Overcharging higher-dimensional black holes with point particles

Karl Simon Revelar* and Ian Vega

National Institute of Physics
University of the Philippines

*karevelar@upd.edu.ph

Capra 20, UNC Chapel Hill

Overcharging and Overspinning using test objects

Cosmic censorship: curvature singularities arising from gravitational collapse are hidden by an event horizon

Important studies that used test objects to subvert censorship:

Wald (1974): Extremal Kerr-Newman black hole cannot be overspun/overcharged by test objects

Hubeny (1999): For near-extremal RN BH, there exists region in $\{E, m, q\}$ space of test charge that yields overcharged state

Jacobson, Sotiriou (2009): Region in $\{\delta E, \delta J\}$ parameter space for test body exists that yields overspun state of near-extremal KN BH

Zimmerman, Vega, Hass, & Poisson (2013): EM self-force becomes repulsive near the horizon preventing overcharging in Hubeny scenario

Colleoni, Barack, Shah, & Van de Meent (2015): Gravitational self-force prevents overspinning in Jacobson-Sotiriou scenario

Self-force as a cosmic censor in higher dimensions?

The overspinning scenario has already been analyzed in higher dimensions. Cosmic censorship is found to be upheld in this case. Bouhmadi-Lopez, Cardoso, Nerozzi & Rocha (2010)

Studies of the self-force in higher dimensions: Harte, Flanagan & Taylor (2016), Taylor & Flanagan (2015), Frolov & Zelnikov (2014), and Beach, Poisson & Nickel (2014).

These motivate us to extend the Hubeny overcharging scenario to higher D.

Review of Hubeny scenario

The Hubeny scenario consists of a test charge with mass m, energy E, and charge q falling radially towards a nearly-extremal RN black hole of mass M, charge Q.

Test particle approximation is imposed by setting $m \sim E \sim q \ll Q < M$.

The particle follows an equation of motion

$$ma^{\alpha} = qF^{\alpha}{}_{\beta}u^{\beta}. \tag{1}$$

To cross the horizon, its velocity $u^{\alpha} = (\dot{t}, \dot{r}, 0, 0)$ must satisfy

1. $\dot{r}^2 > 0, \ \ \forall r \geq r_+$ (No turning point)
2. $\dot{t} > 0 \ \ \forall r > r_+$ (u^{α} future-pointing for $r > r_+$)

Review of Hubeny scenario

The black hole is overcharged after particle absorption when $Q + q > M + E$.

Following these preceding conditions, Hubeny found the constraints to the region in the $\{E, m, q\}$ parameter space that satisfy the overcharging condition.

Hubeny inequalities

$$q > \frac{r_+ - Q}{2}, \tag{2a}$$

$$\frac{qQ}{r_+} < E < q + Q - M, \tag{2b}$$

$$m < Q\sqrt{\frac{2MEq - Q(E^2 + q^2)}{Q(M^2 - Q^2)}}. \tag{2c}$$

Review of Hubeny scenario

Extremal case ($Q = M$)

$q < E < q, \quad m < \infty, \quad q > 0.$ no solution

Nearly-extremal case ($M = 1, Q = 1 - 2\epsilon^2$)

$$\begin{aligned} q &= a\epsilon \\ E &= a\epsilon - 2b\epsilon^2 \\ m &= c\epsilon, \end{aligned} \quad \text{such that} \quad \begin{aligned} &a > 1 \\ &1 < b < a \\ &c < \sqrt{a^2 - b^2} \end{aligned} \tag{3}$$

Charged Schwarzschild-Tangherlini BH

We consider a test charge moving in the charged Schwarzschild-Tangherlini spacetime, $D > 4$-dimensional charged static, asymptotically flat, spherically-symmetric black hole solution of the D-dimensional Einstein-Maxwell equation.

D-dimensional Reissner-Nordström /charged Schwarzschild -Tangherlini line element

$$ds^2 = -\left(1 - \frac{\mu}{r^{D-3}} + \frac{\xi^2}{r^{2(D-3)}}\right) dt^2 + \left(1 - \frac{\mu}{r^{D-3}} + \frac{\xi^2}{r^{2(D-3)}}\right)^{-1} dr^2 + r^2 d\Omega^2_{D-2} \tag{4}$$

with

$$\mu = \frac{16\pi M}{(D-2)\Omega_{(D-2)}}, \quad \xi = \left(\frac{8\pi}{\Omega_{(D-2)}(D-2)(D-3)}\right)^{1/2} Q, \quad \text{and} \quad \Omega_{(D-2)} = \frac{2\pi^{(D-1)/2}}{\Gamma\left((D-1)/2\right)} \tag{5}$$

The black hole supports an EM field $A_\alpha = -Q/\left((D-3)r^{D-3}\right)$ which interacts with the particle's charge.

