Self-force: Foundations and formalism

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What is the self-force?

What is the (net) force that something exerts on itself?

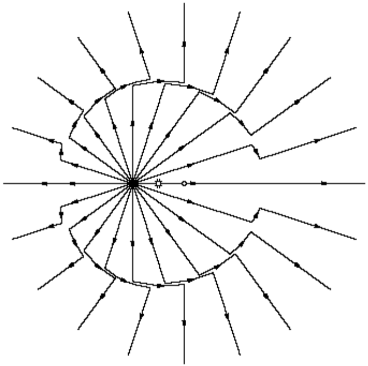
What is the (net) force that something can exert on itself?

A bit vague.

But you know it when you see it. . .

Radiation reaction

Objects coupled to long-range fields can radiate.



They must move in reaction to emitted energy, momentum, etc.

Radiation reaction II

Not only radiation reaction

Momentum carried by radiation implies a (self-) force:

- This can sometimes be used to calculate said force.
- But objects don't really "care" what's happening to fields far away from them: It's indirect.

Also, there can be nonradiative self-forces. . .

What is the self-force not?

Sometimes (misleadingly!) identified with

- Radiation reaction
- 2-body problem, esp. small mass ratios
- Black hole perturbation theory

These are special cases. . .

Self-force is just one aspect of the general problem of motion

But it's an interesting and often challenging aspect.

Look at the general problem of motion. . .

Approaches to motion problems

Consider a compact clump of matter interacting with long-range fields (charged solid in Maxwell EM, star in GR, . . .)

1. Either compute "everything" (numerics)
   - *Many inputs*: detailed matter model, initial and boundary conditions
   - *Complicated output*: detailed density, velocity, temperature fields
   - "Complete"
   - **Describes only very specific systems**

2. ...or focus only on a few "bulk" or "external" quantities (CM etc.)
   - *Simple input*
   - *Simple output*: center of mass, spin, ...
   - Not complete
   - **Can describe large classes of systems simultaneously**

Internal and external variables in celestial mechanics

Ordinary celestial mechanics makes "PDEs $\to$ ODEs:"

External (or bulk) variables

Center of mass positions
Linear momenta
Angular momenta

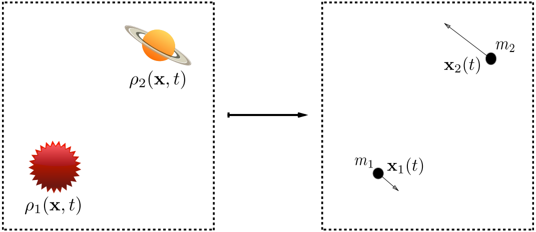
Internal variables

Density distributions
Internal velocities
Thermodynamic variables

Focus on the external variables.

Example: Newtonian N-body problem

N points in $\mathbb{R}^3$, described only by their positions and (constant) masses. Positions evolve via simple **ODEs, not PDEs**.



Tremendous (and useful) simplification over the full continuum mechanics.

Derivation is well-understood.

Generalizing celestial mechanics

Can this be repeated in electromagnetism, GR, ...?

A question

In what sense is it true that $D\dot{z}^a/ds = 0$ for freely-falling masses?

Clearly true in *some* limits.

But interesting regimes require being precise about

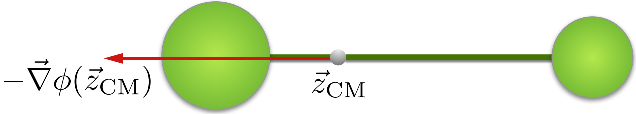
1. $z(s)$,
2. D/ds.

