Evolution of small-mass-ratio binaries with a spinning secondary

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Motivation: compact binaries

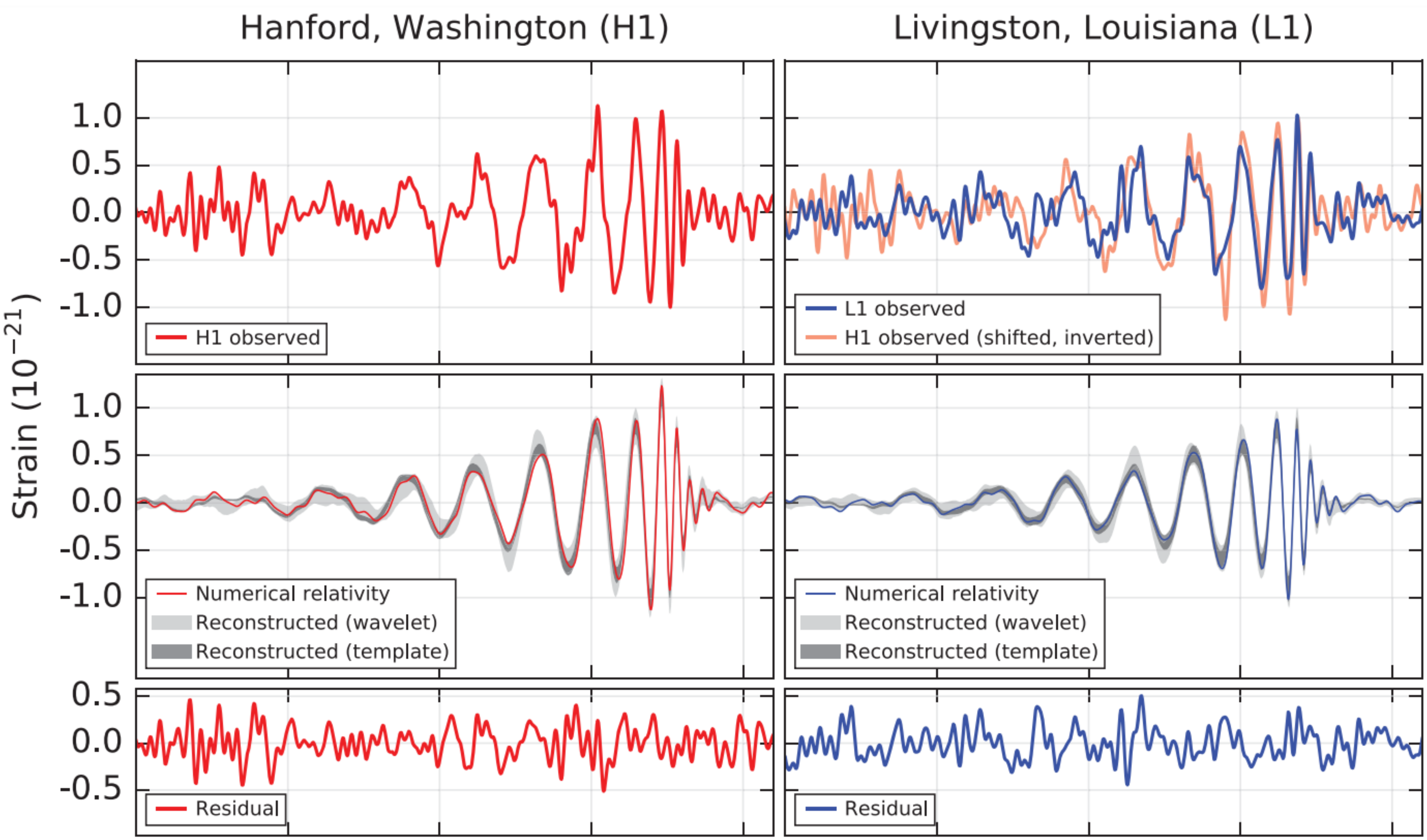
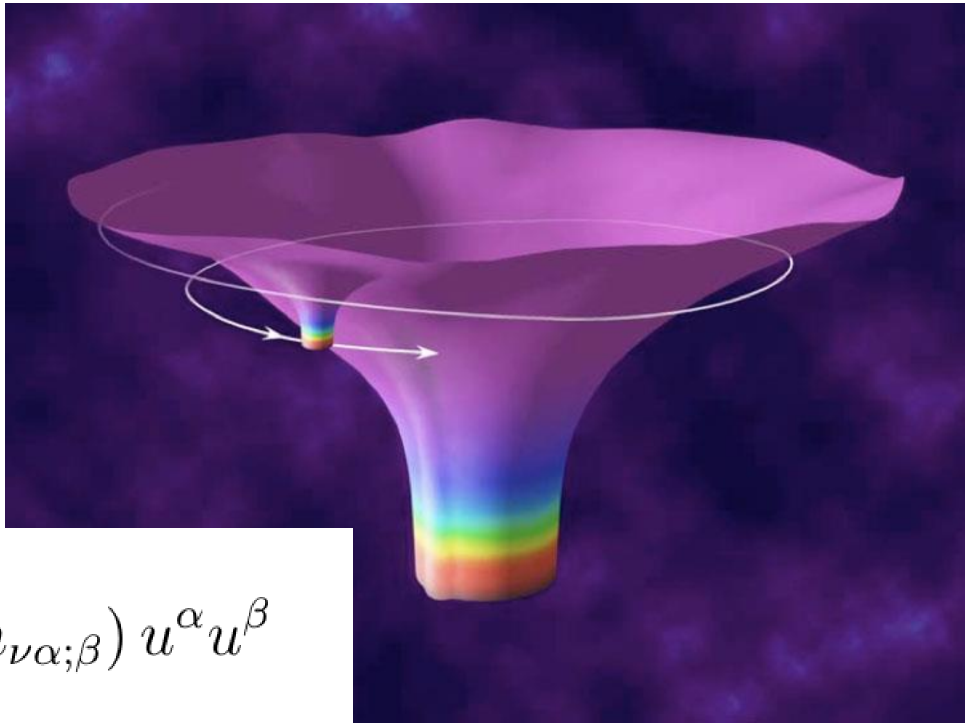


Image credit: LIGO Scientific Collaboration

Motivation: perturbation theory

- The era of gravitational wave astronomy has dawned
- Compact binaries are important sources
- Highly-relativistic small-mass-ratio binaries are not well suited for post-Newtonian or numerical relativity
- Perturb metric in powers of mass-ratio (μ/M)
- Evolve motion with perturbed metric (gravitational self-force)



$$F^{\mu} = \frac{\mu}{2}\left(g^{\mu\nu} + u^{\mu}u^{\nu}\right)\left(h_{\alpha\beta;\nu} - 2h_{\nu\alpha;\beta}\right)u^{\alpha}u^{\beta}$$

Features of EMRI model used here

- First-order self-force (dissipative and conservative)
- Spin-curvature interaction (spin-force)
- Accurate: 7+ digits of force accuracy (track phase to within ~0.1 radians)
- Broad range of orbital parameters (high eccentricity)

Other important effects:

- ~~Kerr spacetime~~ (see talk by van de Meent)
- ~~Second-order self-force~~ (see talks by Pound and Wardell)

