Inspiral into Gargantua

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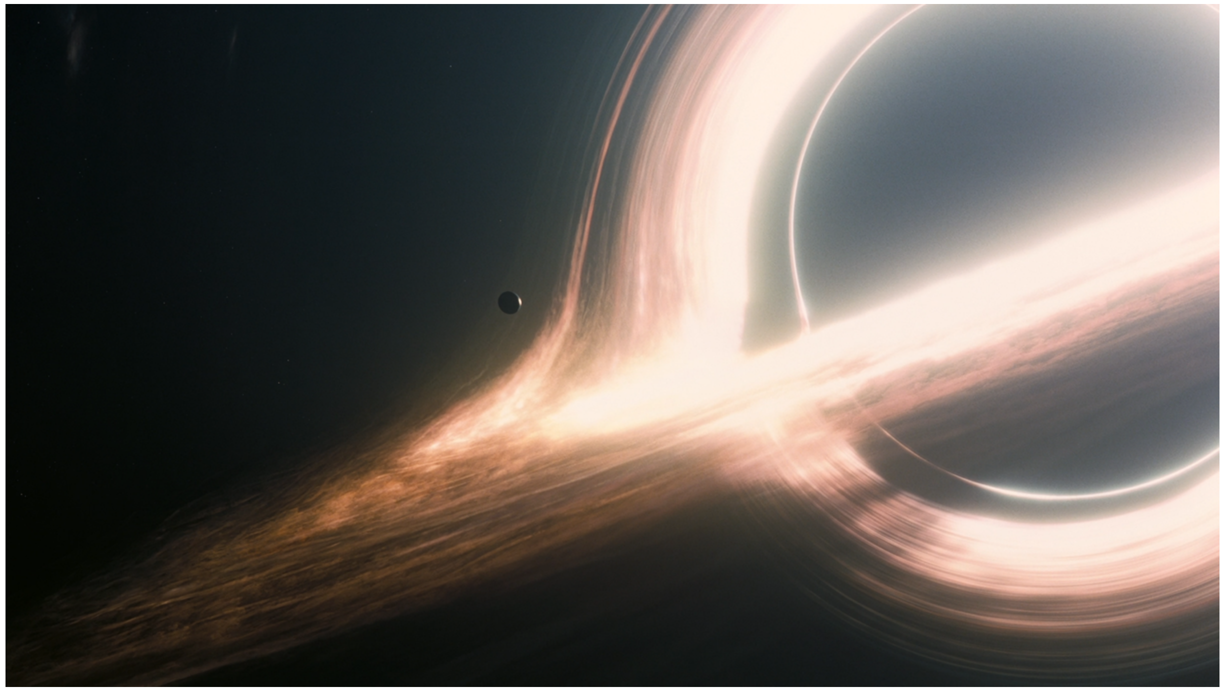
Gargantua?



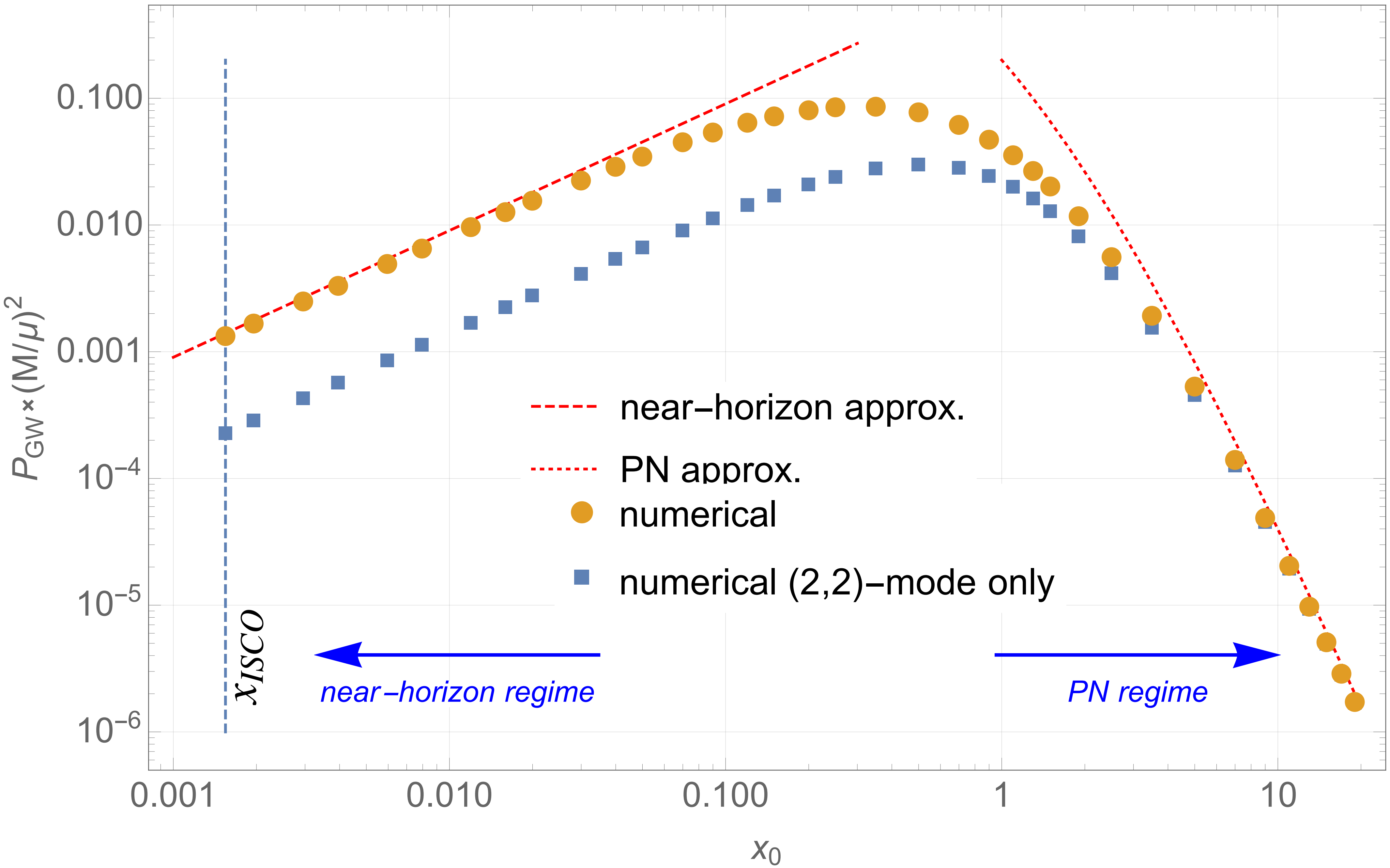
For ‘Interstellar’ Thorne estimates the black hole (Gargantua) must be spinning at

$$a/M \simeq 1 - 10^{-14}$$

This talk: Gravitational wave emission from an inspiral into a near-extremal Kerr (NEK) black hole. $\mathrm{NEK} \implies a/M \gtrsim 0.9999$

Useful notation: $x = \dfrac{r - r_+}{r_+}$ $\qquad \epsilon = \sqrt{1 - a^2/M^2}$

Anatomy of a NEK inspiral



Flux decreases as horizon is approached

New analytic flux approximation

Derivation of main result

$$E = \frac{\mu}{\sqrt{3}}\left[1 + \frac{2x_0}{3}\left(1 - \frac{2\epsilon^2}{x_0^3}\right)\right]$$