Nontrivial even in Newtonian gravity

Using $z(s) \to \vec{z}_{\mathrm{CM}}(t)$,

$$\frac{D\dot{z}^a}{ds} = 0 \quad \longrightarrow \quad \frac{d^2\vec{z}_{\mathrm{CM}}}{dt^2} = -\vec{\nabla}\phi(\vec{z}_{\mathrm{CM}}).$$

But this is false even for an isolated body:



Self-fields require $\phi(\vec{z}_{\mathrm{CM}})$ to be replaced by something else
[adding higher moments doesn't help]

Self-gravitating Newtonian masses can be described by replacing $\phi(\vec{z}_{\rm CM}) \to \phi_{\rm ext}(\vec{z}_{\rm CM})$ in the test body equation.

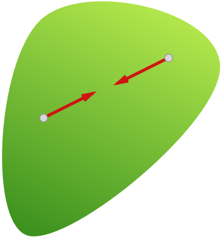
A body $\mathcal{B}$ moves in an "effective field" $\phi_{\rm ext}$ which is *nonlocally related* to the physical one:

$$\begin{aligned}
\phi_{\rm ext}[\phi; \mathcal{B}] &= \phi - \phi_{\rm self} \\
&= \phi - \left(- \int_{\mathcal{B}} \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3\vec{x}' \right) \\
&= \phi - \left(- \frac{1}{4\pi} \int_{\mathcal{B}} \frac{\nabla'^2 \phi}{|\vec{x} - \vec{x}'|} d^3\vec{x}' \right) \\
&= \frac{1}{4\pi} \oint_{\partial\mathcal{B}} \left[\vec{\nabla}'\phi' \left(\frac{1}{|\vec{x} - \vec{x}'|} \right) - \phi' \vec{\nabla}' \left(\frac{1}{|\vec{x} - \vec{x}'|} \right) \right] \cdot d\vec{S}'
\end{aligned}$$

Why? I. No self-force

This works because $\phi_{\rm self}$ exerts no net force (or torque):

$$\begin{aligned}
\vec{F}_{\rm self} &= -\int_{\mathcal{B}} d^3\vec{x}\rho\vec{\nabla}\phi_{\rm self} \\
&= -\int_{\mathcal{B}} d^3\vec{x}\rho\vec{\nabla}\int_{\mathcal{B}} d^3\vec{x}'\rho' G(\vec{x},\vec{x}') \\
&= -\frac{1}{2}\int_{\mathcal{B}} d^3\vec{x}\int_{\mathcal{B}} d^3\vec{x}'\rho\rho'(\vec{\nabla}+\vec{\nabla}')G(\vec{x},\vec{x}') \\
&= -\frac{1}{2}\int_{\mathcal{B}} d^3\vec{x}\int_{\mathcal{B}} d^3\vec{x}'\rho\rho'\mathcal{L}_{\vec{\partial}}G(\vec{x},\vec{x}') \to 0 \\
&= 0.
\end{aligned}$$



Everything cancels.

Why? II. ϕ_{ext} varies slowly

If bodies are well-separated,

$$
\begin{aligned}
\vec{F} &= \vec{F}_{\mathrm{ext}} + \cancelto{0}{\vec{F}_{\mathrm{self}}} \\
&= -\int_{\mathcal{B}} d^3\vec{x}\rho\vec{\nabla}\phi_{\mathrm{ext}} \\
&= -\int_{\mathcal{B}} d^3\vec{x}\rho\vec{\nabla}(\phi_{\mathrm{ext}}^{\mathrm{CM}} + \ldots) \\
&= -m\vec{\nabla}\phi_{\mathrm{ext}}^{\mathrm{CM}} + (\mathrm{quadrupole}) + \ldots
\end{aligned}
$$

Slow variation implies that

The point particle limit of $\vec{\nabla}\phi_{\mathrm{ext}}$ exists, even at $\vec{z}_{\mathrm{CM}}$.