Motivation: high eccentricity and LISA

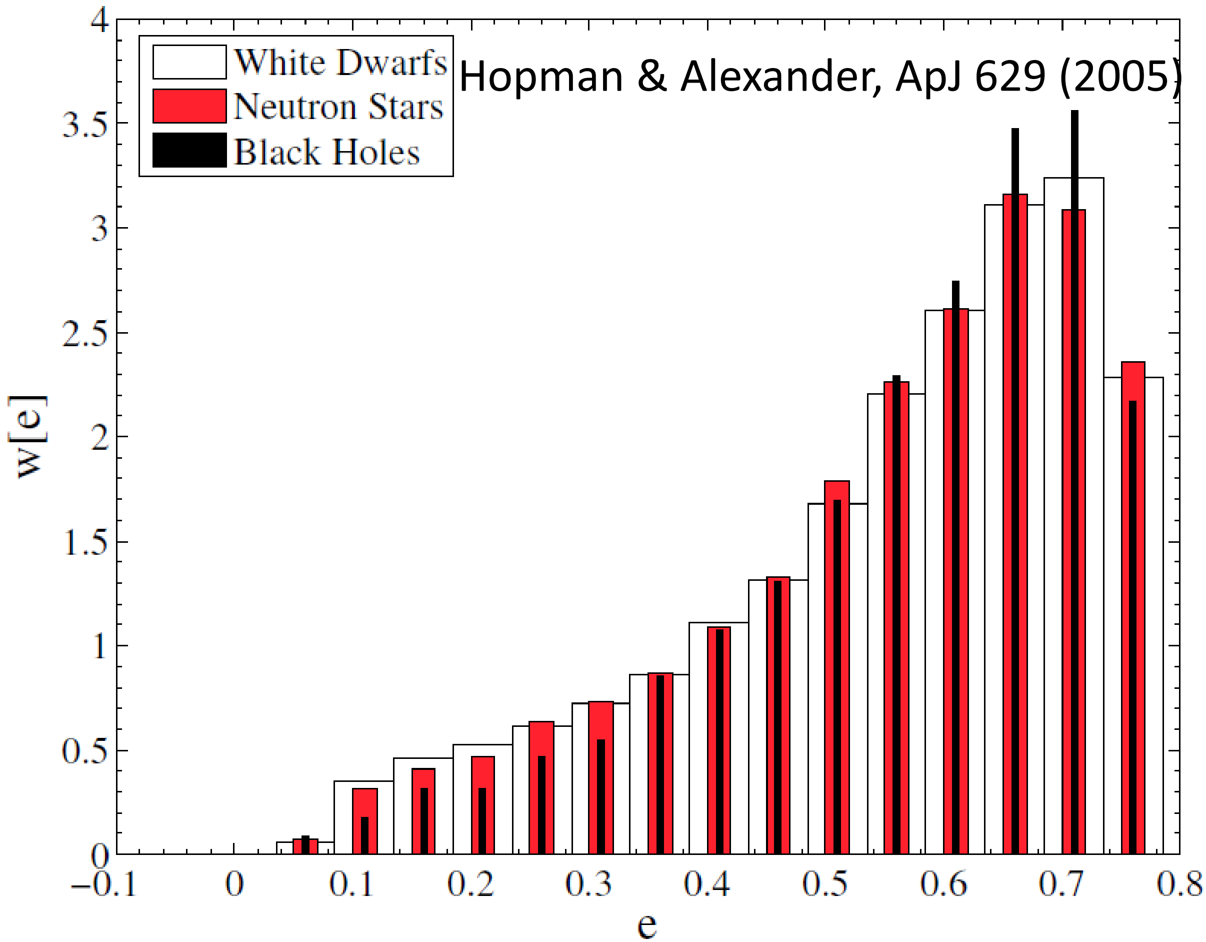
- Objects enter LISA band with eccentricities up to e≈0.8?

- Past gravitational self-force codes limited to e≲ 0.3

Warburton et al.
Phys. Rev. D 85 (2012)

Akcay et al.
Phys. Rev. D 88 (2013)

Challenge: improve eccentricity range



Numerical tool: metric perturbations

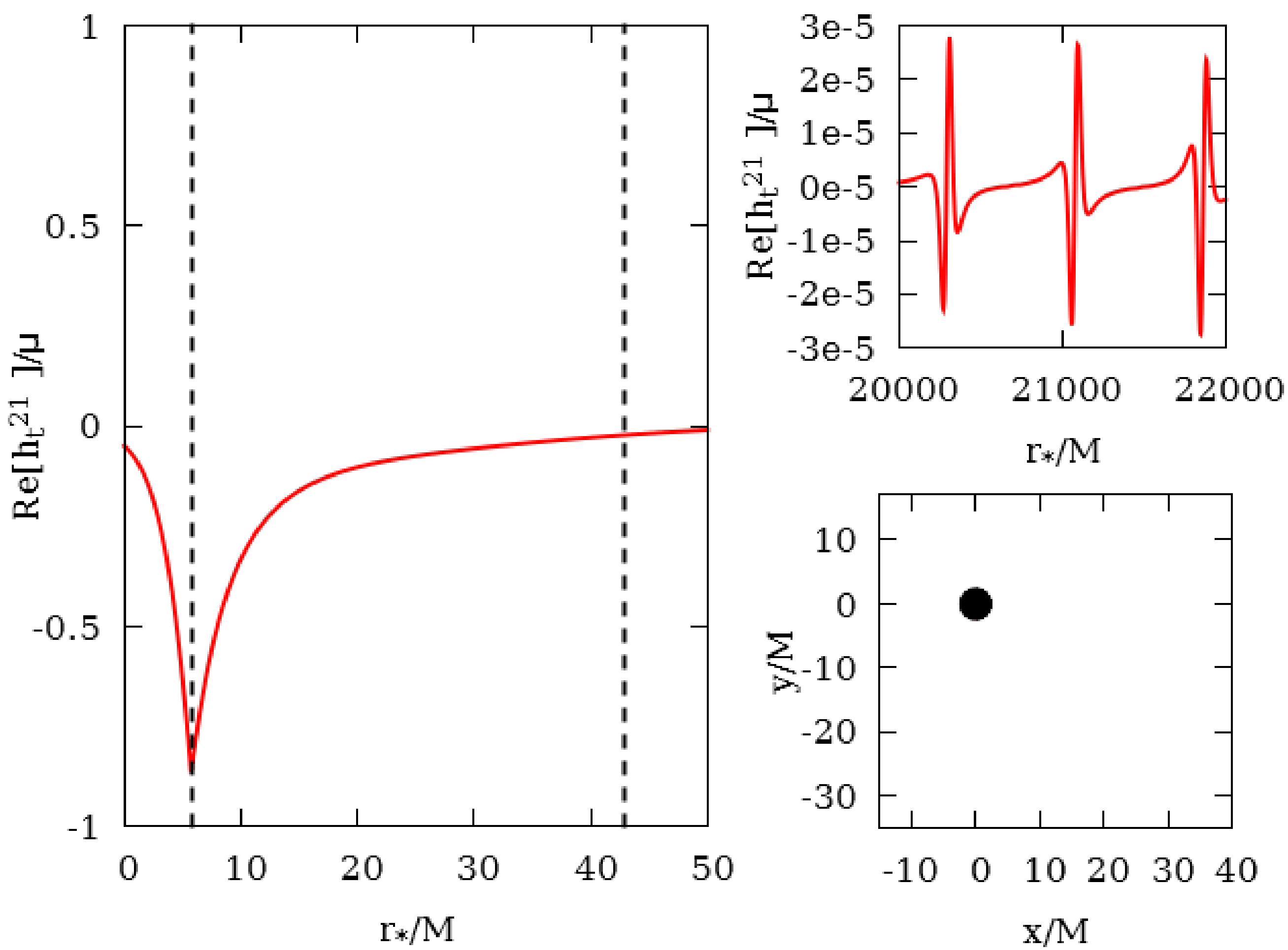
- Lorenz gauge: $\Box \bar{h}_{\mu\nu} + 2R^{\alpha\ \beta}_{\ \mu\ \nu}\bar{h}_{\alpha\beta} = -16\pi T_{\mu\nu}$
- Schwarzschild metric perturbations separable into tensor spherical harmonic and Fourier modes (l,m,n)

$$h_{\mu\nu}(t,r,\theta,\phi) = \sum_{l,m,n,k} \tilde{h}^{(k)}_{lmn}(r)\, e^{-i\omega_{mn}t}\, S^{lm(k)}_{\mu\nu}(\theta)\, e^{im\phi}$$

- Solve up to ~30,000 ODE systems (l,m,n) per orbit
- Eccentricity and separation range limited by ill-conditioning problem and computational cost
- New code developed to handle these problems

