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Regularisation via the Detweiler-Whiting singular field (revisited)

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Outline

- Detweiler-Whiting singular field
- Motivation
- Self-force methods
- To date
- Calculating the singular field
- Testing the singular field
- Summary / still to do

Detweiler-Whiting Singular Field

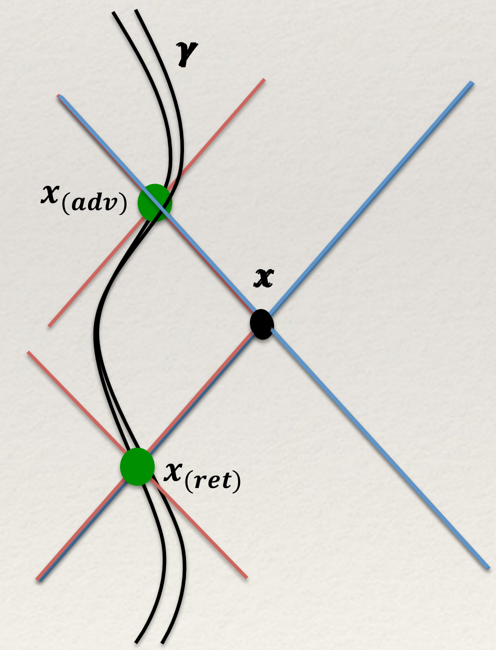
- Wave equation for scalar field

$$(\Box - \xi R)\,\Phi(x) = -4\pi\mu(x), \qquad \mu(x) = q\int_\gamma \delta_4(x,z)\,d\tau$$

- Use Green function to solve

$$\Phi(x) = q\int G(x,z)\,d\tau, \quad \Rightarrow \quad (\Box - \xi R)\,G(x,x') = -4\pi\delta_4(x,x'),$$

- Want to remove singular part of field that
  - Fully contains the singular behaviour
  - Does not affect the self force or EOM



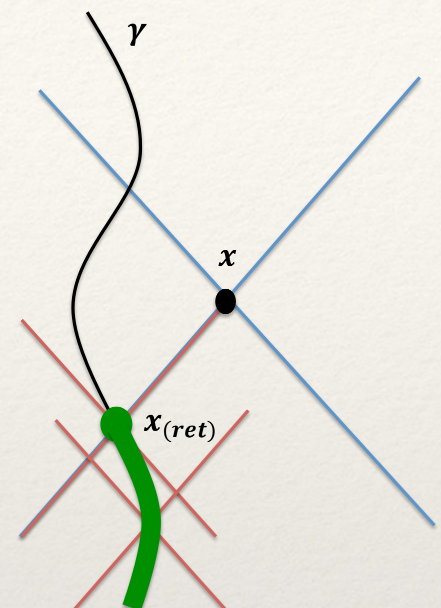
$$\Phi_{(ret)} = \Phi_{(S)} + \Phi_{(R)}.$$

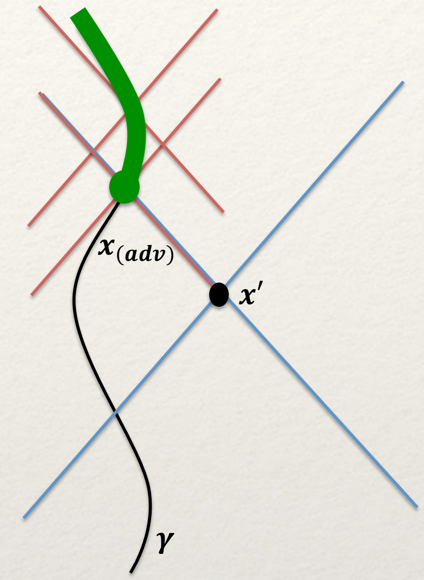
- In flat spacetime:

$$\Phi_{(S)} = \frac{1}{2}\left[\Phi_{(ret)} + \Phi_{(adv)}\right], \qquad \Phi_{(R)} = \frac{1}{2}\left[\Phi_{(ret)} - \Phi_{(adv)}\right].$$

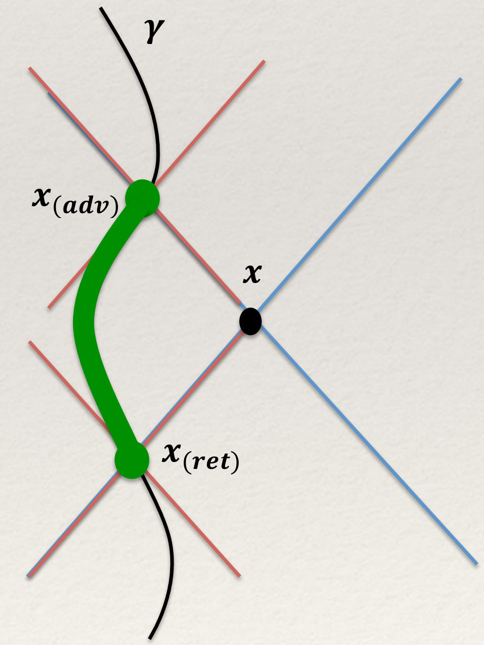
Detweiler-Whiting Singular Field

- But in curved spacetime:
- H must satisfy homogeneous wave equation,
- Must maintain reciprocity in respect to x and x'





$$G_{(ret)}(x,x') = G_{(adv)}(x',x)$$



$$G_{(S)}\left(x,x'\right) = \frac{1}{2}\left[G_{(ret)}\left(x,x'\right) + G_{(adv)}\left(x,x'\right) - H\left(x,x'\right)\right]$$

$$G_{(R)}\left(x,x'\right) = \frac{1}{2}\left[G_{(ret)}\left(x,x'\right) - G(adv)\left(x,x'\right) + H\left(x,x'\right)\right].$$

$$H\left(x,x'\right) = G_{(adv)}\left(x,x'\right), \quad x \in I^{-}(x'),$$

$$H\left(x,x'\right) = G_{(ret)}\left(x,x'\right), \quad x \in I^{+}(x').$$

$H(x,x')$

$$G_{(ret/adv)}(x,x') = U(x,x')\delta_{+/-}(\sigma) - V(x,x')\Theta_{+/-}(-\sigma)$$

$$(\Box - \xi R)\, G(x,x') = -4\pi\delta_4\left(x,x'\right)$$

$$G_{(S)}(x,x') = \frac{1}{2}\left[U(x,x')\delta(\sigma) - V(x,x')\Theta(\sigma)\right]$$

Detweiler-Whiting Singular Field

The scalar singular field and self-force

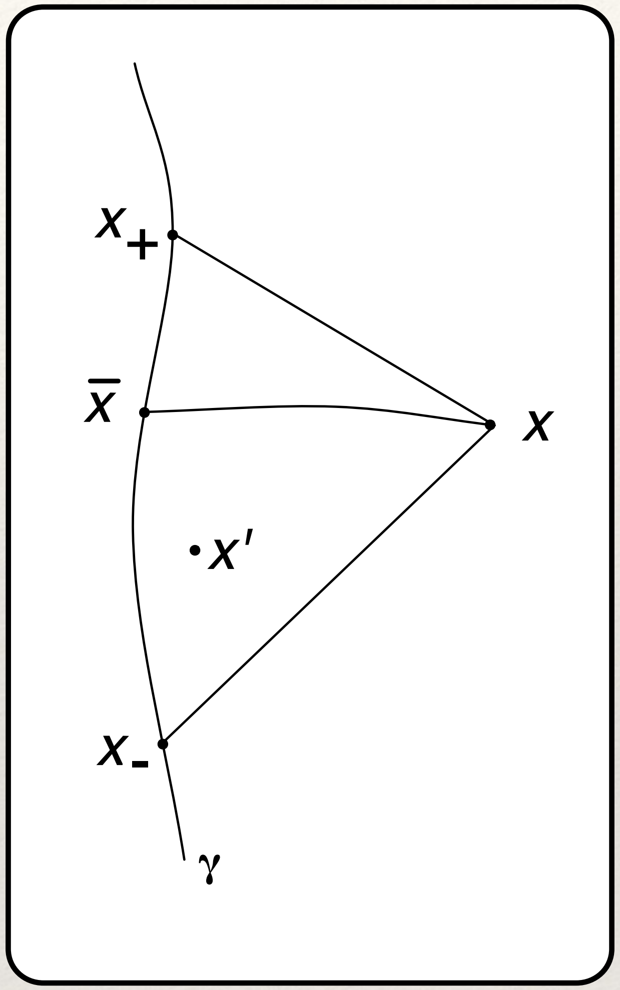
$$\Phi^{(\mathrm{S})}(x) = \left[\frac{U(x,x')}{2\sigma_{a'}u^{a'}}\right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V(x,z(\tau))d\tau$$

