Scalar self-force for highly eccentric orbits in Kerr spacetime

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in collaboration with

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⋆ **may have a solution to Lorenz gauge instabilities** [with Sam Dolan]
$\Rightarrow$ if this works, then **extension to gravitational field looks doable**

Overall Plan of the Computation

Effective-Source (puncture-field) regularization

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- exploit axisymmetry of Kerr background
- can handle (almost) any orbit, including high eccentricity
- separate 2+1-dimensional time-domain (numerical) evolution for each m
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- (fixed) mesh refinement; some (finer) grids follow the worldtube/particle

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Assume a δ-function particle with scalar charge q.

The particle's physical (retarded) scalar field φ satisfies $\Box\varphi = \delta\big(x - x_{\text{particle}}(t)\big)$.
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Instead we choose $\varphi_{\text{puncture}} \approx \varphi_{\text{singular}}$ so that $\varphi_{\text{residual}} := \varphi - \varphi_{\text{puncture}}$ is finite and "differentiable enough" at the particle. It's then easy to see that

$$\Box\varphi_{\text{residual}} = \left\{ \begin{matrix} 0 & \text{at the particle} \\ -\Box\varphi_{\text{puncture}} & \text{elsewhere} \end{matrix} \right\} := S_{\text{effective}}$$

If we can solve this equation for $\varphi_{\text{residual}}$, then we can compute the self-force (exactly!) via $F_a = q(\nabla_a \varphi_{\text{residual}})\big|_{\text{particle}}$.

Puncture field and effective source

We choose $\varphi_{\text{puncture}}$ so that $|\varphi_{\text{puncture}} - \varphi_{\text{singular}}| = \mathcal{O}\big(\|x - x_{\text{particle}}\|^n\big)$ where the puncture order $n \geq 2$ is a parameter.

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The choice of the puncture order n is a tradeoff:
Higher $n \Rightarrow \varphi_{\text{residual}}$ is smoother at the particle (good),
but $\varphi_{\text{puncture}}$ and $S_{\text{effective}}$ are more complicated (expensive) to compute.

We choose $n = 4 \Rightarrow \varphi_{\text{residual}}$ is C^2 at the particle.

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The actual computation of $\varphi_{\text{puncture}}$ and $S_{\text{effective}}$ involves lengthly series expansions in Mathematica, then machine-generated C code. See Wardell, Vega, Thornburg, and Diener, *PRD* 85, 104044 (2012) = arXiv:1112.6355 for details.

Computing $S_{\text{effective}}$ at a single event requires $\sim \frac{1}{2} \times 10^6$ arithmetic operations.

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Solution:
introduce finite worldtube containing the particle worldline

- define "numerical field" $\varphi_{\text{numerical}} = \begin{cases} \varphi_{\text{residual}} & \text{inside the worldtube} \\ \varphi & \text{outside the worldtube} \end{cases}$
- compute $\varphi_{\text{numerical}}$ by numerically solving

$$\Box \varphi_{\text{numerical}} = \begin{cases} S_{\text{effective}} & \text{inside the worldtube} \\ 0 & \text{outside the worldtube} \end{cases}$$

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- $\varphi_{\text{numerical}}$ has a $\pm\varphi_{\text{puncture}}$ jump discontinuity across worldtube boundary
  $\Rightarrow$ finite difference operators that cross the worldtube boundary
  must compensate for the jump discontinuity

m-mode decomposition

Instead of numerically solving $\Box\varphi_{\text{numerical}} = \begin{cases} S_{\text{effective}} & \text{inside the worldtube} \\ 0 & \text{outside the worldtube} \end{cases}$

in 3+1 dimensions, we Fourier-decompose into $e^{im\phi}$ modes and solve for each Fourier mode in 2+1 dimensions via

$$\Box_m \varphi_{\text{numerical},m} = \begin{cases} S_{\text{effective},m} & \text{inside the worldtube} \\ 0 & \text{outside the worldtube} \end{cases} \qquad \begin{bmatrix} \text{numerically} \\ \text{solve this} \\ \text{for each } m \\ \text{in 2+1D} \end{bmatrix}$$

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Advantages (vs. direct solution in $3+1$ dimensions):