Osburn, Forseth, Evans, and Hopper, Phys. Rev. D 90 (2014); **1409.4419**

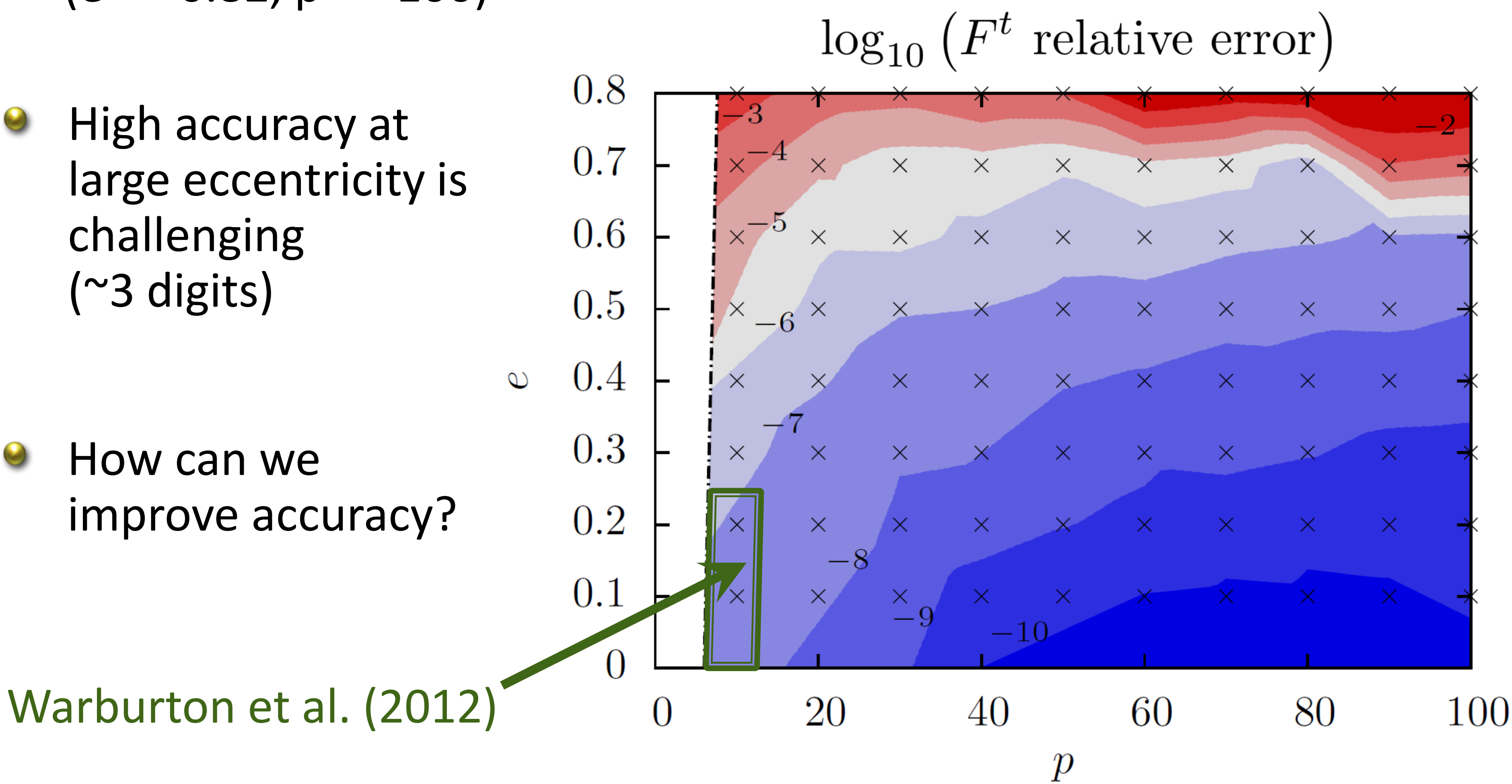
Metric perturbations and self-force



- Self-force: mode-sum regularization $F^{\mu} = \sum_{\ell'} \left[F^{\mu\,\ell'}_{\rm ret} - (2\ell'+1)A - B - \ldots \right]$

Larger domain, accuracy limitations

- We have extended the available domain of orbital parameters (e <= 0.82, p <= 100)
- High accuracy at large eccentricity is challenging (~3 digits)
- How can we improve accuracy?



Hybrid method: higher accuracy

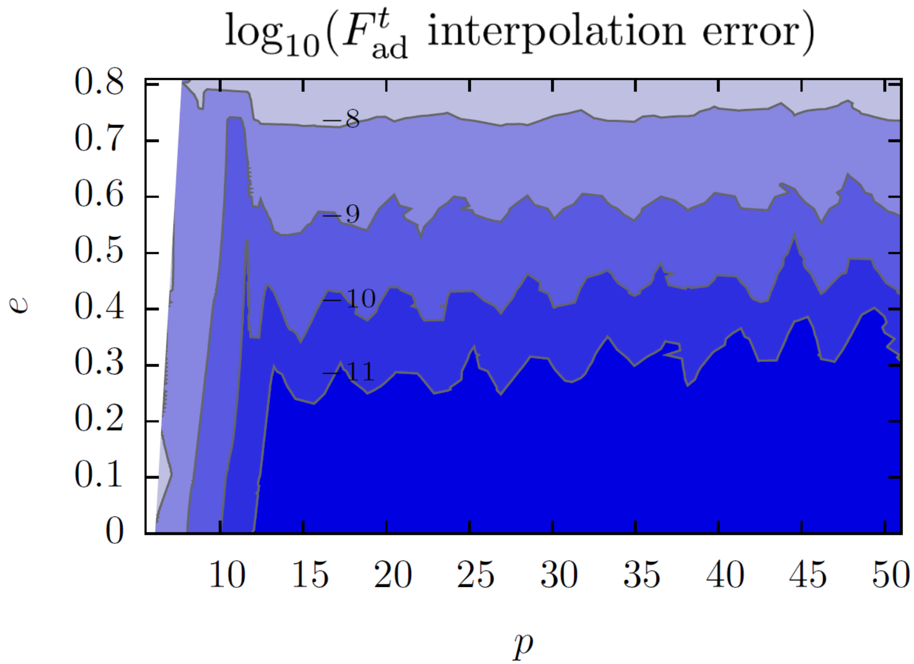
Total accumulated orbital phase: $\Phi = \kappa_0 \left(\frac{\mu}{M}\right)^{-1} + \kappa_1 + \kappa_2 \left(\frac{\mu}{M}\right) + \cdots$

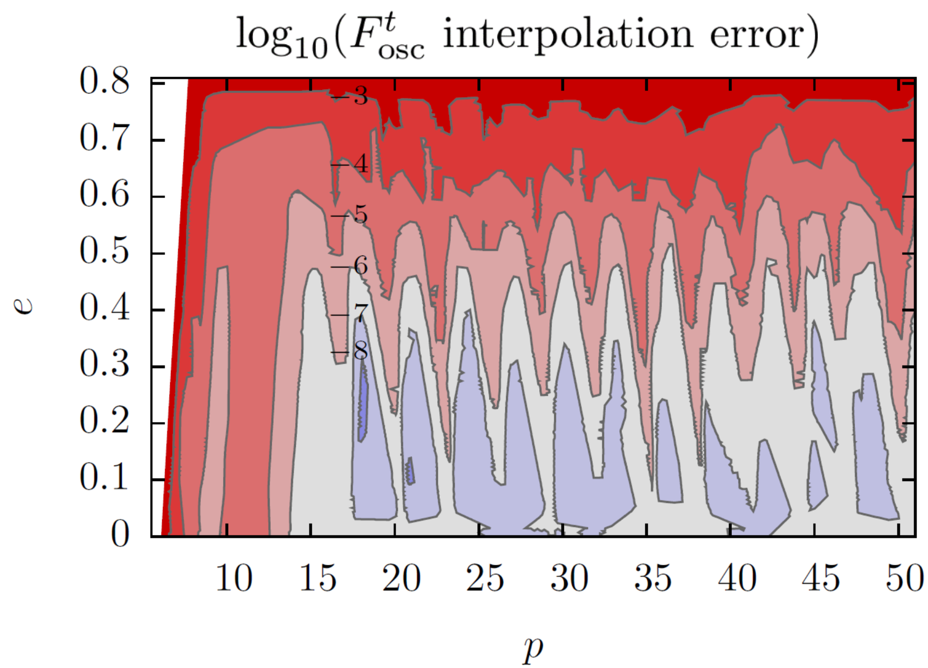
$(\mu/M = 10^{-5})$ adiabatic $\approx 10^6$ rad post-1-adiabatic ≈ 10 rad

- Goal: compute orbital phase to within ~0.1 radians
- Requires self-force accuracy ≤ $(10^{-2}\mu/M) \approx 10^{-7}$
- Very hard to achieve 7+ digits at high eccentricity
- Hybrid method: Use high accuracy flux for adiabatic correction (secular approx.), GSF for post-1-adiabatic
- Carefully replace orbit averaged self-force with flux values computed in RWZ gauge

Osburn, Forseth, Evans, and Hopper, Phys. Rev. D 90 (2014); **1409.4419**

Interpolation error





- Adiabatic part calculated from accurate RWZ gauge fluxes
- Interpolate with data from 43875 orbits (2054 CPU hours)

- Post-1-adiabatic part calculated from Lorenz gauge self-force
- Interpolate with data from 9602 orbits (2308 CPU hours)