Energy of a particle on a circular equatorial orbit

$$P_{\rm GW} = (C_\infty + C_H)x_0$$

Gralla, Porfyriadis, Warburton, Phys. Rev. D 92, 064029, arXiv:1506.0896

$$\left(\frac{dE}{dx_0}\right)^{-1}\frac{dE}{dt} = \frac{dx_0}{dt} = -\frac{x_0}{\tau}\left(\frac{1}{1 - 2\epsilon^2/x_0^3}\right)$$

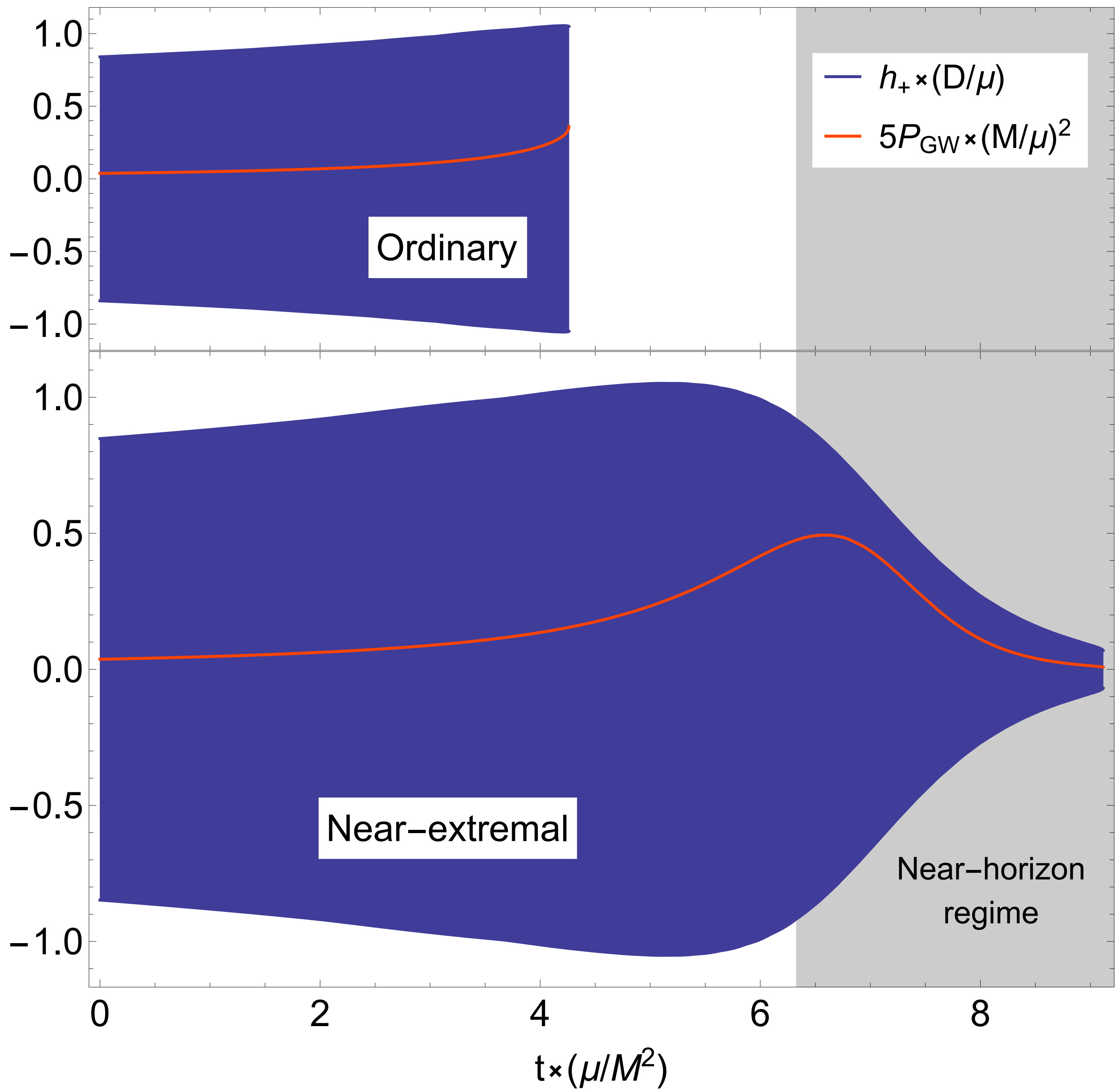
$$\tau = 0.451\mu(M/\mu)^2$$

$$x_0(t) = X_0 e^{-t/\tau} \implies P_{\rm detector} \sim e^{-t/\tau}$$

Similarly:

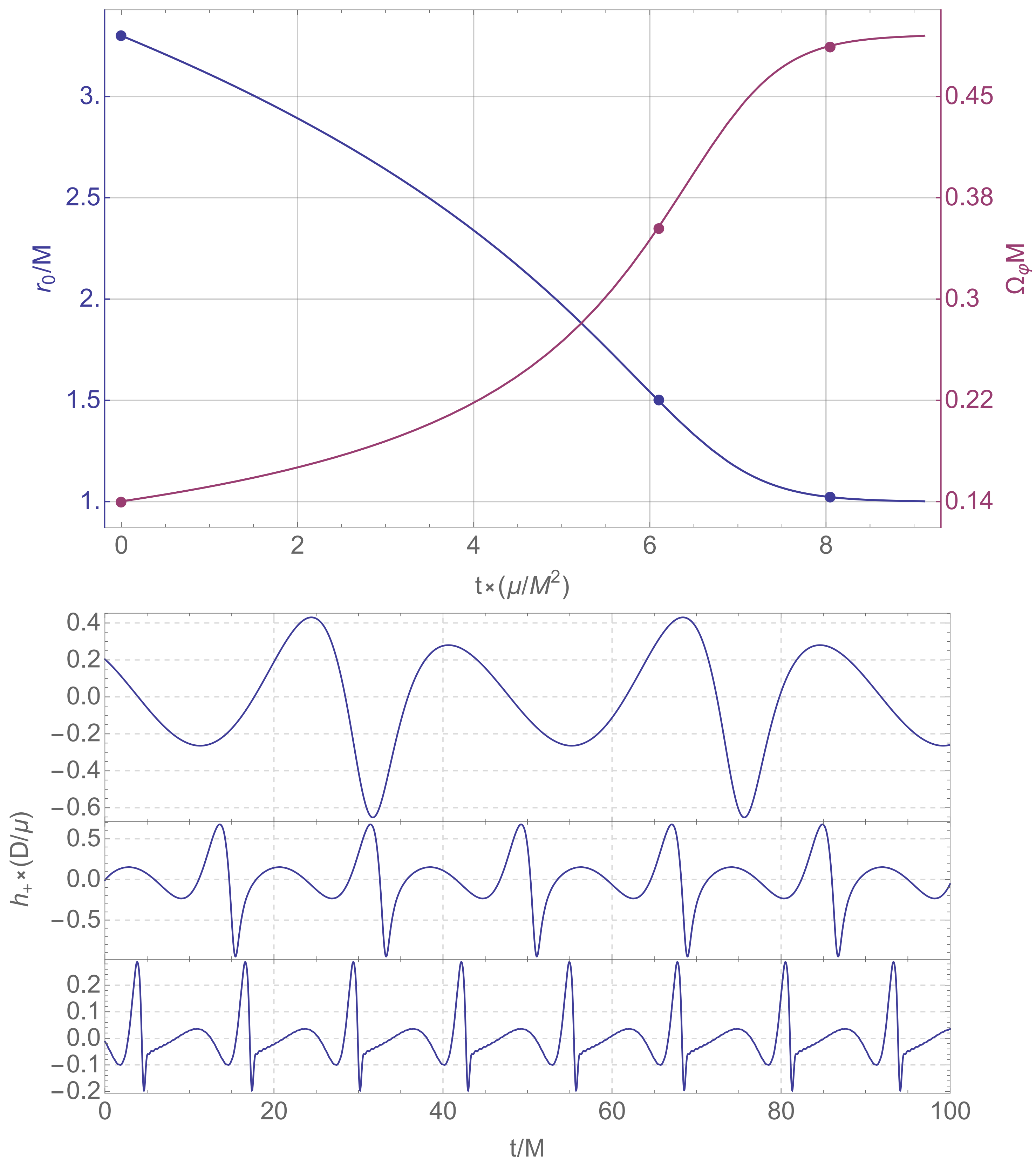
$$f_{\rm detector} \sim 2 \times \Omega_H/2\pi = \frac{1}{2\pi M}$$

