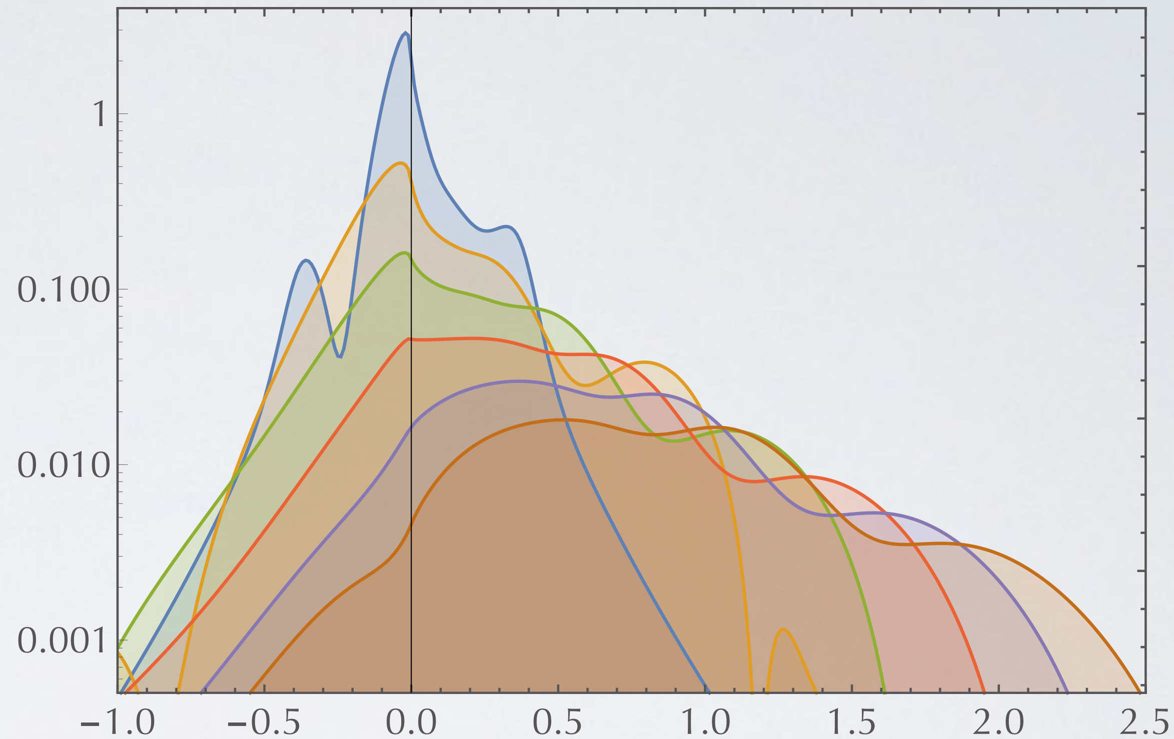
Scattering trajectories in Schwarzschild spacetime

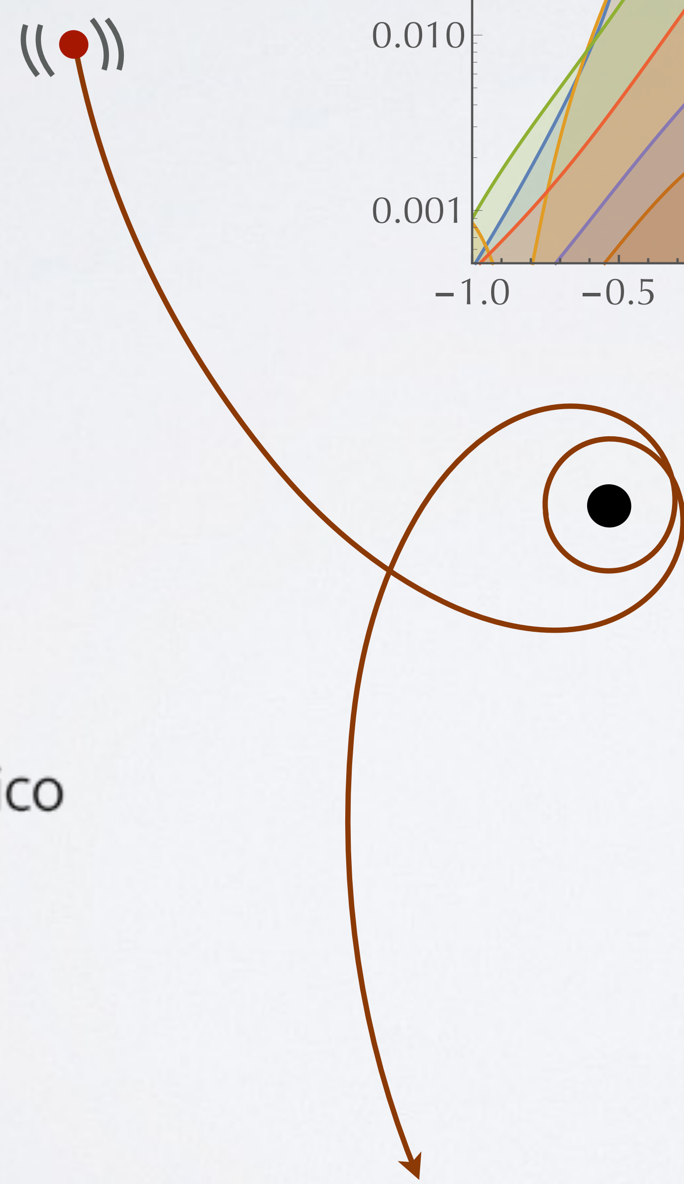


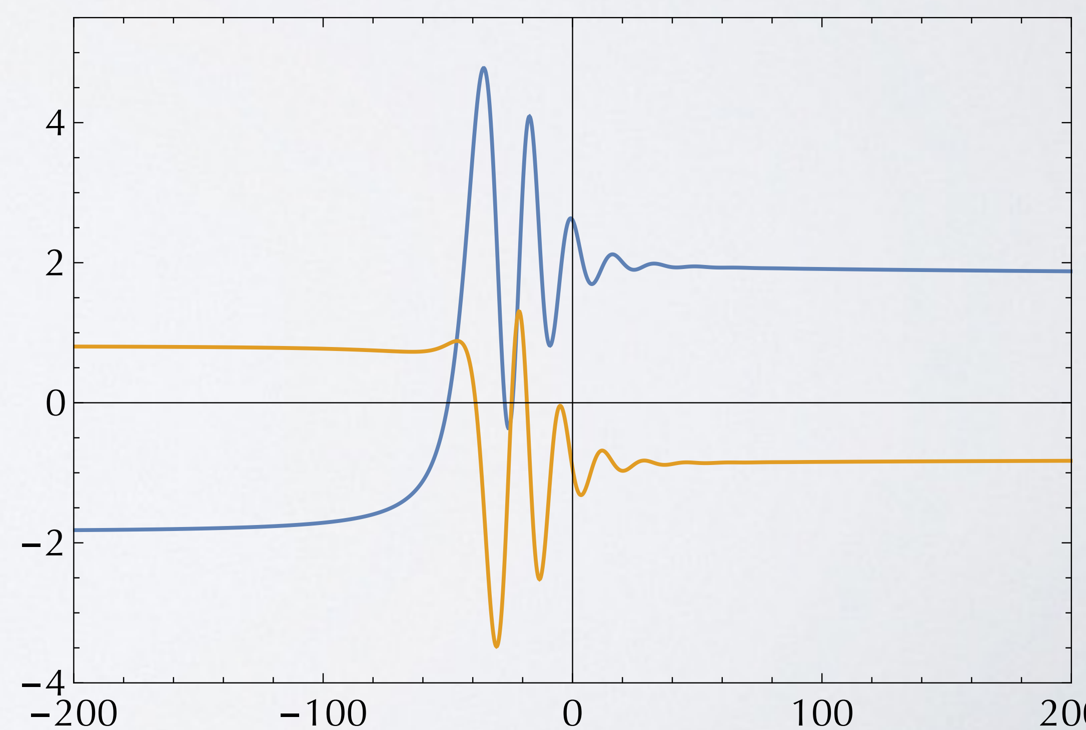
Seth Hopper

Vitor Cardoso

centra

grit
gravitation in técnico



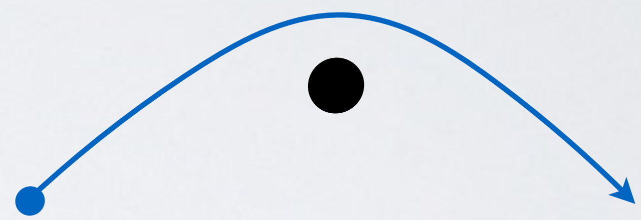


Capra - June 19, 2017

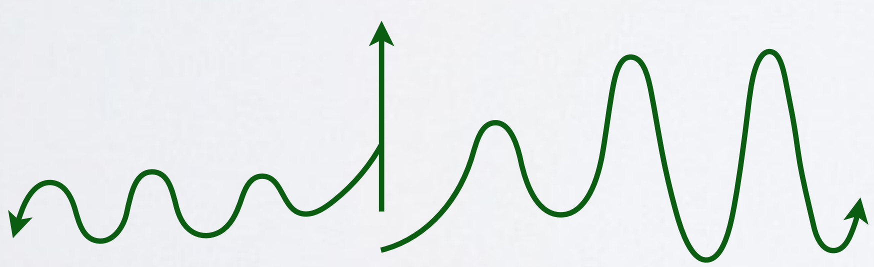
Outline

Bound motion

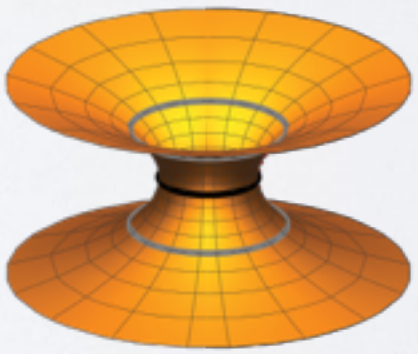
Unbound motion



Local calculations



Wormholes and echoes



Peters & Mathews computed the PN flux from bound eccentric motion

Peters & Mathews, 1963

$$\left\langle \frac{dE}{dt} \right\rangle_{\rm 3PN} = \frac{32}{5}\left(\frac{\mu}{M}\right)^2 x^5 \left(\mathcal{I}_0 + x\,\mathcal{I}_1 + x^{3/2}\,\mathcal{K}_{3/2} + x^2\,\mathcal{I}_2 + x^{5/2}\,\mathcal{K}_{5/2} + x^3\,\mathcal{I}_3 + x^3\,\mathcal{K}_3\right)$$

$$\mathcal{I}_0 = \frac{1}{(1-e_t^2)^{7/2}}\left(1 + \frac{73}{24}\,e_t^2 + \frac{37}{96}\,e_t^4\right)$$

“Enhances” flux from Hulse-Taylor pulsar (e=0.62) by factor of 12

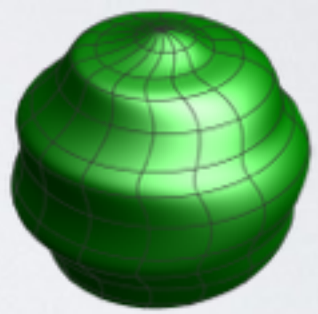
We work in a gauge which simplifies the field equations

Schwarzschild metric

Key assumptions

1. Black hole background: $g_{\mu\nu}^{\rm BG} = g_{\mu\nu}^{\rm BH}$

2. Small deviations: $g_{\mu\nu} = g_{\mu\nu}^{\rm BH} + h_{\mu\nu}$ $\qquad |h_{\mu\nu}| \ll |g_{\mu\nu}^{\rm BH}|$



Metric perturbation

Result

Lorenz gauge: $$\Box \bar{h}_{\mu\nu} + 2R_{\alpha\mu\beta\nu}\bar{h}^{\alpha\beta} = -16\pi T_{\mu\nu}$$

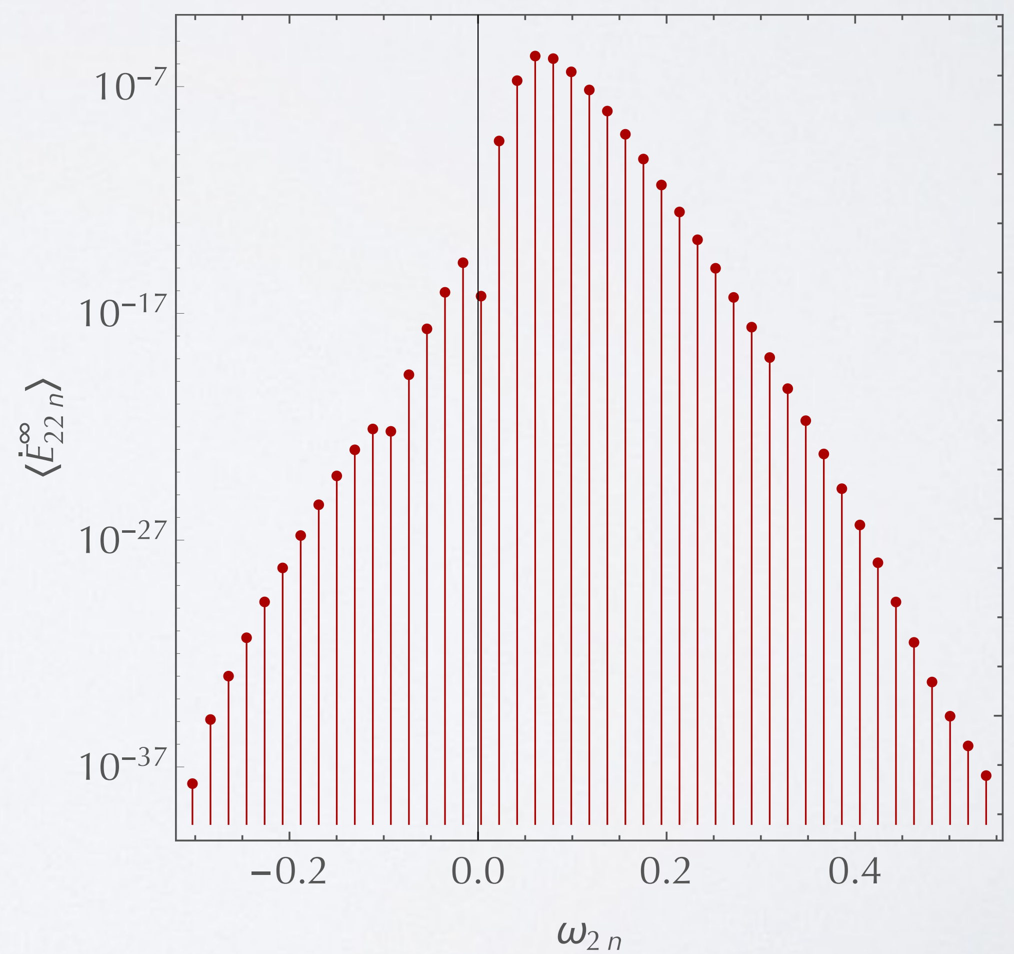
Regge-Wheeler gauge: $$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_\ell(r)\right]\Psi_{\ell m}(t,r) = G_{\ell m}(t)\,\delta[r - r_p(t)] + F_{\ell m}(t)\,\delta'[r - r_p(t)]$$

Periodic motion implies a discrete spectrum

$$\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}-V_\ell(r)\right]\Psi_{\ell m}(t,r)=S_{\ell m}(t,r) \longrightarrow \left[\frac{d^2}{dr_*^2}+\omega_{mn}^2-V_\ell(r)\right]X_{\ell mn}(r)=Z_{\ell mn}(r)$$

$$\Psi_{\ell m}(t,r)=\sum_{n=-\infty}^{\infty}X_{\ell mn}(r)\,e^{-i\omega_{mn}t} \qquad S_{\ell m}(t,r)=\sum_{n=-\infty}^{\infty}Z_{\ell mn}(r)\,e^{-i\omega_{mn}t}$$

$(p,e)=(10,0.2)$



The particular solution follows from integrating over the source

Time domain

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r_*^2} - V_\ell(r)\right] \Psi_{\ell m}(t,r) = G_{\ell m}(t)\, \delta[r - r_p(t)] + F_{\ell m}(t)\, \delta'[r - r_p(t)]$$

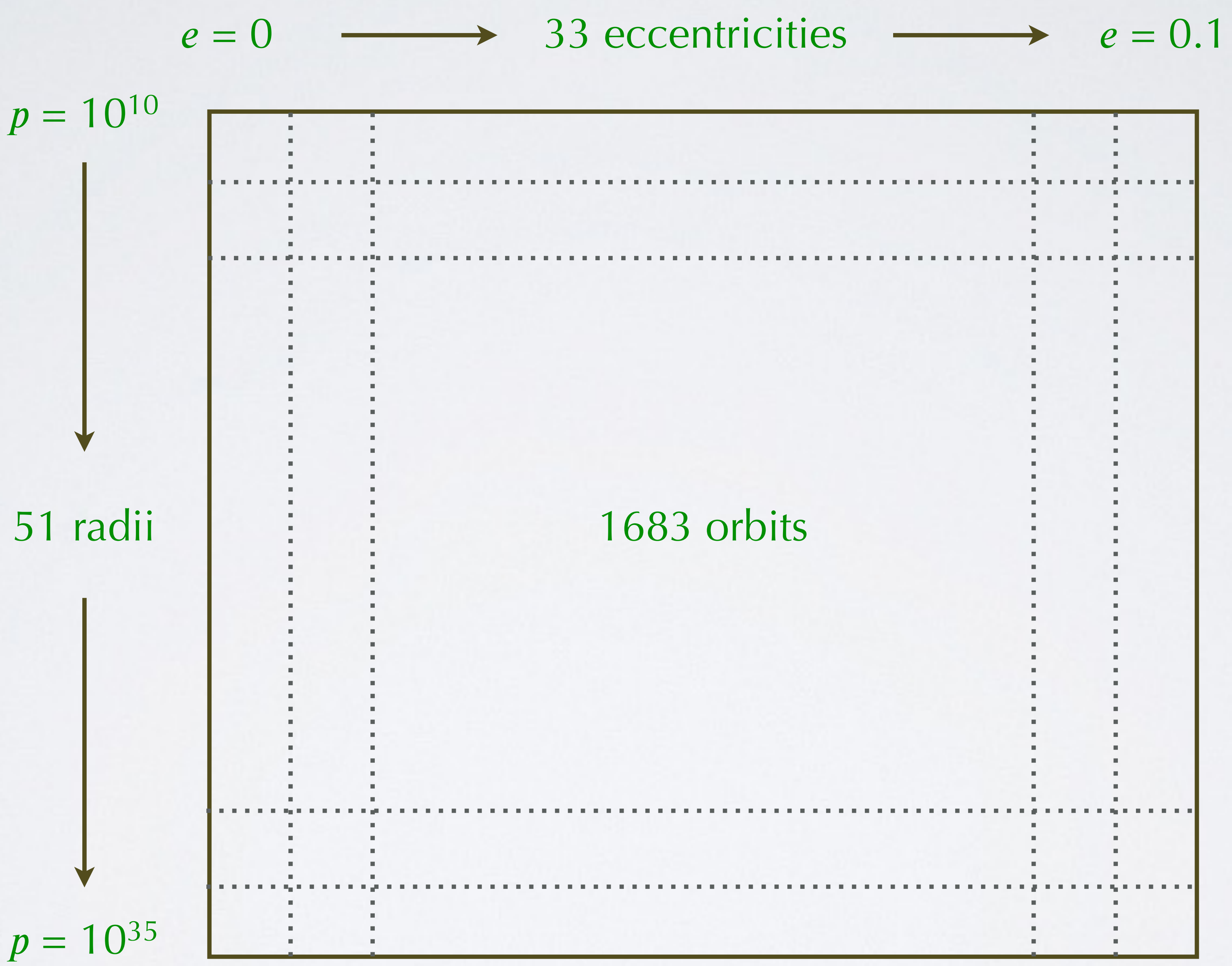
Frequency domain

$$\left[\frac{d^2}{dr_*^2} + \omega^2 - V_\ell(r)\right] X_{\ell m\omega}(r) = Z_{\ell m\omega}(r).$$

$$C^{\pm}_{\ell m\omega} \sim \int_0^{T_r} dt \left(\hat{X}^{\mp}_{\ell m\omega} G_{\ell m} + \frac{d\hat{X}^{\mp}_{\ell m\omega}}{dr} F_{\ell m} \right)$$

$$\Psi^{\pm}_{\ell m}(t,r) \equiv \sum_{n=-\infty}^{\infty} C^{\pm}_{\ell mn} \hat{X}^{\pm}_{\ell mn}(r)\, e^{-i\omega_{mn} t}$$

