Dynamical Tides in General Relativity

Steinhoff etal, arXiv:1608.01907; Hinderer etal, PRL **116** (2016) 181101

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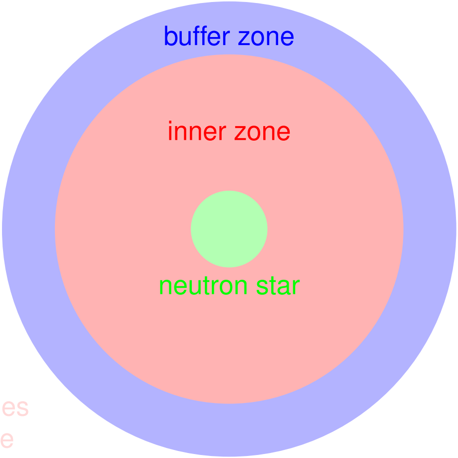
Zones, separation of scales, and effective theory

various zones → separation of scales:

- inner zone
  → scale of the neutron star
- outer/orbit zone
  → scale of the orbit
- wave zone/scale

outer/orbit zone



scales continue down the star:
→ fluid, nucleons, quarks, ?

The physics at "smaller" scales admits an effective (field) theory description!

Here: Effective theory for dynamical tides
→ dynamical, time-dependent response
(of the inner zone to perturbations from the outer zone)
→ harmonic oscillator effective theory

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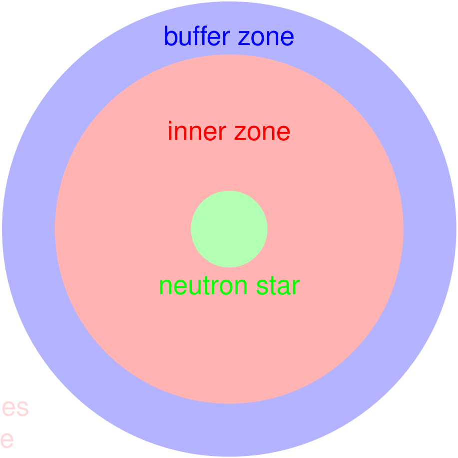

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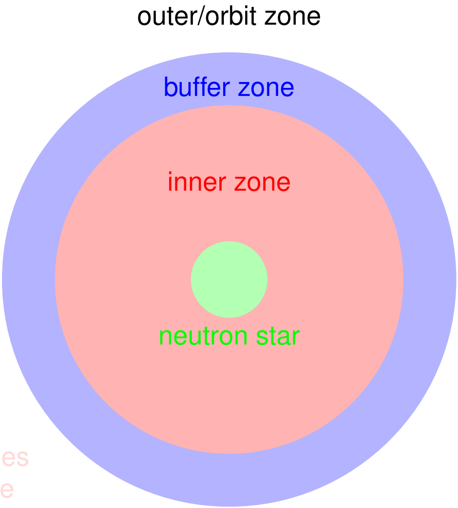

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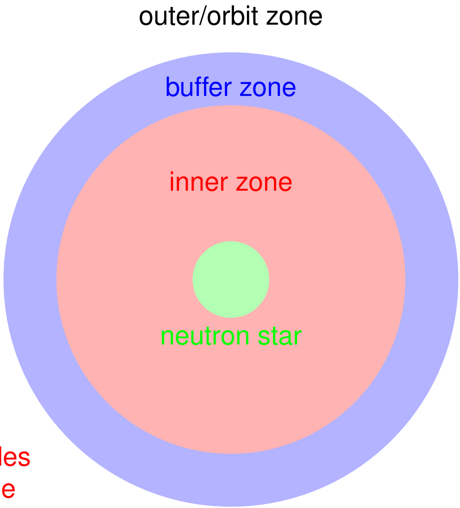

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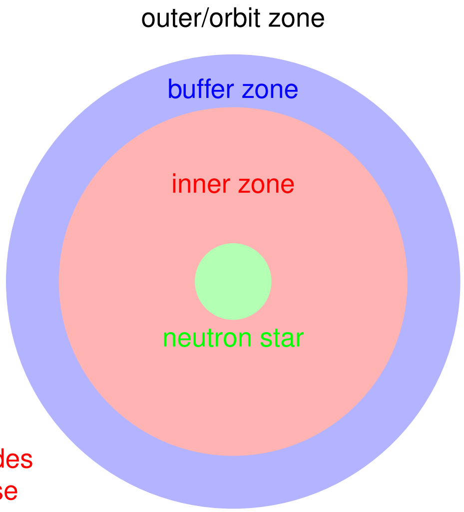

Effective field theory and matching

Effective field theory (EFT):

- Precursors
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Matching:

- Matched asymptotic expansion in harmonic gauge
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- multipole moments: "macroscopic state variables of the inner zone"
- dynamical tides $\rightarrow$ effective theory of dynamical multipoles