- can use different numerical parameters for different m
  - ⋆ (this is crucial for our hoped-for solution to the Lorenz-gauge instabilities in the gravitational case)
- each individual m's evolution is smaller $\Rightarrow$ test/debug code on laptop
- get moderate parallelism "for free" (run different m's evolutions in parallel)

Moving the worldtube

We actually do m-mode decomposition *before* introducing worldtube
$\Rightarrow$ worldtube "lives" in (t, r, θ) space, not full spacetime

The worldtube must contain the particle in (r, θ).
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Large eccentricity:

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- recall that our numerically-evolved field is

$$\varphi_{\text{numerical}} := \begin{cases} \varphi - \varphi_{\text{puncture}} & \text{inside the worldtube} \\ \varphi & \text{outside the worldtube} \end{cases}$$

  this means then if we move the worldtube, we must adjust the evolved $\varphi_{\text{numerical}}$: add $\pm\varphi_{\text{puncture}}$ at spatial points which change from being inside the worldtube to being outside, or vice versa

Code Validation

Comparison with frequency-domain results kindly provided by Niels Warburton

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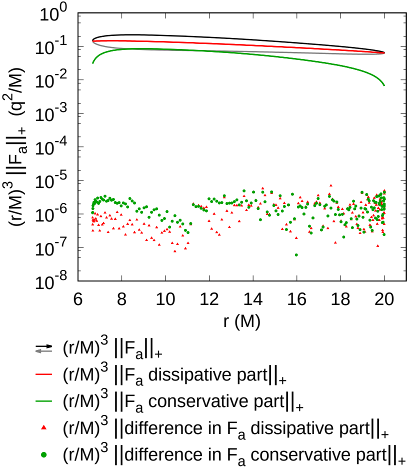
Typical example:
$(a, p, e) = (0.9, 10M, 0.5)$
$\Rightarrow$ results agree to $\sim 10^{-5}$ relative error

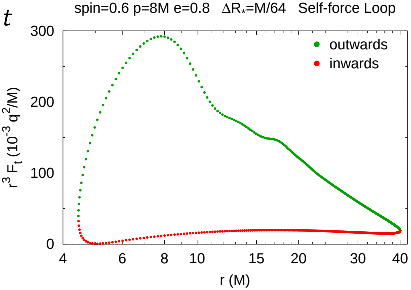
We have also compared a variety of other configurations, with fairly similar results



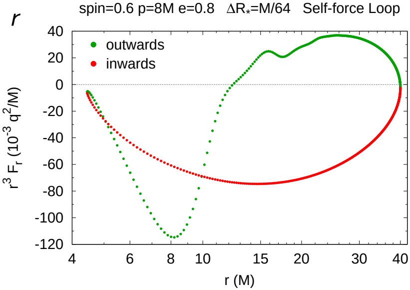
$e = 0.8$ orbit

$(a, p, e) = (0.6, 8M, 0.8)$

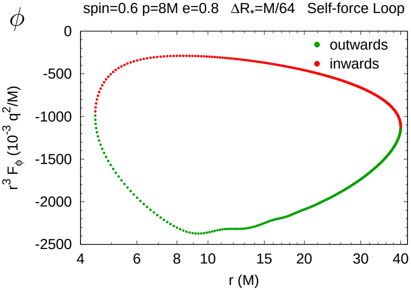
t



r



ϕ



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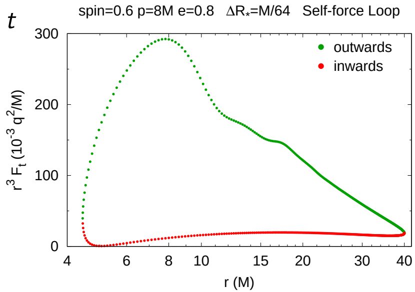
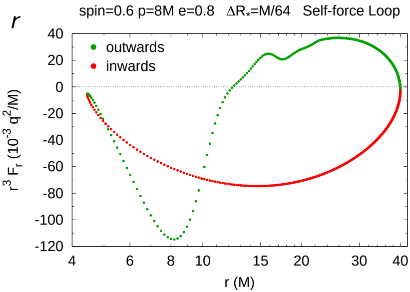
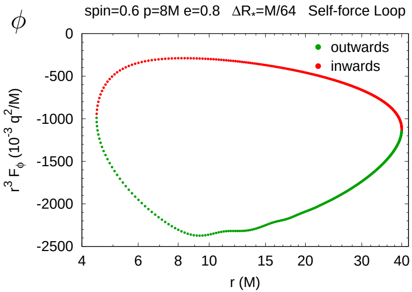
Notice the "bump" in F_r
near $r = 15M$ moving outwards

Maybe a caustic crossing?