$$f^a = g^{ab}\Phi^{(\mathrm{R})}{}_{,b}.$$

The EM singular field and self-force

$$A_a^{\mathrm{S}} = \left[\frac{u^{a'}U^a{}_{a'}(x,x')}{2\sigma_{a'}u^{a'}}\right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V^a{}_{a'}(x,z(\tau))u^{a'}d\tau$$

$$f^a = g^{ab}u^c A^{(\mathrm{R})}{}_{[c,b]}.$$



The gravitational singular field and self-

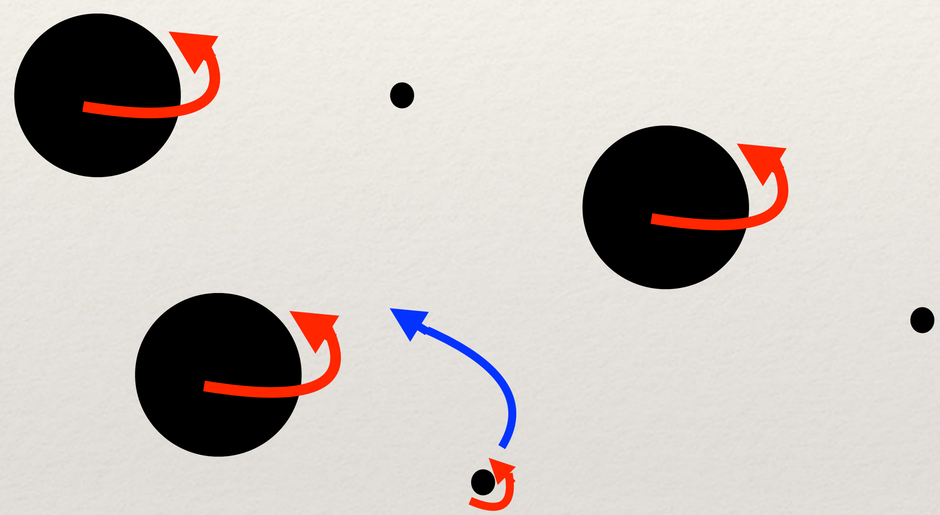
$$\bar{h}^S_{ab} = \left[\frac{u^{a'}u^{b'}U^{ab}{}_{a'b'}(x,x')}{2\sigma_{a'}u^{a'}}\right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V^{ab}{}_{a'b'}(x,z(\tau))u^{a'}u^{b'}d\tau \qquad \text{and} \qquad f^a = k^{abcd}\bar{h}^{(R)}_{bc;d},$$

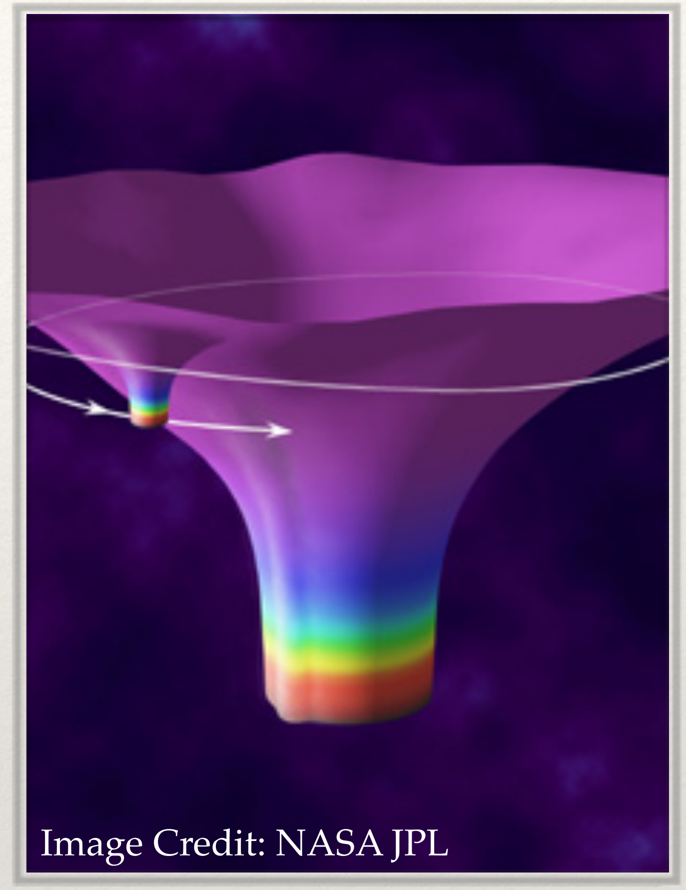
where

$$k^{abcd} \equiv \frac{1}{2}g^{ad}u^bu^c - g^{ab}u^cu^d - \frac{1}{2}u^au^bu^cu^d + \frac{1}{4}u^ag^{bc}u^d + \frac{1}{4}g^{ad}g^{bc}.$$

Motivation

- Extreme Mass Ratio Inspirals
- SF: Evolved spinning black holes, generic orbit
  - Kerr
  - Inclined
  - Non-geodesic





- Regularization: Numerical efficiency → waveform template bank
  - Higher order
  - → Increased efficiency

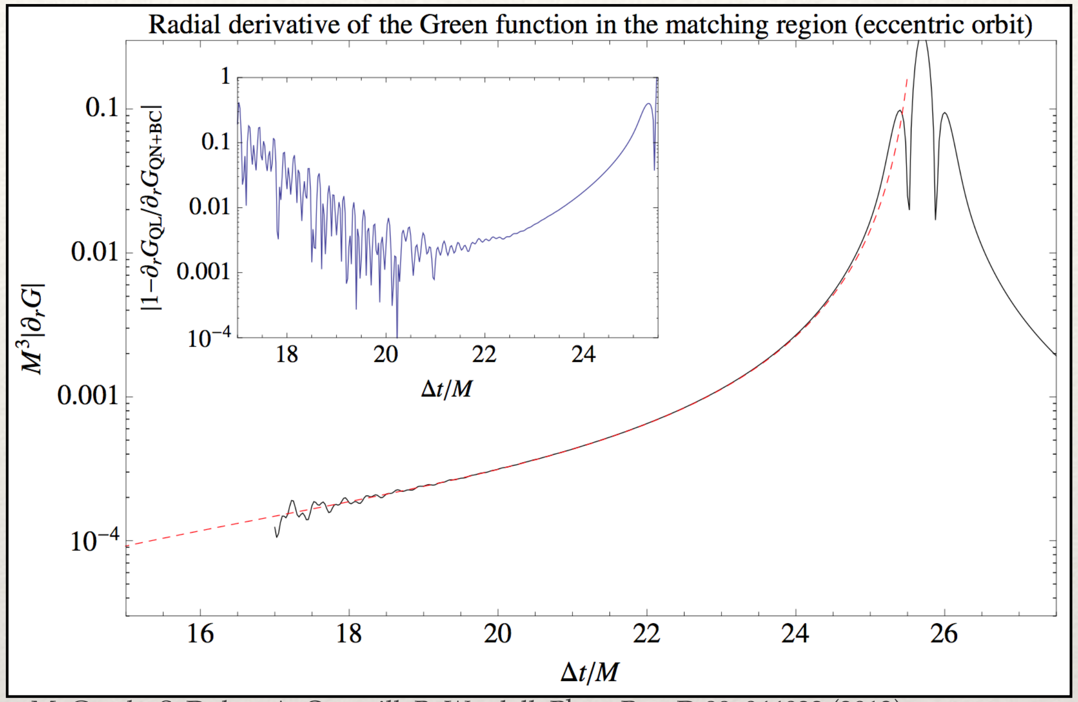
	$l_{\max} = 25, n = 12$		$l_{\max} = 80, n = 50$	
RPs used	abs.	rel.	abs.	rel.
AB	$1.3784482573 \times 10^{-5}$	1.2×10^{-10}	$1.37844825756674 \times 10^{-5}$	3.7×10^{-14}
ABD	$1.37844825757 \times 10^{-5}$	5.0×10^{-12}	$1.378448257566791 \times 10^{-5}$	3.3×10^{-15}
$ABDF$	$1.378448257567 \times 10^{-5}$	4.2×10^{-13}	$1.378448257566793 \times 10^{-5}$	1.7×10^{-15}
$ABDFH$	$1.37844825756675 \times 10^{-5}$	3.0×10^{-14}	$1.3784482575667951 \times 10^{-5}$	5.5×10^{-16}
CPU time	155s		4247s	

