Extracting the redshift factor in binary black hole simulations

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Two body problem

Distance

Mass Ratio

Self-force

Numerical Simulations

Post-Newtonian
Two body problem

- Self-force
- Numerical Simulations
- Post-Newtonian

Mass Ratio

Distance

Szilagyi et al. (2015)
Comparisons between SF and NR

• Want to compare (pseudo-)invariants
• Independent verification of SF predictions
• Discovery: test regime of validity
• Compute higher order SF effects
• Calibrate analytic waveform models
What do we get from NR?

\[ \Omega \hat{\ell} = \frac{\vec{r} \times \dot{\vec{r}}}{r^2} \]

\[ \mathcal{R}_1, \mathcal{X}_1 \]

\[ \mathcal{R}_2, \mathcal{X}_2 \]

\[ A_1, \bar{\mathcal{X}}_1 \]

\[ A_2, \bar{\mathcal{X}}_2 \]

\[ \mathcal{R}_1, \mathcal{X}_1 \]

\[ h_{\ell m}, \dot{E}, \dot{J}, \dot{P} \]
Periastron Precession

\[ \Delta \Phi = 2\pi \left( \frac{\Omega_\phi}{\Omega_r} - 1 \right) \]

FIG. 1. Periastron advance extracted from numerical simulations.

A second important effect enters through the magnitude of the eccentricity, \( K \).

Residuals of the polynomial fits.

Upper panel: The dashed curves show that the fitting function is designed to capture, one of them with a non-zero spin. Each symbol represents a separate numerical binary black hole evolution. The results shown here were computed at the orbital frequency of the configuration.

Lower panel: The solid lines show polynomial fits to the eccentricities, eventually the eccentricity-dependent corrections increase roughly inversely proportionally to the eccentricity of the orbit. With decreasing eccentricity, this oscillatory term will be increasingly hard to isolate.

Le Tiec et al. (2013)
Redshift factor $z$

- Invariant quantity in SF and PN theories
- Wealth of connections: SF, PN, EOB
- Sims have extended bodies
- Interface w/ NR: connect to surface grav...
Redshift and surface gravity

\[ K^\mu = (\partial_t)^\mu + \Omega (\partial_\phi)^\mu \]

\[ K^t = 1 \]
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\[ \chi^\mu = K^\mu \]

\[ \chi^\mu \nabla_\mu \chi^\nu = \kappa \chi^\nu \]

\[ \bar{\chi}^\mu = u^\mu \]

\[ \bar{\chi}^\mu \nabla_\mu \bar{\chi}^\nu = \bar{\kappa} \bar{\chi}^\nu \]
Redshift and surface gravity

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\[ \bar{\chi}^\mu \nabla_\mu \bar{\chi}^\nu = \bar{\kappa} \bar{\chi}^\nu \]
\[ z = \frac{\kappa}{\bar{\kappa}} \]

\[ u^\mu = z^{-1} K^\mu \]
Redshift in NR

- Normalization absent
- Consider rescaling
  \[ l^\mu \rightarrow \alpha l^\mu \quad \kappa(l) \rightarrow \alpha \kappa(l) \]
- Rescaling invariant \( z \)
  \[ z = \frac{\kappa(l)}{l^t \bar{\kappa}} \quad z \rightarrow z \]
- Don’t access EH: use AH

\[ \langle z \rangle = \frac{\int dA \ z(\theta^B)}{A} \]
Approximate HKV

• Quasicircular inspirals only have approximate HKV

\[ \nabla_{(\mu} l_{\nu)} \neq 0 \]

• Generators shear, horizon grows

\[ \sigma_{\mu\nu} \sigma^{\mu\nu} \sim \dot{m} \]

\[ \dot{m} \sim 10^{-9} \implies |\sigma| \sim 10^{-4} - 10^{-5} \]

• Other errors: no corotation, nonadiabatic

\[ \frac{\dot{\Omega}}{2\Omega^2} \sim 10^{-2} - 10^{-4} \quad \quad \Omega_H^2 \sim \Omega^2 \sim 10^{-2} - 10^{-4} \]
Redshift factor in NR

![Diagram showing redshift factor as a function of $m\Omega_{\text{ISCO}}$. The $z$ axis represents the redshift, ranging from 0.65 to 0.95, and the $m\Omega$ axis ranges from 0.02 to 0.10. Various lines indicate different mass ratios: $m_1/m = 7/9$, $m_1/m = 5/7$, $m_1/m = 1/2$, $m_2/m = 2/7$, and $m_2/m = 2/9$. There are also symbols indicating the points where $m_2 = 0$.](image-url)
Redshift vs PN

\[ \frac{\Delta z}{z} \times 10^3 \]

- \( m_1/m = 1/2 \)
- \( m_1/m = 5/7 \)
- \( m_2/m = 2/7 \)
- \( m_1/m = 7/9 \)
- \( m_2/m = 2/9 \)

\[ m \Omega \]

\[ m \Omega_{\text{ISCO}} \]
First law of binary black holes

• First law: thermodynamic relation for BHs

\[ \delta M - \Omega_H \delta J = \kappa \frac{\delta A}{8\pi} \]

• Modified relations for circular binaries

\[ \delta M - \Omega_H \delta J = \kappa_1 \frac{\delta A_1}{8\pi} + \kappa_2 \frac{\delta A_2}{8\pi} = z_1 \delta m_1 + z_2 \delta m_2 \]

\[ Q = M - 2\Omega J = z_1 m_1 + z_2 m_2 \]

• Connect local and global properties, lower and higher orders in SF
• Used in SF, PN, EOB
• Can test with our numerical z
First law of binary black holes

\[ \Delta Q = z_1 m_1 + z_2 m_2 - (M - 2\Omega J) \]
Redshift vs SF

Preliminary
Redshift vs SF

![Graph showing the relationship between redshift (z) and magnetic field strength (mΩ) with different q values: q = 2/7 (red), q = 1/7 (blue), q = 1/9 (orange), and q = 1/10 (green). The graph is labeled as Preliminary.]
Redshift vs SF

Preliminary

\[ \Delta z \times 10^3 \]

![Graph showing redshift vs SF with different line styles and colors for various parameters: q = 2/7, q = 1/7, q = 1/9, and q = 1/10.](image)
Summary and outlook: Redshift

- Extracted redshift in NR
- Confirmed first law for binaries to 1:1000
- Higher mass ratios, high order SF
- Spinning, eccentric binaries
- Testing and extending first law of binary black holes
Outlook: SF and NR

- Already done by others: periastron advance, E(J)
- Redshift in infancy
- Self torque (hard)
- Other frequency shifts
- Tidal invariants
- Everything with spin, eccentricity
- Pushing to high mass ratios key