Gauge Invariant Perturbations of Petrov Type D Spacetimes

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Overview

- Motivation, I & II
- Schwarzschild & Kerr
- GHP
- Schwarzschild Examples
- New Results: Zerilli variable and equation
- Future

Motivation I

- Conduct a systematic approach to finding gauge invariant quantities in Type D spacetimes.
- Historically, Regge Wheeler and Zerilli variables were found in a specific gauge:
    - Their gauge invariance determined *post facto* by finding how to complete their description starting from the chosen gauge.
- Bardeen and Press equations constructed with gauge and tetrad invariance in mind, using the Newman-Penrose formalism?
- Imaginary part, and time derivative, of Ψ_2 discovered as afterthoughts?
- Steve Detweiler had developed a systematic approach in Schwarzschild.
    - Can we conduct a similar approach for Kerr, or general type D spacetimes?

Motivation II

- Stewart (Proc. Roy. Soc. …) showed that gauge invariant quantities must be zero on the background spacetime.
- Weinberg (Gravitation and Cosmology …) and Stephani et al (Exact Solutions …) both suggest that, in four dimensions, and with at most two derivatives acting on the metric, there should be 14 gauge invariant quantities.
- For what geometries are they linearly independent?
- What are they all, and how might they be useful?
- What perturbation equations might they obey?
- Do they propagate? Are they scalar? Are they real?

Schwarzschild

- The ten perturbed Einstein tensor components are gauge invariant, and are second order in derivatives.
- The perturbed Ψ_0 and Ψ_4 components of the Weyl tensor are gauge invariant, and are second order in derivatives, but are not real.
- The perturbation of the imaginary part of Ψ_2 is gauge invariant, and is second order in derivatives.
- The Regge-Wheeler and Zerilli variables are gauge invariant - but neither is second order in derivatives.

Kerr

- Already know that the perturbed Ψ_0 and Ψ_4 components of the Weyl tensor are gauge invariant, and are second order in derivatives.
    - Can we find others which would simplify self-force calculations in Kerr?
- The imaginary part of Ψ_2 is not zero, and its perturbation is not gauge invariant.
- The time derivative of Ψ_2 is zero, but its perturbation is (apparently) not gauge invariant.
- The geometrical structure of Kerr is similar to that of a large class of Petrov Type D spacetimes.
- Working at a geometrical level may help identify quantities of interest, which would be generally valid.
- The Geroch-Held-Penrose (GHP) formalism is ideal for this.

Geroch-Held-Penrose

- Advantages:
  - Based on Newman-Penrose spinor formalism.
    - Has richer derivatives, and hence fewer variables to consider.
  - Includes ‘, * and ˜ symmetries of the equations, so only need a core of one fourth the total number of equations.
  - Has Spin and Boost weight inherent — only quantities of identical spin and boost weight (ie, Type) can be combined.
    - This simplifies the number of possibilities for combining different quantities to eliminate variables and build up a gauge invariant quantity.
- Disadvantage - not so well known, but can be removed from final results.

Schwarzschild examples

- $$\psi = -\eth(\text{Þ}+\rho)h_{2,4} + \eth(\text{Þ}'+\rho')h_{1,4} + \eth'(\text{Þ}+\rho)h_{2,3} - \eth'(\text{Þ}'+\rho')h_{1,3} \equiv \Im(\dot{\Psi}_2)$$
- obeys $\left[-\eth\eth' + 3\Psi_2 + (\text{Þ} - 4\rho)(\text{Þ}' - 4\rho')\right]\psi = 0,$
- equivalent to the Regge-Wheeler equation, but
- $\psi_{\rm RW} \equiv (-\rho'\text{Þ} - \rho\text{Þ}' + 4\rho\rho')(-\eth\eth h_{4,4} + \eth'\eth' h_{3,3}) + 2(\eth\eth' - 2\Psi_2 - 2\rho\rho')(-\rho'\eth h_{1,4} - \rho\eth h_{2,4} + \rho'\eth' h_{1,3} + \rho\eth' h_{2,3})$
- Note: $\psi_{\rm RW} \sim -\rho'\text{Þ}\psi + \rho\text{Þ}'\psi$, is a time derivative of ψ.

New Results I: the Zerilli Variable

- $\Psi_2^{3/2}(\eth'\eth + \eth\eth' + 2\psi_2 - 4\rho\rho')\psi_z =$
  $(\eth'\eth + \eth\eth' - 4\Psi_2 - 4\rho\rho')(\eth'\eth + \eth\eth')(\rho\text{Þ}' - \rho'\text{Þ})h_{34}$
  $+ (\eth'\eth + \eth\eth' - 4\Psi_2 - 4\rho\rho')(\eth'\eth + \eth\eth')(\rho'^2 h_{11} - \rho^2 h_{22})$
  $+ 2(\rho\rho' - \Psi_2)(\eth'\eth + \eth\eth' - 4\Psi_2 - 4\rho\rho')$
  $(\rho'(\eth' h_{13} + \eth h_{14}) - \rho(\eth' h_{23} + \eth h_{24}))$
  $- 2\rho\rho'(\eth'\eth + \eth\eth' - 4\Psi_2 - 4\rho\rho')$
  $(\eth'\text{Þ}' h_{13} + \eth\text{Þ}' h_{14} - \eth'\text{Þ} h_{23} - \eth\text{Þ} h_{13})$
  $- (\eth'\eth + \eth\eth' + 2\psi_2 - 4\rho\rho')(\rho\text{Þ}' - \rho'\text{Þ})(\eth'\eth' h_{33} + \eth\eth h_{44})$
- notice the operator on the left hand side.

New Results II: the Zerilli Equation

- $(\eth'\eth + \eth\eth' + 2\psi_2 - 4\rho\rho')^2((\rho\text{Þ}' + \rho'\text{Þ})^2 - (\rho\text{Þ}' + \rho'\text{Þ})^2)\psi_Z =$
  $\rho\rho'((\eth'\eth + \eth\eth' + 2\psi_2 - 4\rho\rho')^2(2(\eth'\eth + \eth\eth' + 2\psi_2 - 4\rho\rho') + 5\rho\rho')$
  $+ 48\Psi_2^2(\eth'\eth + \eth\eth' + 2\psi_2 + 2\rho\rho')$
  $+ (\eth'\eth + \eth\eth' + 2\psi_2 - 4\rho\rho')\Psi_2(7(\eth'\eth + \eth\eth' + 2\psi_2 - 4\rho\rho') + 48\rho\rho'))\psi_Z$

- Note: this is really sixth order in derivatives.

- At least three other fourth order gauge invariants are known.

- Are they equivalent (subject to the vacuum perturbation equations)?

- What differential equations might they satisfy?

Future

- There should be two more second order, gauge invariant perturbations for the Kerr geometry.
- Are they unrelated and non-trivial?
- Are they independent from Ψ_0, Ψ_4 and $\delta G_{\mu\nu}$?
- What are they and how can they be found?
- What perturbation equations do they satisfy.
- Useful quantities may involve many more that two derivatives. How many should we be considering?