Results: circular, equatorial



There is no chirp

Results: circular, equatorial inspiral

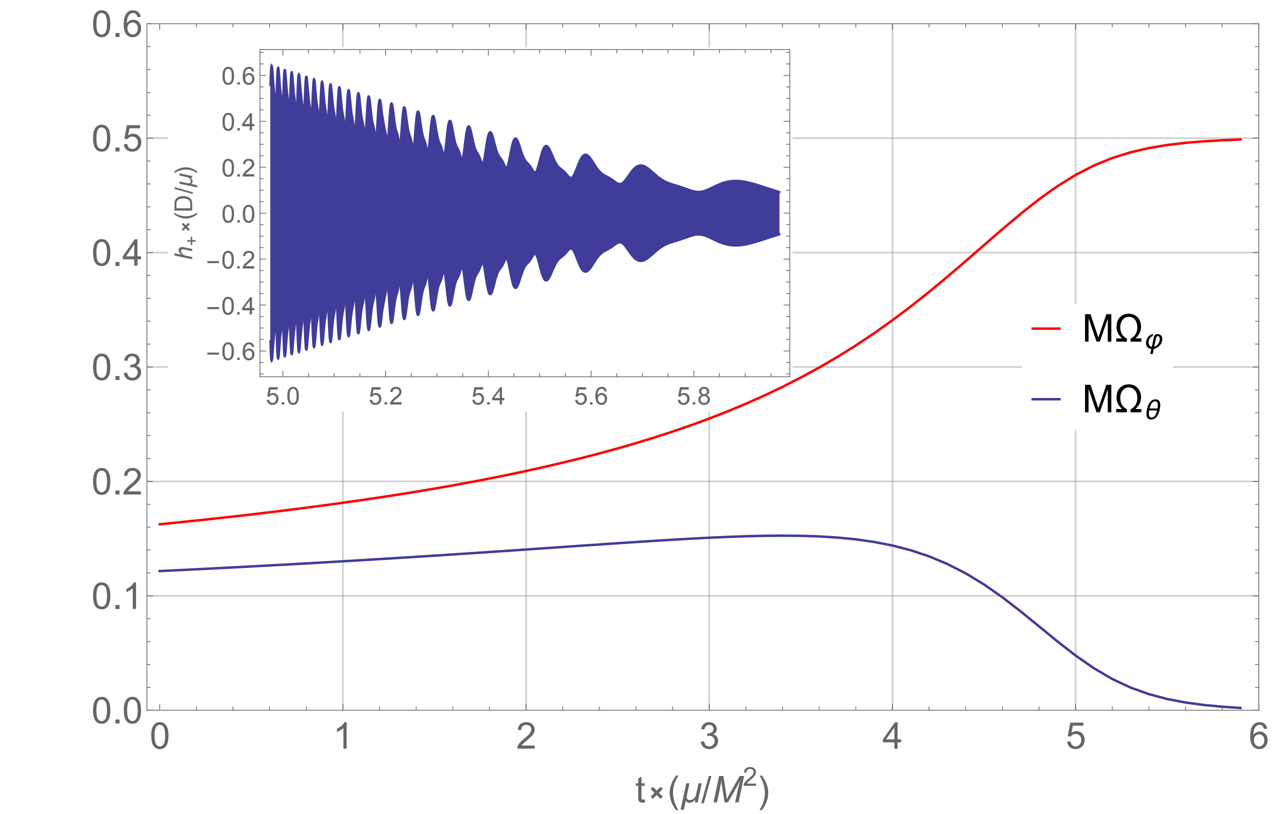


Orbital radius decays slowly near the horizon

Orbital frequency saturates at the horizon frequency

Strong relativistic beaming in the near-horizon regime

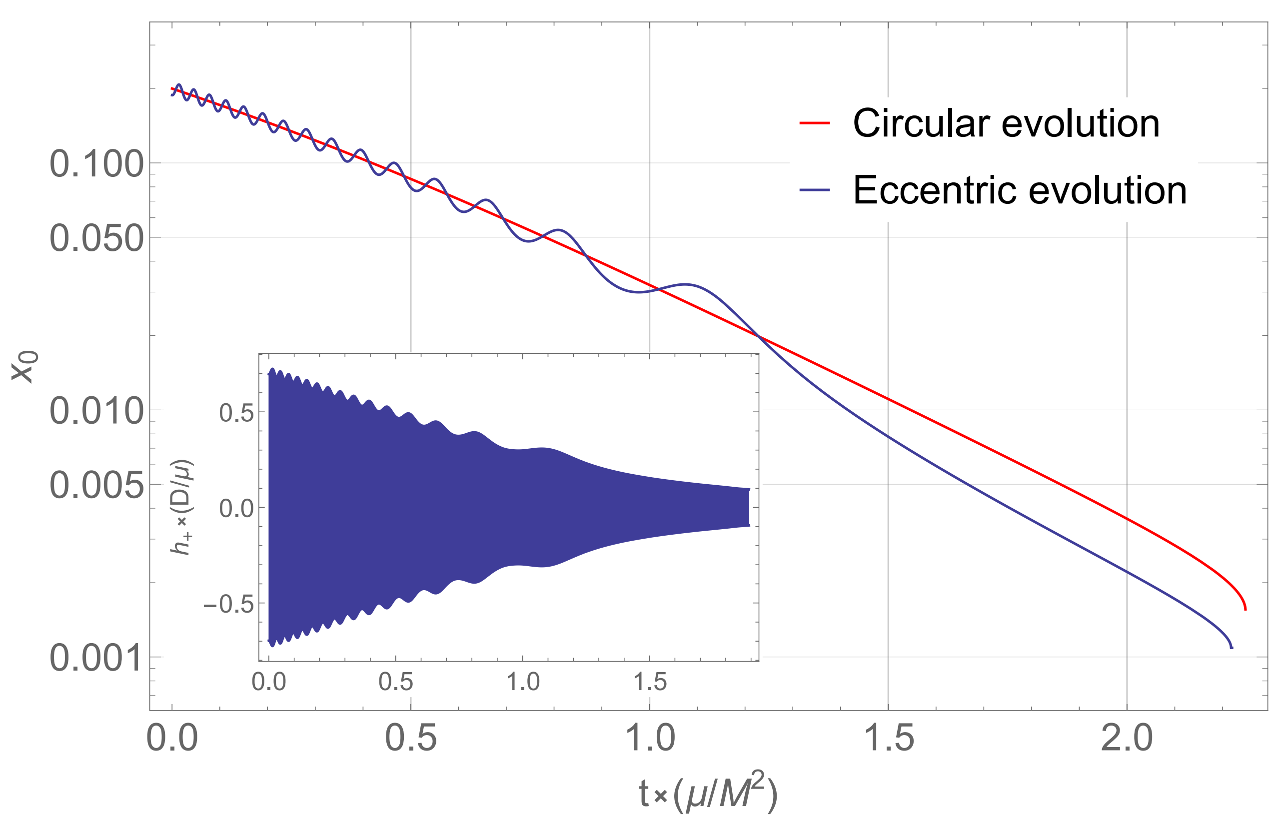
Results: spherical and eccentric, equatorial inspirals