Osburn, Warburton, and Evans, Phys. Rev. D 93 (2014); **1511.01498**

Spin-curvature force

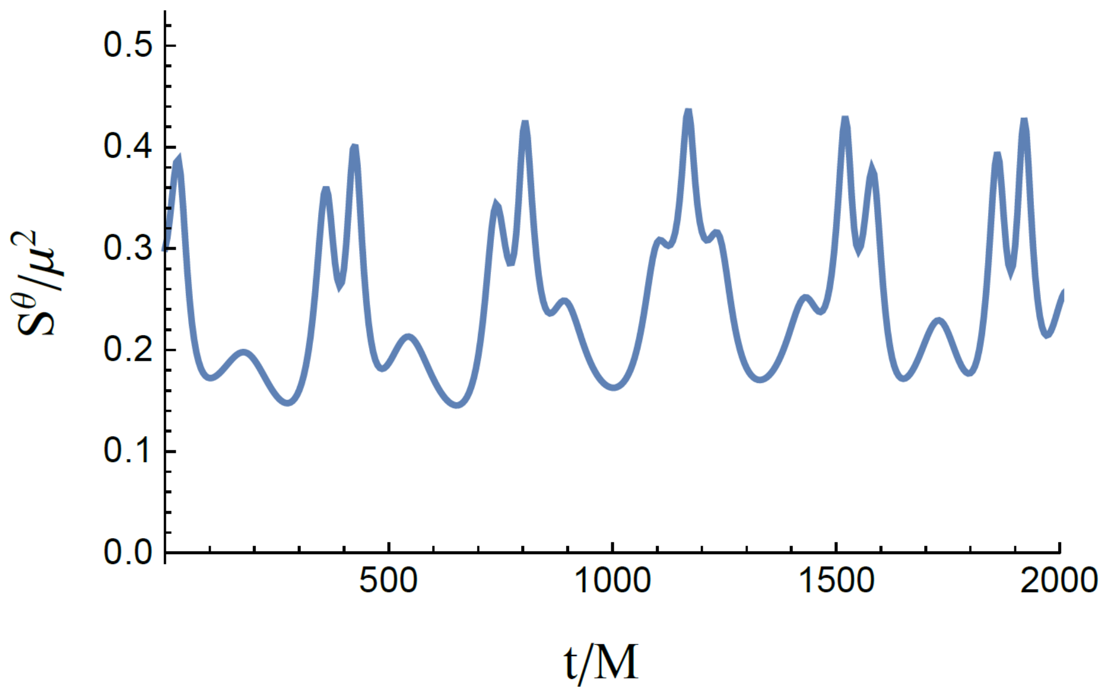
- Approximation 1: first-order expansion in spin magnitude (S)
- Approximation 2: Ignore spin-force when calculating self-force
- Mathisson-Papapetrou-Dixon spin-force:

$$F^{\mu}_{\text{spin}} = -\frac{1}{2} R^{\mu}{}_{\nu\lambda\sigma} u^{\nu} S^{\lambda\sigma}$$

- Geodetic spin precession:

$$u^{\alpha} \nabla_{\alpha} S^{\beta} = 0$$

- F^{θ} introduced, causes orbital plane to precess



Inspirals: osculating elements

- Choose a set of geodesic constants as orbital elements I^A

$$I^A = \{e, p, \chi_0, \iota, \Omega, \Phi, T\}$$

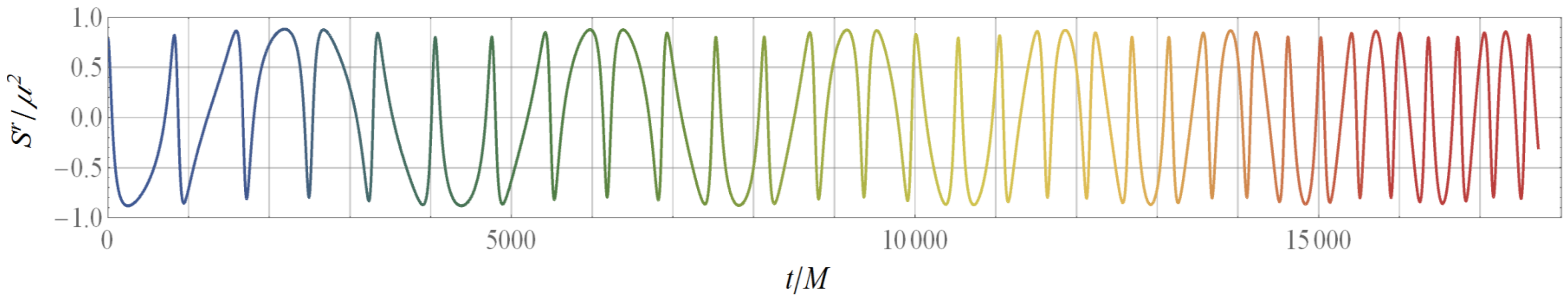
- The equations of motion are re-formulated as an initial value problem for I^A

Pound and Poisson Phys. Rev. D 77 (2008)

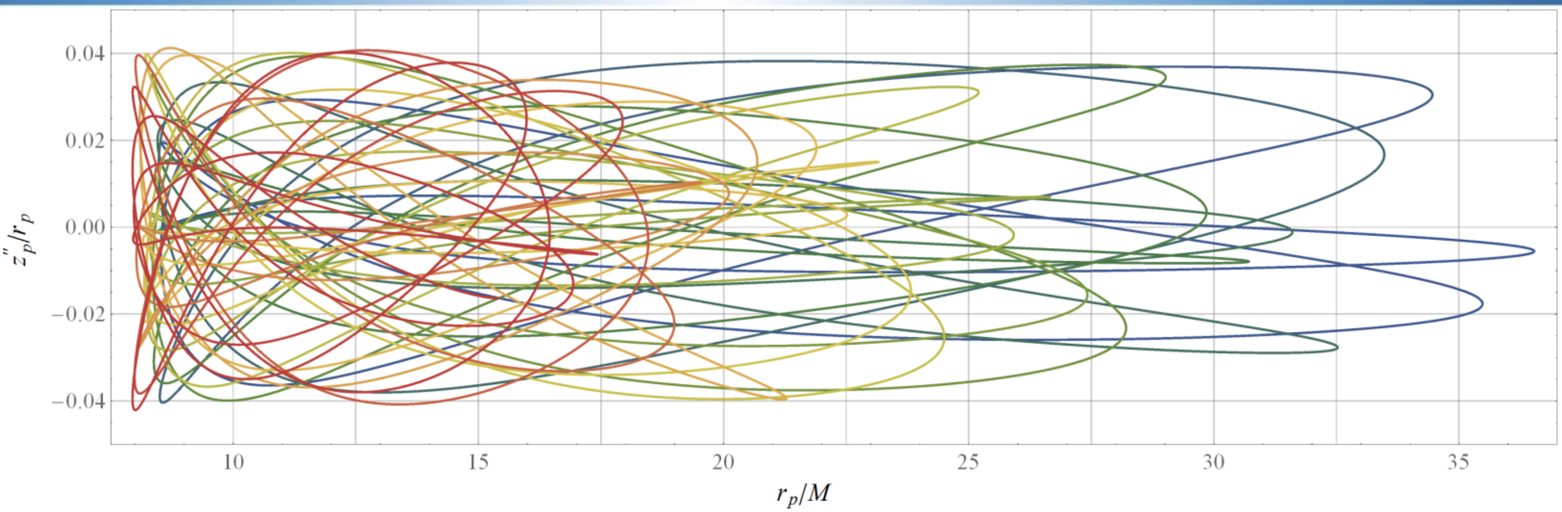
$$\frac{\partial z_G^\alpha}{\partial I^A}\frac{\partial I^A}{\partial \tau} = 0 \qquad \mu\frac{\partial u_G^\alpha}{\partial I^A}\frac{\partial I^A}{\partial \tau} = F^\alpha$$