We spanned the two dimensional space of orbits



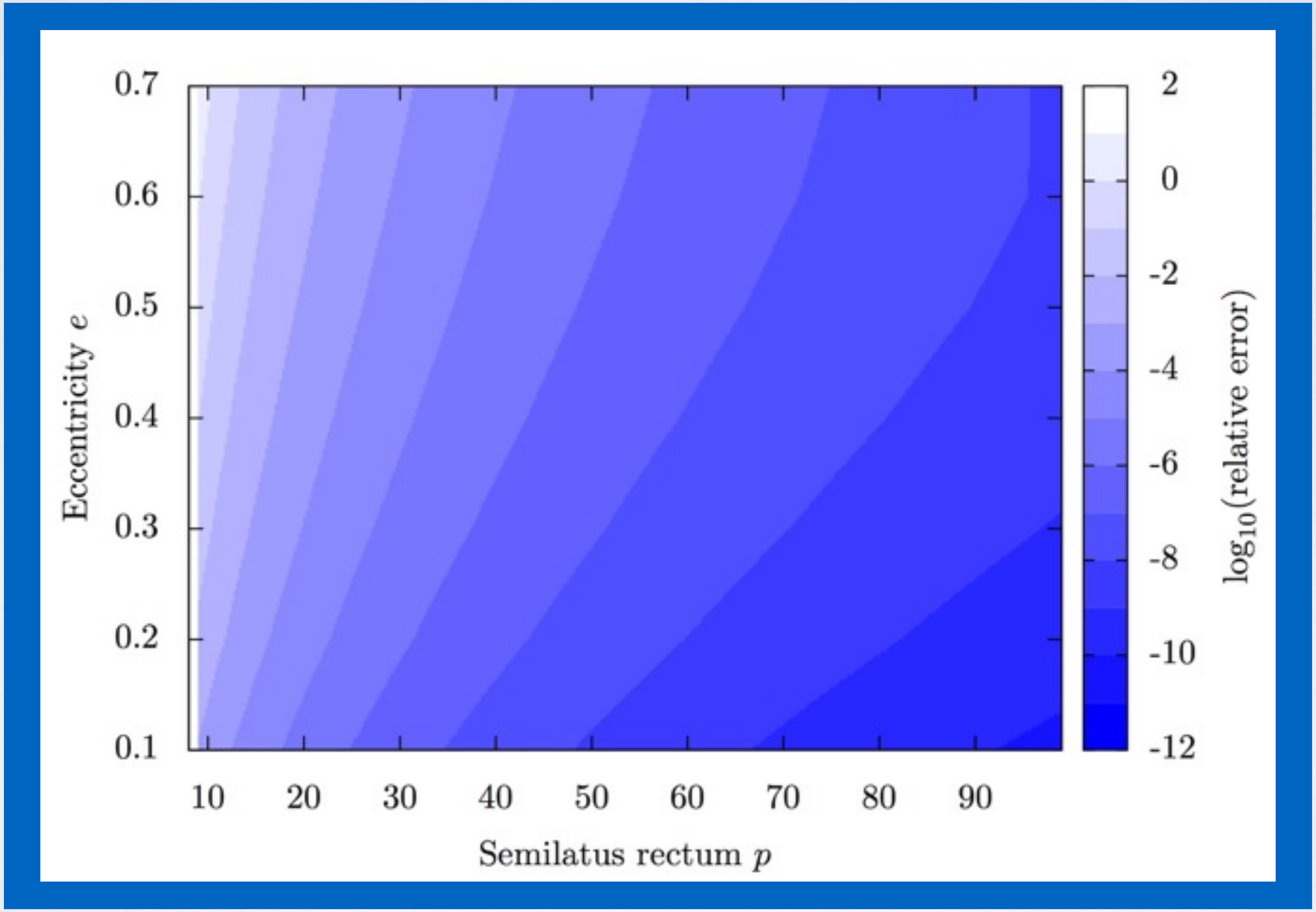
PN parameters involve sums of transcendentals

Found with PSLQ algorithm

$$
\begin{aligned}
\mathcal{L}_4 = \frac{1}{(1-e^2)^{15/2}} \Bigg[& -\frac{323105549467}{3178375200} + \frac{232597}{4410}\gamma_{\text{E}} - \frac{1369}{126}\pi^2 + \frac{39931}{294}\log(2) - \frac{47385}{1568}\log(3) \\
& + \left(-\frac{128412398137}{23543520} + \frac{4923511}{2940}\gamma_{\text{E}} - \frac{104549}{252}\pi^2 - \frac{343177}{252}\log(2) + \frac{55105839}{15680}\log(3) \right) e^2 \\
& + \left(-\frac{981480754818517}{25427001600} + \frac{142278179}{17640}\gamma_{\text{E}} - \frac{1113487}{504}\pi^2 + \frac{762077713}{5880}\log(2) - \frac{2595297591}{71680}\log(3) \right. \\
& \qquad \left. - \frac{15869140625}{903168}\log(5) \right) e^4 \\
& + \left(-\frac{874590390287699}{12713500800} + \frac{318425291}{35280}\gamma_{\text{E}} - \frac{881501}{336}\pi^2 - \frac{90762985321}{63504}\log(2) + \frac{31649037093}{1003520}\log(3) \right. \\
& \qquad \left. + \frac{10089048828125}{16257024}\log(5) \right) e^6 \\
& + d_8 e^8 + d_{10} e^{10} + d_{12} e^{12} + d_{14} e^{14} + d_{16} e^{16} + d_{18} e^{18} + d_{20} e^{20} + d_{22} e^{22} + d_{24} e^{24} + d_{26} e^{26} \\
& + d_{28} e^{28} + d_{30} e^{30} + d_{32} e^{32} + d_{34} e^{34} + d_{36} e^{36} + d_{38} e^{38} + d_{40} e^{40} + \cdots \Bigg]
\end{aligned}
$$

New PN parameters have been confirmed with a separate code

Flux residuals after subtracting new PN parameters

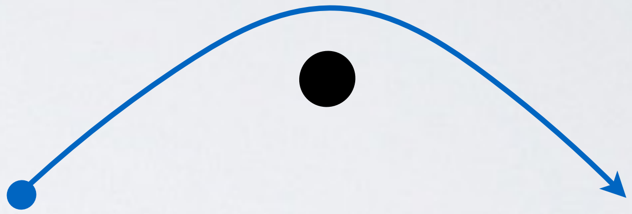


Code by Thomas Osburn

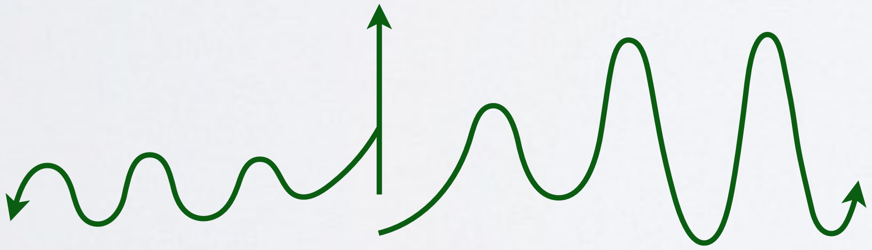
Outline

Bound motion

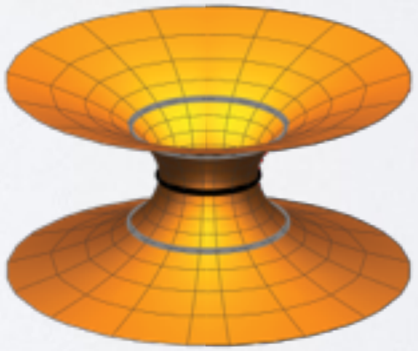
Unbound motion



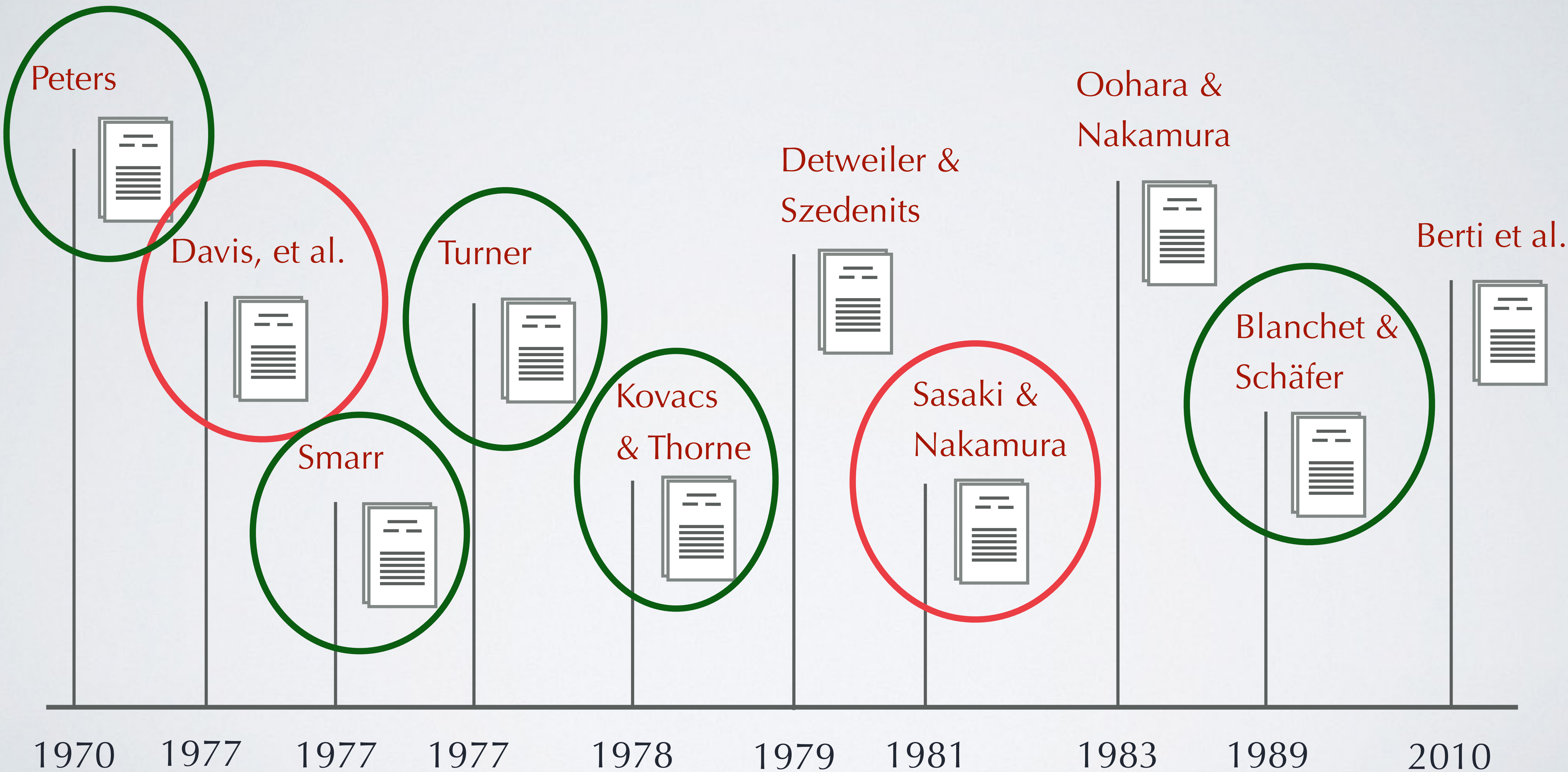
Local calculations



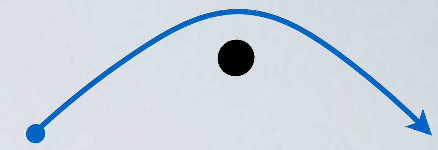
Wormholes and echoes

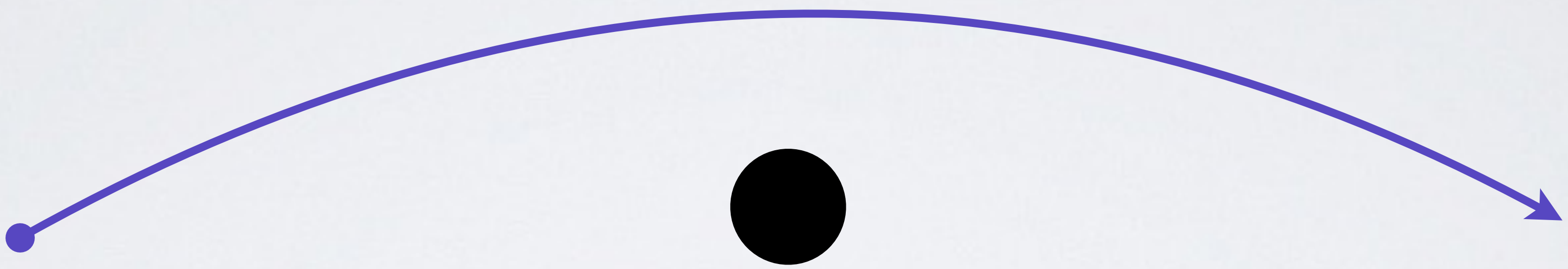


There has been a lot of previous work, but here are a couple highlights



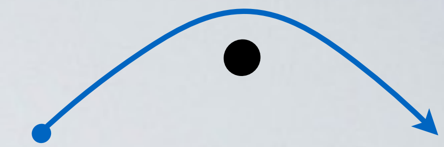
Turner did the 'Peters-Mathews calculation' for scattering



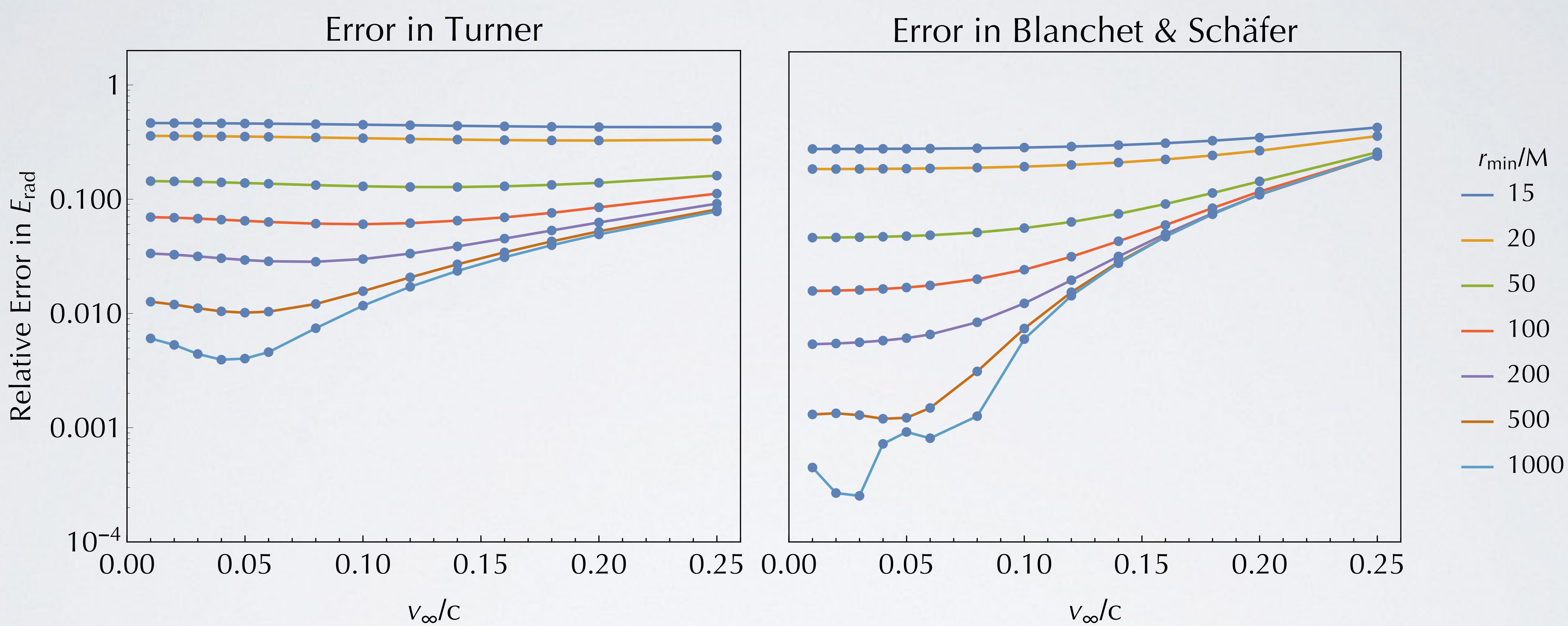


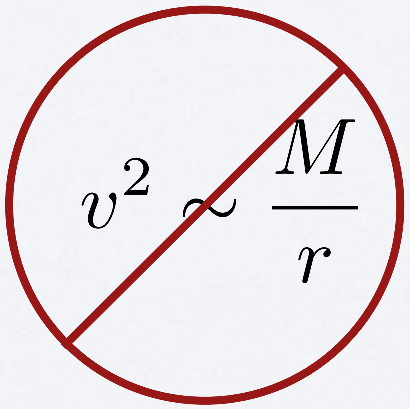
$$E_{\mathrm{rad}} = \frac{8}{15}\frac{M^6\mu^2}{J^7}\left[24\arccos(-1/e)\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)+(e^2-1)^{1/2}\left(\frac{301}{6}+\frac{673}{12}e^2\right)\right], \quad e \geq 1$$

The Turner result has only been extended to 1PN

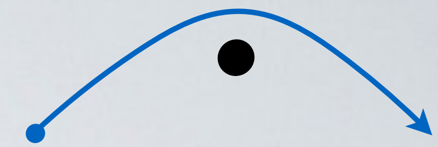


0PN 1PN





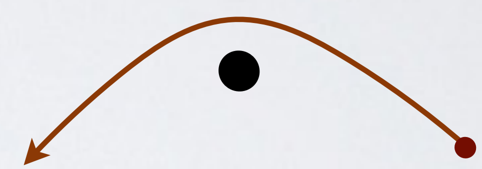
We still use spectral methods in the unbound case



$$\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}-V_\ell(r)\right]\Psi_{\ell m}(t,r)=S_{\ell m}(t,r)$$

Bound

Unbound



$$\Psi_{\ell m}(t,r)=\sum_{n=-\infty}^{\infty}X_{\ell mn}(r)\,e^{-i\omega_{mn}t} \qquad\longrightarrow\qquad \Psi_{\ell m}(t,r)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X_{\ell m\omega}(r)\,e^{-i\omega t}d\omega$$

$$S_{\ell m}(t,r)=\sum_{n=-\infty}^{\infty}Z_{\ell mn}(r)\,e^{-i\omega_{mn}t} \qquad\qquad S_{\ell m}(t,r)=\frac{1}{2\pi}\int_{-\infty}^{\infty}Z_{\ell m\omega}(r)\,e^{-i\omega t}d\omega$$

$$\left[\frac{d^2}{dr_*^2}+\omega_{mn}^2-V_\ell(r)\right]X_{\ell mn}(r)=Z_{\ell mn}(r) \qquad\longrightarrow\qquad \left[\frac{d^2}{dr_*^2}+\omega^2-V_\ell(r)\right]X_{\ell m\omega}(r)=Z_{\ell m\omega}(r).$$

The particular solution follows from integrating over the source

Time domain

$$\left[-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial r_*^2}-V_\ell(r)\right]\Psi_{\ell m}(t,r)=G_{\ell m}(t)\,\delta[r-r_p(t)]+F_{\ell m}(t)\,\delta'[r-r_p(t)]$$

