Self-forces and self-torques in arbitrary dimensions: Part II

Abraham Harte
(with Éanna Flanagan & Peter Taylor)

Max Planck Institute for Gravitational Physics
Albert Einstein Institute
Potsdam, Germany

June 27, 2016

19th Capra Meeting
Meudon

1. Why?!

2. Static to dynamical self-force

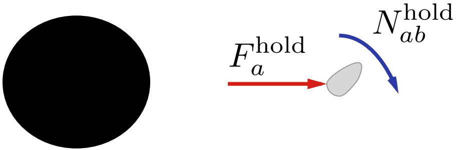
3. Approximations (must depend on dimension)

Why care about $d \neq 4$?

1. Deeper understanding
   - What's really important?
   - Good testing ground.
   - Forces one to focus on different things.
2. Lower dimensions are physical
   - There *are* real systems which are effectively $1+1$ or $2+1$ dimensional.
   - Self-force can be stronger!

Static self-force

Observables for static case: *Which force and torque* must be applied to maintain staticity?



Choosing a mass center satisfying $S_{ab}\tau^b = 0$,

$$F_a^\perp = (F_{\rm hold} + F_{\rm grav} + F_{\rm self})_a^\perp = \hat{m} D_a N,$$

$$N_{ab}^\perp = (N_{\rm hold} + N_{\rm grav} + N_{\rm self})_{ab}^\perp = 0.$$

Review

In terms of generalized forces, everything follows from

$$\mathcal{F}_{\mathrm{hold}}(\xi) = \frac{d\hat{P}_t}{dt} - \int_\Sigma \left(\frac{1}{2} \hat{T}^{ab} \mathcal{L}_\xi g_{ab} - J \mathcal{L}_\xi \hat{\Phi}_{\mathrm{self}} \right) N dV.$$

- Hatted quantities are defined using an auxiliary propagator $G(x, x')$, and there are many possibilities.
- Each possibility corresponds to a different way to compute $\mathcal{F}_{\mathrm{hold}}$.
- While individual terms are a matter of definition, their sum is not. *Choose whichever $G(x, x')$ gives the simplest computations!*

Static propagators

One useful class of possibilities satisfy

1. $G(x, x') = G(x', x)$,
2. $G = G[N, h_{ab}]$, and this dependence is quasilocal,
3. G is a parametrix for a self-adjoint field equation. In EM,

$$D^a(N^{-1} D_a G) = -\omega_n \delta_\perp(x, x') + (\text{smooth}).$$

Hadamard parametrices on (Σ, h_{ab}) work:

$$G \sim (NN')^{1/2} \left[\frac{U}{\sigma_\perp^{\frac{1}{2}(d-3)}} + V \ln(\sigma_\perp/\ell^2) + W \right]$$

Forces exerted by the corresponding Φ_S can only renormalize things.

Dynamical self-force

In non-static systems, similar arguments guarantee that propagators with the following properties would be useful:

1. $G(x, x') = G(x', x)$,
2. $G = G[g_{ab}]$, and this dependence is quasilocal,
3. It is a parametrix for the field equation [e.g. $\Box G = -\omega_n \delta(x, x') + \ldots$],
4. $G(x, x') = 0$ if x, x' are timelike-separated.

These are basically the defining properties of a Detweiler-Whiting $G_{DW,S}$.

Everything is fine if d is even. . .

No Detweiler-Whiting in odd d

Existence fails if d is odd.

Reflections $\sigma \to -\sigma$ don't work, for example:

$$\Box(G_{\mathrm{adv}} + G_{\mathrm{ret}}) \sim \Box \left[\frac{\Theta(-\sigma)}{(-\sigma)^{d/2-1}} \right] \sim \delta \quad \text{but} \quad \Box \left(\frac{\Theta(\sigma)}{\sigma^{d/2-1}} \right) = 0.$$

The above DW constraints are sufficient not necessary: They can be weakened...

Useful constraints for dynamical problems in odd d

1. $G(x, x') = G(x', x)$,
2. $G = G[g_{ab}]$, and this dependence is quasilocal,
3. $\hat{\Phi} := \Phi - \Phi_S$ is well-behaved even "for point particles."
4. $G(x, x') = 0$ if x, x' are timelike-separated,

In even d, using a parametrix $\implies$ condition 4:

- Static cases follow from elliptic regularity theorems.
- Dynamical cases follow from propagation-of-singularity theorems.

But condition 4 $\not\Longrightarrow$ that G must be a parametrix.

Are there other possibilities?

