with SF.

Calibration of the a potential: results

Combine EOB and SF

$$\Omega_F^2 = \Omega_0^2 \left(1 + \nu \left(\partial_u a(1/4) - 2\right)\right)$$
$$\Omega_F^2 = \Omega_0^2 \left(1 - \eta + 6\Delta E + 16\frac{M}{\mu} F^r(4M)\right)$$

Our result

$$\partial_u a(1/4) = 2\left(1 + \frac{\delta\Omega}{\eta\Omega_0}\right) = 3.10(8).$$

Previous result

$$\partial_u a(1/4) = 3.107206... \quad \text{(Akcay et al.)}$$

Conclusions

- We presented a first computation of the IBCO shift via a full GSF calculation along unbound orbits
- The result is consistent with the one obtained by looking at circular orbits and applying the first law of binary black hole mechanics
- Our framework represents a totally independent tool to calibrate EOB and could be used to study hyperbolic-like orbits