Credit: N. Warburton

Self-Force Methods

- Matched Expansions
  (Wiseman, Poisson, 1998)

$$f_\mu = q^2 \nabla_\mu \left[\int_{\tau_m}^{\tau^-} d\tau' V(z(\tau), z(\tau')) + \int_{-\infty}^{\tau_m} d\tau' G_{ret}(z(\tau), z(\tau')) \right]$$



M. Casals, S. Dolan, A. Ottewill, B. Wardell, Phys. Rev. D 88, 044022 (2013)

- Mode-Sum
  (Barck,Ori 2001)

$$f_a^l(r_0, t_0) = \lim_{\Delta r \to 0} \sum_m f_a^{lm}(r_0 + \Delta r, t_0) Y^{lm}(\alpha_0, \beta_0)$$

$$= \frac{2l+1}{4\pi} \lim_{\Delta r \to 0} \int f_a(r_0 + \Delta r, t_0, \alpha, \beta) P_l(\cos\alpha) d\Omega$$

- Effective Source
  Vega, Detweiler, Goldburn, Barack (2007)

$$\varphi^A_{(\mathrm{ret})} = \tilde{\varphi}^A_{(\mathrm{S})} + \tilde{\varphi}^A_{(\mathrm{R})}$$

$$S_{eff} = \mathcal{D}^A{}_B \tilde{\varphi}^B_{(\mathrm{R})} = -4\pi \mathcal{Q} \int u^A \delta_4(x, z(\tau')) d\tau' - \mathcal{D}\tilde{\varphi}^A_{(\mathrm{sing})}$$

To date …

- Barack & Ori (1999-2002)
  - Introduction of mode-sum scheme (direct and tail field split)
  - Regularisation parameters (first 2) for Schwarzschild & Kerr for scalar, EM and gravitational cases
- Detweiler, Whiting & Messaritaki (2003)
  - Regular-singular field split via Detweiler-Whiting singular field
  - Regularisation parameters (first 4) and self-force for Schwarzschild scalar circular orbit case
- Haas & Poisson (2006)
  - Covariant expansion with tetrad decomposition
  - Regularisation parameters (first 4) and self-force in Schwarzschild scalar & EM eccentric orbit case
- Heffernan, Ottewill, Wardell (2012-2014)
  - Covariant and coordinate expansions of singular field in Riemann Normal coordinates
  - Additional 14 regularisation parameters in Schwarzschild & Kerr, scalar, EM & gravity (geodesic)
  - First non-geodesic parameters in Schwarzschild scalar
- Linz, Friedmann, Wisemann (2014)
  - Non-geodesic parameters (first 2) for Schwarzschild & Kerr, Scalar, EM and gravitational

To date …

GEODESIC

Cases	$F_{a[-1]}$	$F_{a[0]}$	$F_{a[2]}$	$F_{a[4]}$	$F_{a[6]}$
Schwarzschild scalar	BO	BO	DMW / HP	HOW	HOW
Schwarzschild electromagnetic	BO	BO	HP	HOW	HOW
Schwarzschild gravity	BO	BO	HOW	HOW	—
Schwarzschild *huu*	—	BO	HOW	HOW	—
Kerr scalar	BO	BO	HOW	HOW	—
Kerr electromagnetic	BO	BO	HOW	—	—
Kerr gravity	BO	BO	HOW	—	—
Kerr *huu*	—	HOW	HOW	—	—

ONLY EQUATORIAL ORBITS CONSIDERED!

To date …

NONGEODESIC

Cases	$F_{a[-1]}$	$F_{a[0]}$	$F_{a[2]}$	$F_{a[4]}$	$F_{a[6]}$
Schwarzschild scalar	HOW	HOW	HOW		
Schwarzschild electromagnetic	LFW	LFW			
Schwarzschild gravity	LFW	LFW			
Schwarzschild huu					
Kerr scalar	LFW	LFW			
Kerr electromagnetic	LFW	LFW			
Kerr gravity	LFW	LFW			
Kerr huu					

The scalar singular field and self-force are

$$\Phi^{(\mathrm{S})}(x) = \left[\frac{U(x,x')}{2\sigma_{a'}u^{a'}}\right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V(x,z(\tau))d\tau$$

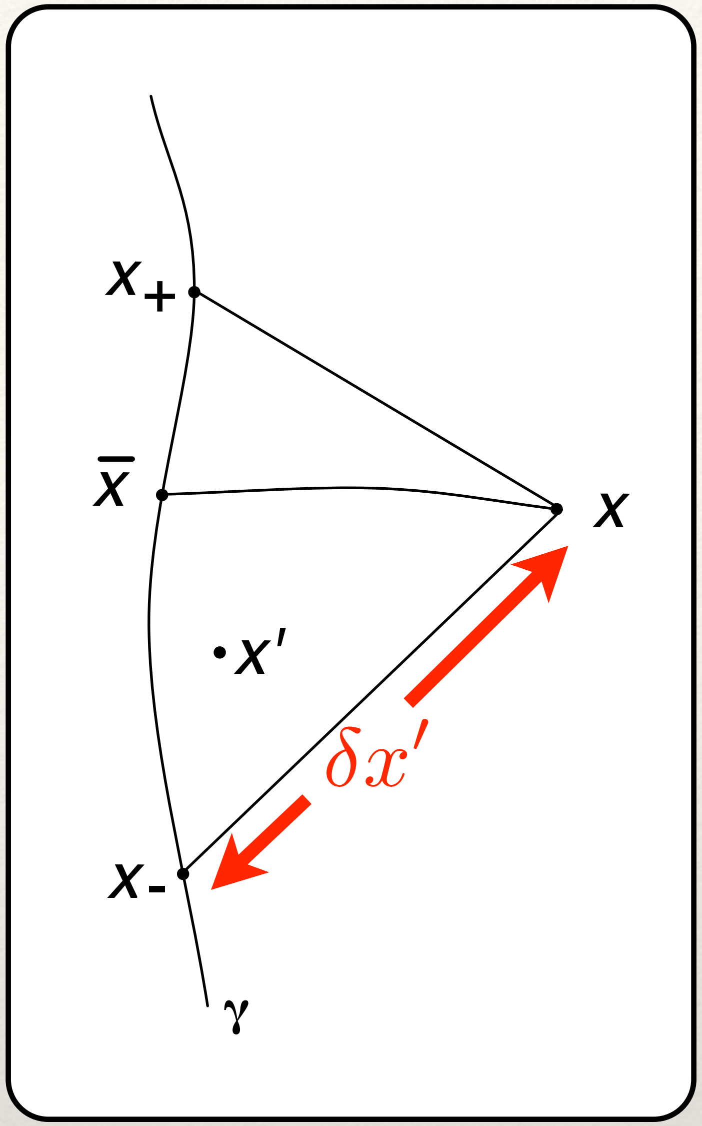
$$f^a = g^{ab}\Phi^{(\mathrm{R})}{}_{,b}.$$