A good propagator for odd d

The following gives good S-fields

$$G \sim \frac{U \ln(\sigma/\ell^2)}{\sigma^{d/2-1}} \Theta(\sigma)$$

We have shown that it

1. produces fields whose forces and torques can only renormalize things (nonperturbatively, for any extended body),
2. has a good point particle limit.

It is *not* a Green function or parametrix.

We don't know why this gives good pp limits.

G can roughly be obtained by:

1. Starting with $\frac{1}{2}(G_{\rm adv} + G_{\rm ret})$ in arbitrary d,
2. replacing $\sigma \to -\sigma$,
3. applying $\partial/\partial d$,
4. fixing d to the physical dimension.

This is reminiscent of dimensional regularization...

DW replacement

In any case, one working 2-point function is

$$G \sim \frac{U \ln(\sigma/\ell^2)}{\sigma^{d/2-1}} \Theta(\sigma).$$

Use it. . .

Approximations

In static or dynamic cases, for any d, we have integral forces and torques with the multipole expansions

$$F_a^{\text{self}} = \sum_{p=0}^{\infty} \frac{1}{p!} Q^{b_1 \cdots b_p} \nabla_a \hat{\Phi}^{\text{self}}_{,b_1 \cdots b_p} = Q \nabla_a \hat{\Phi}_{\text{self}} + \ldots$$

$$F_a^{\text{grav}} = \frac{1}{2} \sum_{p=2}^{\infty} \frac{1}{p!} \hat{I}^{b_1 \cdots b_p cd} \nabla_a g_{cd,b_1 \cdots b_p} = \frac{1}{6} \hat{I}^{bcdf} \nabla_a R_{b(df)c} + \ldots$$

These aren't useful as infinite series. Can they be truncated?

Approximations

Consider *shrinking* 1-parameter families $T^{ab} = T^{ab}(x; \lambda)$ as $\lambda \to 0^+$.

Various constraints:

- Internal spatial scales $\ll$ temporal scales $[(\text{size}) \sim \lambda^1]$
- Internal spatial scales $\ll$ external scales $[(\text{size}) \sim \lambda^1]$
- Mass density remains finite $[\rho_M \sim O(\lambda^0)]$,
- EM or scalar SF is larger than gravitational SF $[m^2 \sim \lambda^2 Q^2]$,
- (Self-energy) $\lesssim$ (total mass) $[(\text{ratio}) \sim O(\lambda^0)]$.

Why dim-dependence?

In a d dimensional spacetime,

$$(\text{self-energy}) \sim \frac{Q^2}{r^{d-3}} \sim m, \qquad \rho_M \sim \frac{m}{r^{d-1}}, \qquad \ldots$$

So scalings depend on d.

Scalings

Physical quantity	Scaling rate	$d = 3$	$d = 4$
2^p-pole holding force	λ^{d+p-1}	λ^{2+p}	λ^{3+p}
EM self-force	$\lambda^{2(d-2)}$	λ^2	λ^4
Grav. self-force	$\lambda^{2(d-1)}$	λ^4	λ^6

$$(\text{EM or scalar self-force}) \sim (2^{d-3}\text{-pole holding force})$$

1. *SF is dominated by extended-body effects* in higher d.
2. But **it's leading-order in** $2+1$**!**

$2+1$ dimensions

In $2+1$, staticity requires a F_{ab}^{hold} satisfying

$$QF_{ab}^{\mathrm{hold}}u^b = \hat{m}D_a \ln N - Q\hat{F}_{ab}^{\mathrm{self}}u^b,$$

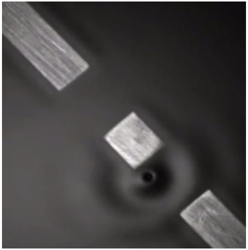
with $\hat{F}_{ab}^{\mathrm{self}} := F_{ab}^{\mathrm{self}} - F_{ab}^{S}$.

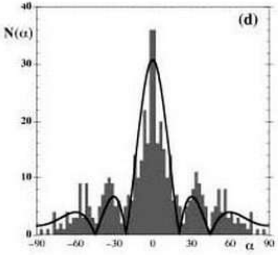
The dynamical case is messier but similar.

Someone should work out $2+1$ SF!

Even in flat spacetime, nontrivial BCs and strong $\int t^{-1}$ tails can probably give interesting phenomenology.f

Tunneling, diffraction through slits, quantized bound states, etc. seen with pilot-wave hydrodynamics [Couder, Bush, . . .]





Conclusions

- Self-force and self-torque now formulated in all d, though its consequences have not yet been explored!
- Detweiler-Whiting has to be modified a bit for even d: Vacuum condition on $\hat{\phi}$ is lost.
- Self-interaction mixes with extended-body effects in an essential way.
- It's more significant in lower dims: Leading order when $d = 3$.