Spherical

- Azimuthal frequency saturates at horizon frequency. Polar frequency tends to zero

- Waveform: exp. decay modulated with polar libration frequency



Eccentric, equatorial

- Considered low eccentricity inspirals (e~0.01)

- Waveform: exp. decay modulated with radial libration frequency

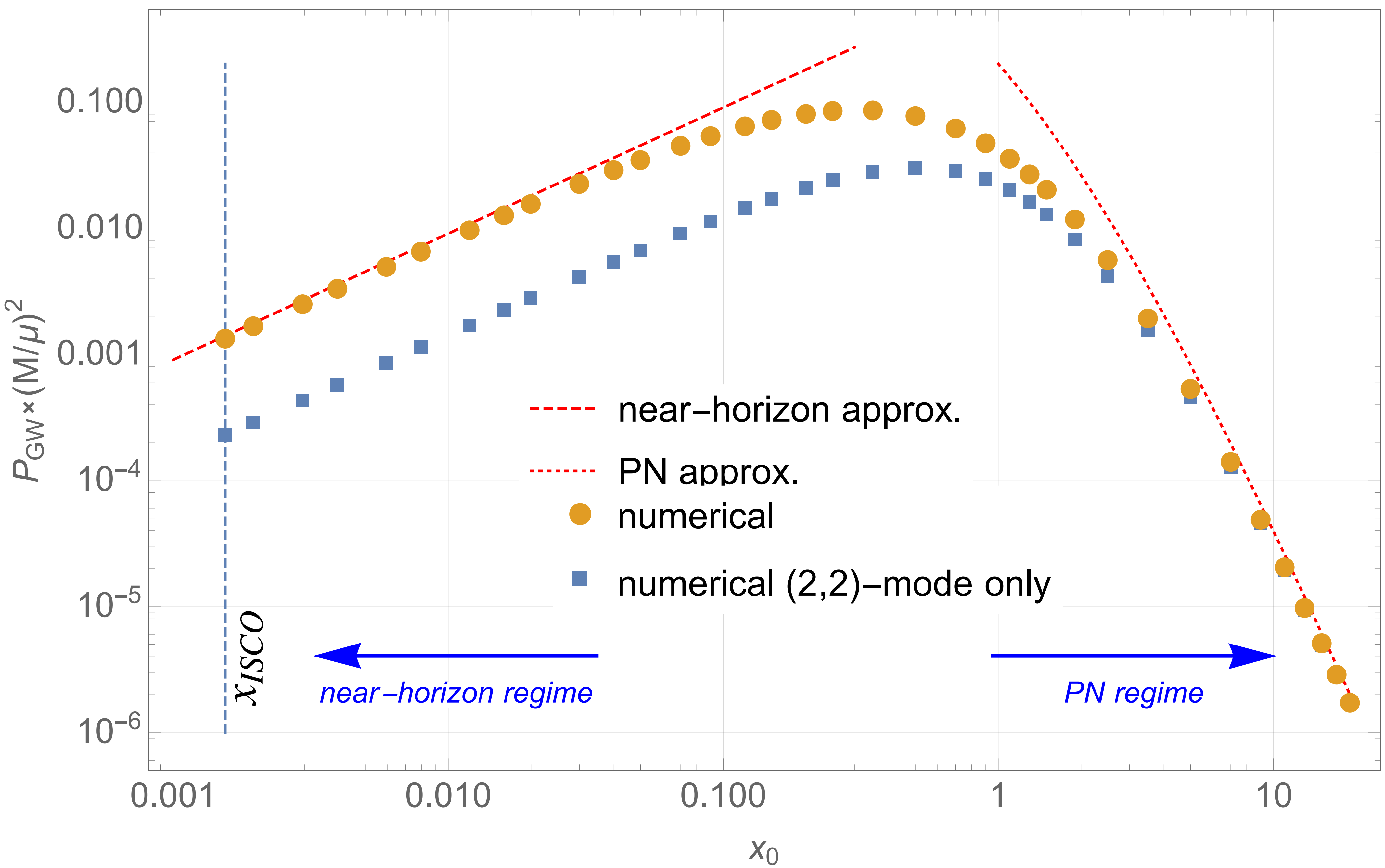
Extra details in the paper

- detectability with eLISA and ground-base detectors
- enough room for extended body near the horizon?
- when is the evolution adiabatic?
- zoom-whirl orbits can have ‘inverted’ behaviour
- confusion with quasi-normal mode ringing?

Extra details not in the paper

Shift in the ISCO location due to the smaller body…

The ISCO gives access to the near-horizon region



Near horizon flux not sensitive to ε.
The location of the ISCO more important.

ISCO shift in Kerr spacetime

Isoyama+, PRL 113, 161101 (2014)

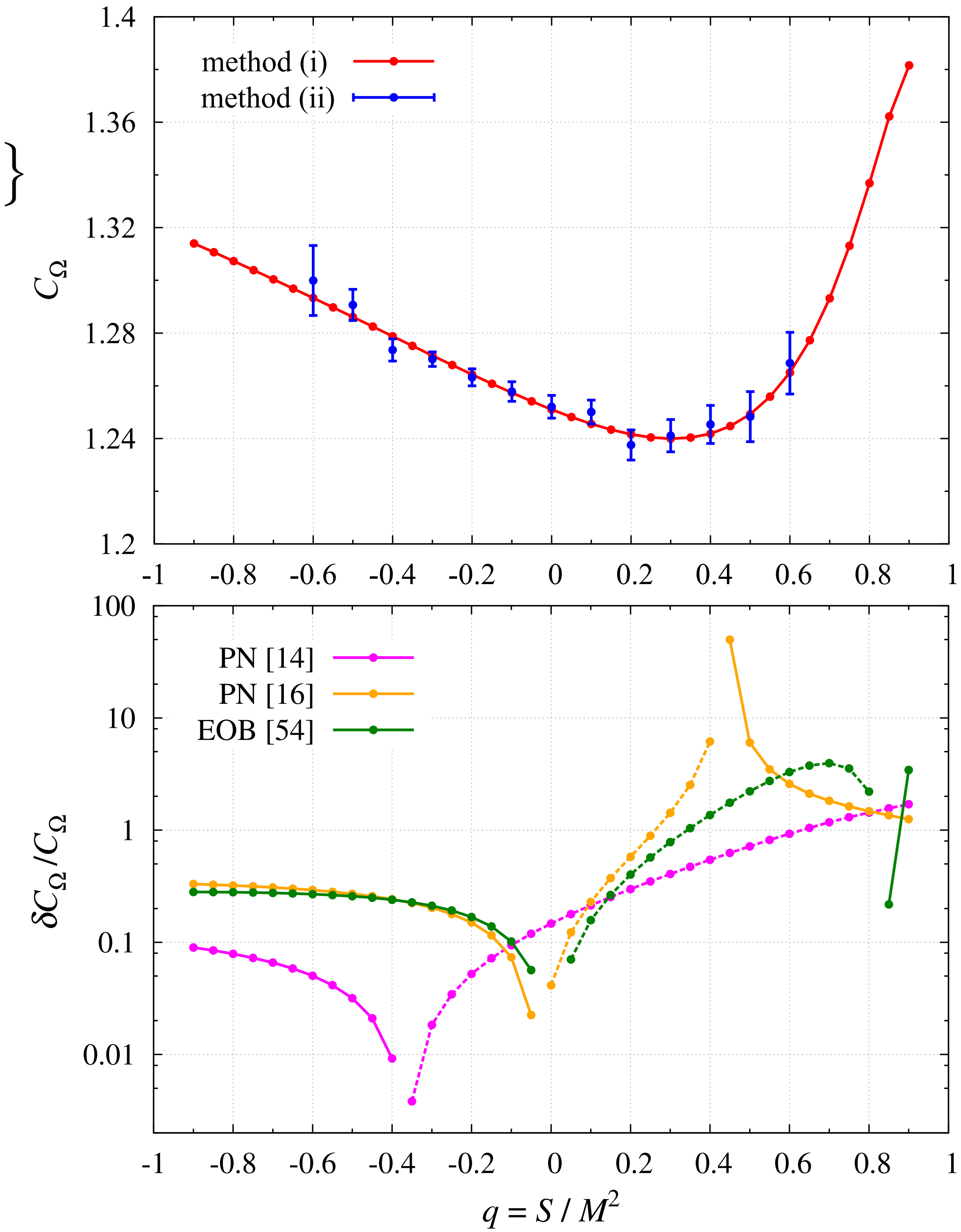
$$(M+\mu)\,\Omega_{\rm isco} = M\Omega^{(0)}_{\rm isco}(q)\,\{1+\eta\, C_\Omega(q) + \mathcal{O}(\eta^2)\}$$