The EM singular field and self-force are

$$A_a^{\mathrm{S}} = \left[\frac{u^{a'}U^a{}_{a'}(x,x')}{2\sigma_{a'}u^{a'}}\right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V^a{}_{a'}(x,z(\tau))u^{a'}d\tau$$

$$f^a = g^{ab}u^c A^{(\mathrm{R})}{}_{[c,b]}.$$

$$2\sigma = \sigma_{a'}\sigma^{a'}$$



The gravitational singular field and self-force are

$$\bar{h}^S_{ab} = \left[\frac{u^{a'}u^{b'}U^{ab}{}_{a'b'}(x,x')}{2\sigma_{a'}u^{a'}}\right]_{x'=x_-}^{x'=x_+} + \int_{\tau_-}^{\tau_+} V^{ab}{}_{a'b'}(x,z(\tau))u^{a'}u^{b'}d\tau \qquad \text{and} \qquad f^a = k^{abcd}\bar{h}^{(R)}_{bc;d},$$

where

$$\sigma(x,x') = \tfrac{1}{2}g_{ab}(x)\delta x^{a'}\delta x^{b'} + A_{abc}(x)\delta x^{a'}\delta x^{b'}\delta x^{c'} + \ldots$$

$$k^{abcd} \equiv \frac{1}{2}g^{ad}u^b u^c - g^{ab}u^c u^d - \frac{1}{2}u^a u^b u^c u^d + \frac{1}{4}u^a g^{bc}u^d + \frac{1}{4}g^{ad}g^{bc}.$$

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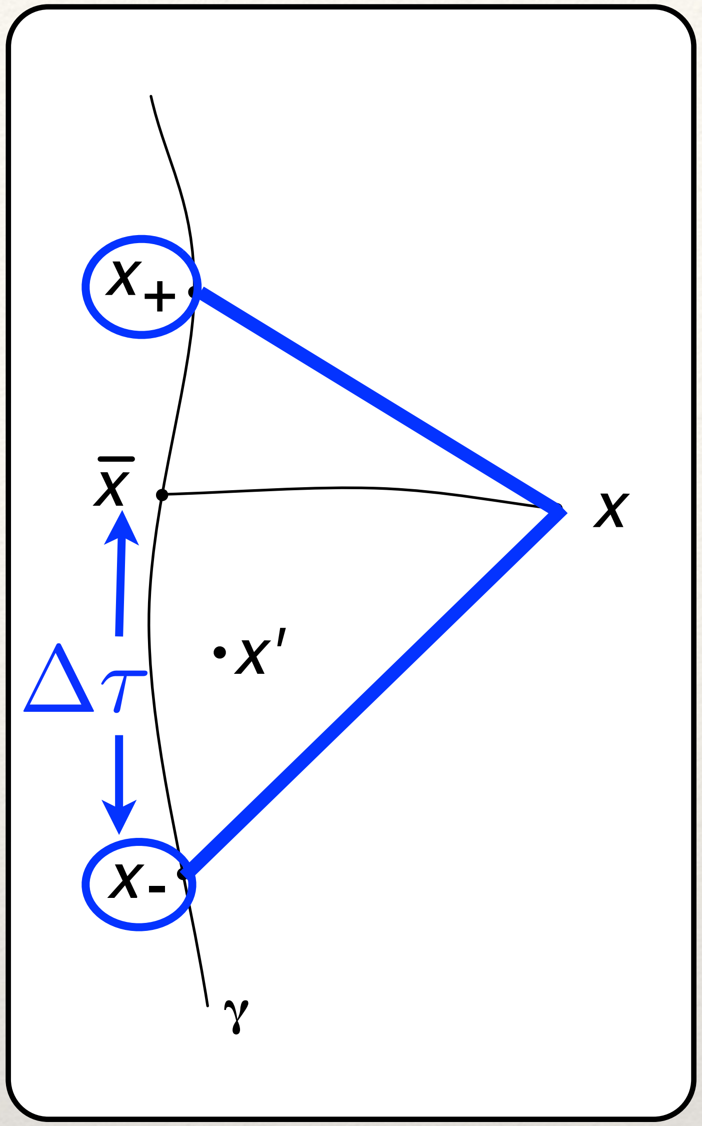
$$f^a = g^{ab}u^c A^{(\mathrm{R})}{}_{[c,b]}.$$

$$\sigma = 0$$



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where

$$x^{a'}(\tau) = x^{\bar{a}} + u^{\bar{a}}\Delta\tau + \frac{1}{2!}\dot{u}^{\bar{a}}\Delta\tau^2 + \cdots$$

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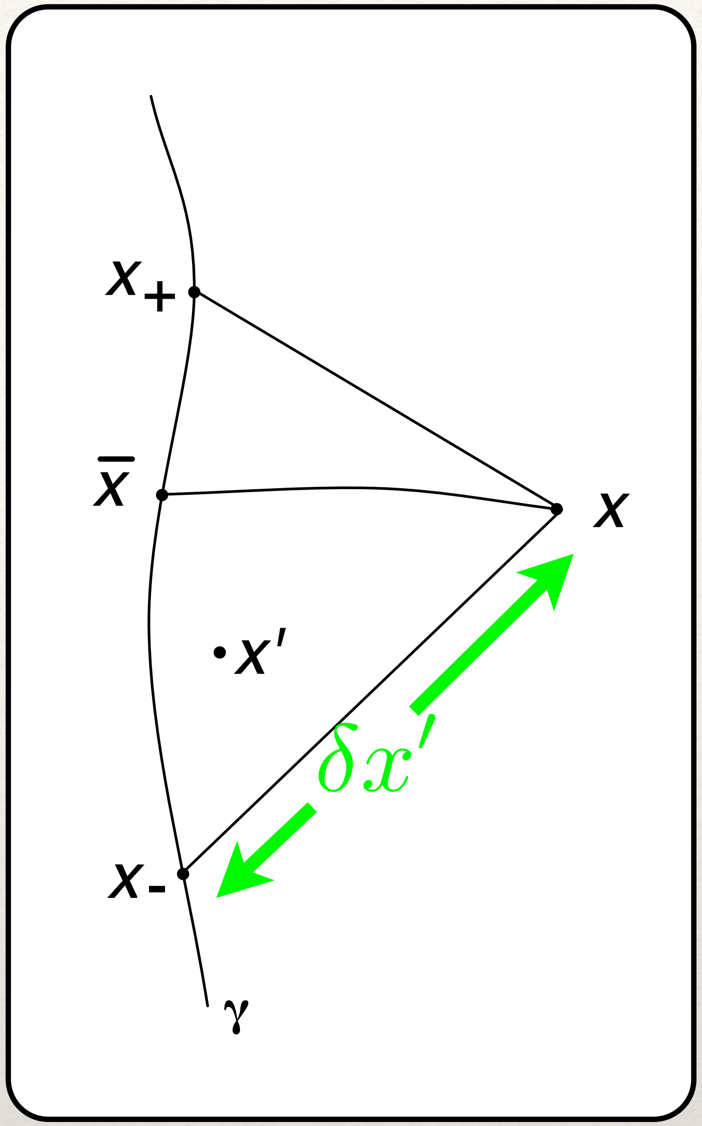
$$f^a = g^{ab}u^cA^{(\mathrm{R})}{}_{[c,b]}.$$

$$g_{ab';c}\sigma^c = 0$$



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where

$$\Delta^{\frac{1}{2}}(x,x') = \left(-\left[-g(x)\right]^{-\frac{1}{2}}\left|-\sigma_{a'b}(x,x')\right|\left[-g(x')\right]^{-\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$k^{abcd} \equiv \frac{1}{2}g^{ad}u^bu^c - g^{ab}u^cu^d - \frac{1}{2}u^au^bu^cu^d + \frac{1}{4}u^ag^{bc}u^d + \frac{1}{4}g^{ad}g^{bc}.$$