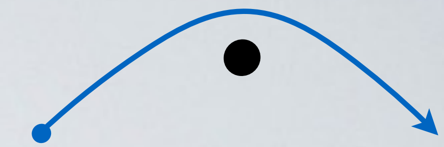
Frequency domain

$$\left[\frac{d^2}{dr_*^2}+\omega^2-V_\ell(r)\right]X_{\ell m\omega}(r)=Z_{\ell m\omega}(r).$$

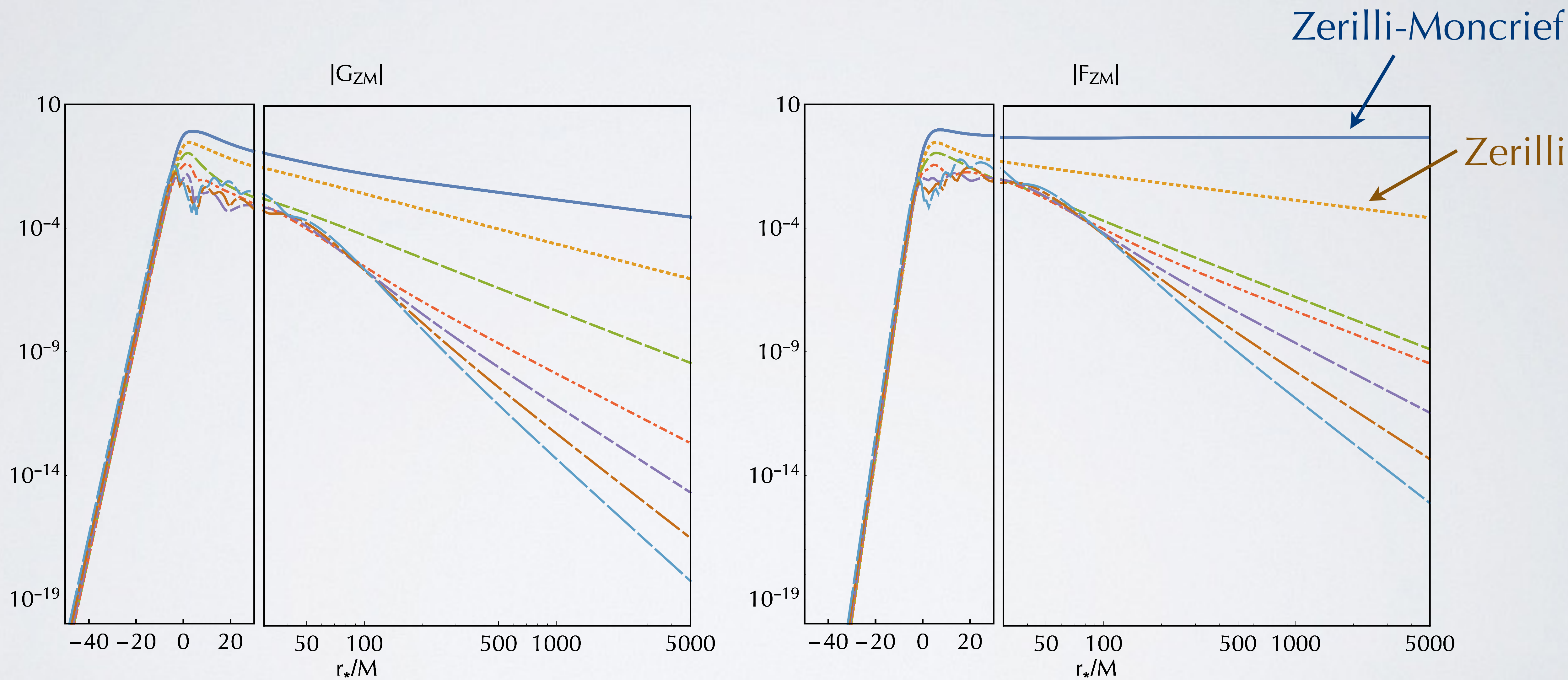
$$C^{\pm}_{\ell m\omega}\sim\int_{-\infty}^{\infty}dt\left(\hat{X}^{\mp}_{\ell m\omega}G_{\ell m}+\frac{d\hat{X}^{\mp}_{\ell m\omega}}{dr}F_{\ell m}\right)$$

$$\Psi^{\pm}_{\ell m}(t,r)\equiv\frac{1}{2\pi}\int_{-\infty}^{\infty}C^{\pm}_{\ell m\omega}\hat{X}^{\pm}_{\ell m\omega}(r)\,e^{-i\omega t}d\omega$$

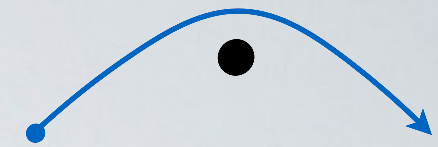
Convergence depends on which master function you choose

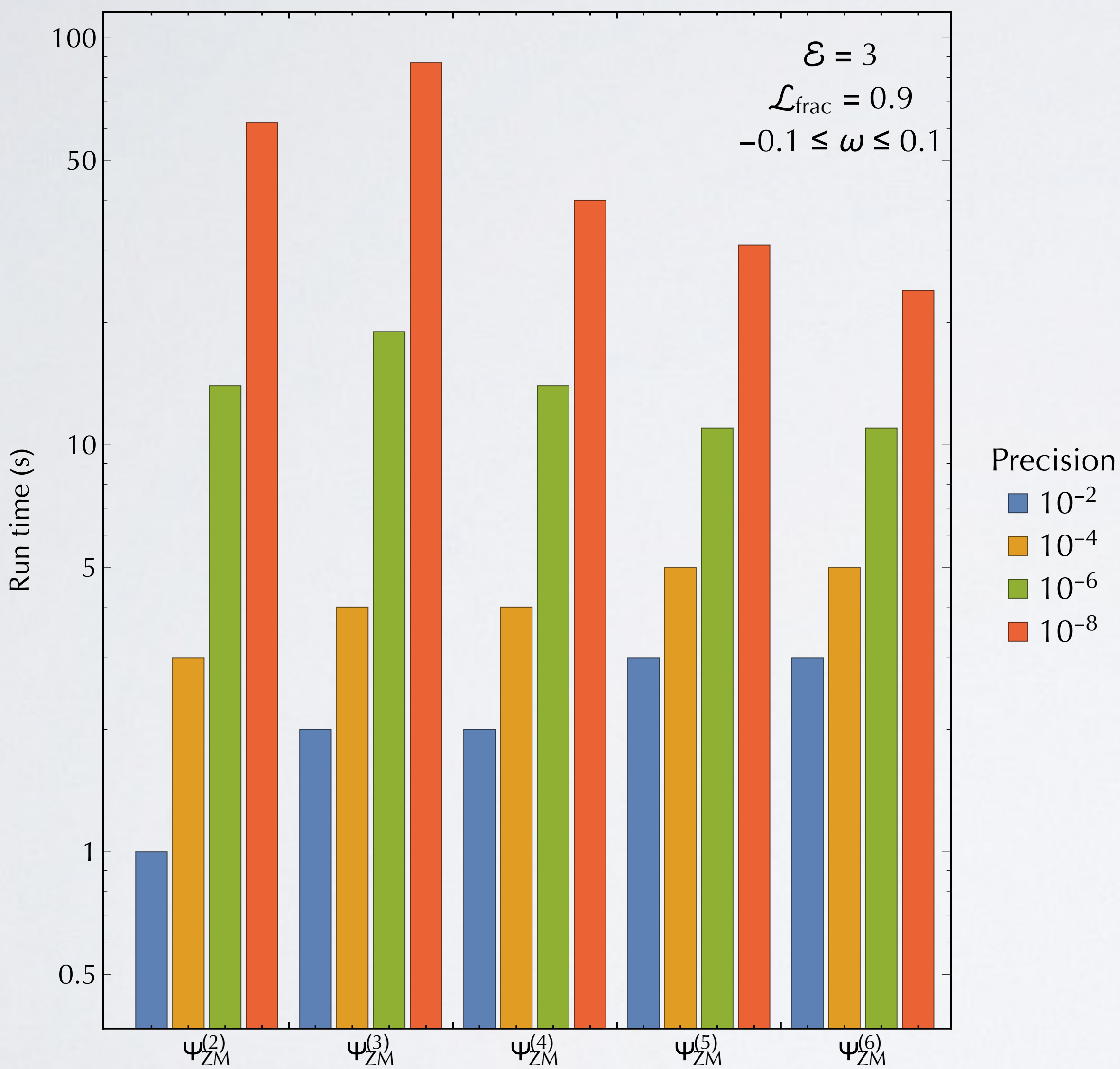


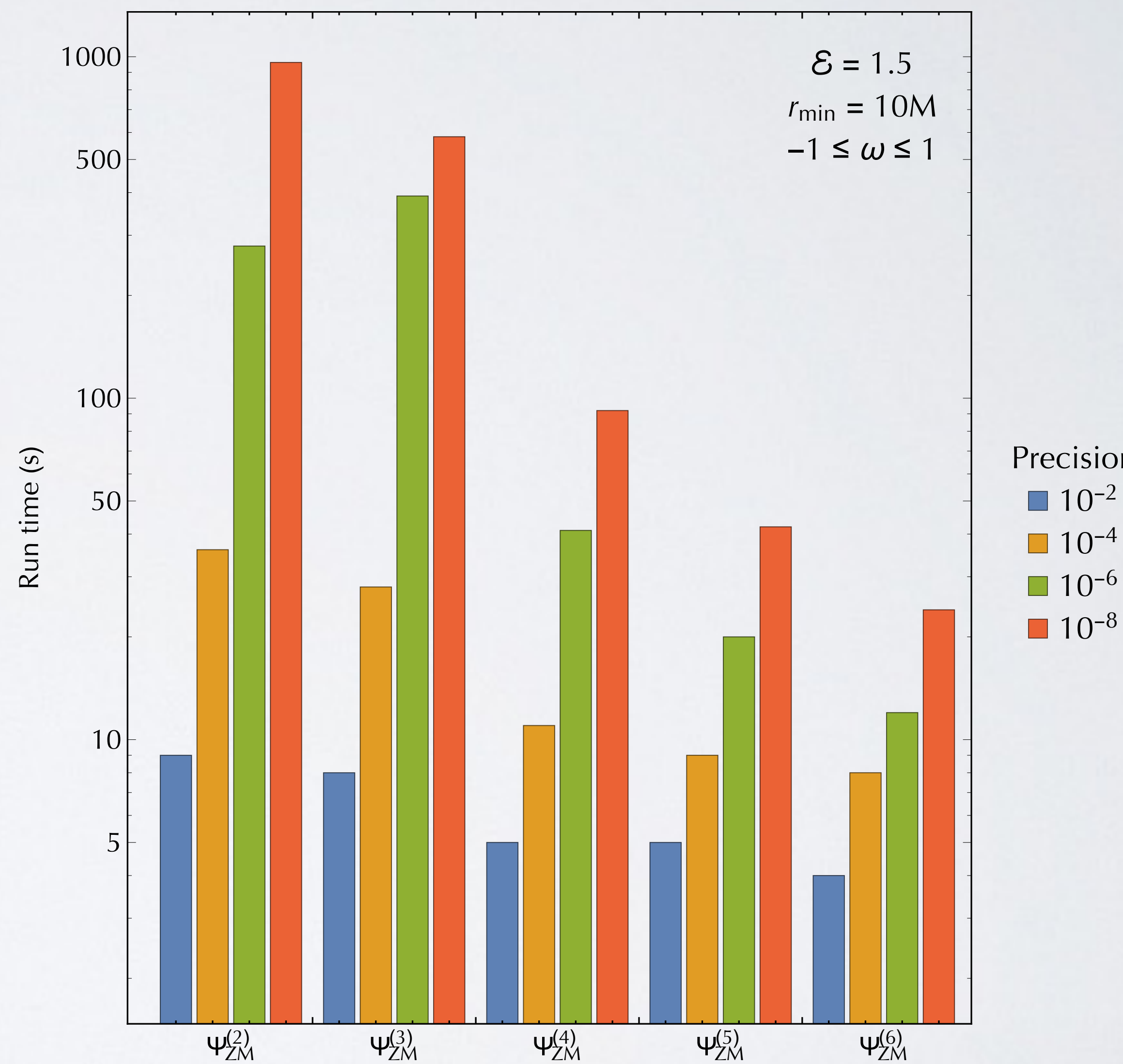
$$C^{\pm}_{\ell m\omega} \sim \int_{-\infty}^{\infty} dt \left(\hat{X}^{\mp}_{\ell m\omega} G_{\ell m} + \frac{d\hat{X}^{\mp}_{\ell m\omega}}{dr} F_{\ell m} \right)$$



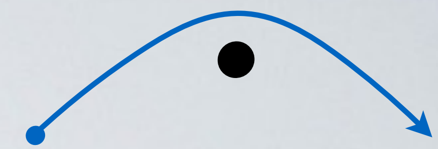
Speed benefits come at large frequencies