- The spin vector is evolved in an ODE system shared with I^A

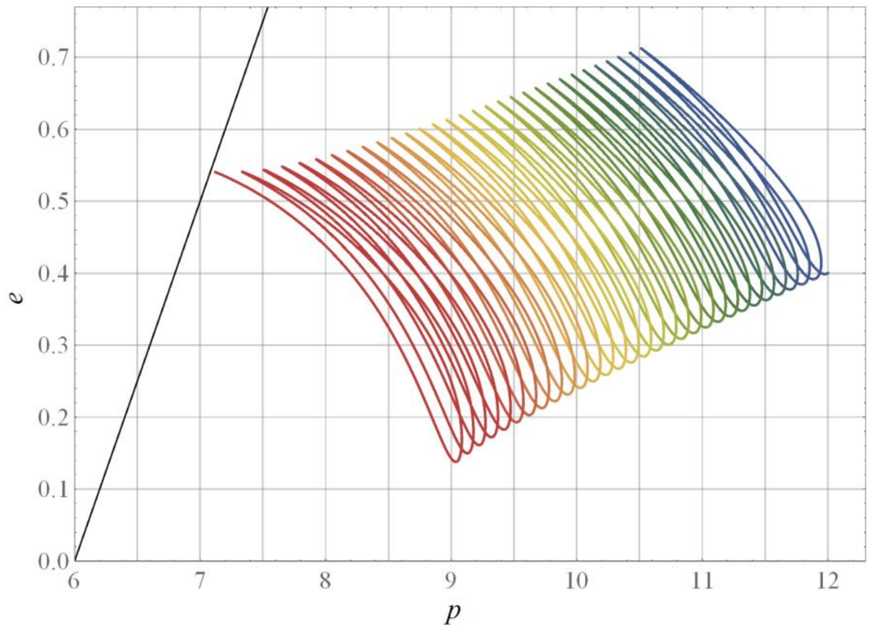
$$u^\alpha \nabla_\alpha S^\beta = 0$$

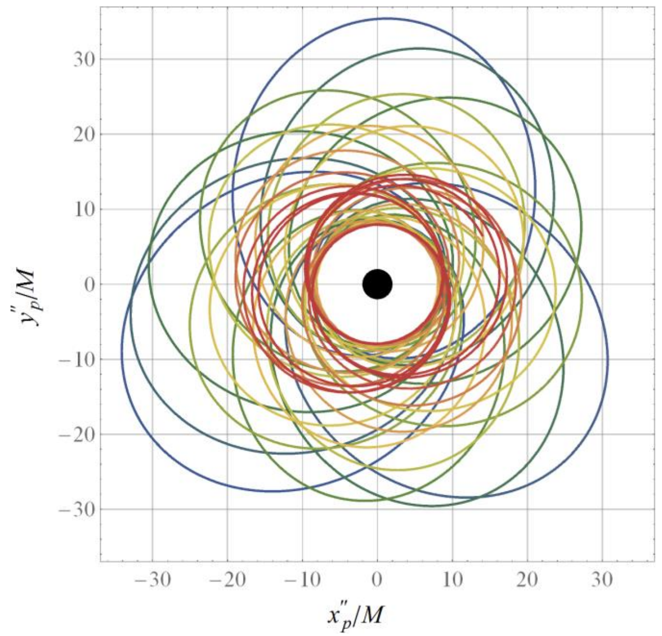


Inspiral with spin-force



$|s| = 1$

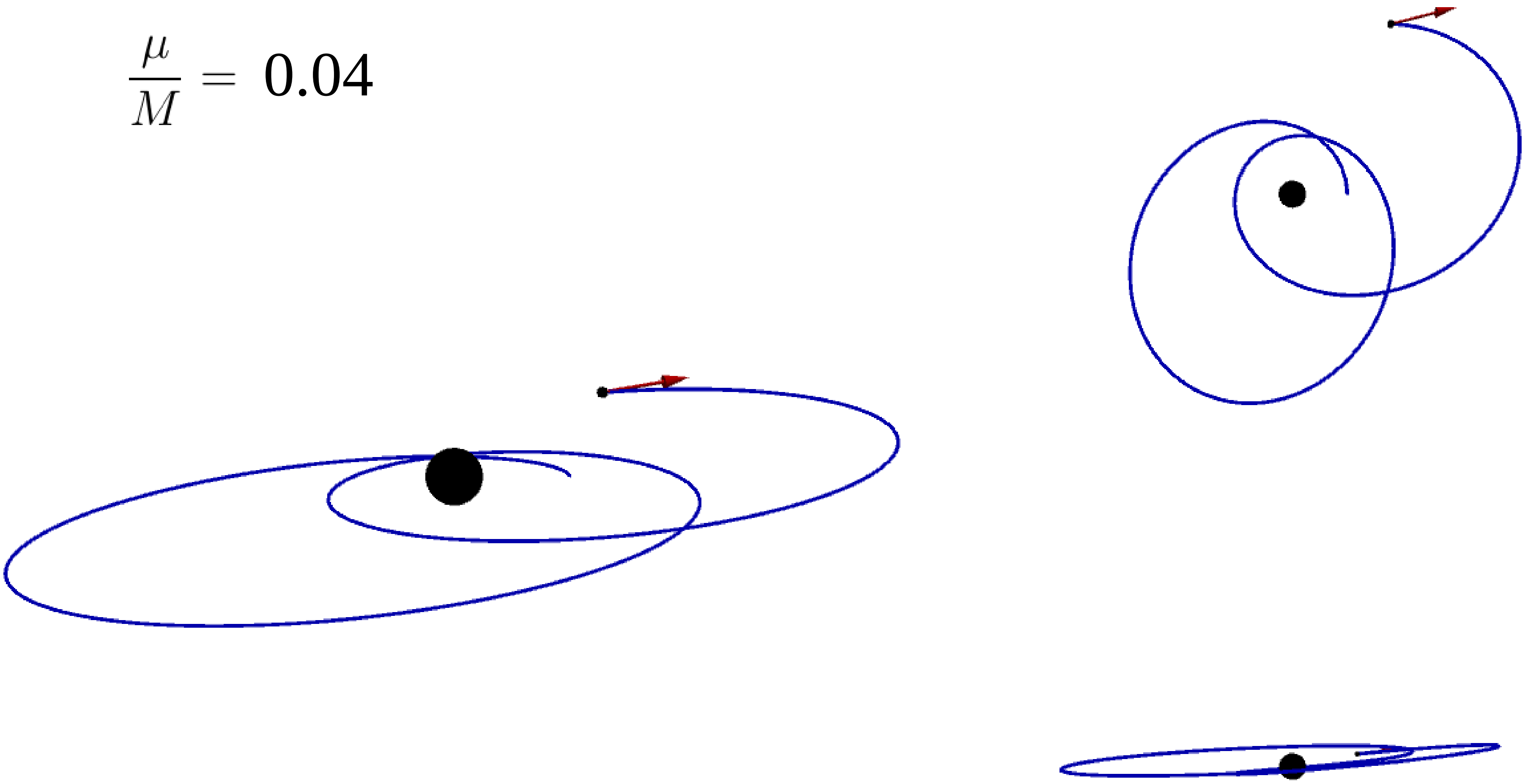




$$\frac{\mu}{M} = 0.08$$

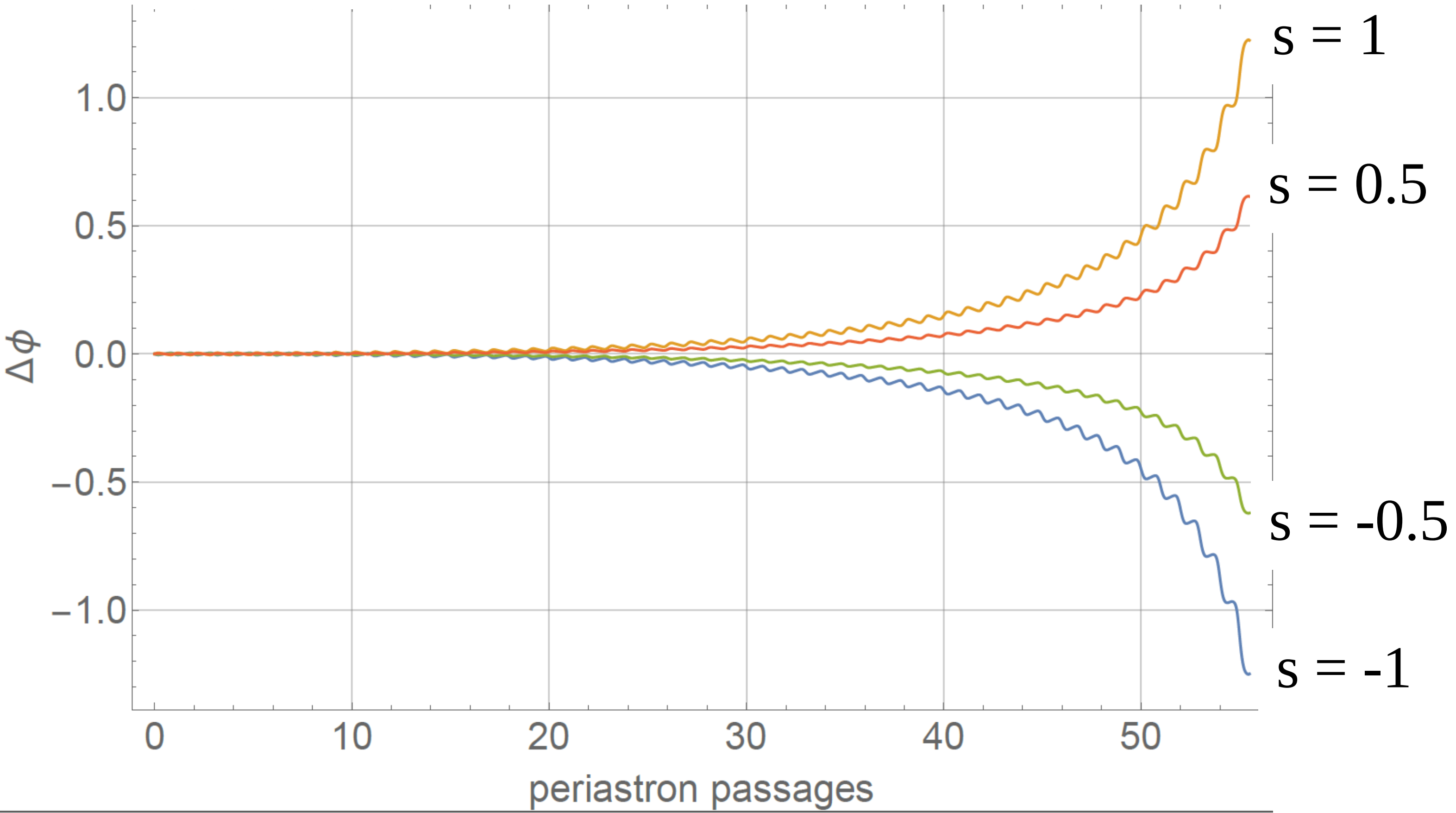
Inspiral with spin-force

$$\frac{\mu}{M} = 0.04$$



Effect of spin on orbital phase

$\frac{\mu}{M} = 0.005$ $e_0 = 0.4$ $p_0 = 10$



Waveform generation

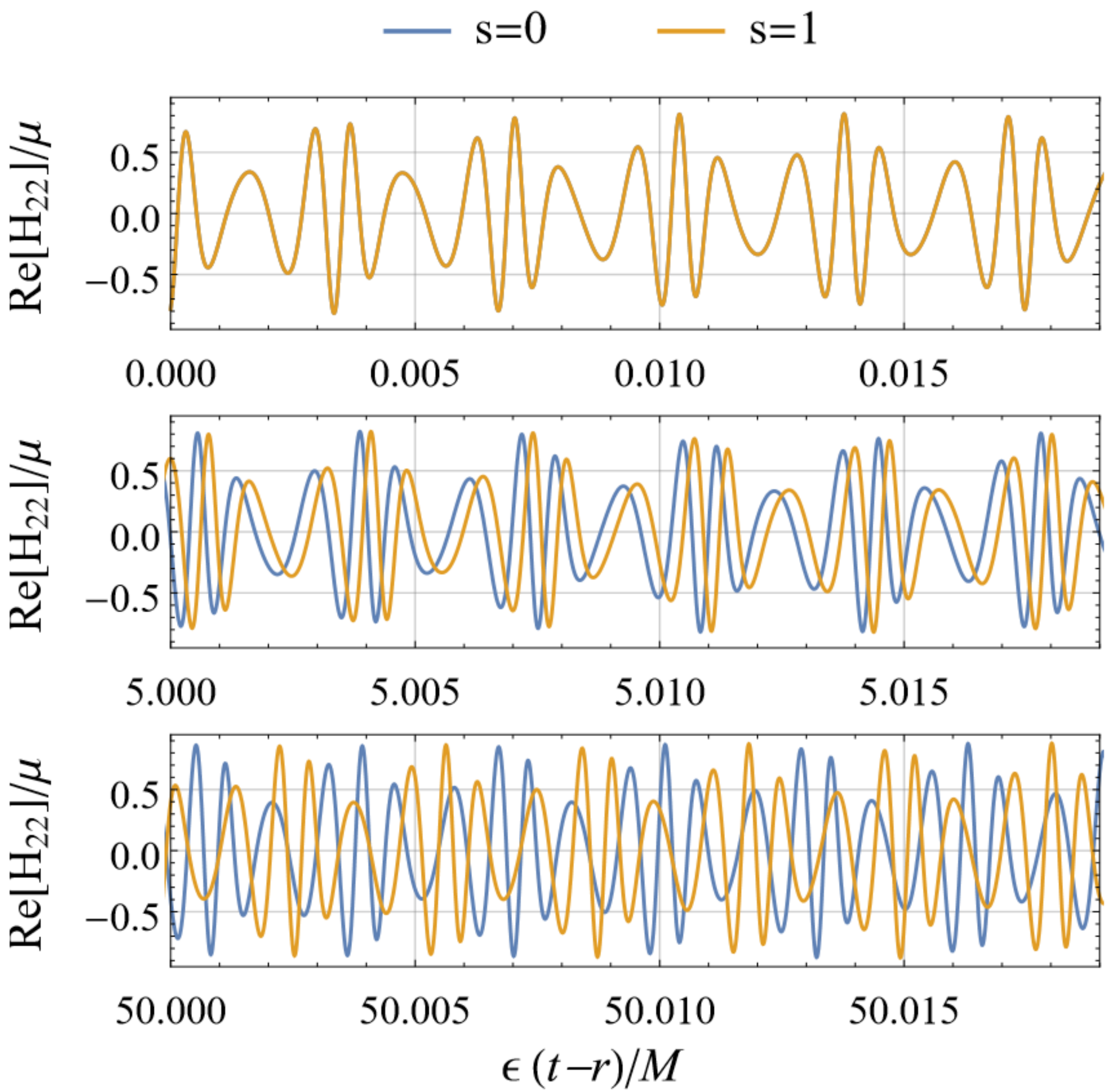
- To calculate the self-force we already solved the Einstein equations
- Snapshot gravitational wave information readily available