C_Ω can be calculated from the redshift invariant computed for a sequence of circular orbits

$$z \equiv 1/u^t$$
$$= z_{(0)} + \eta z_{(1)} + \mathcal{O}(\eta)^2$$

$$z_{(1)} = z_{(0)} H, \quad \text{where} \quad H = \frac{1}{2} h^R_{\alpha\beta} u^\alpha u^\beta$$

$$C_\Omega = 1 - \frac{1}{2}\,\frac{z''_{(1)}(\Omega^{(0)}_{\rm isco})}{\Omega^{(0)}_{\rm isco}\; z'''_{(0)}(\Omega^{(0)}_{\rm isco})}$$



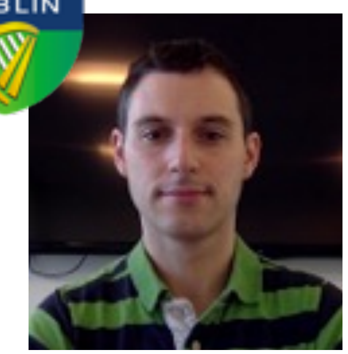
Calculation of H via radiation gauge

MIT KAVLI INSTITUTE

Code to calculate ψ_4 in the near horizon regime

CBPF

UCD DUBLIN



Code to calculate H from ψ_0

$$\psi_4 = \frac{1}{(r - ia\cos(\theta))^4} \sum_{lm\omega} {}_{-2}R_{lm}\ {}_{-2}S^{a\omega}_{lm}(\theta) e^{im\varphi} e^{-i\omega t}$$

$$\psi_0 = \sum_{lm\omega} {}_2R_{lm}\ {}_2S^{a\omega}_{lm}(\theta) e^{im\varphi} e^{-i\omega t}$$

Calculation of H via radiation gauge

$$_{-2}R_{lm} = Z_- \;{}_{-2}\tilde{R}^-_{lm}\Theta(r_0 - r) + Z_+ \;{}_{-2}\tilde{R}^+_{lm}\Theta(r - r_0)$$
$$_{2}R_{lm} = Y_- \;{}_{-2}\tilde{R}^-_{lm}\Theta(r_0 - r) + Y_+ \;{}_{2}\tilde{R}^+_{lm}\Theta(r - r_0)$$

$$Y_+ = \frac{\bar{C}}{4\omega^4}Z_+$$
$$Y_- = 64(2Mr_+)^4 ik(k^2 + 4\epsilon^2)(4\epsilon - ik)C^{-1}Z_-$$

Using Teukolsky-Starobinsky identities

$$_{-2}\tilde{R}_{lm} \leftrightarrow {}_{2}\tilde{R}_{lm}$$

$$\epsilon = \frac{\sqrt{M^2 - a^2}}{4Mr_+}$$
$$C = \sqrt{|C|^2 - (12M\omega)^2} + i12M\omega(-1)^{l+m}$$

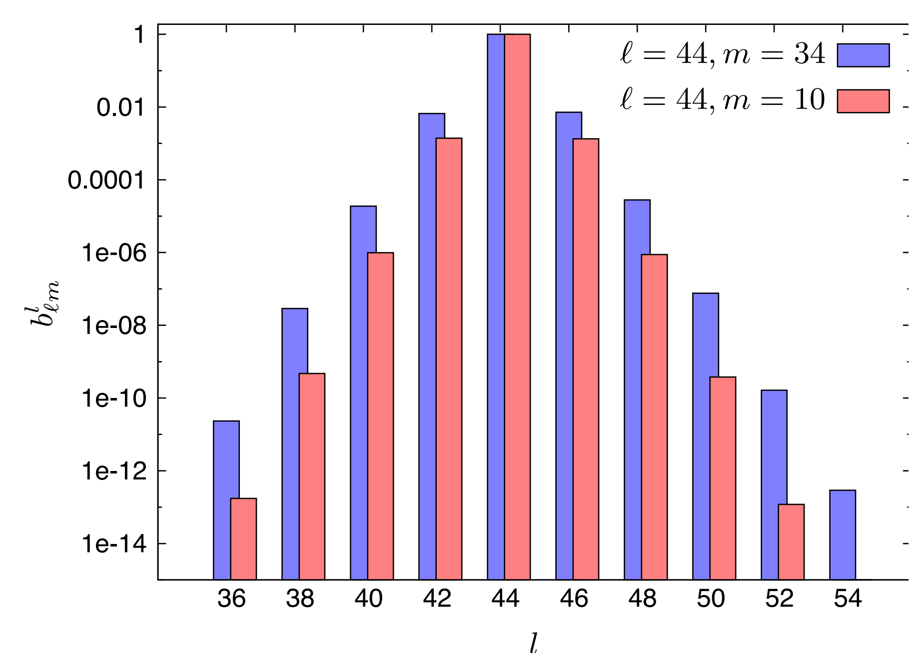
This term was missing from Press and Teukolsky 1974

Regularize spheroidal-harmonic modes directly

$$H^R = \sum_l \left(H^{\rm ret}_l - B_H\right) \qquad S_{\ell m}(\theta)e^{im\phi} = \sum_{l=0}^{\infty} b^{\ell}_{lm}\, Y_{lm}(\theta,\phi)$$