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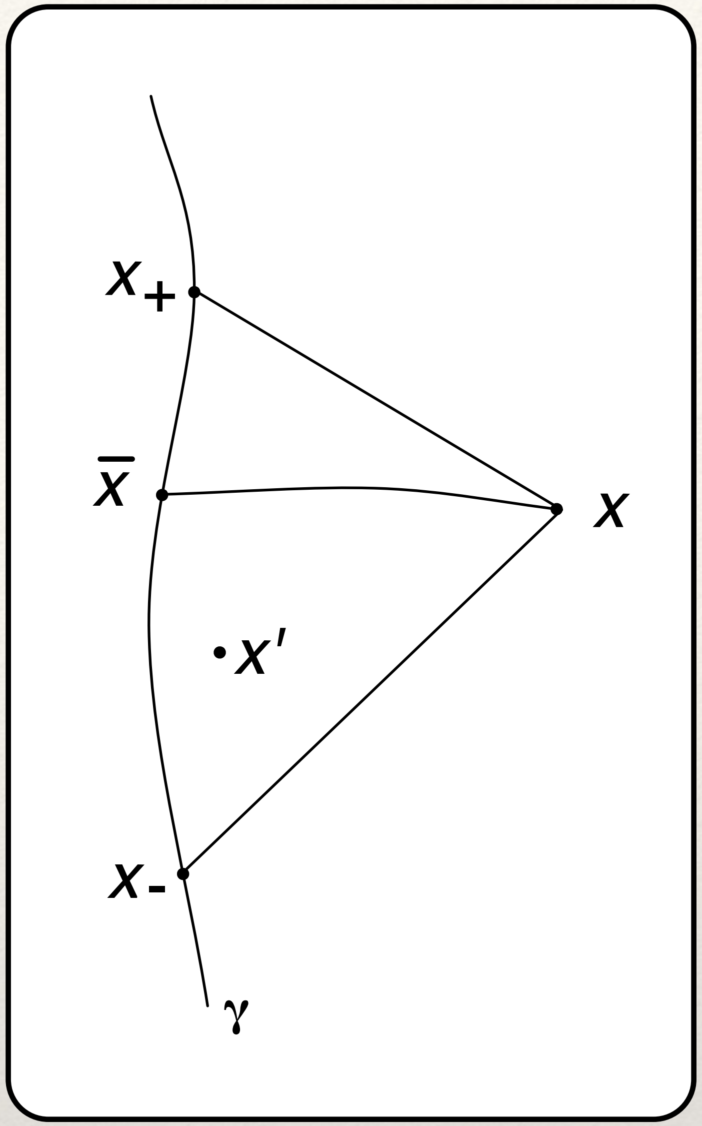
$$f^a = g^{ab}u^c A^{(\mathrm{R})}{}_{[c,b]}. \quad V^{AB'}(x,x') = \sum_{n=0}^{\infty} V_n{}^{AB'}(x,x')\,\sigma^n(x,x')$$



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where

$$\sigma^{;\alpha'}\left(\Delta^{-1/2}V_n^{AB'}\right)_{;\alpha'} + (n+1)\,\Delta^{-1/2}V_n^{AB'} + \frac{1}{2n}\Delta^{-1/2}\mathcal{D}^{B'}{}_{C'}V_{n-1}^{AC'} = 0$$

$$k^{abcd} \equiv \frac{1}{2}g^{ad}u^b u^c - g^{ab}u^c u^d - \frac{1}{2}u^a u^b u^c u^d + \frac{1}{4}u^a g^{bc}u^d + \frac{1}{4}g^{ad}g^{bc}.$$

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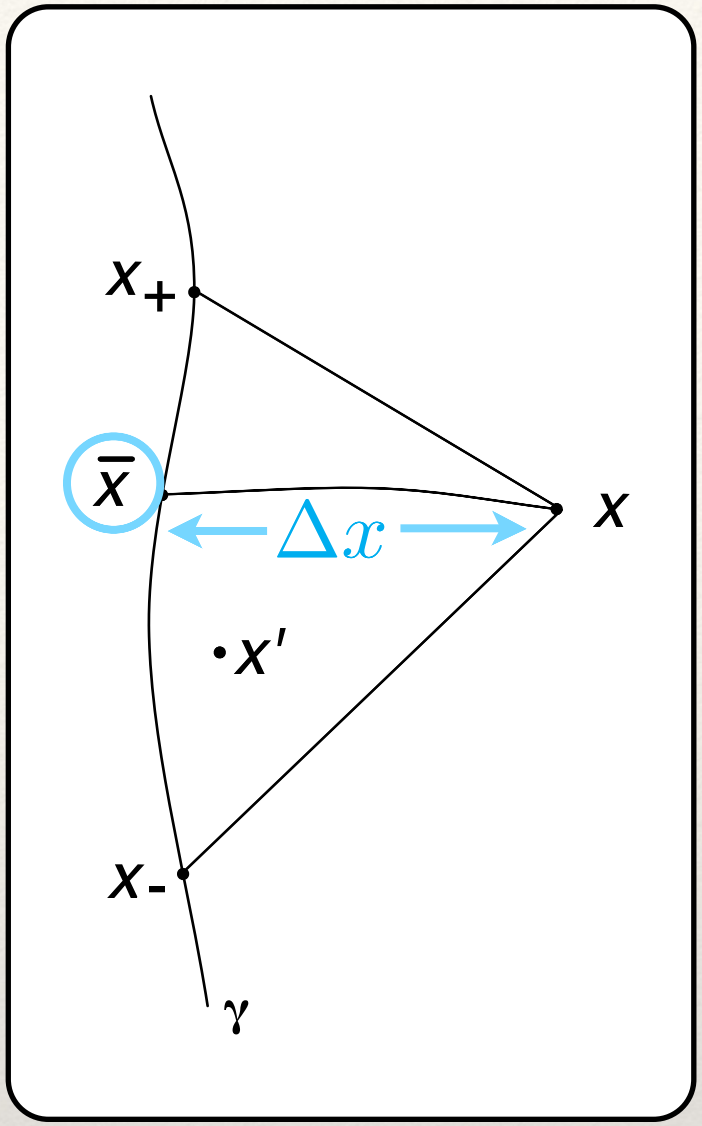

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Non-geodesic motion: $u^\beta\nabla_\beta(mu^\alpha) = F^\alpha = \mu a^\alpha + \frac{dm}{d\tau}u^\alpha$

Scalar Singular Field

- Covariant: $$A(\tau) = A(\bar\tau) + \dot A(\bar\tau)(\tau-\bar\tau) + \frac{1}{2}\ddot A(\bar\tau)(\tau-\bar\tau)^2 + \cdots \,,$$

$$\Delta\tau_\pm = (\bar r \pm \bar s) \mp \frac{(\bar r \pm \bar s)^2}{6\,\bar s} R_{u\sigma u\sigma} \mp \frac{(\bar r \pm \bar s)^2}{24\,\bar s}\left[(\bar r \pm \bar s) R_{u\sigma u\sigma|u} - R_{u\sigma u\sigma|\sigma}\right] + \mathcal{O}(\epsilon^5).$$