$$h_+ - ih_\times = \frac{1}{r} \sum_{l=2}^{l_{\max}} \sum_{m=-l}^{l} H_{lm}(t,r)\, {}_{-2}Y_{lm}(\theta,\varphi)$$

$$H_{lm}(t,r) \equiv \sum_{n=n_{\min}}^{n_{\max}} \frac{1}{2}\sqrt{(l+2)(l+1)l(l-1)}\left(C^{\text{even}}_{lmn} - iC^{\text{odd}}_{lmn}\right) e^{-i\omega_{mn}(t-r)}$$

- The *C* coefficients are amplitudes of Regge-Wheeler master functions and depend only on shape of orbit (*e* and *p*)
- We calculated and interpolated the *C*s over same orbital parameter space as the self-force
- Waveforms generated by updating the *C*s as the orbit evolves

Effect of spin on waveforms



$e_0 = 0.3$ $p_0 = 10$

$$\epsilon = \frac{\mu}{M} = 10^{-5}$$

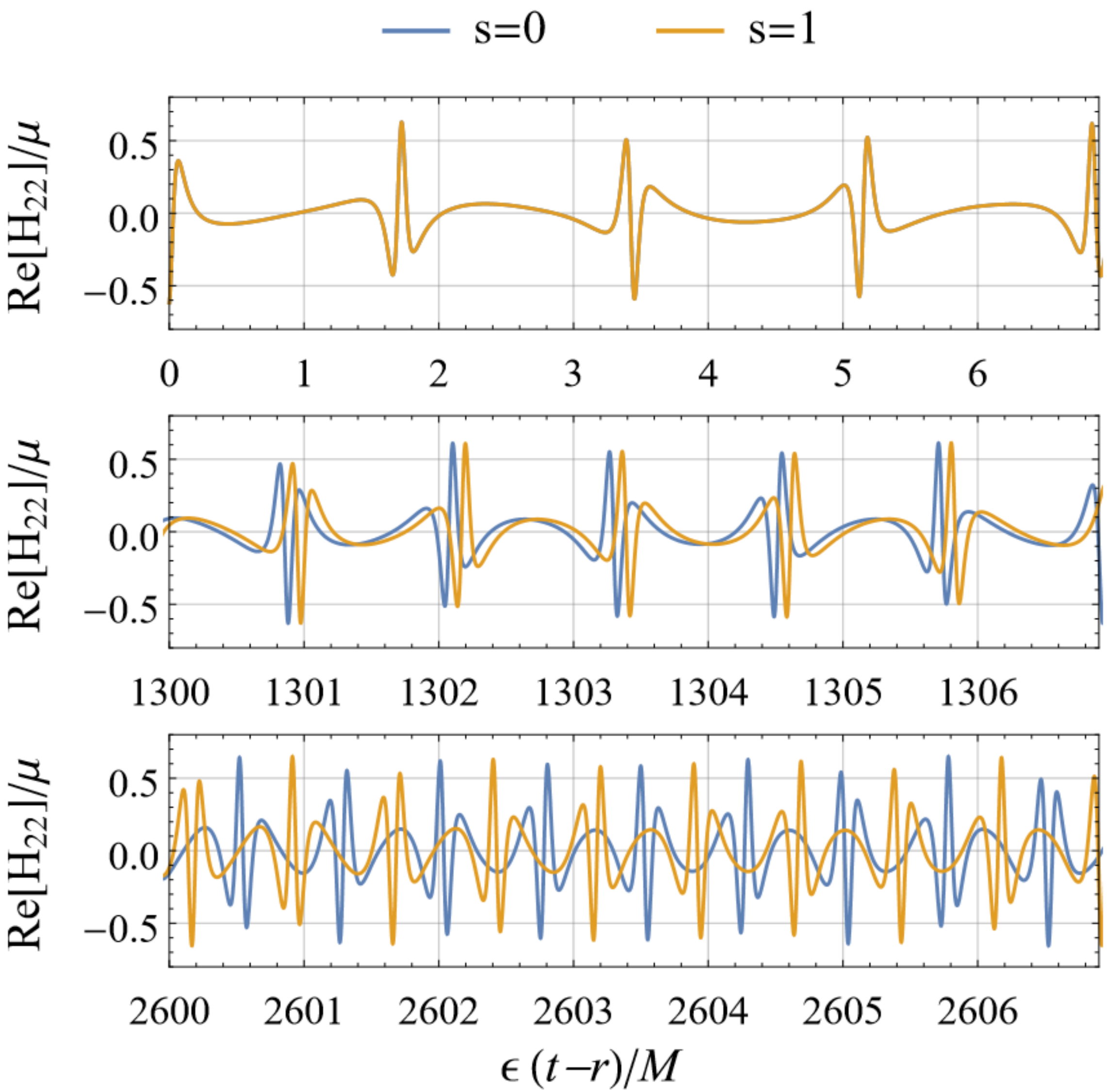
Near plunge stats:

~2400 radial oscillations

Total time ~ $90\,\frac{M}{\epsilon}$

Total dephasing ~ 44 rad

Effect of spin on waveforms



$e_0 = 0.7$ $\quad$ $p_0 = 20$

$$\epsilon = \frac{\mu}{M} = 10^{-3}$$

Near plunge stats:

~700 radial oscillations

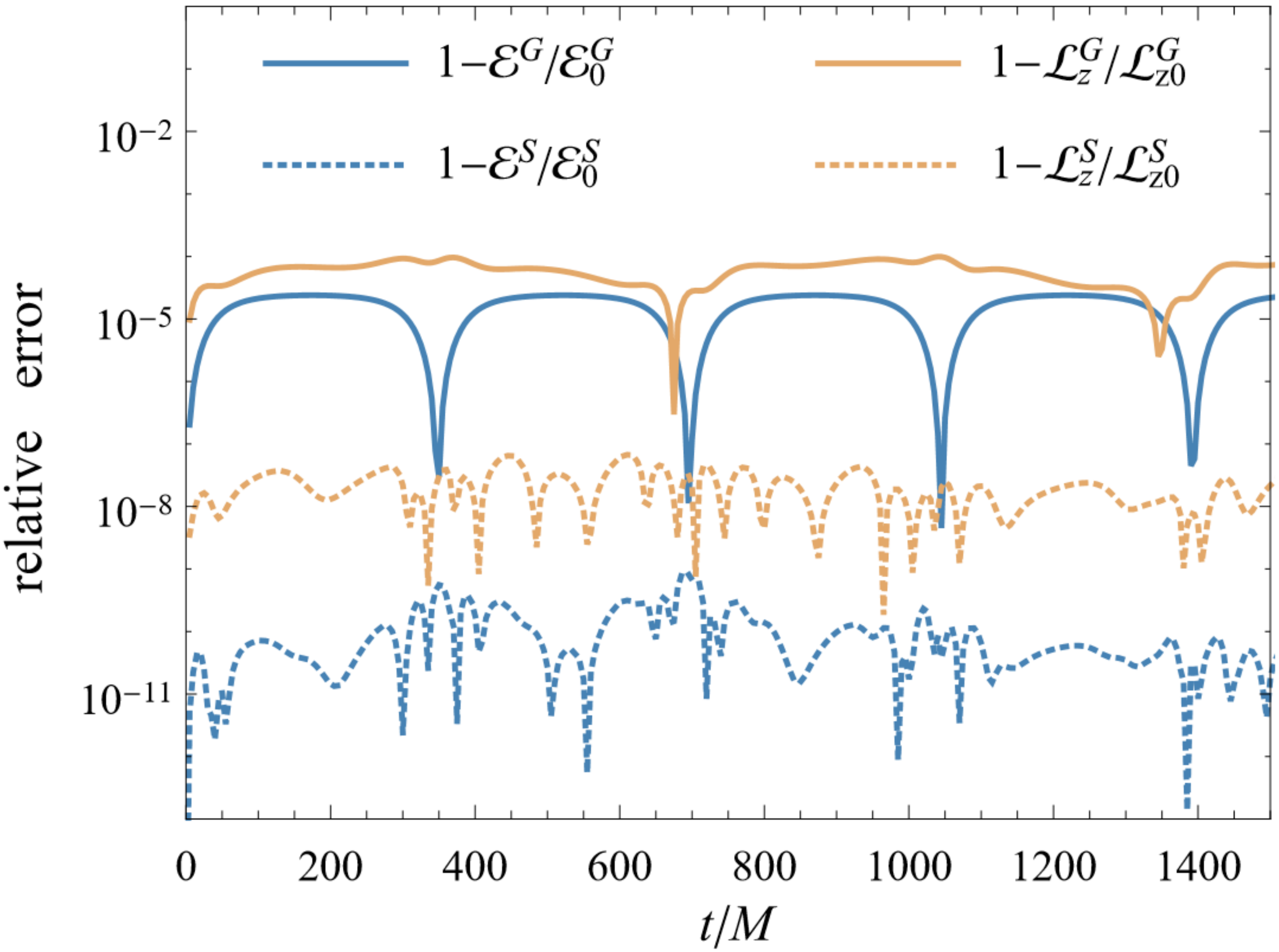
Total time ~ $3700 \frac{M}{\epsilon}$

Total dephasing ~ 15 rad

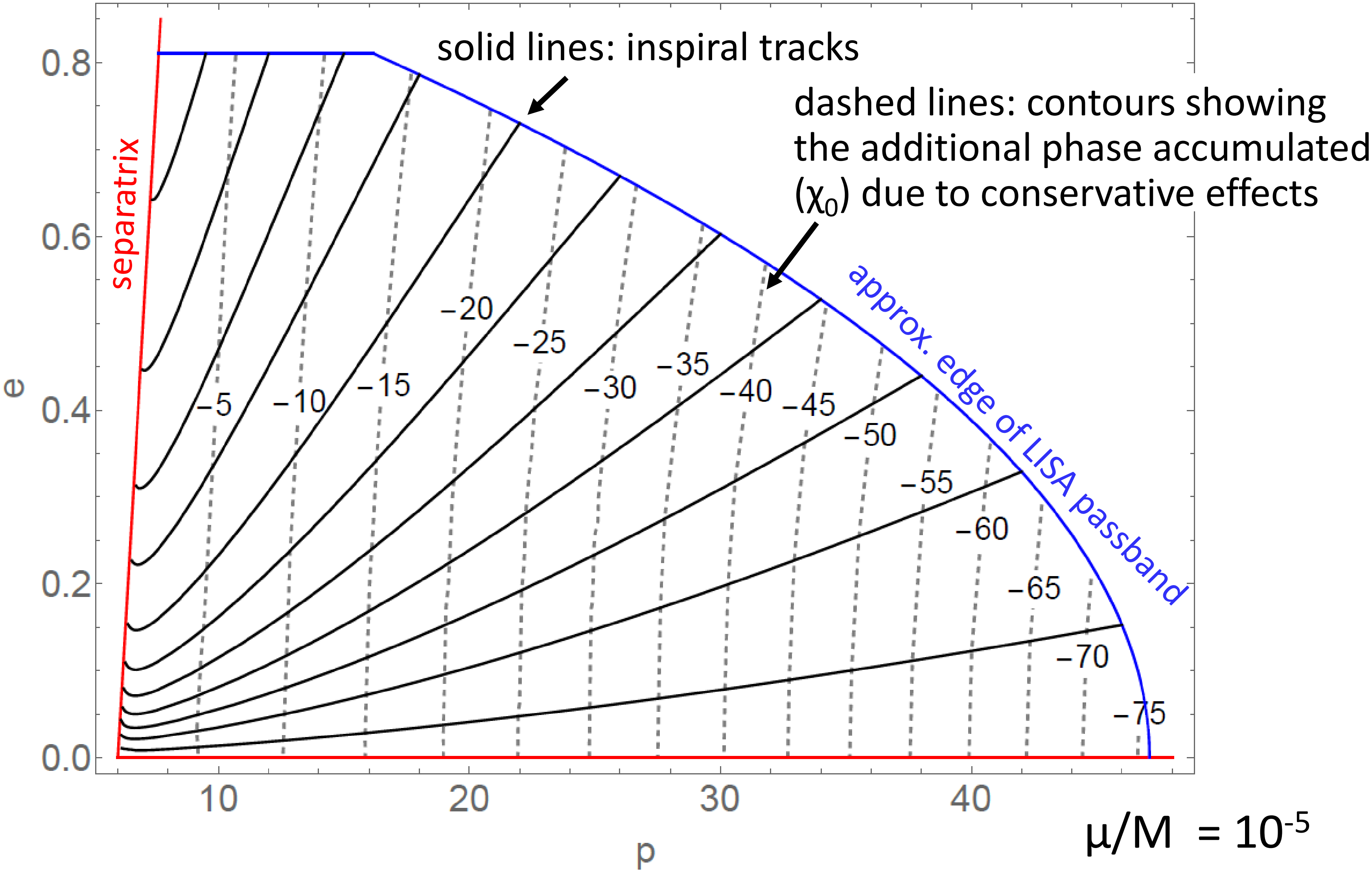
Conclusions and future work

- Important problems for gravitational wave astronomy:
  - Extreme/intermediate mass ratio binaries
  - High eccentricity
  - High accuracy (7+ digits)
  - Spin-curvature coupling
- We accomplish this with the following tools:
  - Hybrid (accurate fluxes for adiabatic) self-force code
  - Add module for spin-curvature coupling (spin-force)
  - Osculating elements code generalized for inclined orbits
- Future work:
  - Kerr background
  - Second order perturbation theory