Metric after absorption

Following previous works, (Hubeny, 1999), (Bouhmadi-Lopez et al, 2010), (Jacobson & Sotiriou, 2009), the ADM mass M and charge Q of the black hole goes to $M + E$ and $Q + q$, respectively.

After absorption, the metric function becomes

$$f(r) = 1 - \frac{16\pi(M+E)}{(D-2)\Omega_{(D-2)} r^{D-3}} + \frac{8\pi(Q+q)^2}{\Omega_{D-2}(D-2)(D-3) r^{2(D-3)}} \tag{6}$$

The event horizon would then be located at

$$r_+^{D-3} = \frac{M+E}{(D-3)\omega_D^2}\left(1 + \sqrt{1 - \frac{\omega_D^2 (Q+q)^2}{(M+E)^2}}\right), \quad \omega_D = \sqrt{\frac{(D-2)}{(D-3)}\frac{\Omega_{(D-2)}}{8\pi}}. \tag{7}$$

Overcharging condition

The metric thus describes a naked singularity when

$$Q + q > \omega_D^{-1}(M + E). \tag{8}$$

This overcharging condition can be written as follows to provide an upper bound for E:

$$E < \omega_D (Q + q) - M. \tag{9}$$

Kinematics of radial infall

For the particle with velocity $u^\alpha = (\dot{T}, \dot{R}, 0, \ldots, 0)$ to cross the horizon during infall,

$$\dot{R}^2 > 0 \;\; \forall r \geq r_+ \tag{10}$$

$$\dot{T} > 0 \;\; \forall r > r_+ \tag{11}$$

Equation of motion

$$m a^\alpha = q F^\alpha{}_\beta u^\beta \tag{12}$$

$$E = -p_\alpha \xi^\alpha_{(t)} = m f \dot{T} + \frac{qQ}{(D-3) r^{D-3}} \quad \text{is conserved.} \tag{13}$$

Thus we get

$$\dot{T} = \frac{1}{mf} \left(E - \frac{qQ}{(D-3) r^{D-3}} \right) \tag{14}$$

$$\dot{R}^2 = \frac{1}{m^2} \left(E - \frac{qQ}{(D-3) r^{D-3}} \right)^2 - f(r) \tag{15}$$

Crossing conditions

We can enforce $\dot{T} > 0 \;\; \forall r > r_+$ if

$$E > \frac{qQ}{(D-3)r_+^{D-3}}, \tag{16}$$

which is a lower bound on E.

We can enforce $\dot{R}^2 > 0 \;\; \forall r \geq r_+$ if

$$m < \omega_D Q \sqrt{\frac{2MEq - Q(E^2 + \omega_D^2 q^2)}{Q(M^2 - \omega_D^2 Q^2)}}. \tag{17}$$

Combining lower and upper bounds for E gives

$$\frac{qQ}{(D-3)r_+^{D-3}} < E < \omega_D \left(Q + q\right) - M. \tag{18}$$

For this interval in E to exist, we need to require

$$q > r_+^{D-3} \left(\frac{M - \omega_D Q}{\omega_D r_+^{D-3} - Q/(D-3)} \right). \tag{19}$$

Generalized Hubeny inequalities for $D > 4$

The set of inequalities constraining the $\{E, m, q\}$ parameter space for a D-dimensional RN black hole (M, Q) are then

$$q > r_+^{D-3}\left(\frac{M - \omega_D Q}{\omega_D r_+^{D-3} - Q/(D-3)}\right) \tag{20}$$

$$\frac{qQ}{(D-3)r_+^{D-3}} < E < \omega_D\left(Q + q\right) - M \tag{21}$$

$$m < \omega_D Q\sqrt{\frac{2MEq - Q(E^2 + \omega_D^2 q^2)}{Q(M^2 - \omega_D^2 Q^2)}}, \tag{22}$$

where

$$\omega_D = \sqrt{\frac{(D-2)}{(D-3)}\frac{\Omega_{(D-2)}}{8\pi}}, \tag{23}$$

is a dimensionless constant.

These reduce to the Hubeny inequalities when $D = 4$.

Extremal case for $D > 4$

In the **extremal case**, we generalize Wald's result to $D > 4$.

The generalized Hubeny inequalities become

$$q > 0 \tag{24a}$$

$$m < \infty \tag{24b}$$

$$q < E < q \tag{24c}$$

This has no solution. Thus, extremal RN black holes for $D > 4$ cannot be overcharged, just as in $D = 4$.