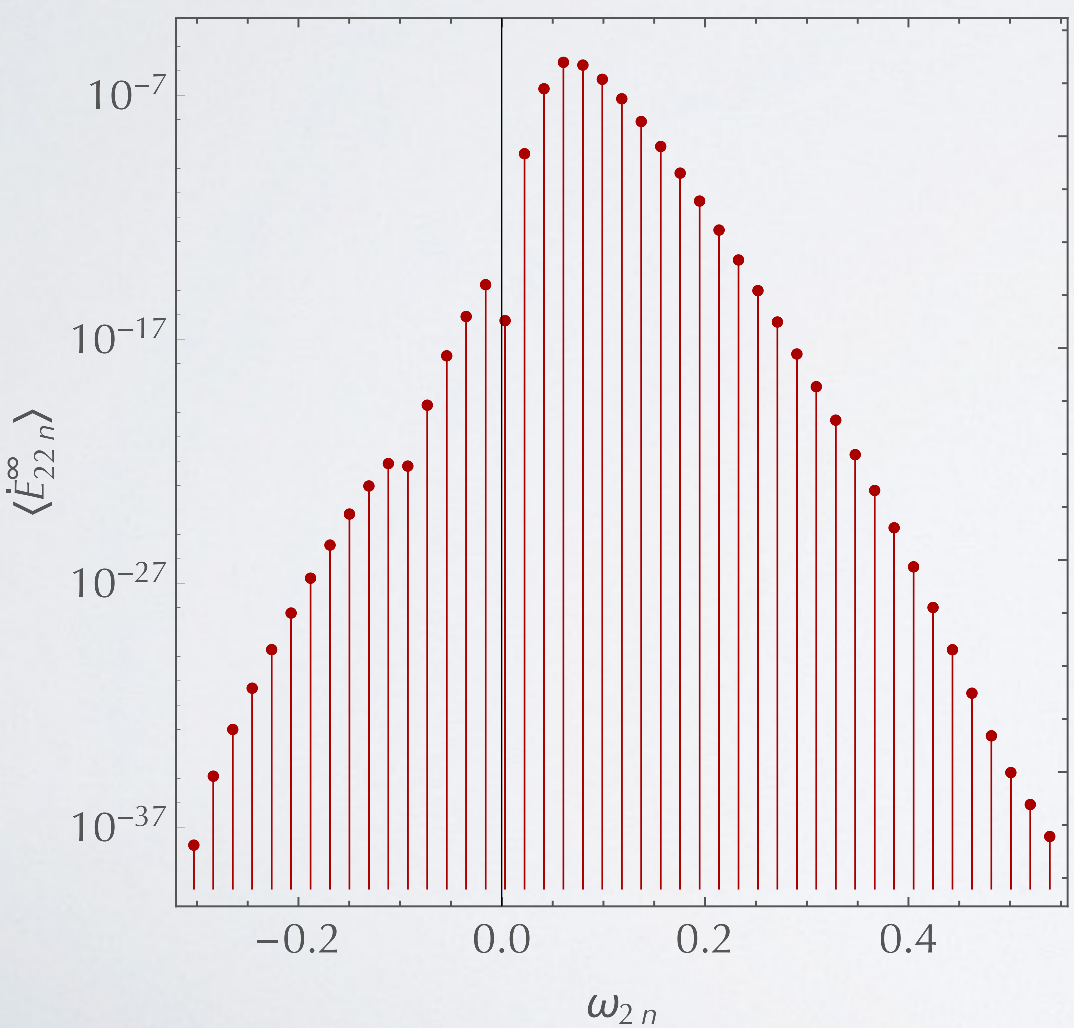




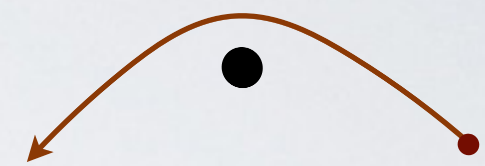
The unbound spectrum is dense

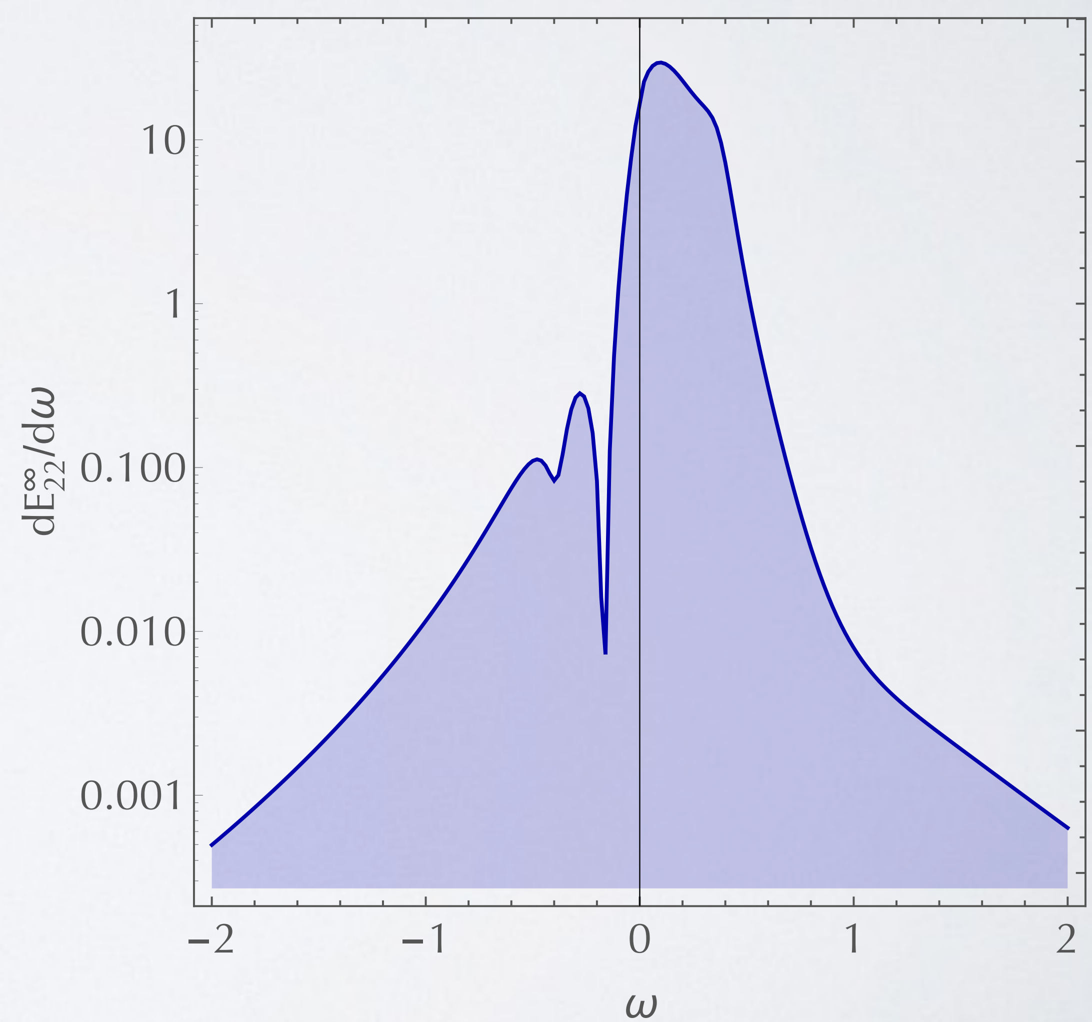


$(p,e)=(10,0.2)$

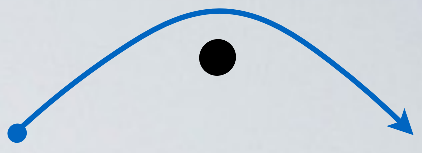


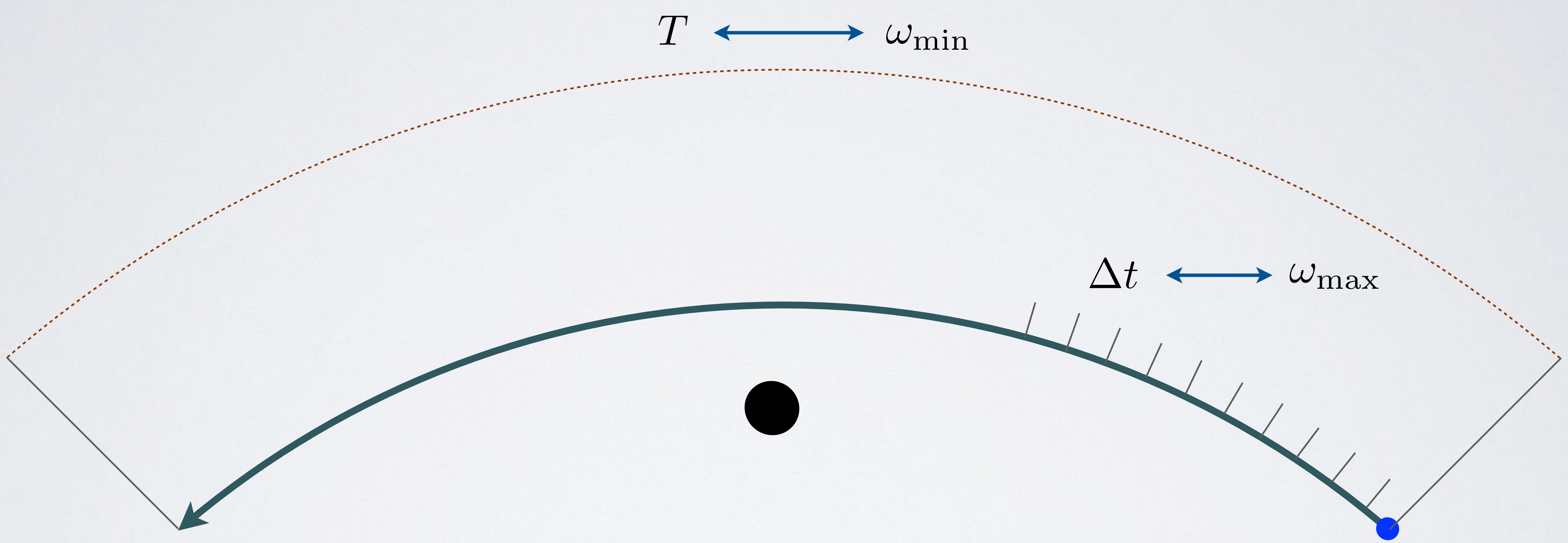
$(\mathcal{E},r_{min})=(1.5,4.3M)$



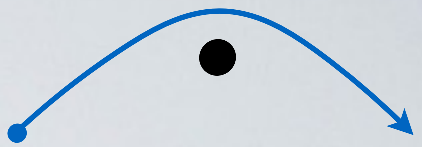


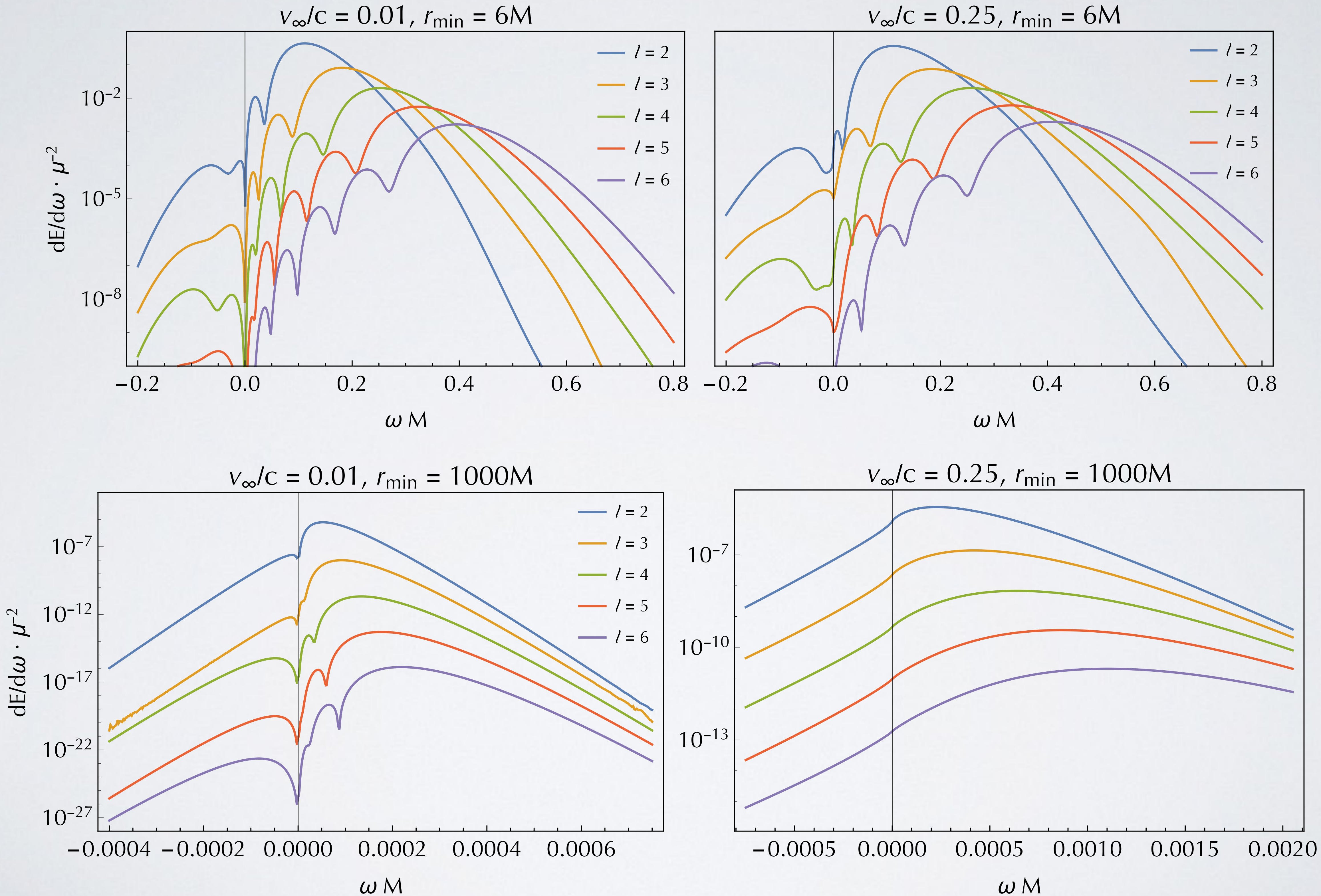
The time domain is dense, too



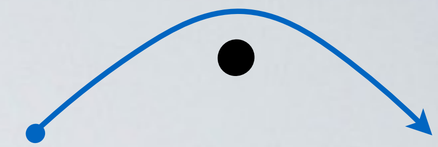


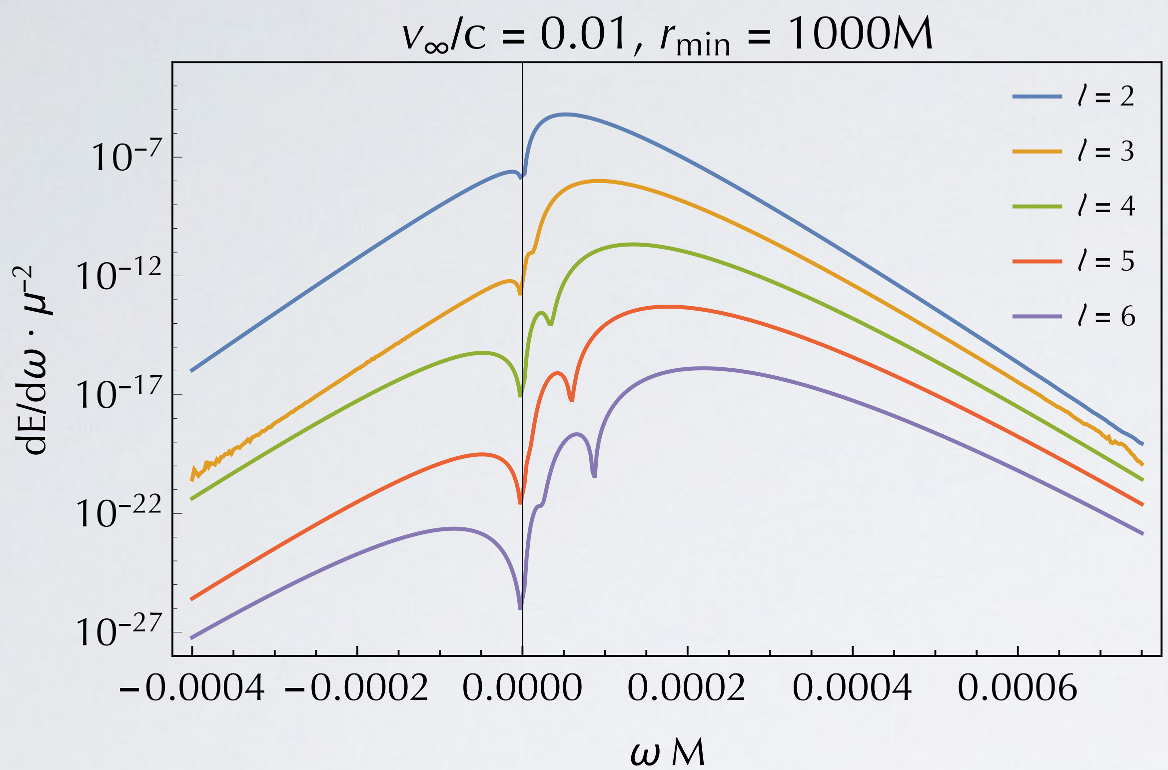
The character of spectra changes with r-min

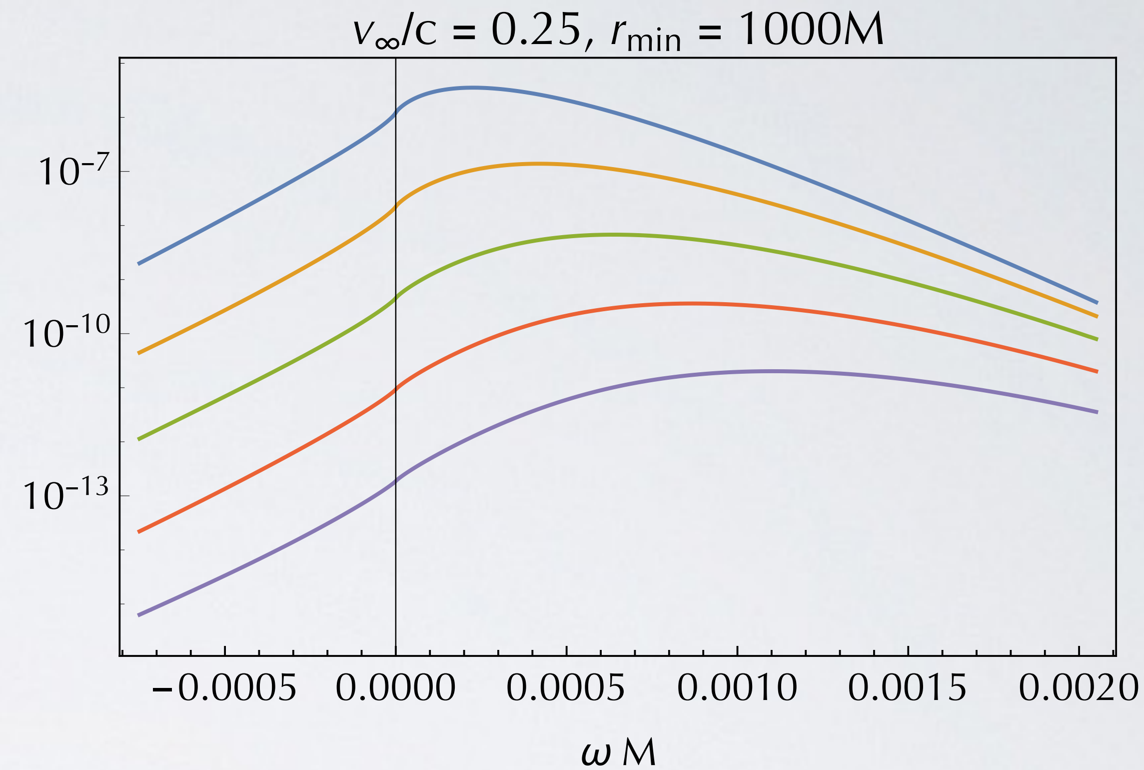




Smarr predicted the zero-frequency-limit to be non-zero for unbound motion

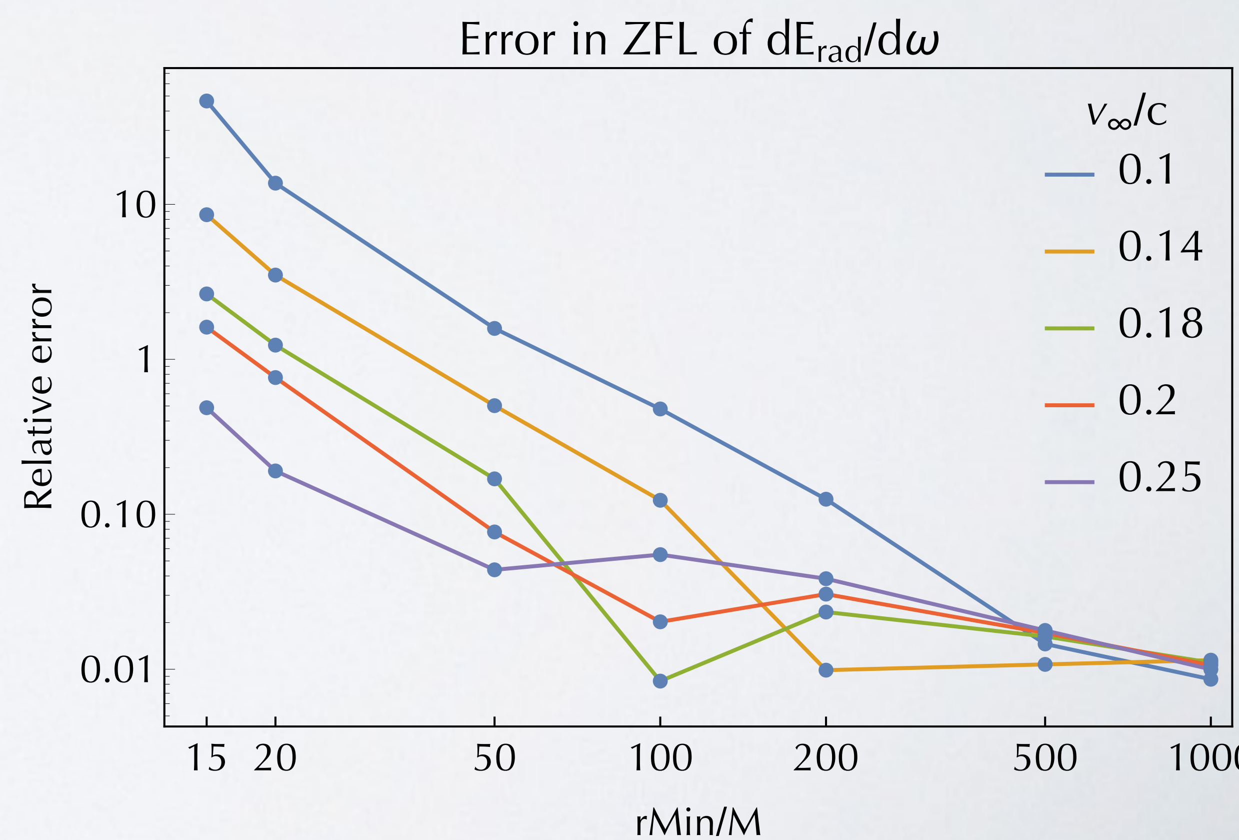




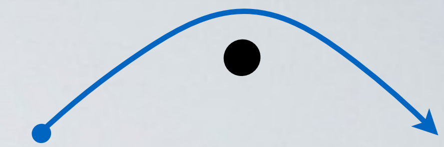


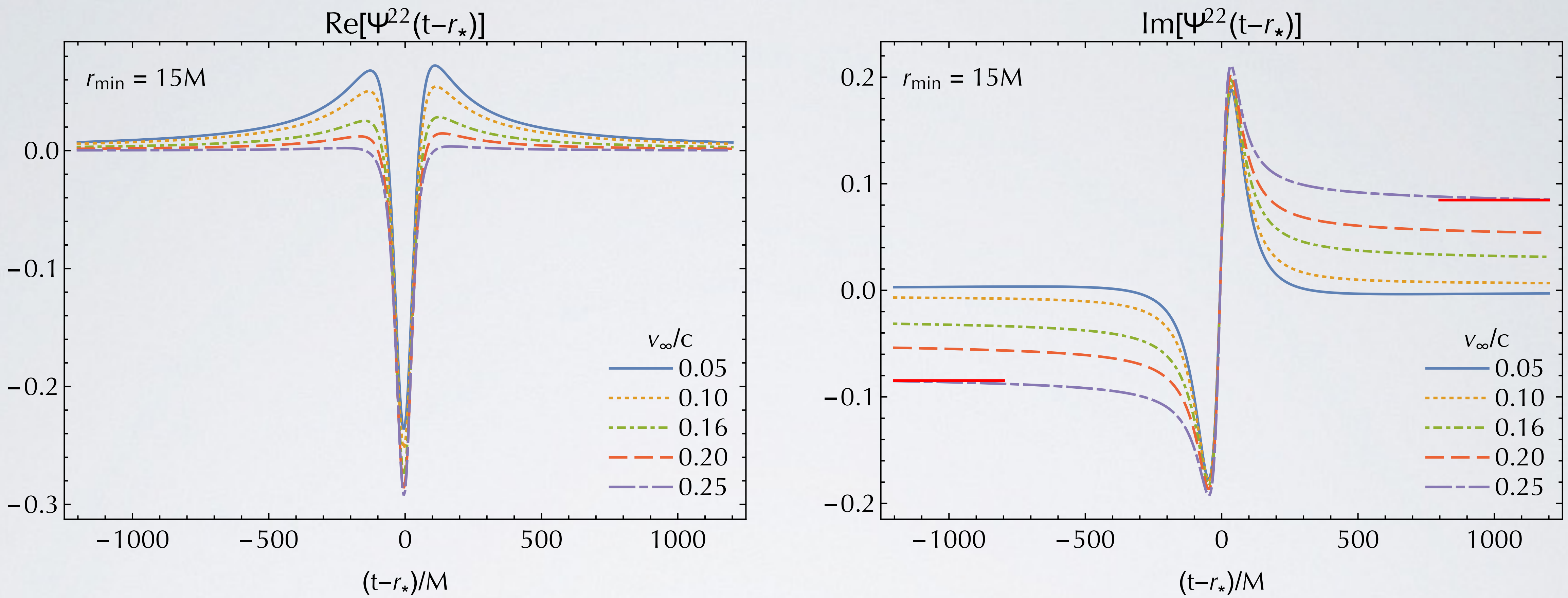
Smarr, 1977:

$$\left(\frac{dE}{d\omega}\right)_{\omega\to 0} = \frac{4}{\pi}\frac{\mu^2 M^2 \mathcal{E}^2}{b^2}\frac{(1+v^2)^2}{v^4}\left[2-\frac{16}{3}v^2+\left(3v-\frac{1}{v}\right)\log\left(\frac{1+v}{1-v}\right)\right]$$