Conserved quantities with spin

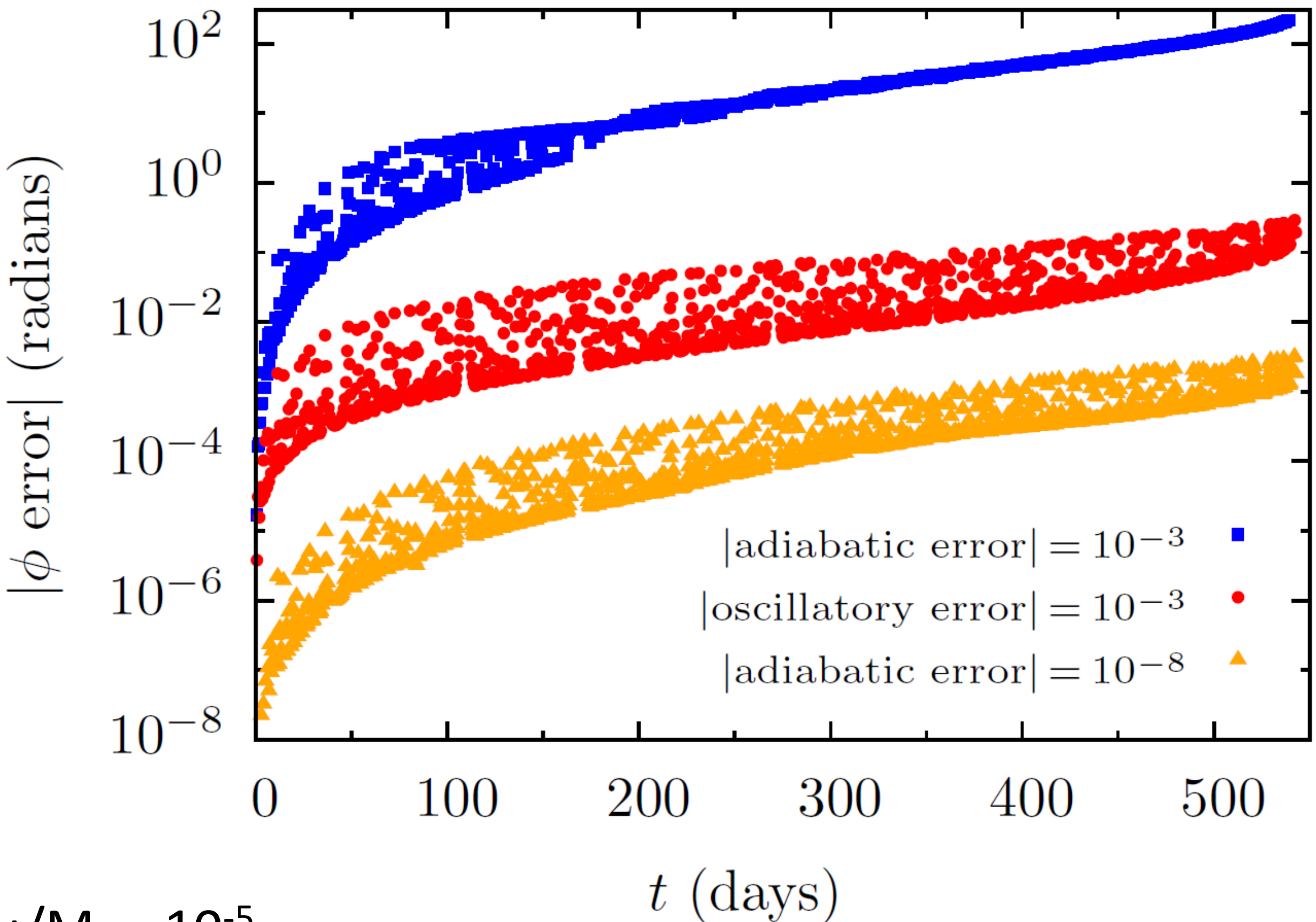


Importance of conservative effects



$\mu/M = 10^{-5}$

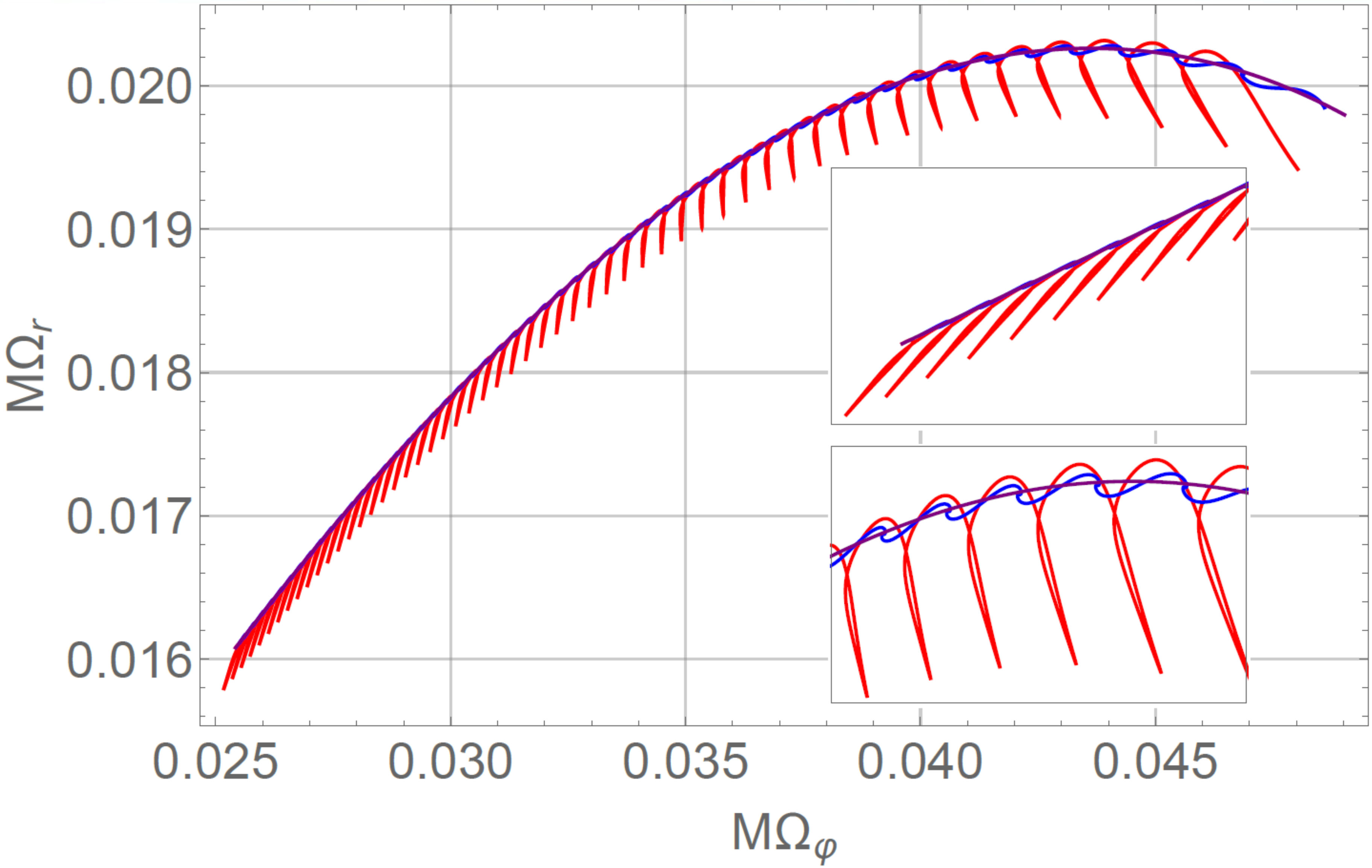
Sensitivity test of hybrid self-force



$\mu/M = 10^{-5}$

$M = 10^6 M_\odot$

Evolution of gauge invariant freqs



Intermediate mass ratio inspiral