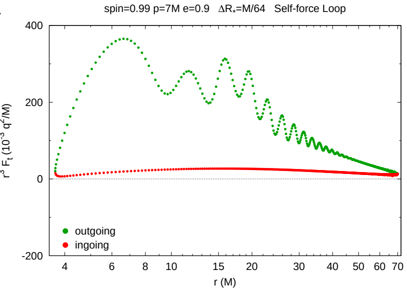




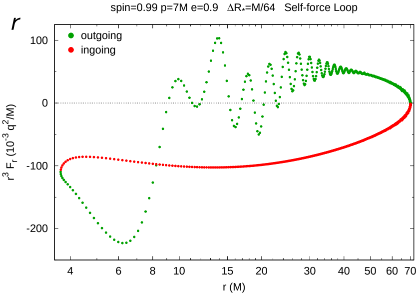
Wiggles!

Higher-eccentricity orbit:
$(a, p, e) = (0.99, 7M, 0.9)$

t



r



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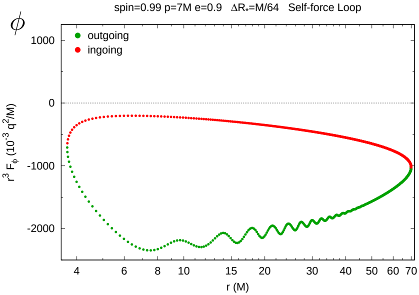

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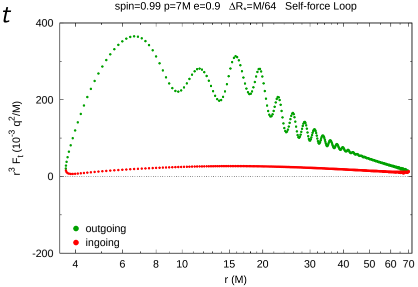
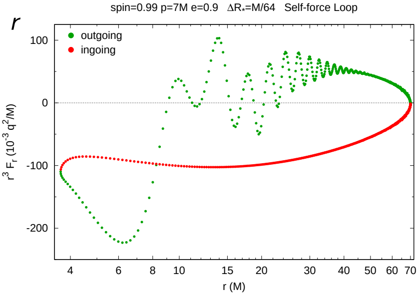
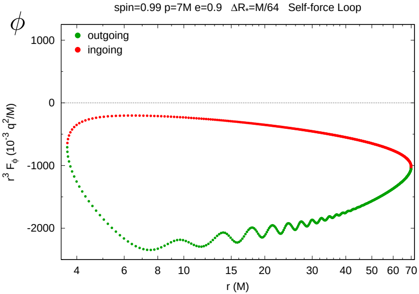
Notice:

- wiggles on **outgoing** leg of orbit
- wiggles **not** seen on **ingoing** leg





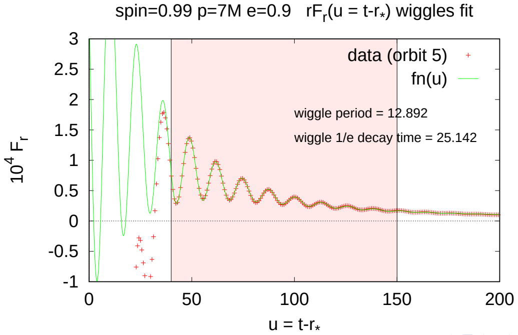


Wiggles as Kerr Quasinormal Modes: Mode Fit

Leor Barack suggested that wiggles might be quasinormal modes of the (background) Kerr spacetime, excited by the particle's close flyby. Test this by fitting decay of wiggles to a damped sinusoid with corrections for motion of the observer (particle):

$$rF_r^{[m=1]}(u) = \text{background}(u) + A \exp\left(-\frac{u-u_0}{\tau}\right) \sin\left(\phi_0 + 2\pi \frac{u-u_0}{T}\right)$$

where $u := t - r_*$



Wiggles as Kerr Quasinormal Modes: Mode Frequencies

Now compare wiggle-fit complex frequency $\omega := 2\pi/T - i/\tau$
vs. known Kerr quasinormal mode frequencies computed by Emanuele Berti.