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- Newtonian dynamical tides from f$_{\text{undamental}}$-mode oscillations
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$$L_Q = \frac{1}{4\lambda\omega_f^2}\left[\dot{Q}^{ij}\dot{Q}^{ij} - \omega_f^2 Q^{ij} Q^{ij}\right] - \frac{1}{2}E_{ij}Q^{ij}, \qquad E_{ij} = \partial_i\partial_j\Phi$$

Q^{ij}: quadrupole λ: tidal deformability ω_f: f-mode frequency Φ: Newtonian potential

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Finally: Dynamical tides in general relativity

Their description through an effective action

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$$u^\mu = \dot{x}^\mu, \qquad z = \sqrt{-u^\mu u_\mu} \quad \text{(is the redshift for } \sigma = t\text{)}$$

- plus regularization/renormalization
- K linked to (almost) completeness of modes: $K \approx 0$
- identify ω_f with real part of quasi-normal-mode frequency

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Mathisson-Papapetrou-Dixon equations of motion

Their completion through the effective theory for dynamical tides

Vary the action! Result:

$$\frac{Dp_\mu}{d\sigma} = \frac{1}{2} S_Q^{\alpha\beta} R_{\alpha\beta\rho\mu} u^\rho - \frac{1}{6} \nabla_\mu R_{\alpha\rho\beta\sigma} J_Q^{\alpha\rho\beta\sigma}$$

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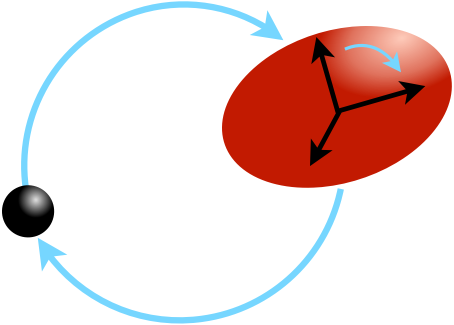

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Test-particle and effective-one-body Hamiltonians

Test-particle Hamiltonian 101:

- get mass-shell constraint: $0 = \mu^2 + p^\mu p_\mu +$ tidal terms, $p_\mu = \dfrac{\partial L}{\partial u^\mu}$
- solve for the energy $H \equiv -p_0$

Absorb interaction into the metric $\rightarrow g_{\text{eff}}^{\mu\nu}$:

- notice $E \propto p^2$
- factorize p^2 terms: $0 = \underbrace{[\mu + H_{\text{oszi}}]^2}_{\mu_{\text{eff}}^2} + \underbrace{\left[g^{\mu\nu} + \frac{1}{\mu} R^{\alpha\mu\beta\nu} Q_{\mu\nu}\right]}_{g_{\text{eff}}^{\mu\nu}} p_\mu p_\nu$
- also works for higher multipoles

When used for EOB: no pole at the light ring in H

pole can be always by removed Akcay, etal, PRD 86 (2012) 104041

but also no gauge-invariant centrifugal radius

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Effective-one-body Hamiltonian

for 1PN dynamical tides, see also Hinderer etal, PRL **116** (2016) 181101

- effective test-particle Hamiltonian (point-mass potentials A, D)

$$H_{\text{eff}} = \sqrt{(A + \mathcal{E}_{ij} Q^{ij}) \left[\mu^2 \left(1 + \frac{2}{\mu} z_c H_o + \mathcal{C}_{ij} Q^{ij} \right) + \frac{p_\phi^2}{r^2} + \frac{p_r^2}{D} + \mathcal{O}(p_r^4) \right]} + f_{\text{DT}}$$

- oscillator Hamiltonian: $H_o = \lambda \omega_f^2 P_{ij} P_{ij} + \frac{Q^{ij} Q^{ij}}{4\lambda}$
- 1PN tidal force $X_A = m_A / M, \quad M = m_1 + m_2, \quad \nu = X_1 X_2, \quad \mu = M\nu, \quad u = M/r$

$$\mathcal{E}_{ij} = -\frac{3 G m_2}{\mu r^3} n^i n^j \left\{ 1 - [2X_2 - (1 - c_1)\nu] u \right\}$$

$$\mathcal{C}_{ij} = \frac{3 G m_2}{\mu^3 r^3} \left\{ \frac{L^i L^j}{r^2} + [1 + (c_2 - 2c_1)\nu] n^i p_j p_r + \left[(1 - c_1) p^2 + (5c_1 - c_2) p_r^2 \right] \nu n^i n^j \right\}$$

- gauge parameters c_1, c_2. blue term: no gauge parameters!
- redshift factor (normalized to 1 for $m_1 \ll m_2$)

$$z_c = 1 + \frac{3}{2} X_1 u + \frac{\nu}{2}(1 + 2c_1) \left[\frac{p^2}{\mu^2} - u \right]$$