The Newtonian $\phi \to \phi_{\mathrm{ext}}$ suggests that in GR, objects fall on geodesics which are *not* determined by ∇_a. Use some "effective external" connection $\nabla_a \to \hat{\nabla}_a$ instead:

$$\frac{\hat{D}\dot{z}^a}{ds} = 0 \quad \text{with} \quad \frac{\hat{D}}{ds} = \dot{z}^b \hat{\nabla}_b \neq \dot{z}^b \nabla_b$$

This *can* be vacuous:

- For *any* $z^\mu(s)$, there exist $\Gamma^\mu_{\nu\lambda}$ st $\ddot{z}^\mu + \Gamma^\mu_{\nu\lambda} \dot{z}^\mu \dot{z}^\nu = 0$.
- Infinitely many possible connections and infinitely many sources...

But it can be useful when coupled with a "nice," precisely-defined $\hat{\nabla}_a$.

An organizing principle

In many contexts, self-force results are usefully summarized by

Detweiler-Whiting scheme [Detweiler & Whiting (2002)]

1. Start with test-body equation of motion
2. Replace all potentials/metrics by $\phi \to \hat{\phi} := \phi - \phi_S$ for some *particular* ϕ_S

Direct analog of the Newtonian result, no reference to boundary conditions or initial conditions.

Very general!

Equivalent representations exist in special cases, but nothing else works so broadly and simply:

- Exact for Newtonian gravity & electrostatics [??]
- Point charges in SR [Dirac (1938)]
- Point particles coupled to scalar, EM, linearized gravity in curved backgrounds [Detweiler & Whiting (2002)]
- Small masses through 2nd order in GR [Pound (2009-)]
- Exact for general extended bodies in scalar, EM, GR [AIH (2008-)]
- Exact for spin DOFs for general extended bodies [AIH (2008-)]
- All dimensions (with some modification) [AIH, Taylor, Flanagan (2016)]

Examples

Self-force in GR:

$$\frac{D}{ds}\dot{z}^a = 0 \qquad \longrightarrow \qquad \frac{\hat{D}}{ds}\dot{z}^a = 0$$

Self-torque in GR:

$$\frac{D}{ds}S_a = 0 \qquad \longrightarrow \qquad \frac{\hat{D}}{ds}S_a = 0$$

Electric charge:

$$m\ddot{z}^a = qF^a{}_b\dot{z}^b \qquad \longrightarrow \qquad m\ddot{z}^a = q\hat{F}^a{}_b\dot{z}^b$$

Also works with all higher multipole moments...

Making this precise

None of this is useful without specifying the maps $\phi \mapsto \hat{\phi}$:

1. Always nonlocal: $\hat{\phi}(x)$ depends on ϕ away from x.
   $\hookrightarrow$ Use propagators $G_S(x, y, \ldots)$

2. Usually linear: $\hat{\phi} = \phi - \phi_S[\phi]$ with $\phi_S[\phi]$ linear.
   $\hookrightarrow$ 2-point propagators $G_S(x, y)$:

$$\phi_S(x) = \int G_S(x, y)\rho(y) dy$$

3. Usually nonvacuum $\rightarrow$ vacuum: $\Box\hat{\phi} = 0$ despite $\Box\phi \neq 0$.
   $\hookrightarrow$ Use *some* Green function

Why vacuum fields?

Sufficient to "imply" slow variations:

No singularities in the point particle limit

Singularity propagation theorems [Hörmander, . . .] for hyperbolic PDEs
$\Longrightarrow$ singularities move along along null geodesics.

No singularities in initial data mean no singularities anywhere.

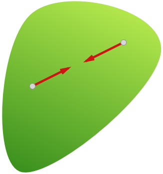
Other possibilities do exist. . .

Is that it?

No! Nonsingular behavior is not sufficient.
(actually meaningless for *individual* physical systems)

Also need something for which $F_a[\phi_S]$ is "ignorable"

Generalize the cancellations of Newton's 3rd law...



$F[\phi_S] \sim 0$

$\hookrightarrow$ More complicated than Newtonian: $F_S = F[\phi_S] \neq 0$,
but it's nevertheless ignorable.

Renormalization

One can define ϕ_S so that F_S may be absorbed into (finite!) redefinitions of mass, spin, ...