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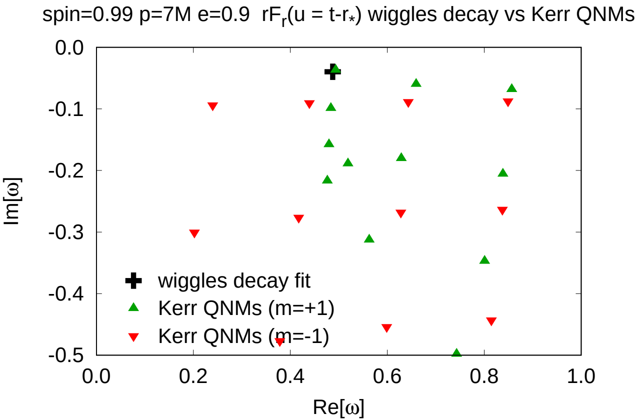
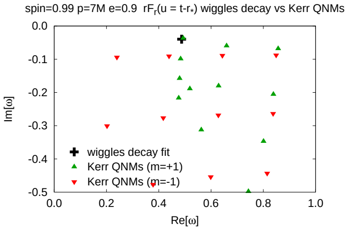
$\Rightarrow$ Nice agreement with least-damped corotating QNM!



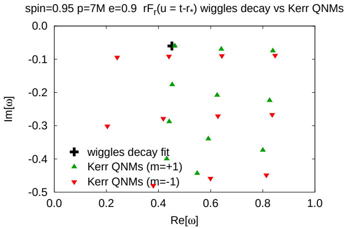
Wiggles as Kerr Quasinormal Modes: Varying BH Spin

Repeat wiggle-fit procedure for other Kerr spins (0.99, 0.95, 0.9, and 0.8)
$\Rightarrow$ Nice agreement with least-damped corotating QNM for all BH spins!