The ZFL also predicts the memory effect



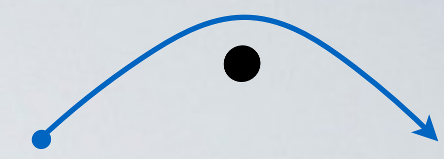


$$\Psi_{\ell m}(u, r_* \to \infty) = \frac{1}{2\pi}\int_{-\infty}^{\infty} C^{+}_{\ell m \omega}\, e^{-i\omega u} d\omega$$

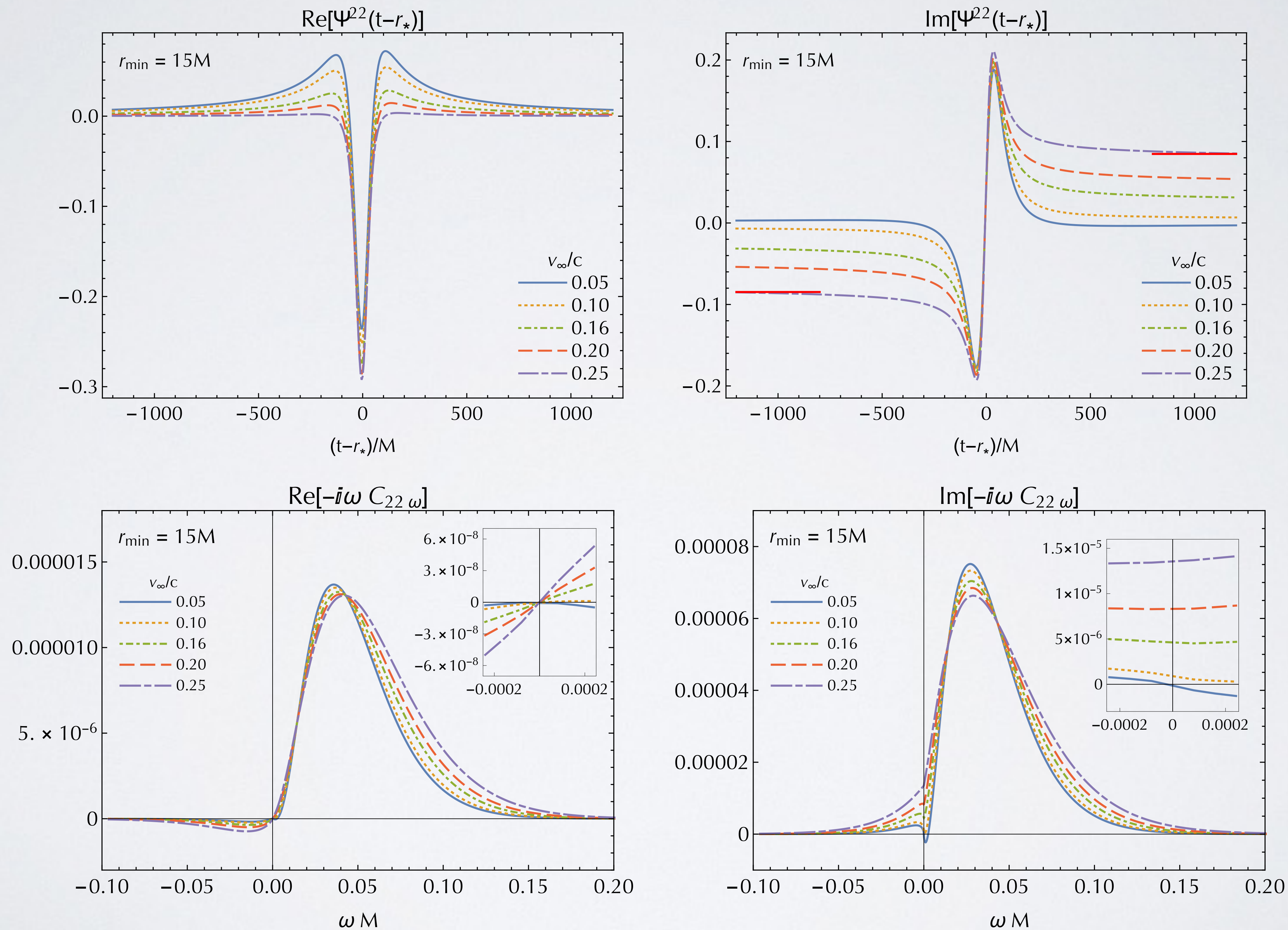
$$\Psi_{\ell m}(\infty, r_*) - \Psi_{\ell m}(-\infty, r_*) = \lim_{\omega \to 0} \int_{-\infty}^{\infty} e^{i\omega u} \partial_u \Psi_{\ell m}(u, r_*) du$$

$$[\![\Psi_{\ell m}]\!] = \lim_{\omega \to 0} \int_{-\infty}^{\infty} e^{i\omega u} \partial_u \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} C^{+}_{\ell m \omega'}\, e^{-i\omega' u} d\omega' \right) du = - \lim_{\omega \to 0} i\omega\, C^{+}_{\ell m \omega}$$

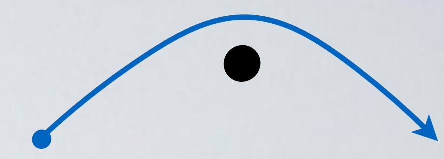
The ZFL also predicts the memory effect



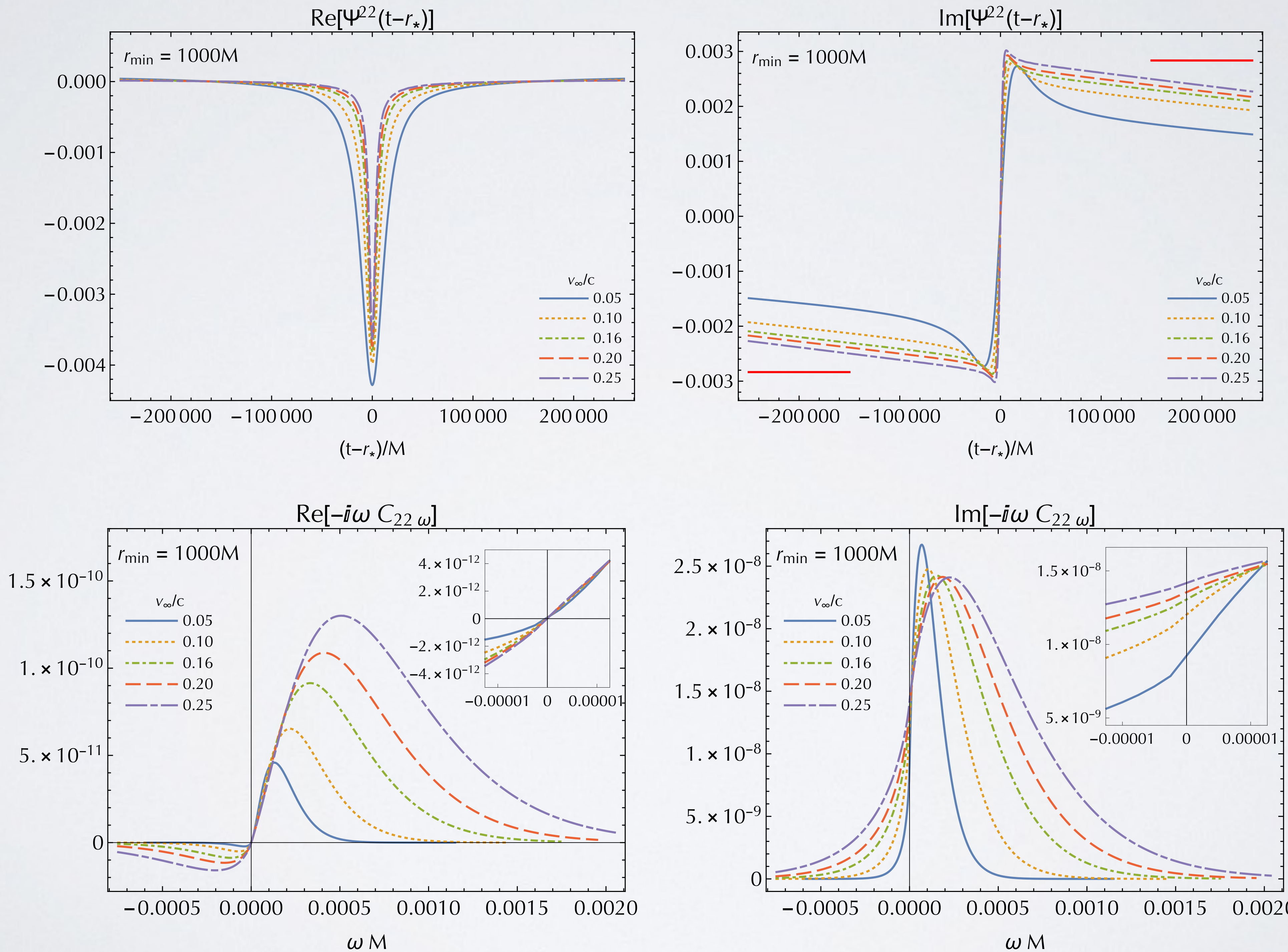
$$[\![\Psi_{\ell m}]\!] = -\lim_{\omega \to 0} i\omega\, C^{+}_{\ell m \omega}$$



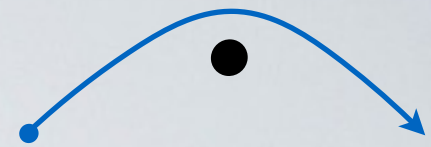
The code struggles with small frequencies

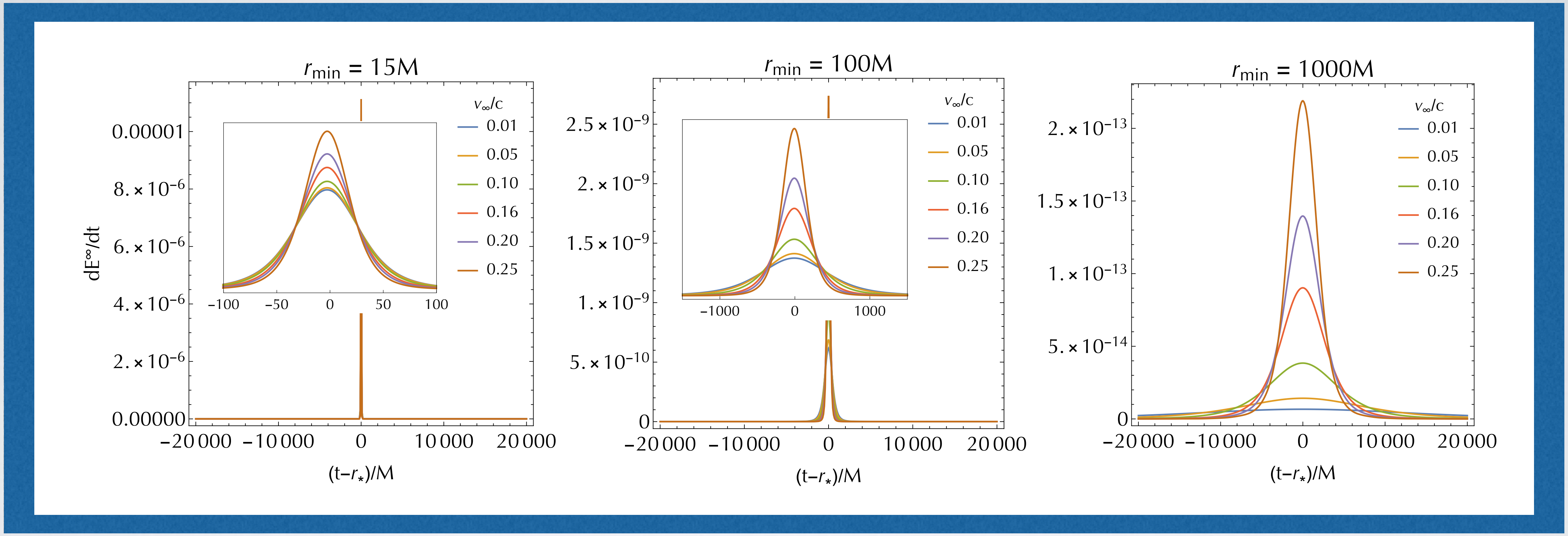


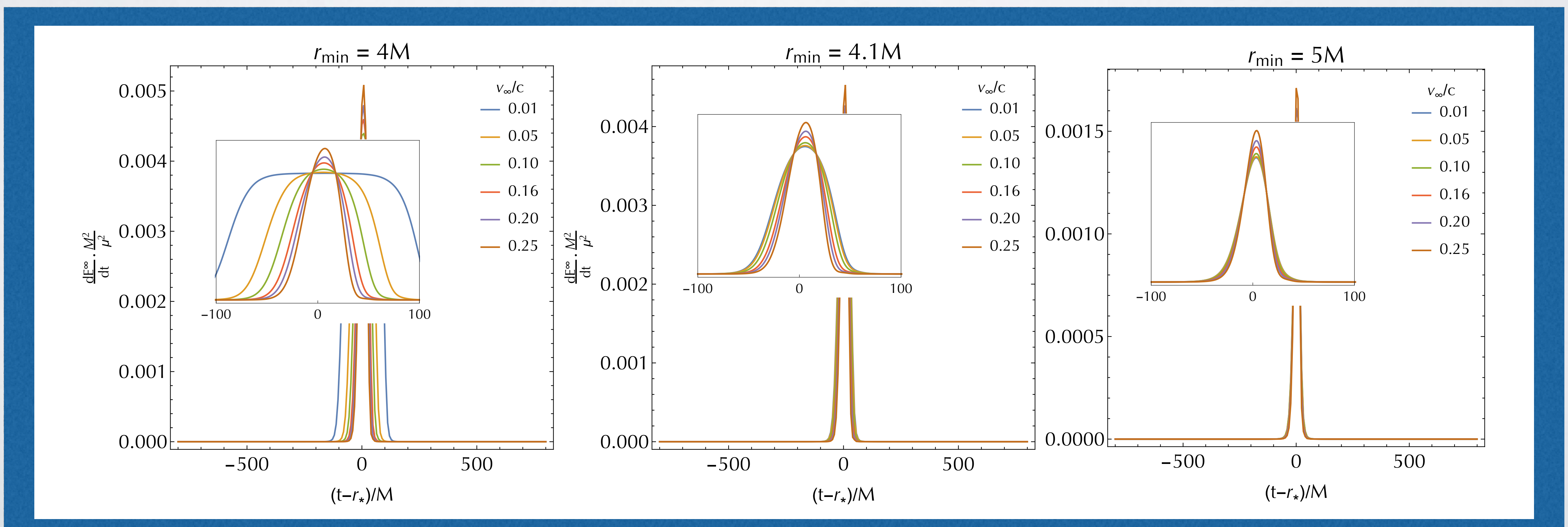
$$[\![\Psi_{\ell m}]\!] = -\lim_{\omega\to 0} i\omega\, C^{+}_{\ell m\omega}$$



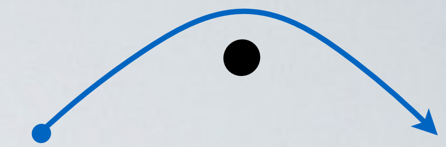
The energy flux is interesting in the large and small limits