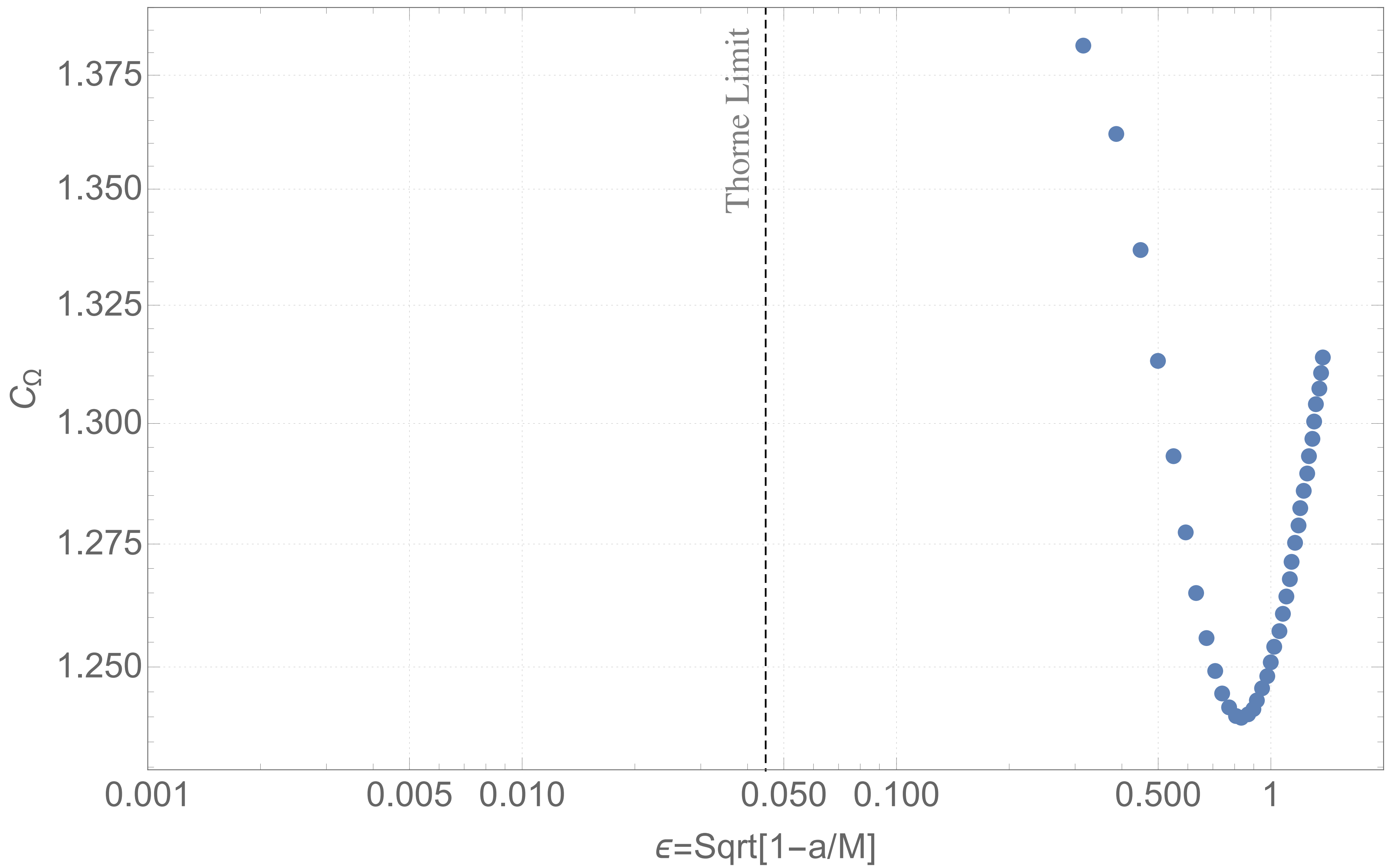
Old way: reg. spherical-harmonic modes

New way: reg. spheroidal-harmonic modes

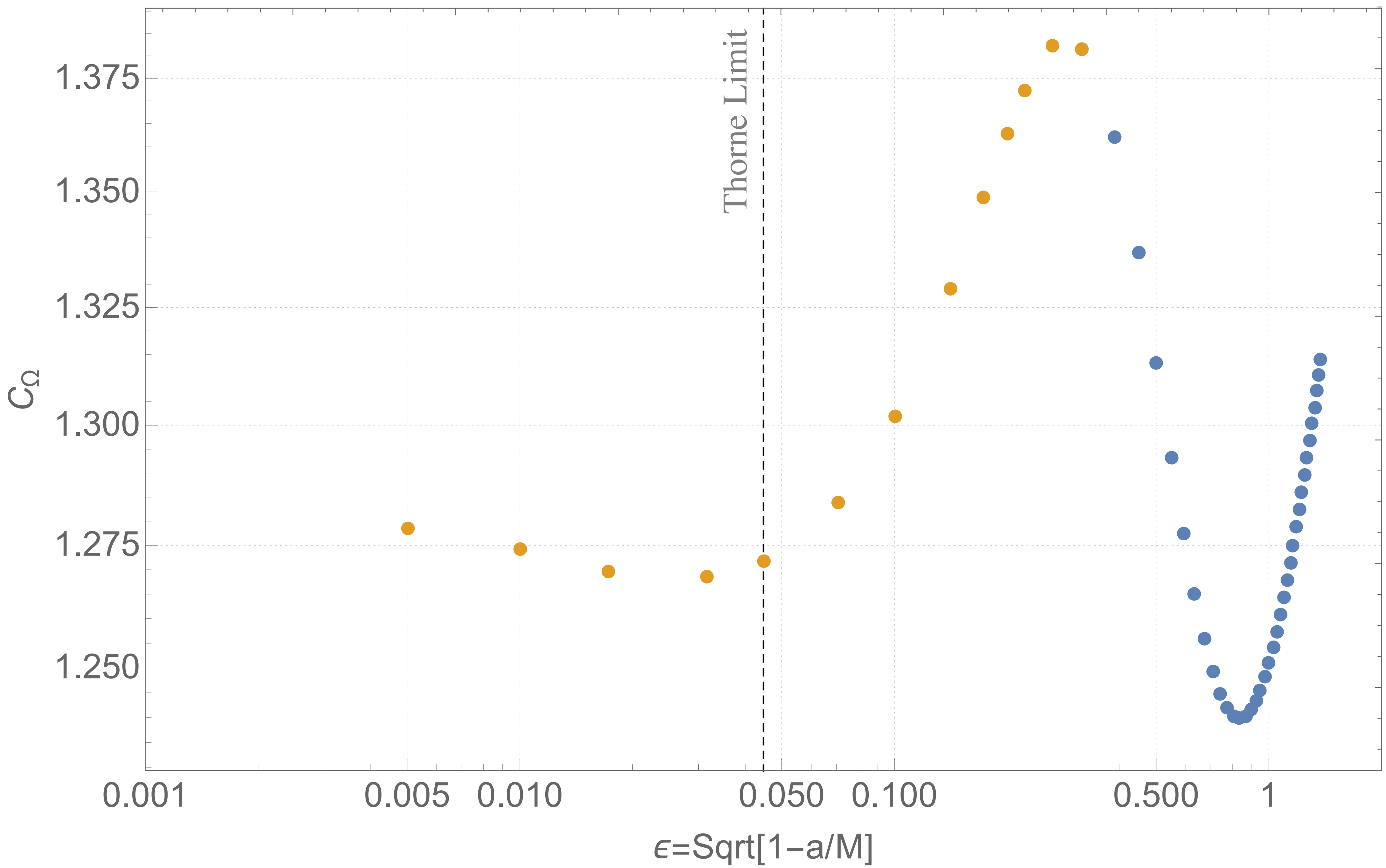


For more details see Kavanagh's talk on Thursday

Near-extremal Kerr ISCO shift



Near-extremal Kerr ISCO shift



Can we say anything about the extremal limit?