- Covariant coordinate:

$$\sigma(x,x') = \tfrac{1}{2} g_{ab}(x)\delta x^{a'}\delta x^{b'} + A_{abc}(x)\delta x^{a'}\delta x^{b'}\delta x^{c'} + B_{abcd}(x)\delta x^{a'}\delta x^{b'}\delta x^{c'}\delta x^{d'} +$$

- Singular Field

$$\Phi_S = q\left\{\frac{1}{\bar s} + \frac{\bar r^2 - \bar s^2}{6\bar s^3} C_{u\sigma u\sigma} + \frac{1}{24\bar s^3}\left[(\bar r^2 - 3\bar s^2)\bar r C_{u\sigma u\sigma;u} - (\bar r^2 - \bar s^2) C_{u\sigma u\sigma;\sigma}\right] + \mathcal{O}(\epsilon^3)\right\}$$

- Non-geodesic

$$\Phi_{[-1]} = \frac{1}{\rho} \qquad \rho^2 = (\bar u_{ab} + g_{ab})\,\Delta x^{ab}$$

$$\Phi_{[0]} = \frac{1}{\rho^3}\left(\frac{1}{2}\bar A^b \Delta x_{bcd}\bar u^{cd} - \frac{1}{2}\Gamma_{bcd}\Delta x^{bcd} - \frac{1}{2}\Gamma_{cde}\Delta x_b^{de}\bar u^{bc}\right) - \frac{1}{\rho}\bar A^b \Delta x_b$$

Scalar Singular Field

$$\Phi_{[1]} = \rho\left[\frac{1}{2}\dot{\bar{A}}^b\bar{u}_b + \frac{3}{8}\bar{A}_b\bar{A}^b - \frac{1}{12}\Gamma_{fde}\left(\Gamma_{bc}{}^f + \Gamma^f{}_{bc}\right)\bar{u}^{bcde}\right]$$
$$+ \frac{1}{\rho}\Big[\dot{\bar{A}}^b\left(\bar{u}_b{}^{cd}\Delta x_{cd} + \frac{1}{2}\Delta x_{bc}\bar{u}^c\right) + \frac{1}{6}\left(\Gamma_{bde,c} + 2\Gamma_{dbe,c} - \Gamma_{bcd,e} - 2\Gamma_{dbc,e} + \Gamma^f{}_{bc}\Gamma_{fde} - \Gamma^f{}_{bd}\Gamma_{fce}\right)\bar{u}^{bc}\Delta x^{de}$$
$$+ \bar{A}^b\left(\frac{3}{8}\bar{A}^c\Delta x_{bc} + \frac{3}{4}\bar{A}_b\bar{u}^{cd}\Delta x_{cd} - \frac{1}{4}\Delta x^{cd}\Gamma_{bcd}\right) + \frac{1}{4}\left(4\Gamma_{cdf,e} - 4\Gamma_{cde,f} + \Gamma_{fc}{}^i\Gamma_{ide} + \Gamma^i{}_{cd}\Gamma_{ief}\right)\bar{u}^{bcde}\Delta x_b{}^f\Big]$$
$$+ \frac{1}{\rho^3}\Big\{ - \frac{1}{6}\dot{\bar{A}}^b\bar{u}^{cde}\Delta x_{cde}\left(\Delta x_b + \bar{u}_b{}^f\Delta x_f\right)$$
$$- \frac{1}{8}\bar{A}^b\Delta x_{cde}\left[\bar{u}^{de}\left(6\bar{A}^c\Delta x_b + \bar{A}_b\bar{u}^c\right) + 2\bar{u}^{cd}\Delta x_c\left(3\Gamma_d{}^{ef}\Delta x_b + \Gamma_b{}^{ef}\right) + 2\Gamma_c{}^{de}\Delta x_b\right]$$
$$+ \frac{1}{12}\bar{u}^{bcde}\Delta x_{bc}\Big[2\Delta x^{fi}\left(\Gamma_{jei}\Gamma^j_{df} - \Gamma_{jfi}\Gamma^j_{de} - \Gamma_{dpi,e} - 2\Gamma_{fid,e} + \Gamma_{def,i} + 2\Gamma_{fde,i}\right)$$
$$+ \bar{u}^{fi}\Delta x_d\left(\Gamma_{njk}\left(\Gamma_{fi}{}^n + \Gamma^n{}_{fi}\right)\bar{u}^{jk}\Delta x_e + \Delta x^j\left(\Gamma_{je}{}^k\Gamma_{kfi} + \Gamma_{kij}\Gamma^k{}_{ef} - 4\Gamma_{efj,i} + 4\Gamma_{efi,j}\right)\right)\Big]$$
$$+ \frac{1}{2}\bar{u}^{bc}\Delta x^{def}\left[\frac{1}{3}\Delta x_b\left(\Gamma_{ief}\Gamma^i{}_{cd} - \Gamma_{def,c} - \Gamma_{cde,f} + \Gamma_{dce,f}\right) - \frac{1}{4}\Gamma_{bde}\Gamma_{cfi}\Delta x^i\right]\Big\}$$
$$+ \frac{1}{8\rho^5}\Big\{3\bar{A}^b\bar{u}^d\Delta x_{bcd}\left[\bar{A}^c\bar{u}^{refi}\Delta x_f - 2\bar{u}^c\left(\Gamma^{refi}\Delta x_{cf} + \Gamma^f{}_{ij}\bar{u}_{ef}\Delta x^j\right)\right]$$
$$+ 3\Delta x^{bfi}\left(\Gamma_{bcd}\Gamma_{efi}\Delta x^{cde} + 2\Gamma^c{}_{de}\Gamma_{fij}\bar{u}_{bc}\Delta x^{dej}\Gamma^d{}_{fi}\Gamma^e{}_{lk}\bar{u}_{bcde}\Delta x^{cfk} + \right)\Big\}$$

Scalar Singular Field

$$\Phi_{[2]} = \Phi_{(2,1)}\rho + \Phi_{(2,-1)}\rho^{-1} + \Phi_{(2,-3)}\rho^{-3} + \Phi_{(2,-5)}\rho^{-5} + \Phi_{(2,-7)}\rho^{-7}$$

$$\mathrm{Ab}^{2b} \left(-\frac{\Delta x_b}{8} - \frac{5}{8}\, \mathrm{ub}_b\, \mathrm{ub}^c\, \Delta x_c\right) + \mathrm{Ab}^b$$

$$\Big(-\frac{5}{4}\, \mathrm{Ab}^{1c}\, \mathrm{ub}_c\, \Delta x_b - \frac{15}{16}\, \mathrm{Ab}_b\, \mathrm{Ab}^c\, \Delta x_c - \frac{5}{4}\, \mathrm{Ab}^1{}_b\, \mathrm{ub}^c\, \Delta x_c + \frac{1}{48}\, \mathrm{ub}^c\, \mathrm{ub}^d\, \Delta x^e\, \big(43\, \Gamma[\nabla]_{ec}{}^{f}\, \Gamma[\nabla]_{fbd} + 23\, \Gamma[\nabla]_{eb}{}^{f}\, \Gamma[\nabla]_{fcd} - 11\, \Gamma[\nabla]_{fde}\, \Gamma[\nabla]^{f}{}_{bc} + 5\, \Gamma[\nabla]_{fcd}\, \Gamma[\nabla]^{f}{}_{be} -$$

$$24\, \Gamma[\nabla]_{cde,b} - 6\, \Gamma[\nabla]_{ecd,b} - 24\, \Gamma[\nabla]_{bce,d} - 24\, \Gamma[\nabla]_{cbe,d} + 6\, \Gamma[\nabla]_{ebc,d} + 24\, \Gamma[\nabla]_{bcd,e} + 48\, \Gamma[\nabla]_{cbd,e}\big) + \mathrm{ub}^c\, \mathrm{ub}^d\, \mathrm{ub}^e\, \mathrm{ub}^f\, \Big(\frac{5}{24}\, \Gamma[\nabla]_{ief}\, \big(\Gamma[\nabla]_{cd}{}^{i} + \Gamma[\nabla]^{i}{}_{cd}\big)$$