Renormalization: An example

Consider a small charged particle with retarded BCs in flat spacetime [Abraham-Lorentz-Dirac]:

$$m\ddot{z}_a = qF^{\text{ext}}_{ab}\dot{z}^b + \frac{2}{3}q^2 P_{ab}\dddot{z}^b - \delta m\ddot{z}_a$$

$$(m + \delta m)\ddot{z}_a = q(F^{\text{ext}}_{ab} + \frac{4}{3}q\dot{z}_{[a}\dddot{z}_{b]})\dot{z}^b$$

Define $\hat{m}$ and $\hat{F}_{ab}$ s.t.

$$\hat{m}\ddot{z}_a = q\hat{F}_{ab}\dot{z}^b$$

Final definition for ϕ_S

$G_S(x, y)$ defines field per charge at x due to y. Demand that

1. This is a Green function: Slow variation
2. $G_S(x, y) = G_S(y, x)$: Reciprocity
3. $G_S(x, y) = 0$ if x, y are timelike-separated: Locality

These imply that G_S is constructed quasilocally from g_{ab}.

Detweiler-Whiting Green function

$$G_S = U\delta(\sigma) + V\Theta(\sigma)$$

Final Detweiler-Whiting scheme

1. Compute physical field ϕ.
2. Use G_S to determine $\hat{\phi}$.
3. Plug $\hat{\phi}$ into test-body equations.

This isolates an appropriate "effective external field"

MiSaTaQuWa self-force

Using DW metric perturbation in linearized GR with retarded BCs,

$$\frac{\hat{D}\dot{z}^a}{ds} = 0$$

turns into [Detweiler & Whiting (2002)]

$$\frac{\bar{D}\dot{z}^a}{ds} = \frac{1}{2} P^{ab} (h^{\rm tail}_{bcd} - 2h^{\rm tail}_{cdb}) \dot{z}^c \dot{z}^d,$$

with

$$h^{\rm tail}_{cab} = 4m \lim_{\epsilon \to 0^+} \int_{-\infty}^{\tau-\epsilon} \bar{\nabla}_c G^{\rm ret}_{aba'b'} \dot{z}^{a'} \dot{z}^{b'} d\tau'.$$

Self-torque

Using DW metric perturbation in linearized GR with retarded BCs,

$$\frac{\hat{D} S_a}{ds} = 0$$

turns into [AIH (2012)]

$$\frac{\bar{D} S_a}{ds} = -2m \dot{z}^b \dot{z}^c \bar{R}_{abc}{}^d S_d + \frac{1}{2} \dot{z}^b S^c (h^{\rm tail}_{cab} - 2 h^{\rm tail}_{(ab)c}).$$

Different derivations:

Perturbative

GR: [Pound, Gralla, Wald,. . .]

Electromagnetism: [Many!]

- Black holes ok
- Closer to "practical" things
- Complicated calculations
- Difficult to modify

Nonperturbative

[AIH, Flanagan, Taylor]

- Exact
- General toolbox
- Physical intuition
- Easy calculations
- No black holes
- Still need to solve field eqns

Also various heuristic motivations. . .

Summary

Laws of motion (including self-interaction) can be summarized by subtracting appropriate S-fields from physical fields.

1. The effects of ϕ_S can all be absorbed into local redefinitions.
2. What remains is slowly varying—even in a point particle limit—and therefore has the same effect as the external field acting on a test body.

None of this depends on point particles or singularities. . .

Some comments

1. Self-force is one aspect of the problem of motion
2. It's more about what *doesn't matter* than what does.
3. Still haven't talked about solving field equations [Hard!]

Future directions

1. Computational tools, phenomenology, etc.
2. New interesting observables
3. Nonperturbative methods and nonlinearity
4. Self-interaction in other theories
   - For which types of theories do similar results hold?
   - Other physical systems (fluid mechanics, ...)