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Maybe…

$$C_\Omega = 1 - \frac{1}{2}\frac{z''_{(1)}(\Omega^{(0)}_{\rm isco})}{\Omega^{(0)}_{\rm isco}\, z'''_{(0)}(\Omega^{(0)}_{\rm isco})}$$

$$z'''_{(0)}(\Omega^{(0)}_{\rm isco}) \sim \epsilon^{-2/3}$$

For $z''_{(1)}(\Omega^{(0)}_{\rm isco})$ take inspiration from Colleoni+ (arXiv:1508.04031)

and split result into completion piece $z''^{\rm compl}_{(1)}(\Omega^{(0)}_{\rm isco}) \sim \epsilon^{-2/3}$

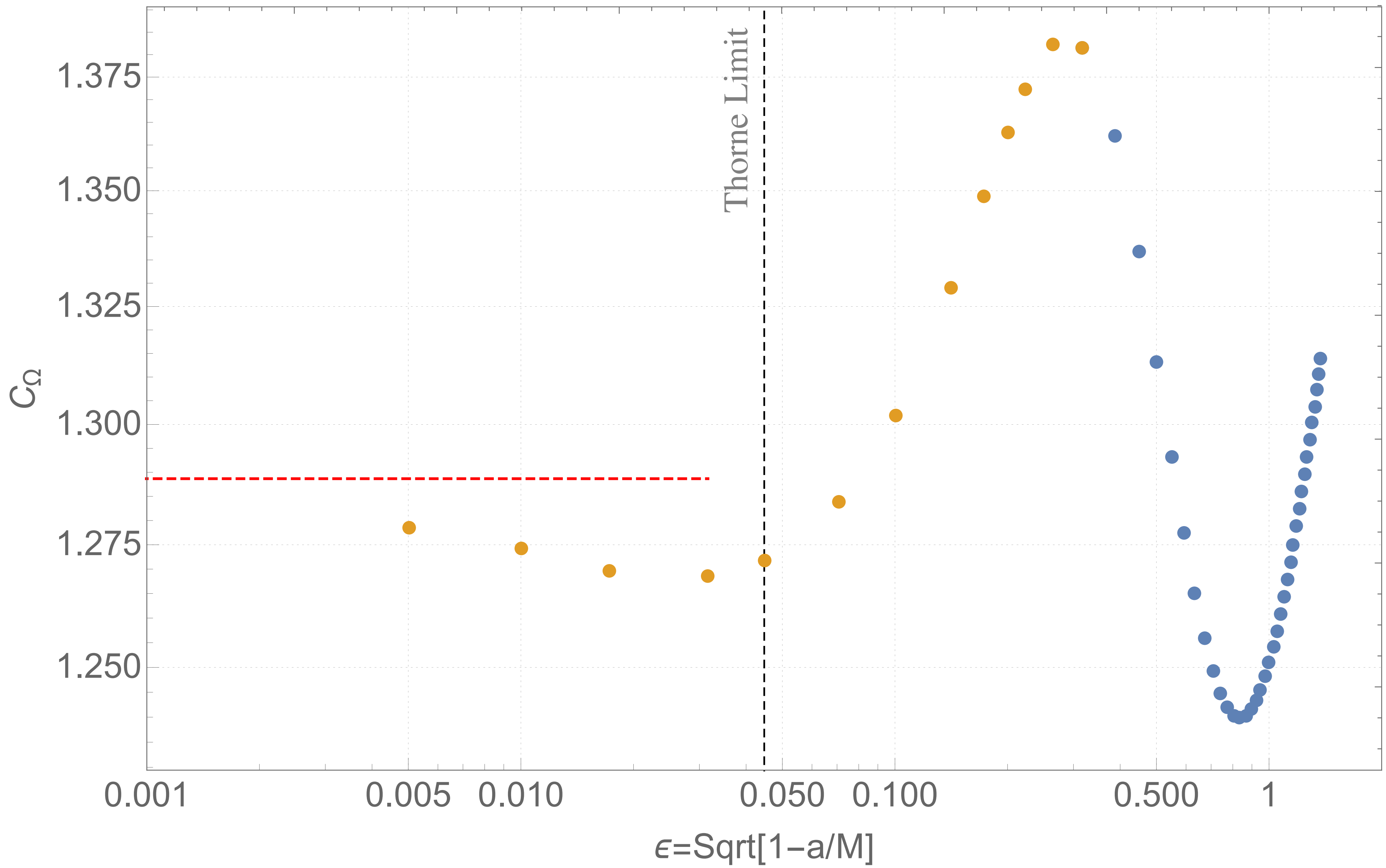
singular piece $z''^{S}_{(1)}(\Omega^{(0)}_{\rm isco}) \sim \rm const$

reconstruction piece

Preliminary numerical results suggest that $z''^{\rm recons}_{(1)}(\Omega^{(0)}_{\rm isco})$

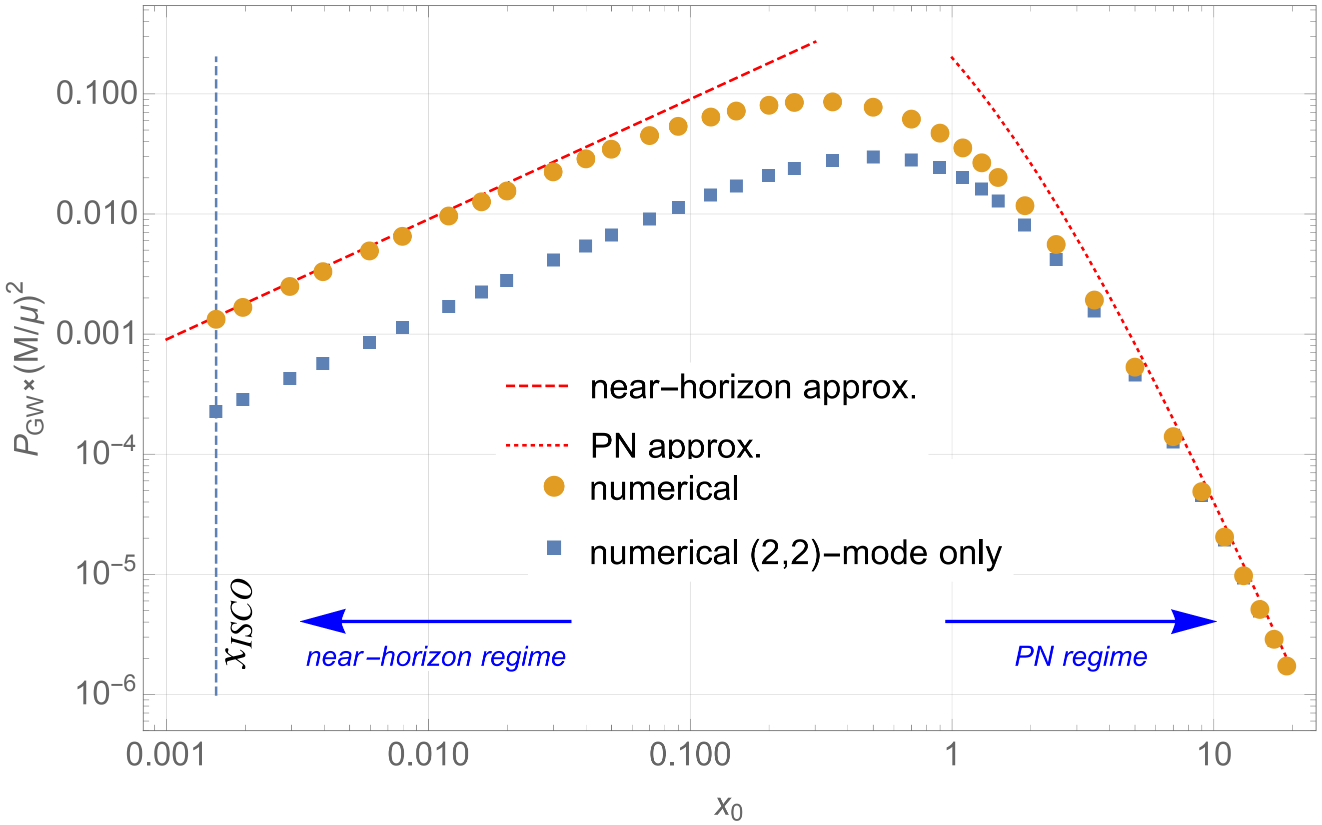
is finite, or at least doesn't diverge as fast as $\epsilon^{-2/3}$