- frame dragging terms $\sim$ spin-orbit + corotating frame, "$S_Q = Q \times P$"

$$f_{\text{DT}} = -\vec{S}_Q \cdot \vec{L} \frac{1}{\mu^2 r^2} \left\{ 1 + [3X_1 - 5 - (1 + c_2)\nu] \frac{u}{2} - (1 - c_2 \nu) \frac{p^2}{2\mu^2} - c_2 \nu \frac{p_r^2}{\mu^2} \right\}$$

Effective-one-body Hamiltonian

for 1PN dynamical tides, see also Hinderer etal, PRL **116** (2016) 181101

- effective test-particle Hamiltonian (point-mass potentials A, D)

$$H_{\text{eff}} = \sqrt{(A + \mathcal{E}_{ij} Q^{ij}) \left[\mu^2 \left(1 + \frac{2}{\mu} z_c H_o + \mathcal{C}_{ij} Q^{ij} \right) + \frac{p_\phi^2}{r^2} + \frac{p_r^2}{D} + \mathcal{O}(p_r^4) \right]} + f_{\text{DT}}$$

- oscillator Hamiltonian: $H_o = \lambda \omega_f^2 P_{ij} P_{ij} + \frac{Q^{ij} Q^{ij}}{4\lambda}$
- 1PN tidal force $X_A = m_A/M, \quad M = m_1 + m_2, \quad \nu = X_1 X_2, \quad \mu = M\nu, \quad u = M/r$

$$\mathcal{E}_{ij} = -\frac{3Gm_2}{\mu r^3} n^i n^j \{ 1 - [2X_2 - (1 - c_1)\nu] u \}$$

$$\mathcal{C}_{ij} = \frac{3Gm_2}{\mu^3 r^3} \left\{ \frac{L^i L^j}{r^2} + [1 + (c_2 - 2c_1)\nu] \, n^i p_j p_r + \left[(1 - c_1) p^2 + (5c_1 - c_2) p_r^2 \right] \nu n^i n^j \right\}$$

- gauge parameters c_1, c_2. blue term: no gauge parameters!
- redshift factor (normalized to 1 for $m_1 \ll m_2$)

$$z_c = 1 + \frac{3}{2} X_1 u + \frac{\nu}{2} (1 + 2c_1) \left[\frac{p^2}{\mu^2} - u \right]$$

- frame dragging terms $\sim$ spin-orbit + corotating frame, "$S_Q = Q \times P$"

$$f_{\text{DT}} = -\vec{S}_Q \cdot \vec{L} \frac{1}{\mu^2 r^2} \left\{ 1 + [3X_1 - 5 - (1 + c_2)\nu] \frac{u}{2} - (1 - c_2\nu) \frac{p^2}{2\mu^2} - c_2 \nu \frac{p_r^2}{\mu^2} \right\}$$

Conclusions

All you need is λ! ?

pics/allLove

Almost, need more coefficients
linked to dynamical tides!

λ, ω_f, K, $\ldots$

Dynamical tides become important
close to resonance with ω_f

effective theory of tides:

- can cope with complicated situations:
  dynamical tides, nonlinear tides
- profits from/enables physical intuition

Dynamical tides are important for accurate waveform models

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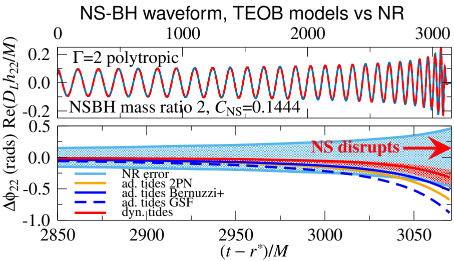

Hinderer etal, PRL **116** (2016) 181101

Bini, Damour, Faye, PRD **85** (2012) 124034

Bernuzzi, etal, PRL **114** (2015) 161103

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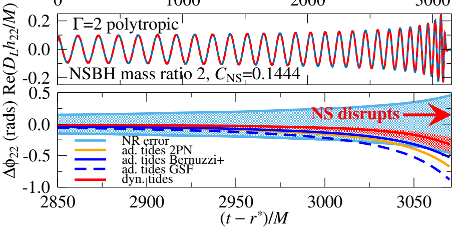

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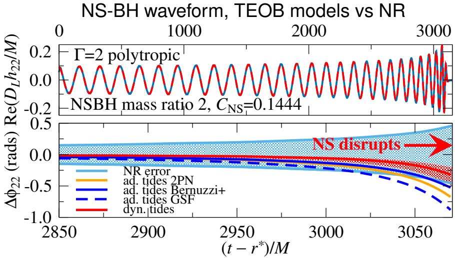

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