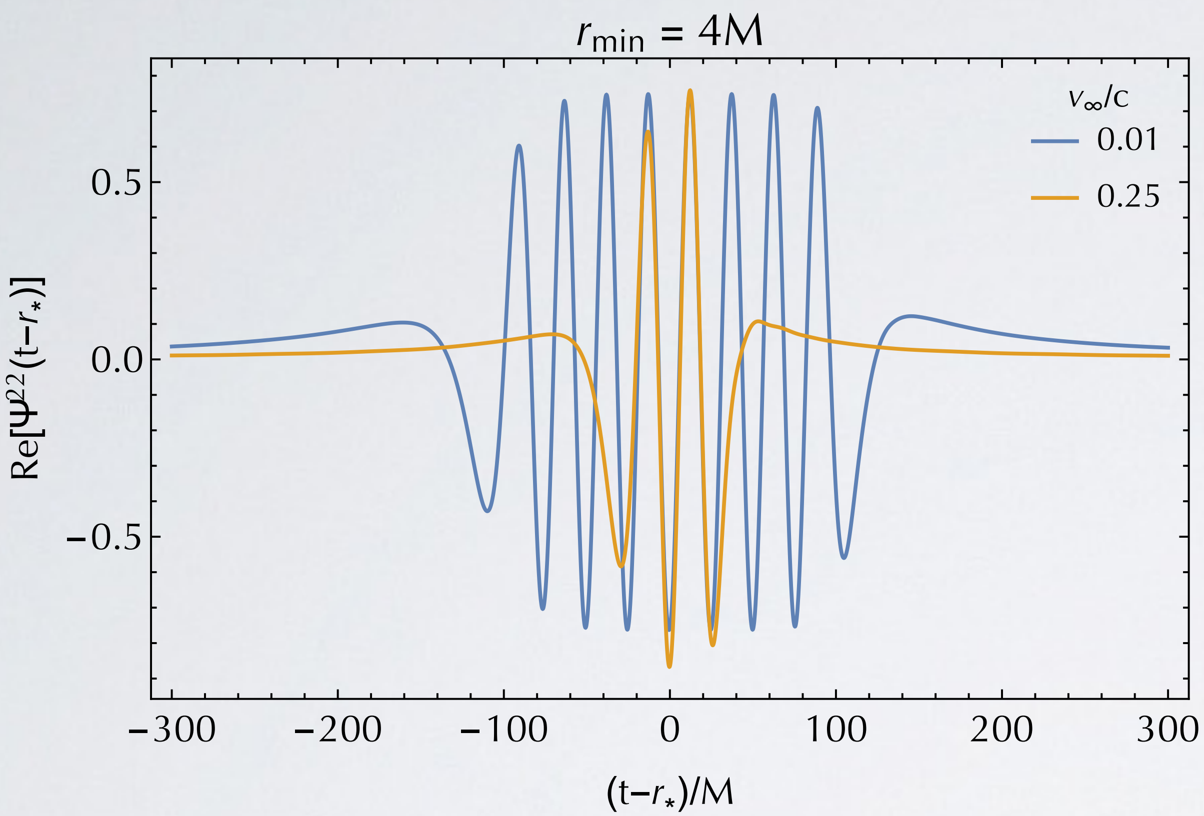


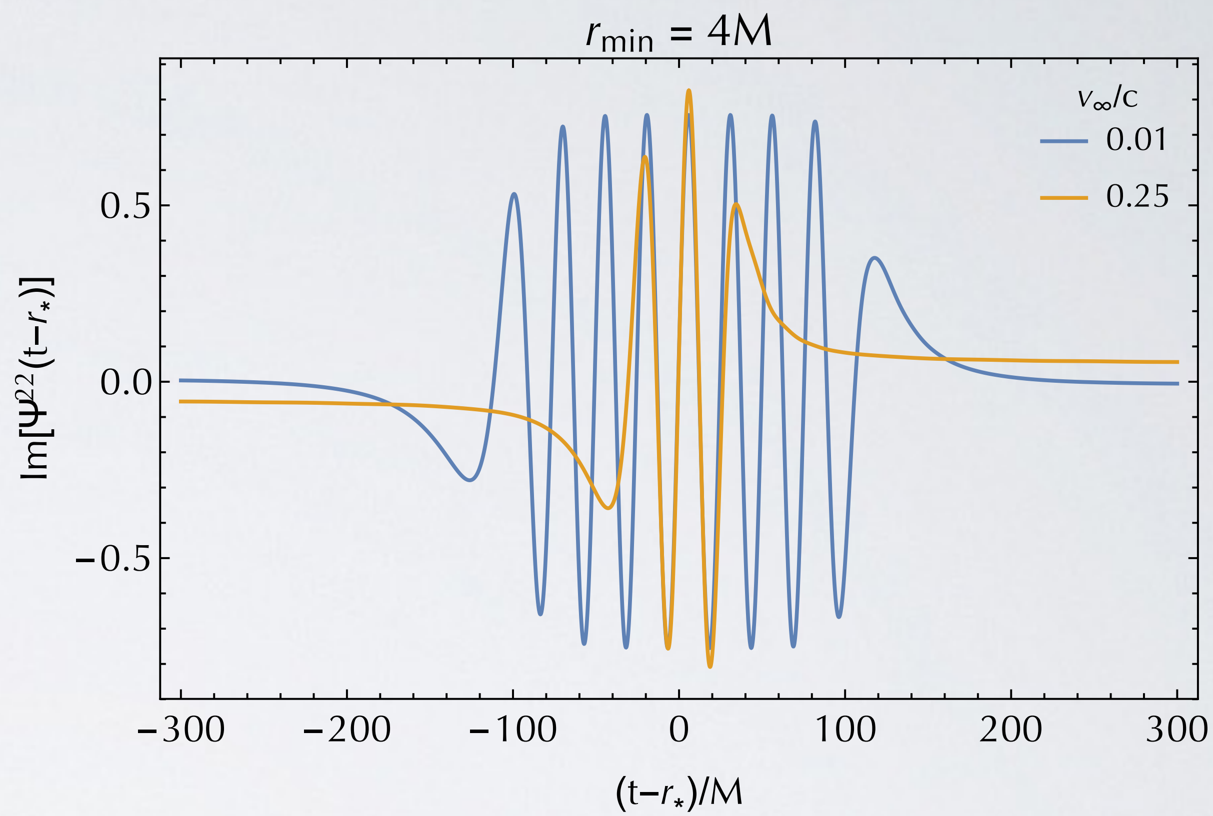


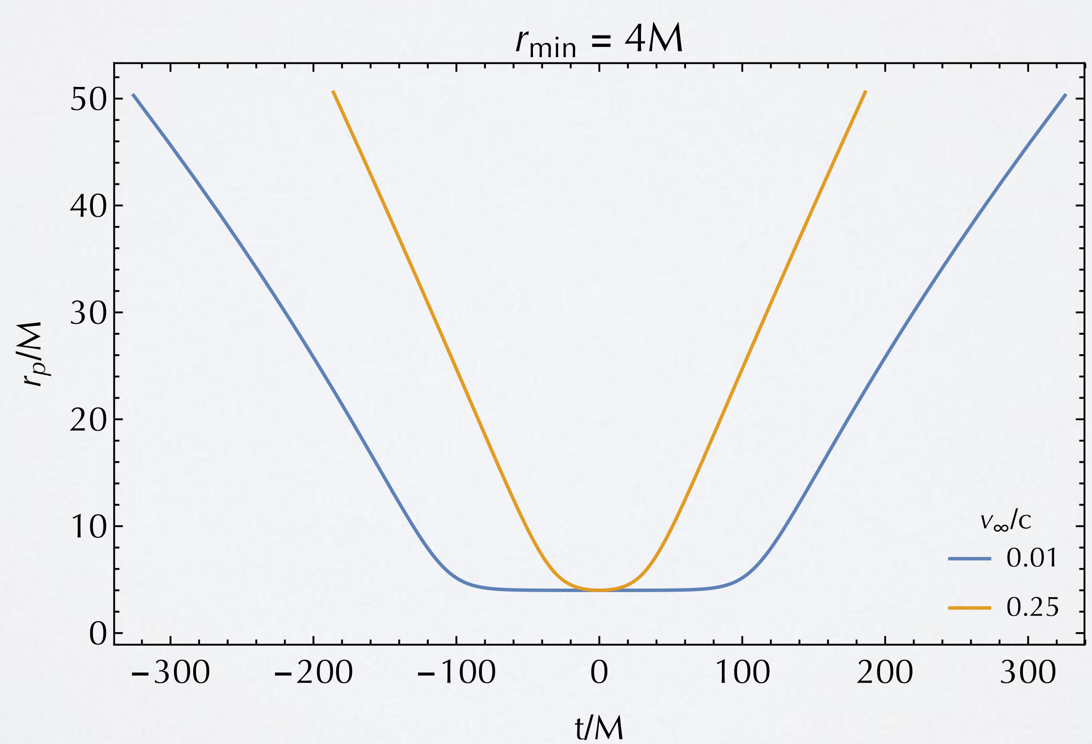


At the critical surface the particle will radiate forever

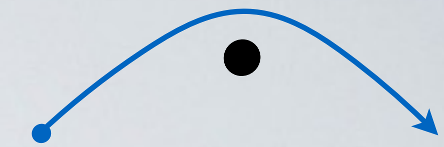


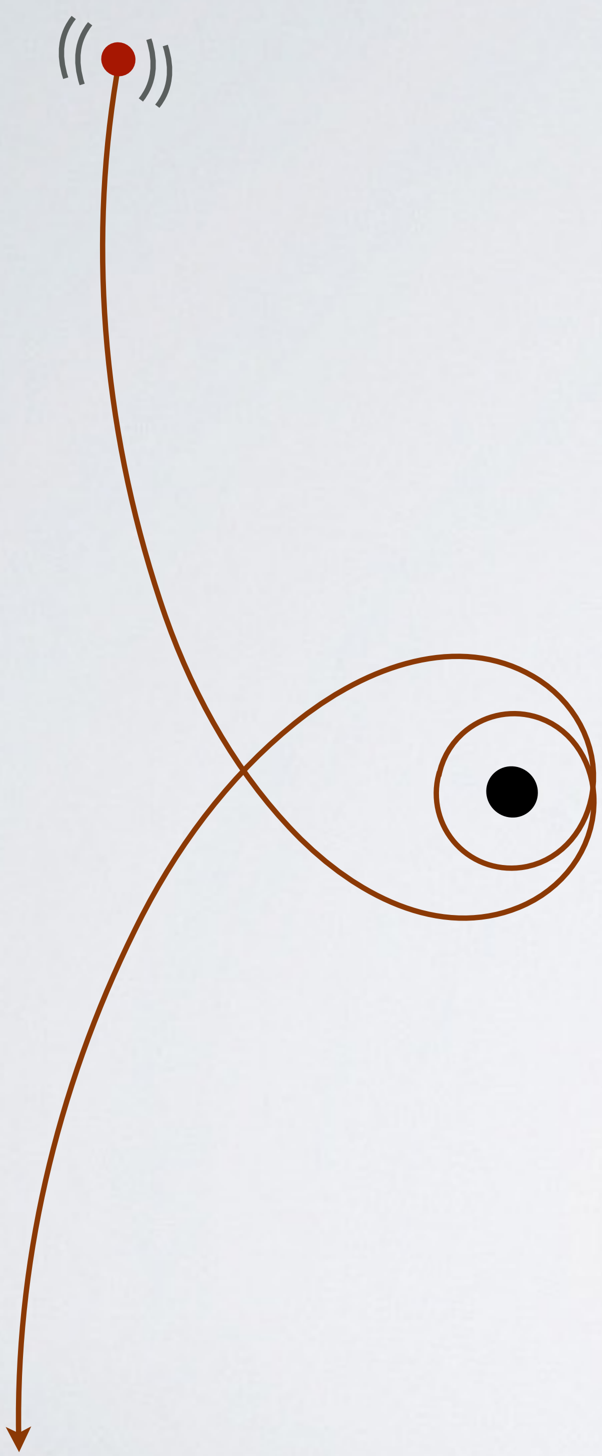


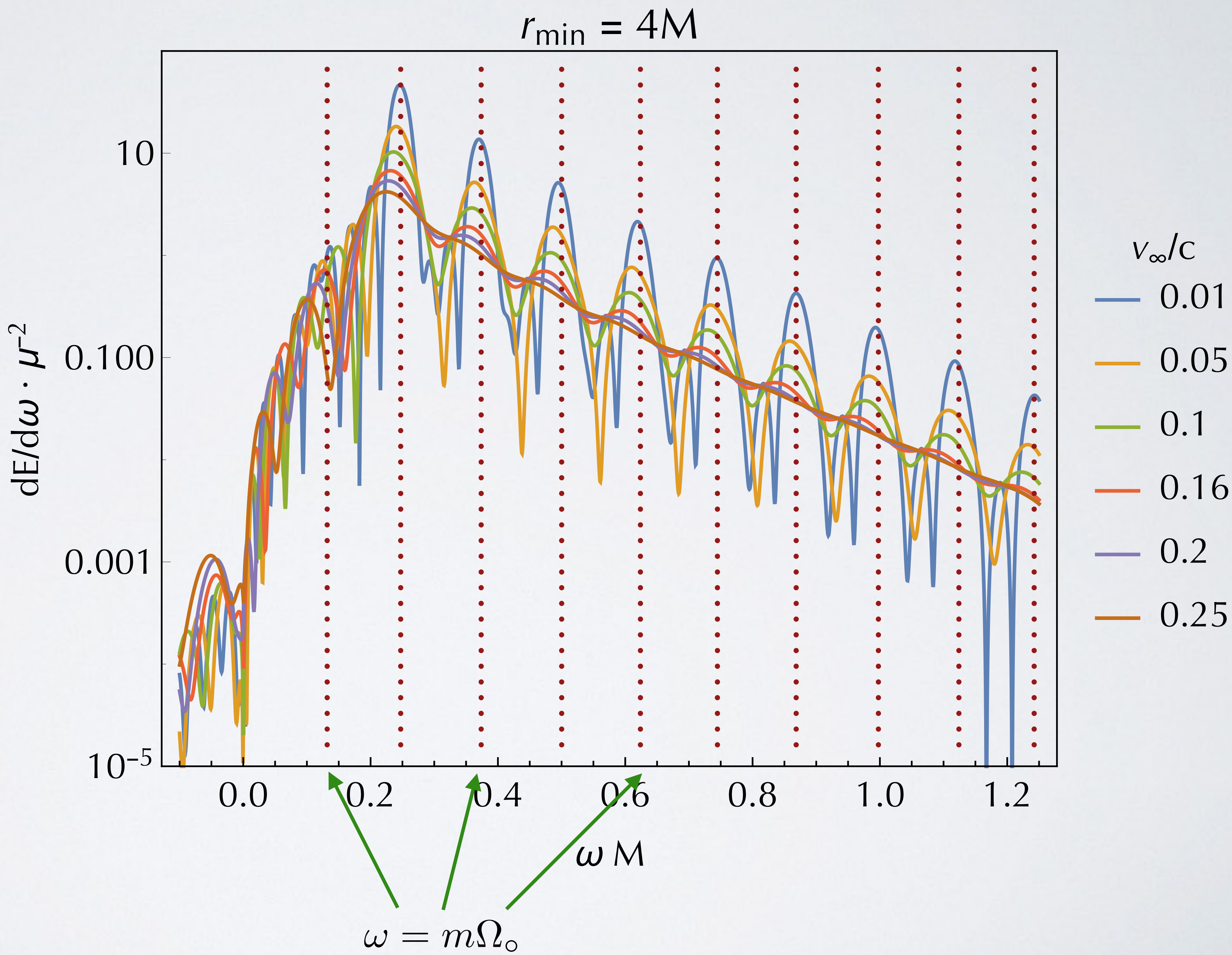




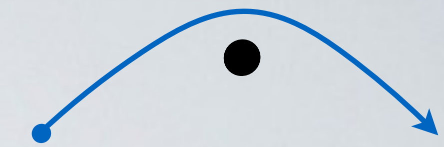
At the critical surface the harmonics are evident



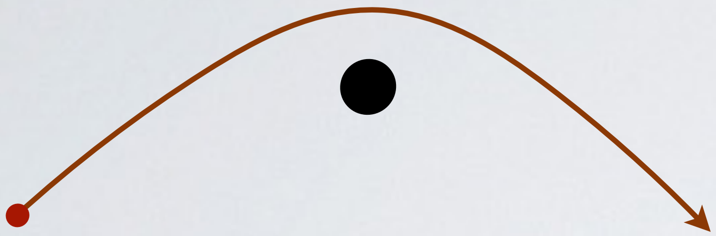


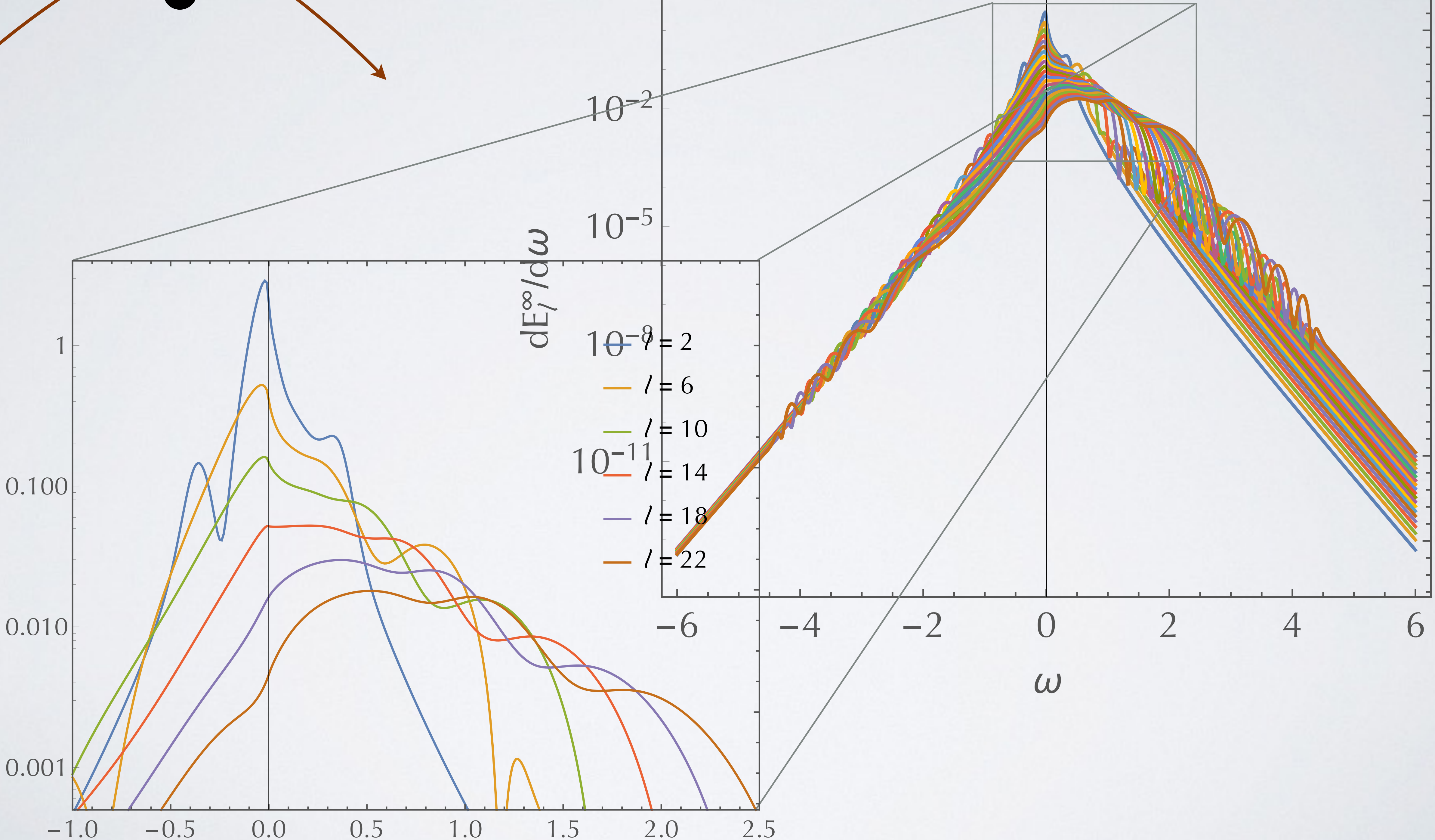


Frequency domain allows high Lorentz factor scatters

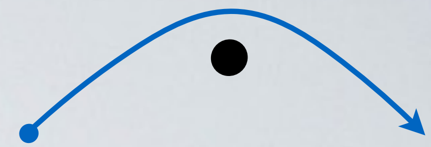


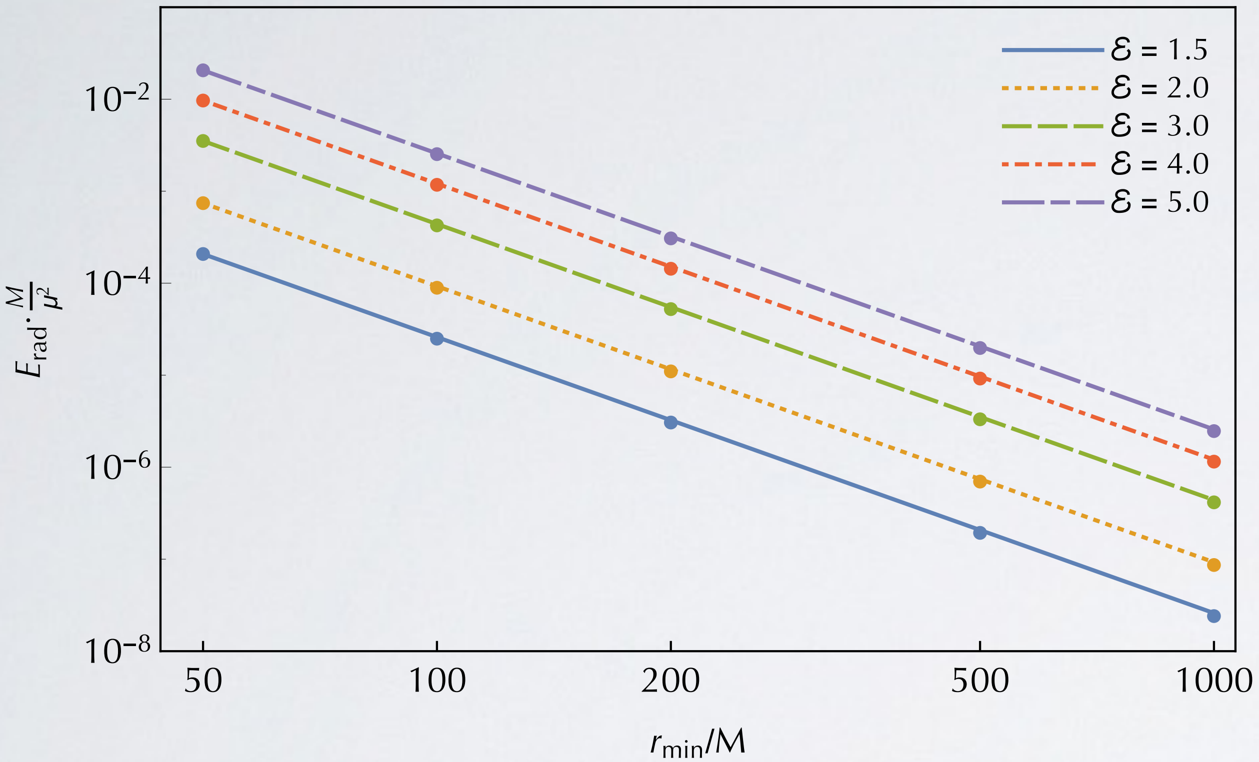
$(\mathcal{E}, b) = (10, 20M)$, v/c = 0.995