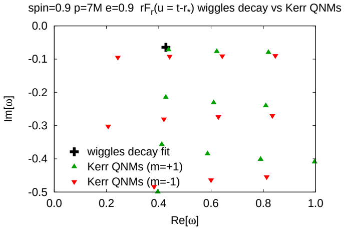
$a = 0.99$



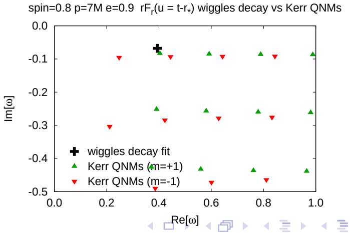
$a = 0.95$



$a = 0.9$



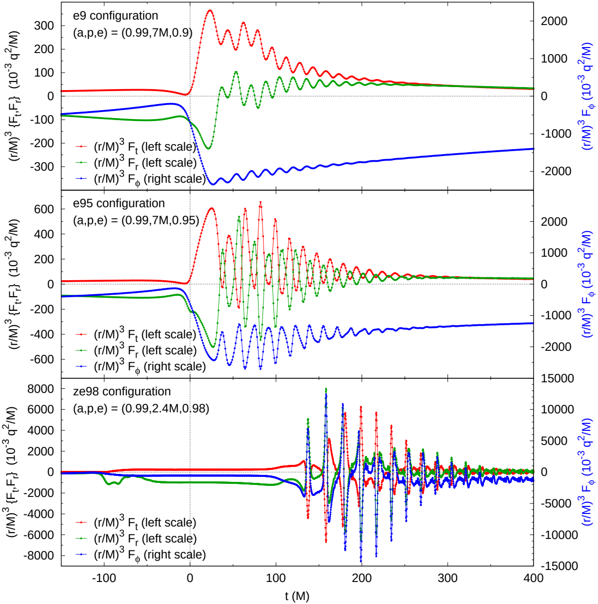
$a = 0.8$



More wiggles

We now see wiggles for many configurations with

- high Kerr spin
- highly-eccentric prograde orbit with close-in periastron



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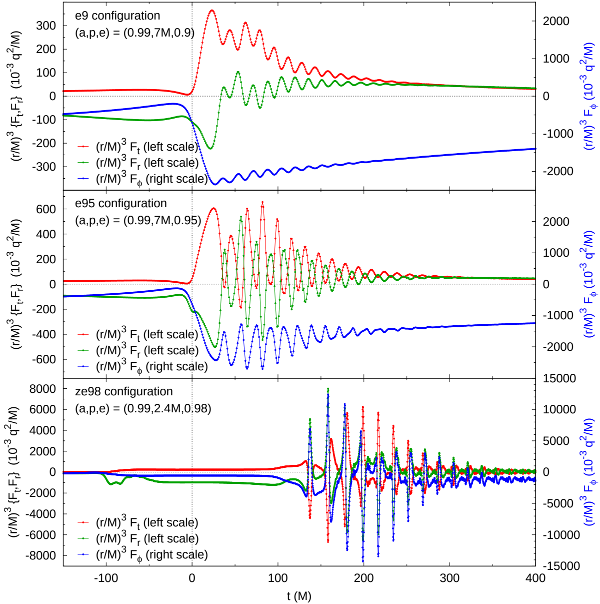
Note non-sinsoidal wiggle shapes
$\Rightarrow$ multiple QNMs?

maybe caustic crossings are also important? (Ottewill & Wardell)



Gravitation: Unstable Lorenz Gauge Modes

How to extend this work to the full gravitational field?

- Work in Lorenz gauge $\Rightarrow$ metric perturbation is isotropic at the particle.
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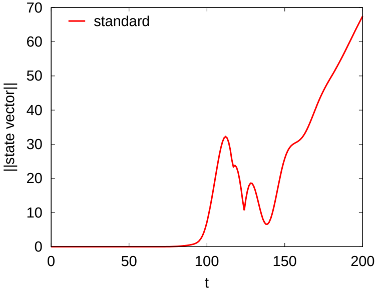
- The effective-source (puncture-field) regularization works fine for the gravitational case.
- Dolan and Barack [*PRD* **87**, 084066 (2013) = arXiv:1211.4586] found that the $m = 0$ and $m = 1$ modes have Lorenz gauge instabilities. These are modes which are consistent with the Lorenz gauge condition at any finite time, but blow up as $t \to \infty$. They were able to stabilize the $m = 0$ mode using a dynamically-driven generalized Lorenz gauge (analogous to the $\mathbb{Z}^4$ evolution system in full nonlinear numerical relativity).
- They were not able to stabilize the $m = 1$ mode. The instability is linear in time.



(Maybe) Stabilizing the $m = 1$ Lorenz Gauge Mode

Basic idea (schematic): [Joint work with Sam Dolan]

- First define (choose) an inner product on state vectors $\mathbf{u}$.
- Then compute the growing mode $G(t, x^i) := t\mathbf{u}_{\text{growing}}(x^i)$. Notice that this is a homogeneous solution of the evolution equation. $\mathbf{u}_{\text{growing}}$ depends only on the spacetime, not on the initial data or particle orbit.

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- Choose (update) λ "every so often"
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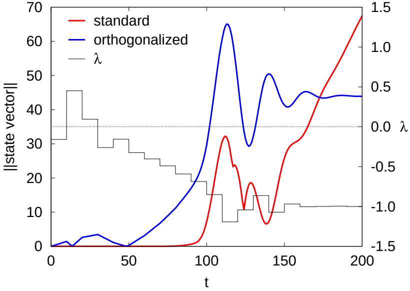

This seems to work: evolutions are stable!

- Now trying it for sourced evolution...

Conclusions

Methods

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Gravitation [Joint work with Sam Dolan]

⋆ **May** have found a way to stabilize Lorenz gauge modes
$\Rightarrow$ can do gravitational self-force etc.