$$\Delta x_b + \frac{1}{48}\, \Delta x_c\, \big(255\, \Gamma[\nabla]_{de}{}^{i}\, \Gamma[\nabla]_{ibf} + 22\, \Gamma[\nabla]_{bd}{}^{i}\, \Gamma[\nabla]_{ief} + 123\, \Gamma[\nabla]_{db}{}^{i}\, \Gamma[\nabla]_{ief} + 40\, \Gamma[\nabla]_{ief}\, \Gamma[\nabla]^{i}{}_{bd} - 30\, \Gamma[\nabla]_{def,b} + 30\, \Gamma[\nabla]_{dbe,f}\big)\Big)\Big) +$$

$$\mathrm{ub}^e\, \Big(\frac{1}{48}\, \mathrm{ub}^b\, \mathrm{ub}^c\, \mathrm{ub}^d\, \Delta x^f\, \big(-23\, \Gamma[\nabla]_{fij}\, \Gamma[\nabla]^{i}{}_{bc}\, \Gamma[\nabla]^{j}{}_{de} + \Gamma[\nabla]_{ifj}\, \Gamma[\nabla]^{i}{}_{bc}\, \Gamma[\nabla]^{j}{}_{de} - 22\, \Gamma[\nabla]^{i}{}_{bc}\, \Gamma[\nabla]_{jei}\, \Gamma[\nabla]^{j}{}_{df} -$$

$$26\, \Gamma[\nabla]_{idj}\, \Gamma[\nabla]^{i}{}_{bc}\, \Gamma[\nabla]^{j}{}_{ef} + 12\, \Gamma[\nabla]^{i}{}_{bf}\, \Gamma[\nabla]_{cdi,e} + 30\, \Gamma[\nabla]^{i}{}_{bc}\, \Gamma[\nabla]_{dfi,e} + 13\, \Gamma[\nabla]^{i}{}_{bc}\, \Gamma[\nabla]_{fdi,e} + 26\, \Gamma[\nabla]^{i}{}_{bf}\, \Gamma[\nabla]_{icd,e} +$$

$$\Gamma[\nabla]_{fb}{}^{i}\, \big(-26\, \Gamma[\nabla]_{jei}\, \Gamma[\nabla]^{j}{}_{cd} - 70\, \Gamma[\nabla]_{icj}\, \Gamma[\nabla]^{j}{}_{de} + 26\, \Gamma[\nabla]_{icd,e}\big) + 43\, \Gamma[\nabla]^{i}{}_{bc}\, \Gamma[\nabla]_{idf,e} - 12\, \Gamma[\nabla]_{bcf,d,e} - 12\, \Gamma[\nabla]_{fbc,d,e} -$$

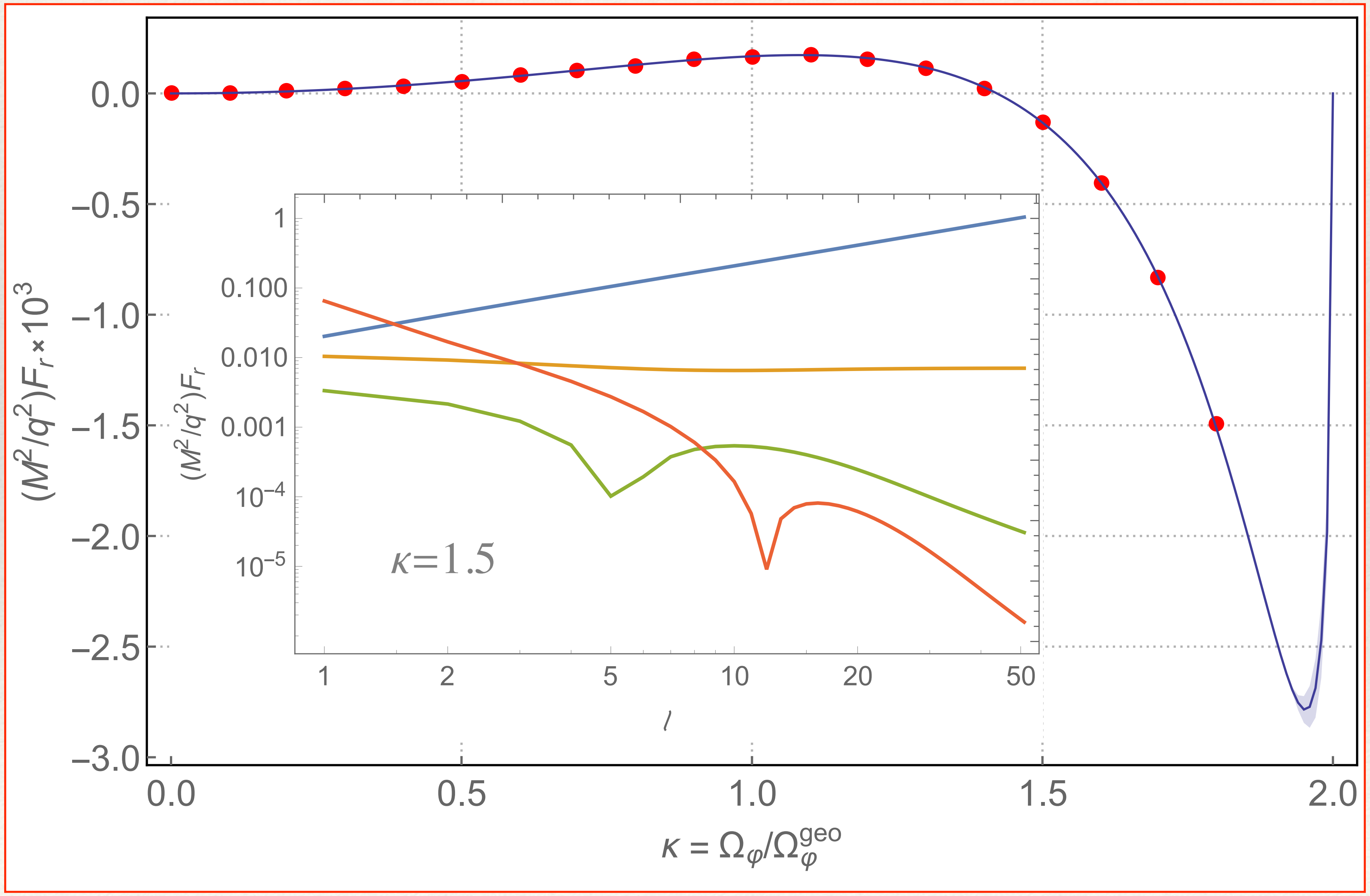
$$42\, \Gamma[\nabla]^{i}{}_{bc}\, \Gamma[\nabla]_{dei,f} - 30\, \Gamma[\nabla]^{i}{}_{bc}\, \Gamma[\nabla]_{ide,f} + 12\, \Gamma[\nabla]_{bcd,e,f} - 12\, \Gamma[\nabla]^{i}{}_{bf}\, \Gamma[\nabla]_{cde,i} + 12\, \Gamma[\nabla]^{i}{}_{bc}\, \Gamma[\nabla]_{def,i}\big) +$$

$$\frac{1}{48}\, \mathrm{ub}^b\, \mathrm{ub}^c\, \mathrm{ub}^d\, \mathrm{ub}^f\, \mathrm{ub}^i\, \Delta x_b\, \big(-287\, \Gamma[\nabla]_{jek}\, \Gamma[\nabla]^{j}{}_{cd}\, \Gamma[\nabla]^{k}{}_{fi} - 123\, \Gamma[\nabla]_{cjk}\, \Gamma[\nabla]^{j}{}_{de}\, \Gamma[\nabla]^{k}{}_{fi} + 57\, \Gamma[\nabla]^{j}{}_{cd}\, \Gamma[\nabla]_{efj,i} -$$

$$6\, \Gamma[\nabla]_{cd}{}^{j}\, \big(19\, \Gamma[\nabla]_{kij}\, \Gamma[\nabla]^{k}{}_{ef} + 61\, \Gamma[\nabla]_{jek}\, \Gamma[\nabla]^{k}{}_{fi} - 19\, \Gamma[\nabla]_{jef,i}\big) + 171\, \Gamma[\nabla]^{j}{}_{cd}\, \Gamma[\nabla]_{jef,i} - 60\, \Gamma[\nabla]_{cde,f,i}\big)\Big)$$