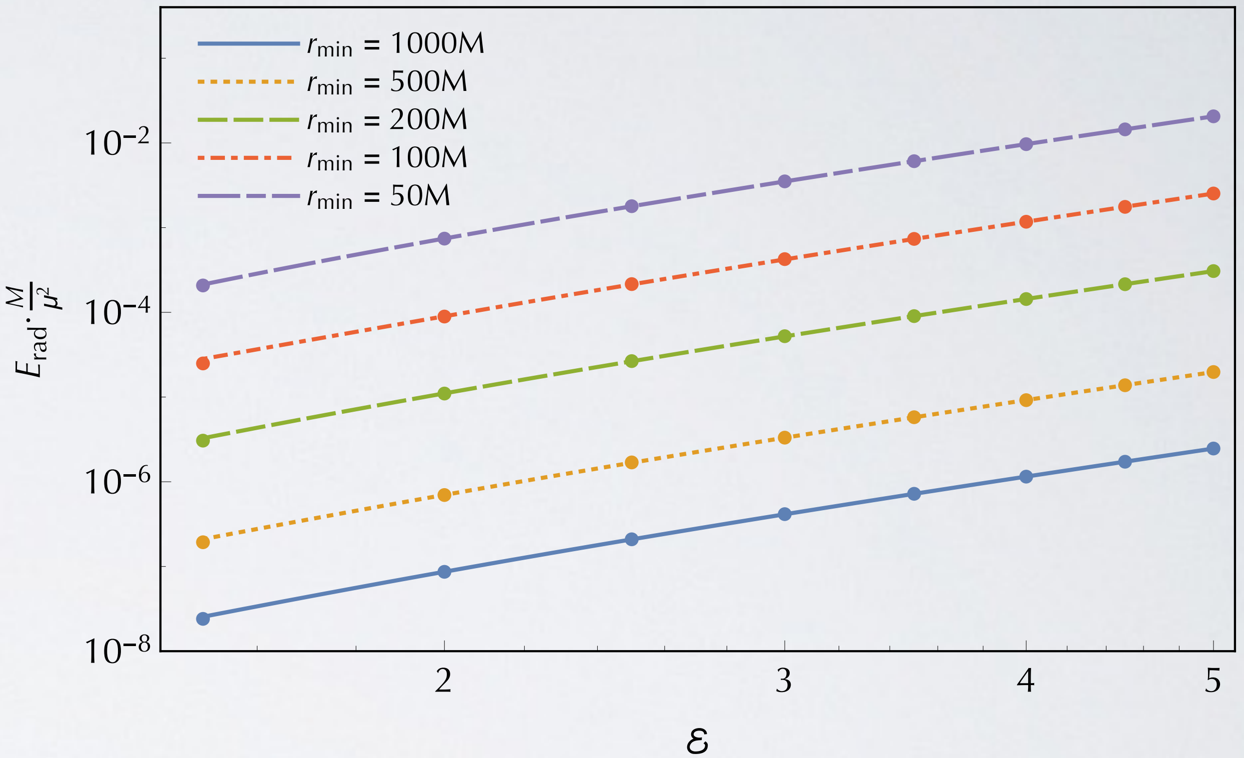




At high energies we agree with Peters' predictions







$$v \ll c$$

$$\frac{E_{\rm rad}}{M} = \frac{37\pi}{15}\frac{G^3}{c^5}\left(\frac{\mu}{M}\right)^2 \frac{v}{(r_{\rm min}/M)^3}$$

$$\mathcal{E} \gg 1$$

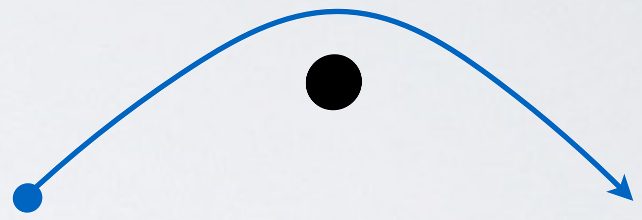
$$\frac{E_{\rm rad}}{M} \sim \frac{G^3}{c^4}\left(\frac{\mu}{M}\right)^2 \frac{\mathcal{E}^3}{(r_{\rm min}/M)^3}$$