Nearly-extremal case for $D > 4$

Near-extremality can be parameterized as

$$M \equiv 1 \tag{25a}$$

$$Q \equiv \omega_D^{-1} - 2\epsilon^2 \tag{25b}$$

Rewriting inequalities in terms of ϵ and then taking a series expansion around $\epsilon = 0$, we can find a solution similar to Hubeny's.

$$\begin{aligned} q &= A\epsilon \\ E &= \omega_D(A\epsilon - 2B\epsilon^2) \\ m &= C\epsilon, \end{aligned} \quad \text{such that} \quad \begin{aligned} &A > \omega_D^{-1/2} \\ &1 < B < \sqrt{\omega_D}A \\ &C < \sqrt{A^2\omega_D^2 - B^2\omega_D} \end{aligned} \tag{26}$$

The main difference is the presence of the D-dimensional factor ω_D.

Therefore, we can also overcharge nearly-extremal black holes for any $D > 4$. The caveat is that D can't be very large.

Large D limit

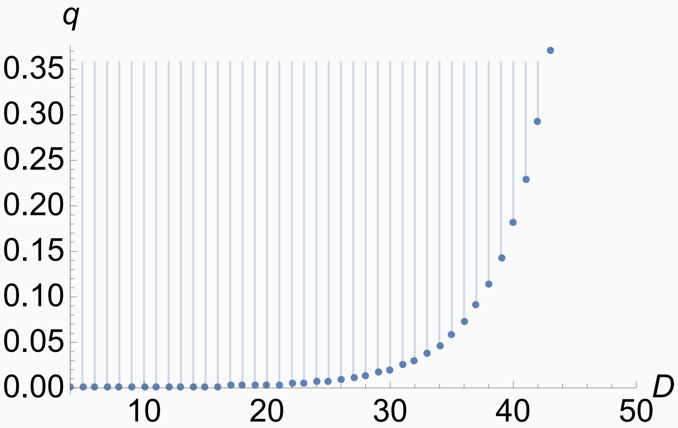

Figure 1: Allowed parameter space in q for nearly-extremal BH with $\epsilon = 0.001$. Notice that as $D \to \infty$, $q_{min} \to \infty$. Therefore, as D increases, the required charge eventually becomes too large so that the test particle assumption breaks down.

Large D limit

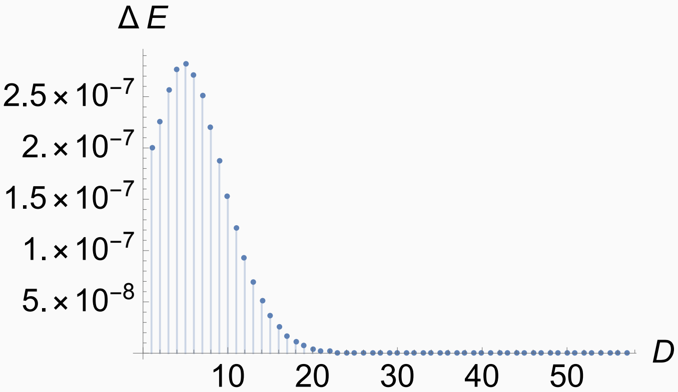

Figure 2: Width of parameter space in E, $\Delta E = E_{max} - E_{min}$, for nearly-extremal BH with $\epsilon = 0.001$. (shaded region) As $D \to \infty$, $\Delta E \to 0$.

Large D limit

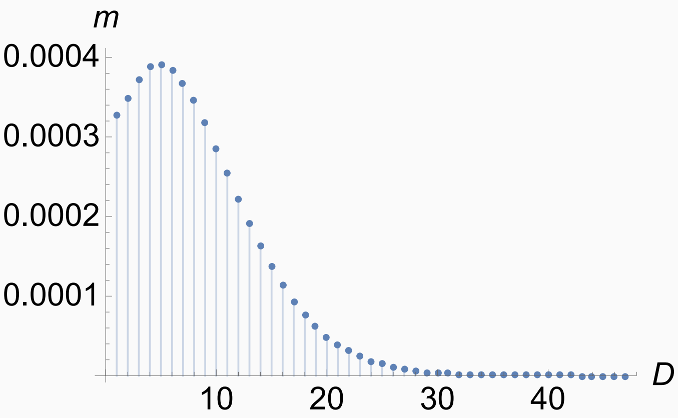

Figure 3: Allowed parameter space in m for nearly-extremal BH with $\epsilon = 0.001$. As $D \to \infty$, $m \to 0$.

Summary and Outlook

Summary

1. Extremal higher-dimensional black holes cannot be overcharged. (Generalization of Wald's result in $D = 4$)
2. Nearly-extremal higher-dimensional black holes can be overcharged. (Generalization of Hubeny in $D = 4$)
3. In the large D limit, overcharging becomes systematically difficult to achieve.

Work in progress

Does Jacobson-Sotiriou overspinning generalize to nearly-extremal Myers-Perry black holes?

Outlook

Can the higher-dimensional self-force act as a cosmic censor?

Thank you