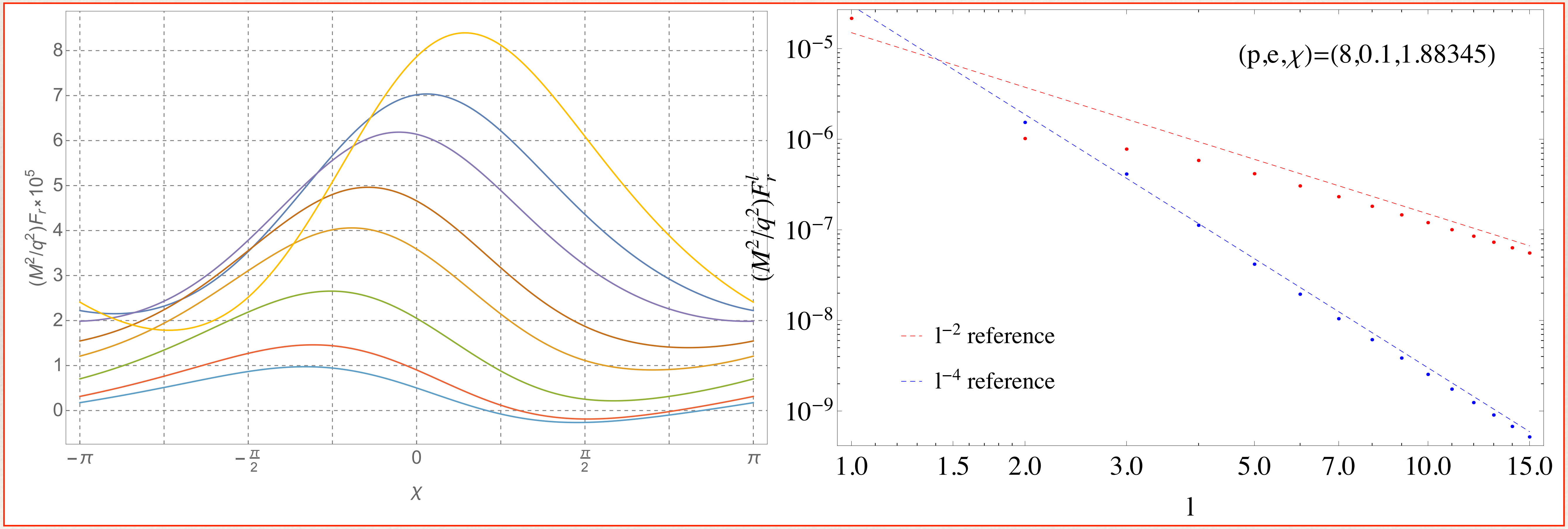
Testing

- Retrieved A & B Schwarzschild non-geodesic RPs
- Successfully regularises circular orbit with constant acceleration



Testing

- Successful regularises eccentric orbits (and more novel orbits)

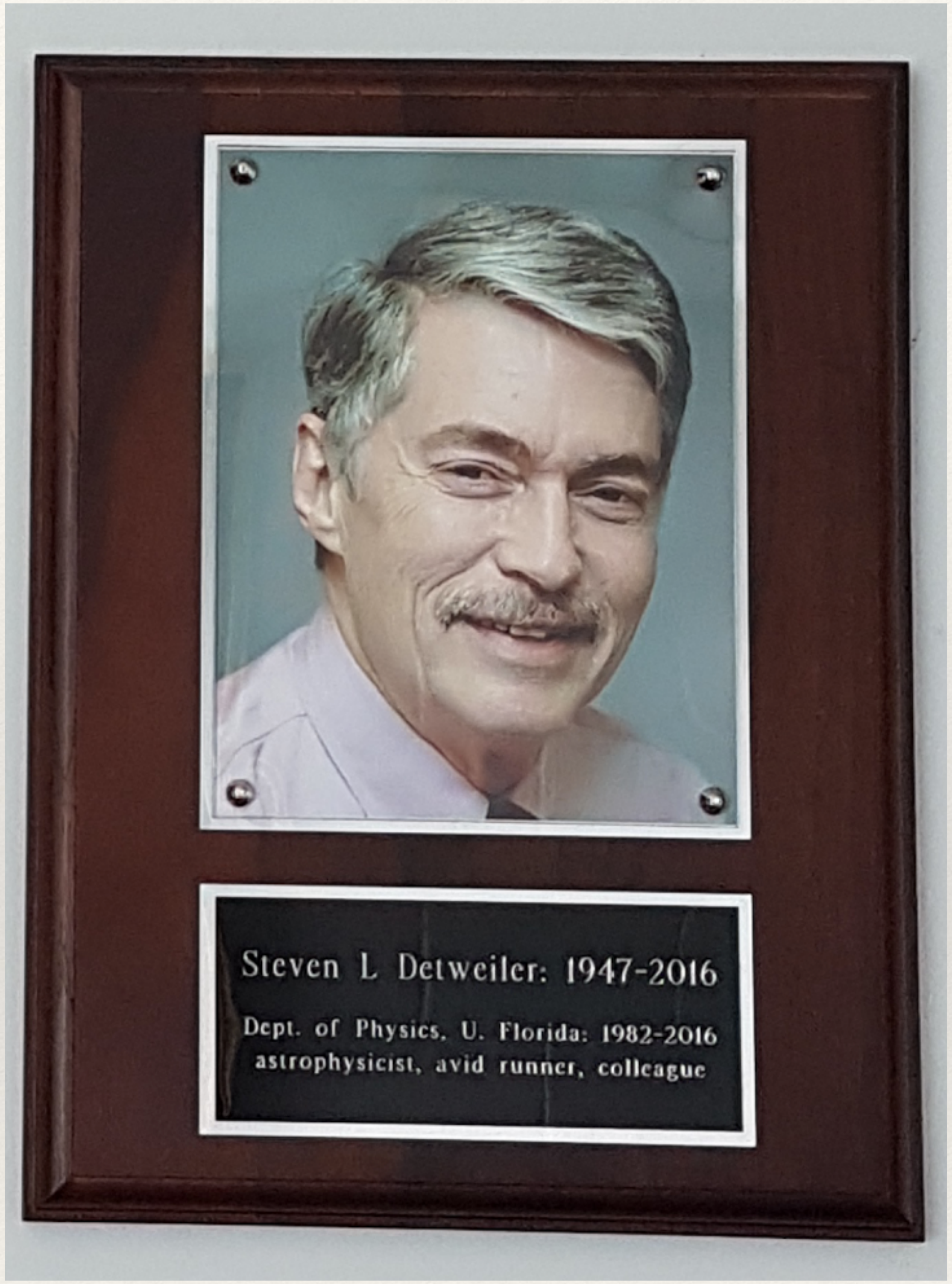


$$dt/d\chi = k \cdot dt/d\chi_{\text{geo}} \qquad k = \{0.9, 1, 1.1, 1.3, 1.5, 2, 3, 4\}$$

Summary

- Covariant Singular field for non-geodesic motion in scalar case
  - easily extendible to gravitational case
- A and B retrieved and tested for Schwarzschild (NG)
  - D requires coordinate conversion
- A and B retrieved and tested for Kerr (G)
  - D requires coordinate conversion
  - Currently no test available for non-geodesic Kerr (soon)

Thank you!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