28 ± 2

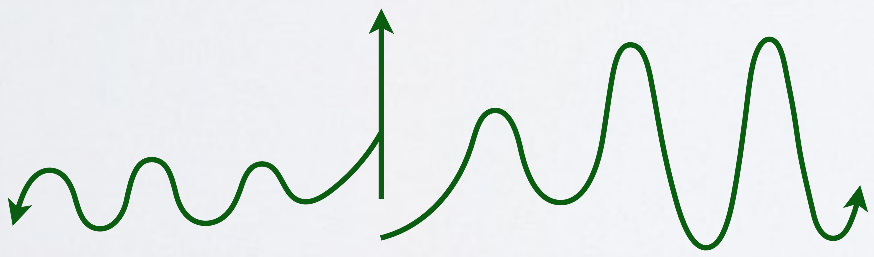
Outline

Bound motion

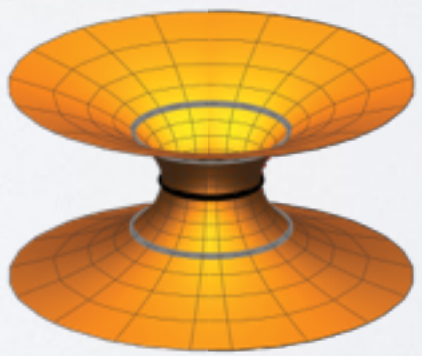
Unbound motion



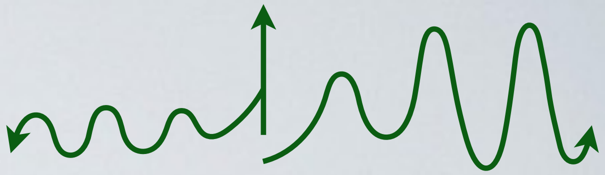
Local calculations

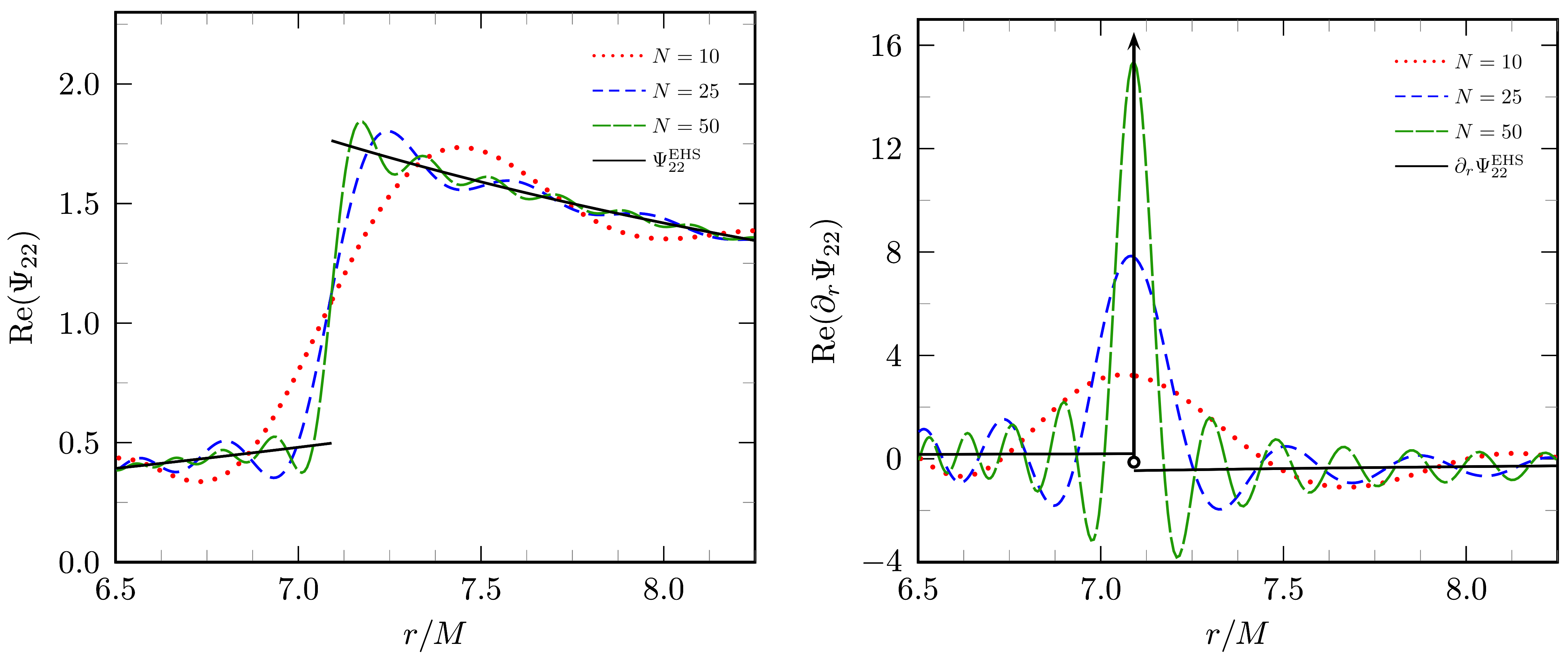


Wormholes and echoes

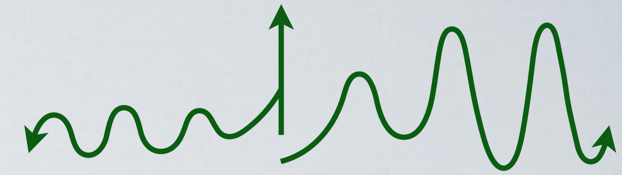


The point particle can cause a Gibbs phenomenon



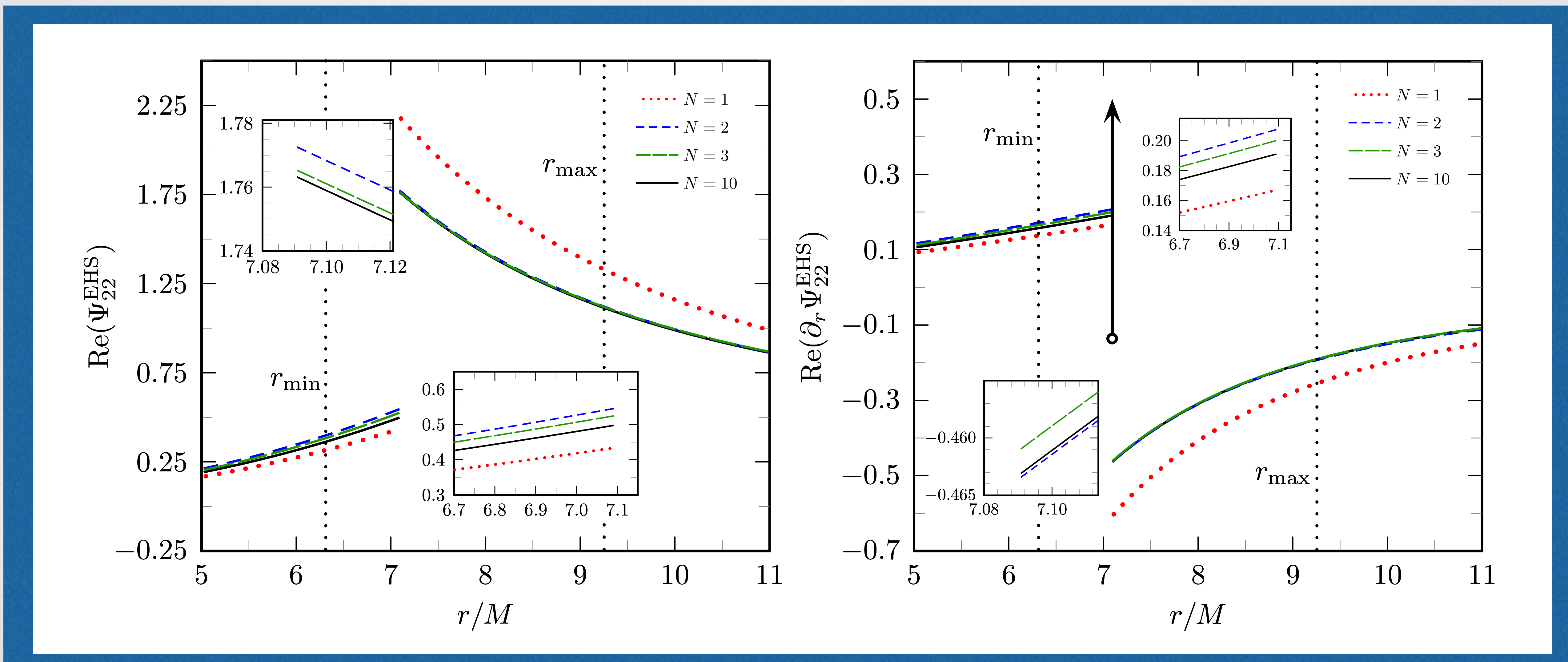


‘Extended homogeneous solutions’ avoids this problem

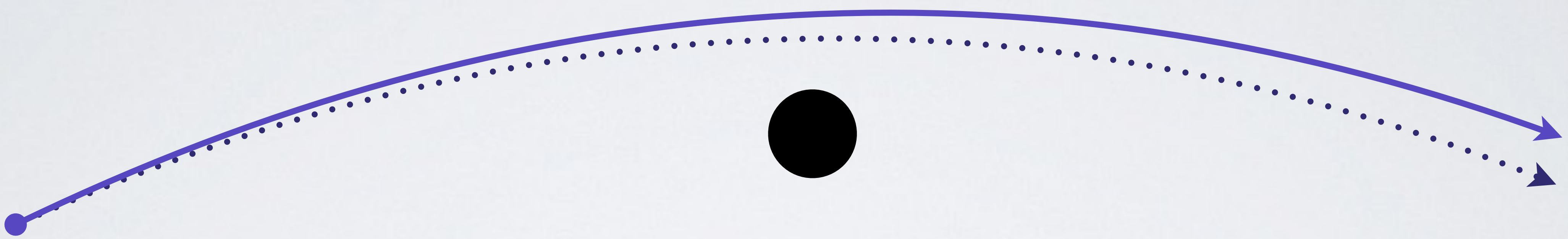


Barack, Ori, Sago

Hopper, Evans



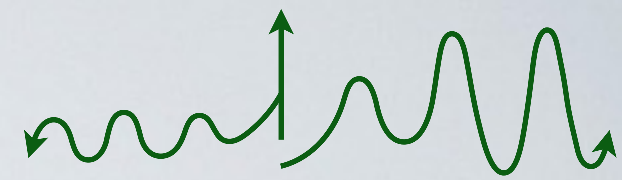
Scatters also have gauge invariants

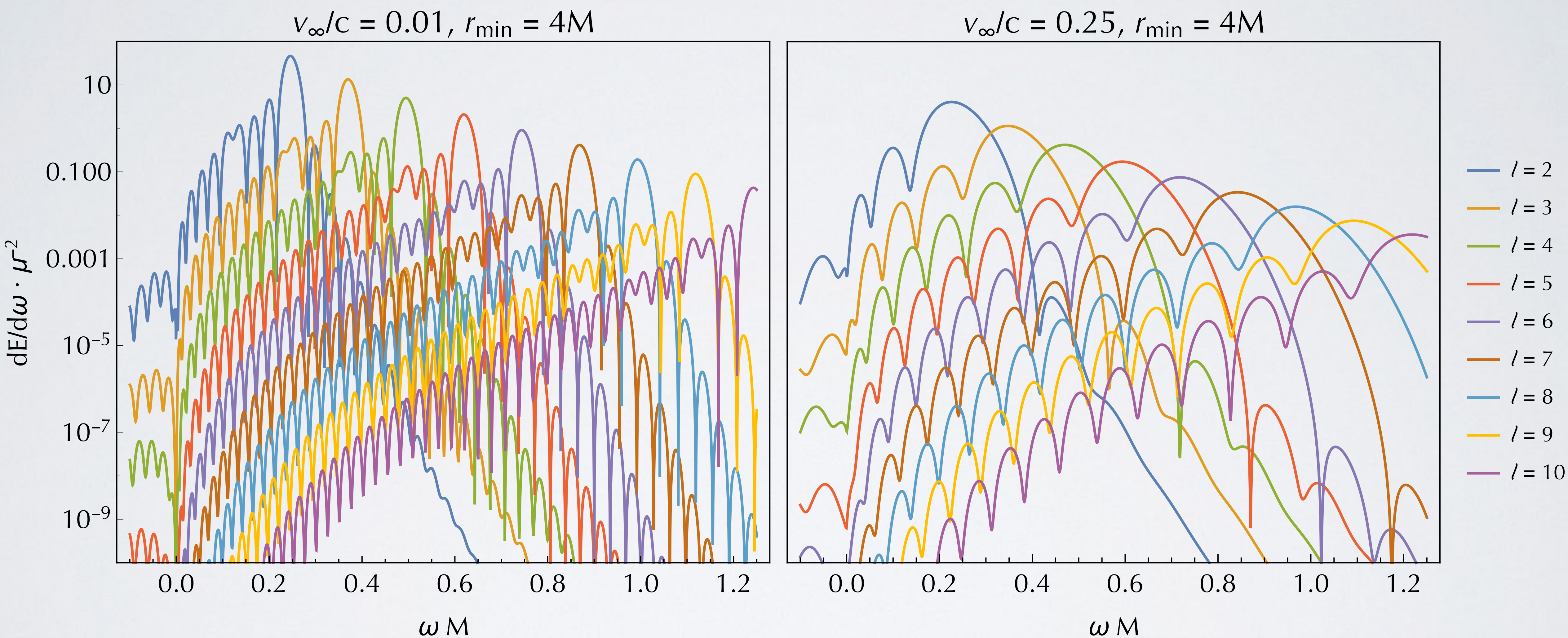


Generalized redshift: $\delta\left(\frac{T}{\mathcal{T}}\right)$

Angle of deflection: $\delta\left(\Delta\theta\right)$

Local calculations are hard, any way you cut it

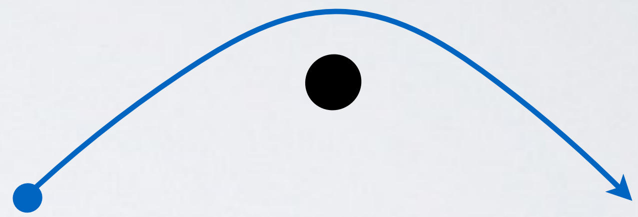




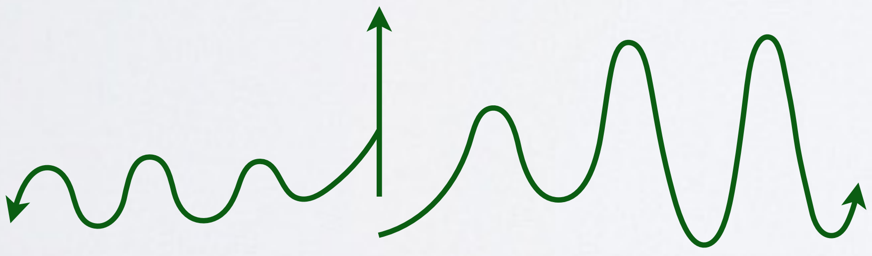
Outline

Bound motion

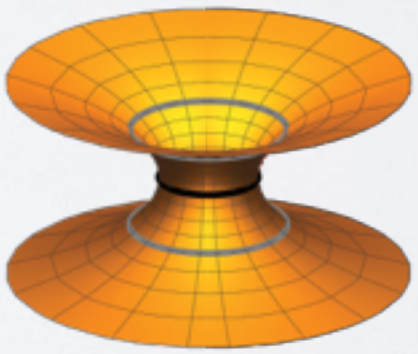
Unbound motion



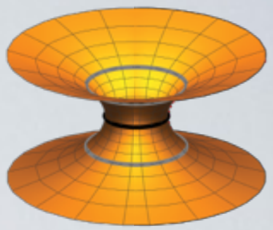
Local calculations

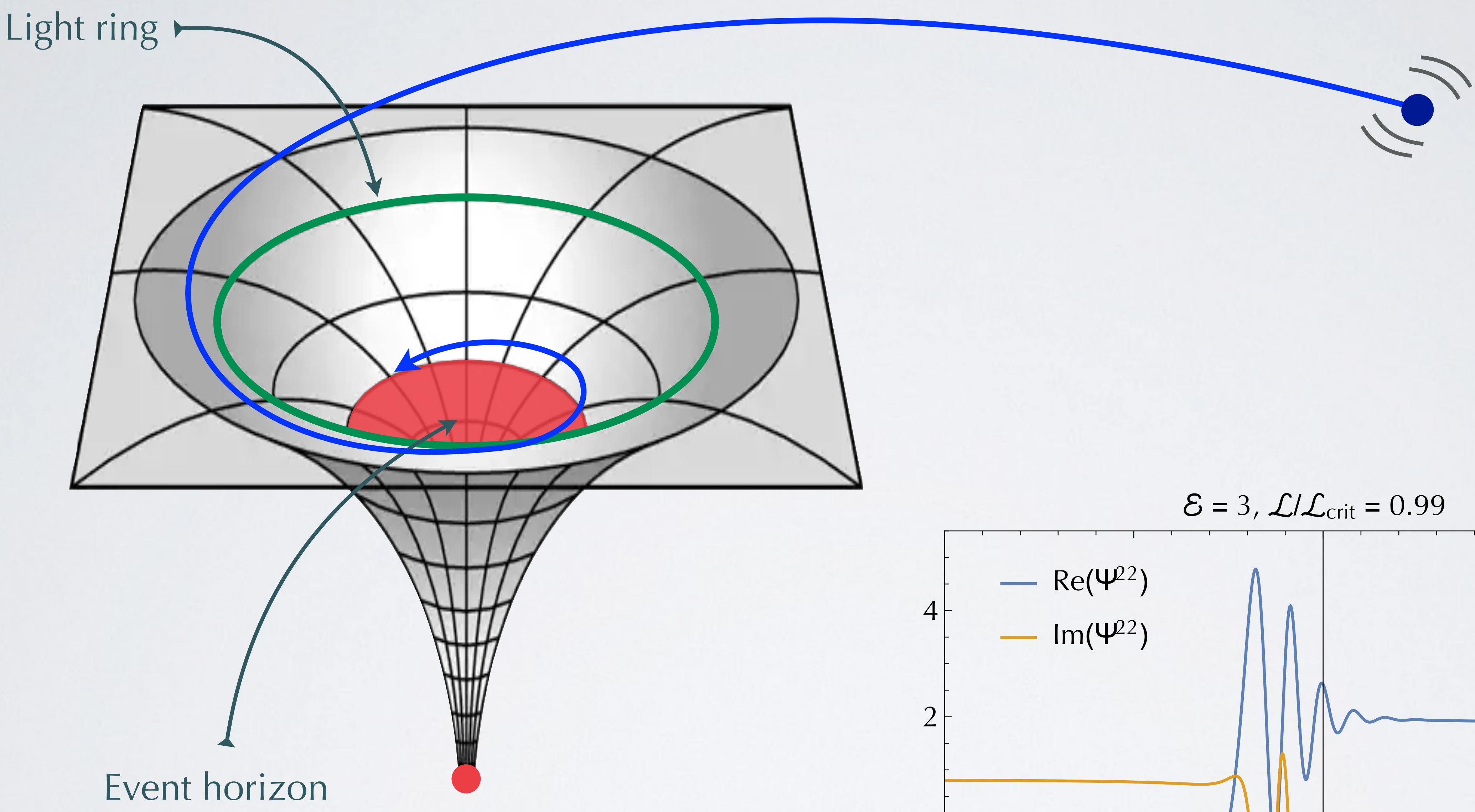


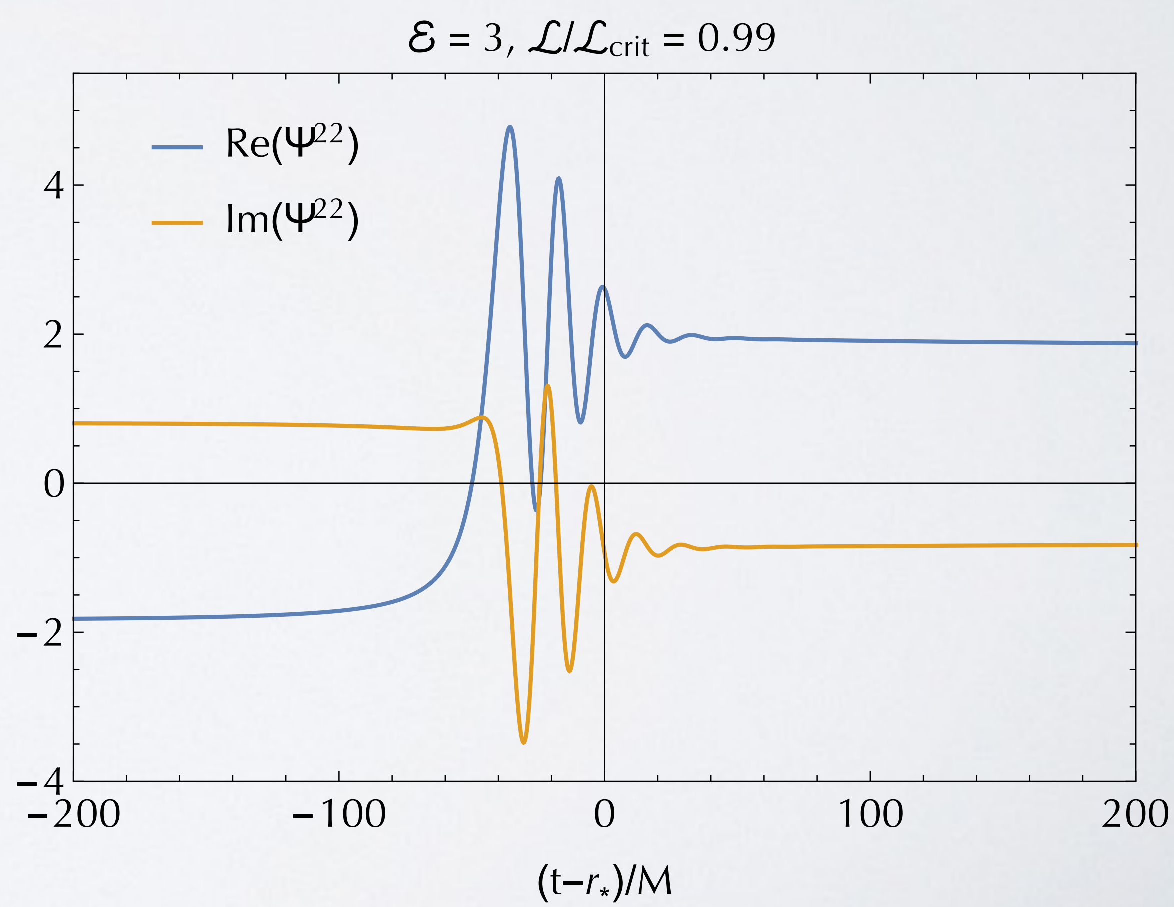
Wormholes and echoes



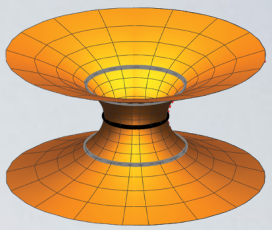
Black holes ring because they have “light rings”

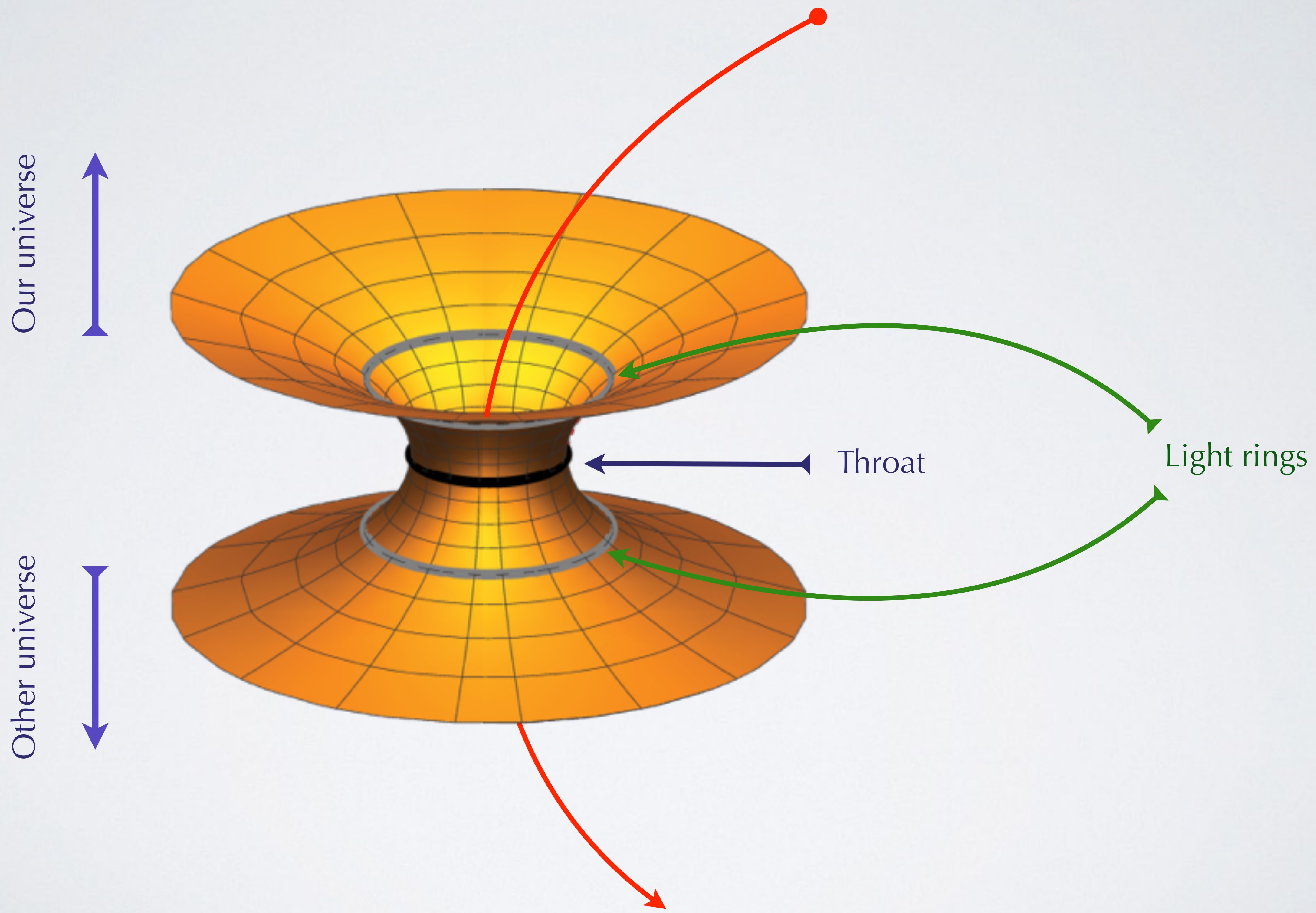




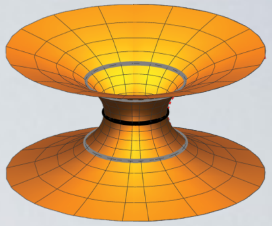


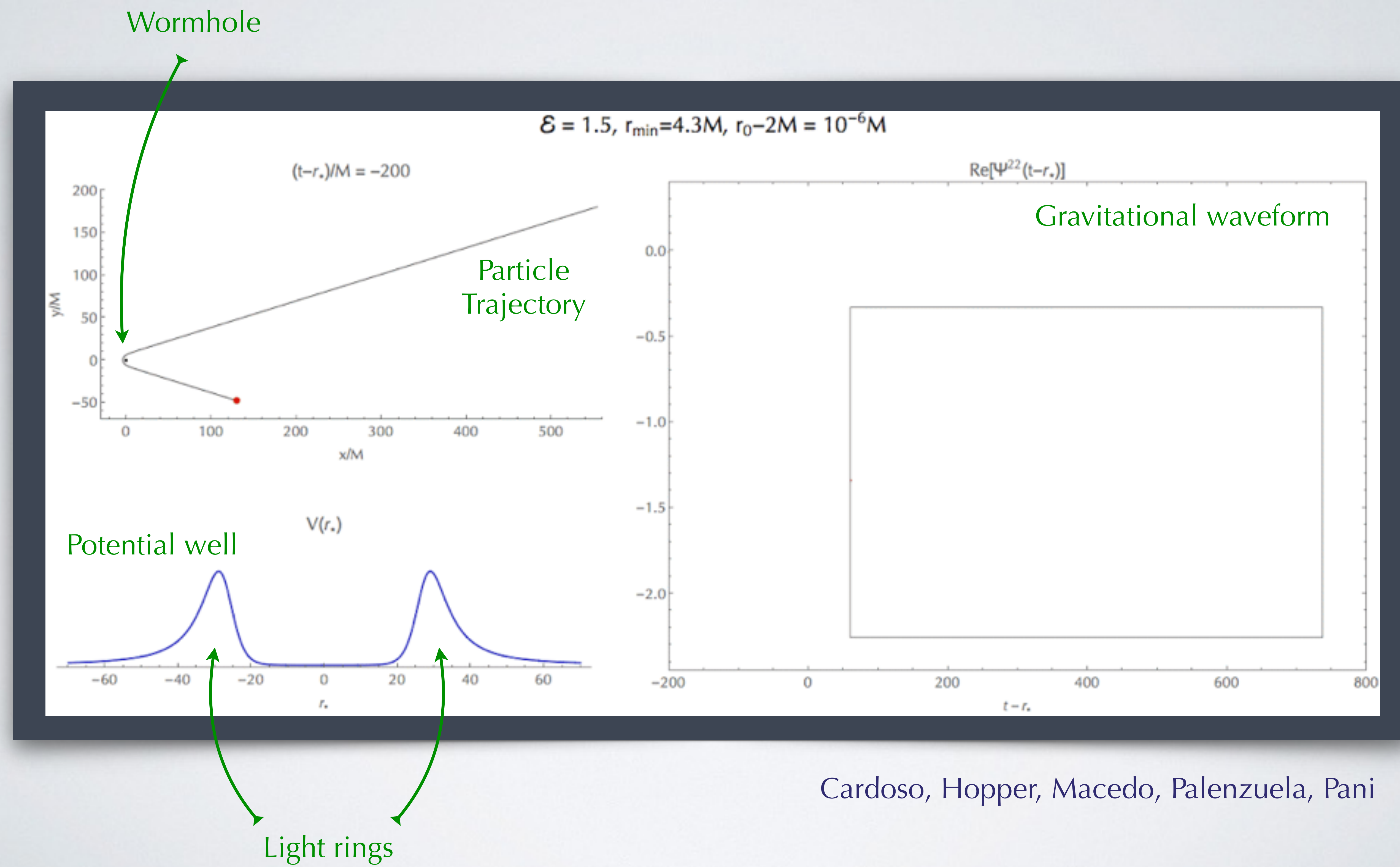
A wormhole can have two light rings





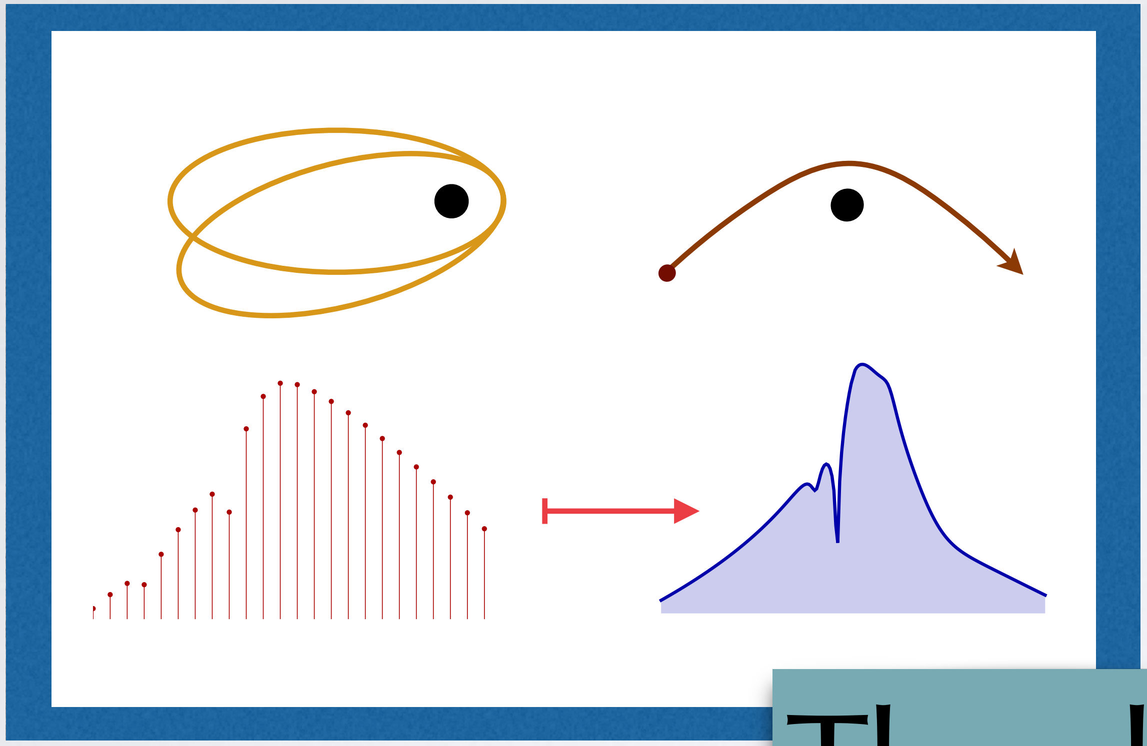
A wormhole rings like a black hole at first, but echoes later

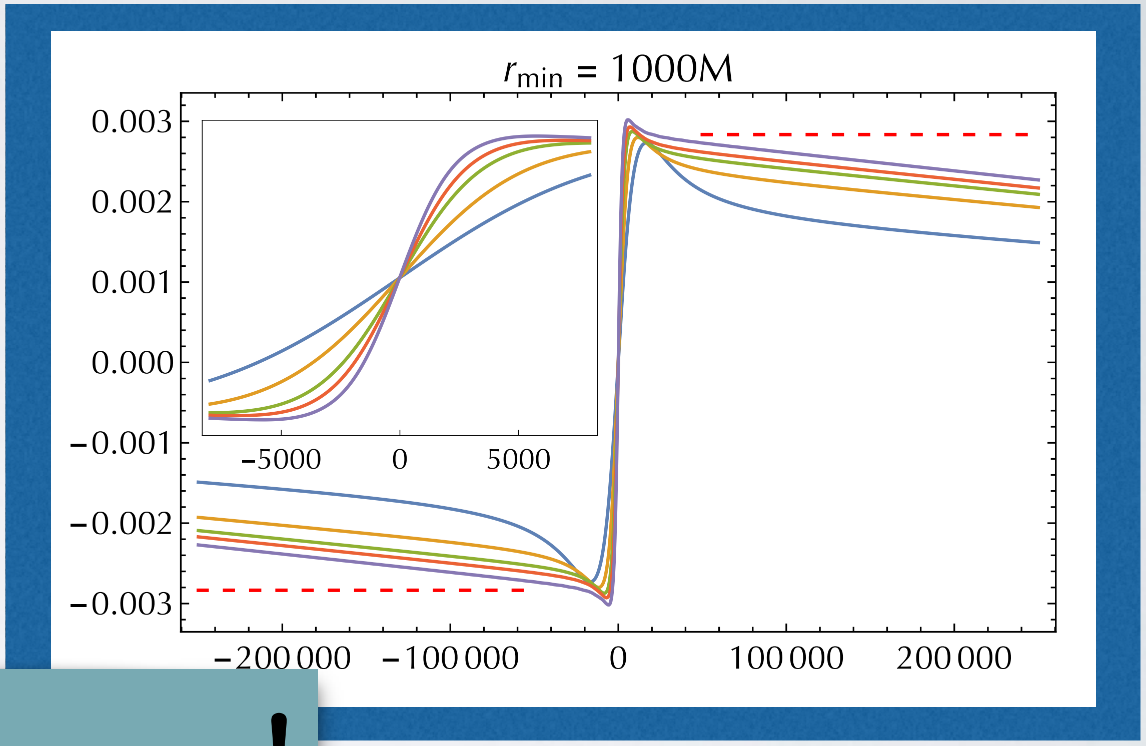




Cardoso, Hopper, Macedo, Palenzuela, Pani

These are the main points





Thank you!