Near-extremal Kerr ISCO shift

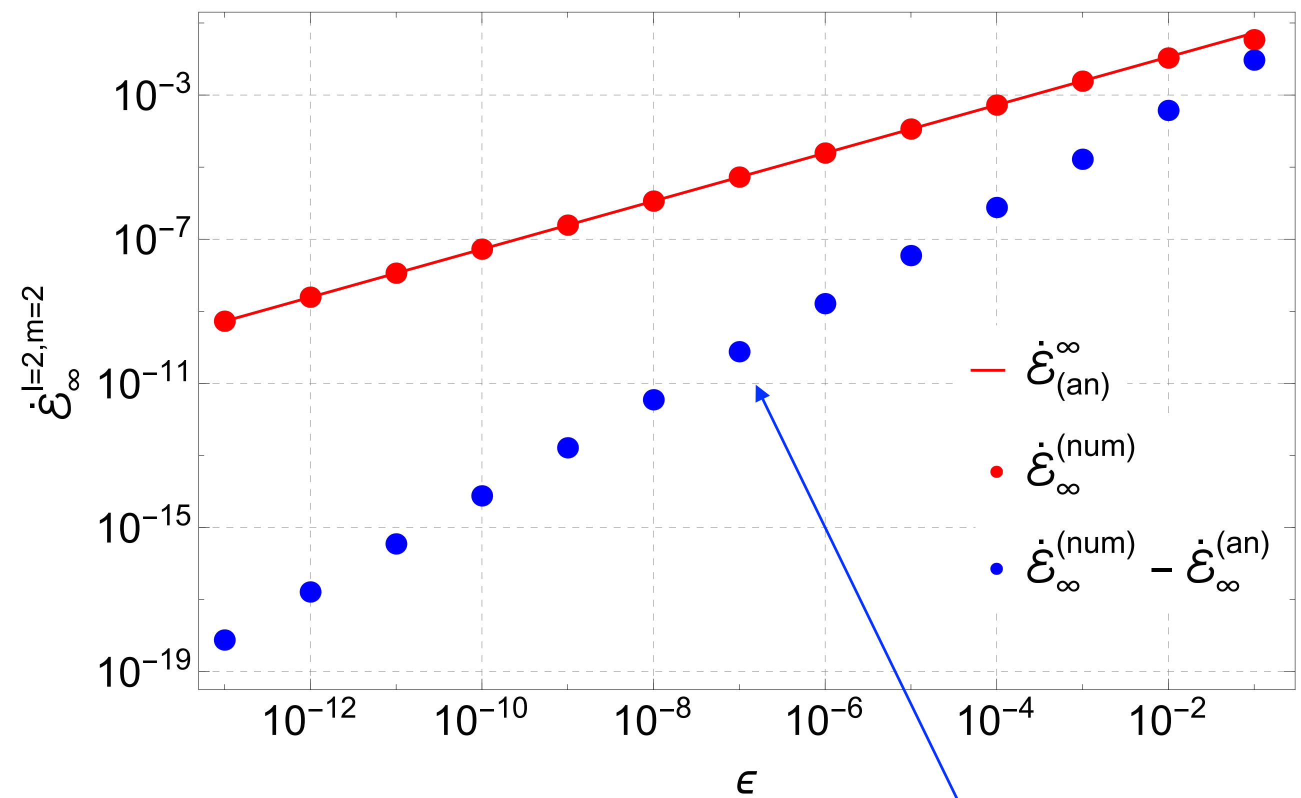


$$C_\Omega(\epsilon = 0) = 1 + \frac{1}{2\sqrt{3}}$$

Future directions



Gralla, Porfyriadis, NW, Phys. Rev. D 92, 064029



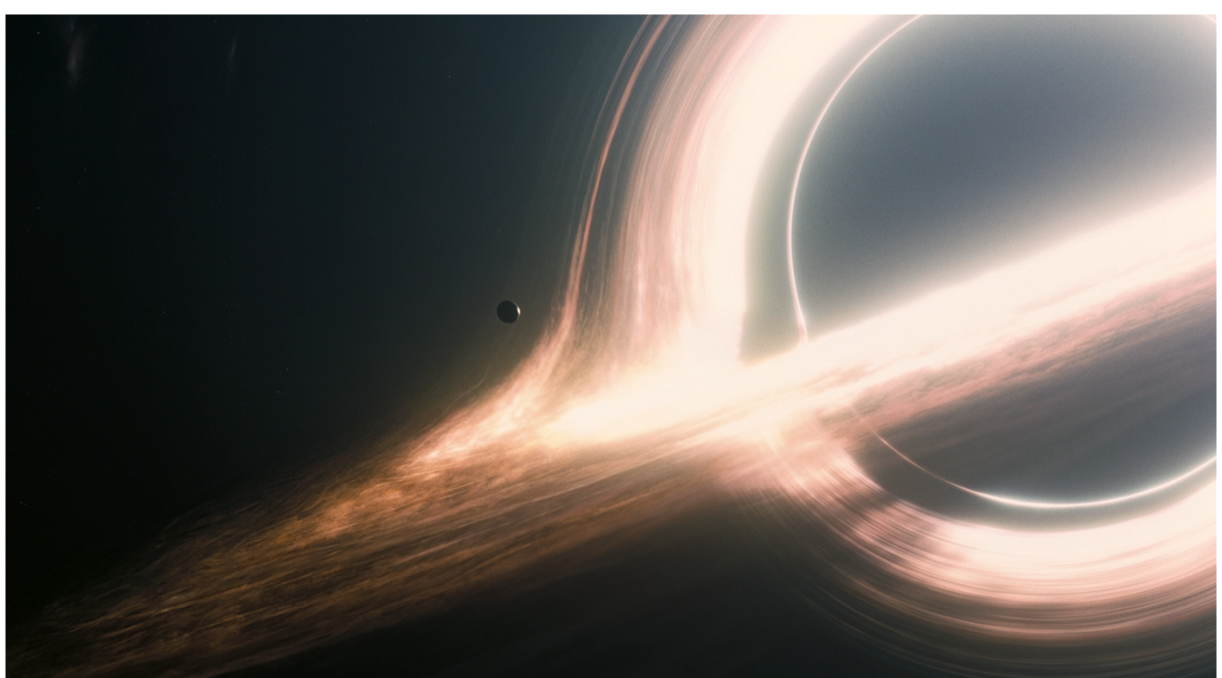
Analytically calculate next term: $P_{GW} = (C_\infty + C_H) x_0 + \mathcal{O}(x_0)^2$

Analytically calculate the ISCO shift

Add plunge and ringdown

Gravitational wave emission from an inspiral into a near-extremal Kerr black hole

- Characteristic exponential decay in waveform amplitude and rapid rise in the frequency: there is no chirp
- Signal robust to perturbations of the inspiral away from circular, equatorial
- Candidate source for eLISA and possibly ground based detectors
- Calculated the conservative ISCO shift for a near-extremal Kerr black hole



If Gargantua is out there eLISA just might find it