Effective-one-body modeling of binary black holes in the era of gravitational-wave astronomy

Andrea Taracchini

(Max Planck Institute for Gravitational Physics, Albert Einstein Institute — Potsdam, Germany)

[https://dcc.ligo.org/G1701130]

Introduction

Outline

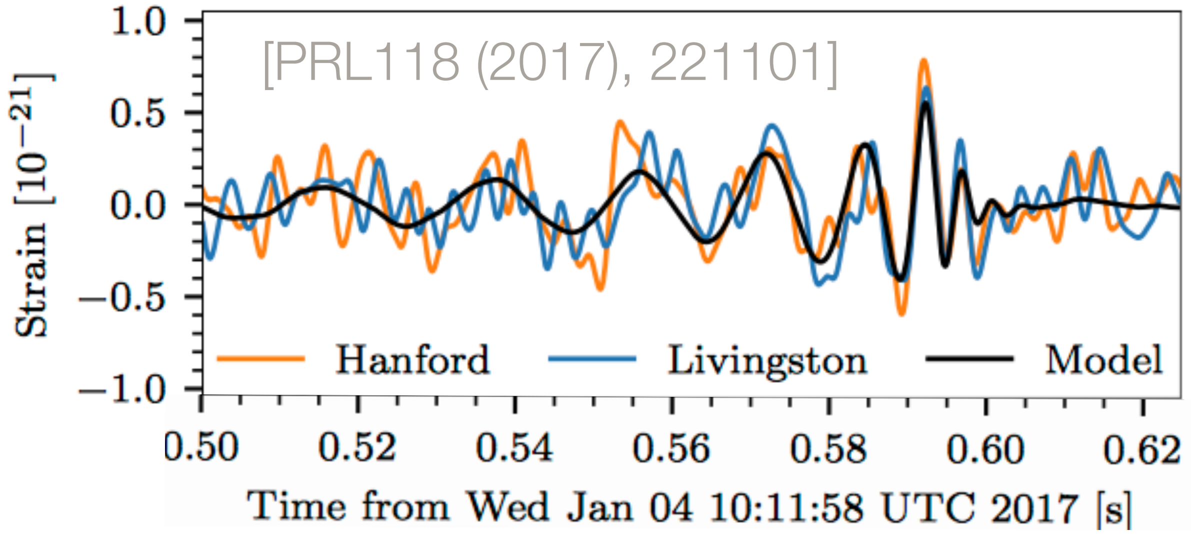
I. *Effective-one-body model*

1) Conservative dynamics of EOB model
2) Inputs from conservative gravitational self-force
    2.1) ISCO shift
    2.2) Periastron advance
    2.3) First law and redshift
    2.4) Gyroscopic precession
3) Waveforms and inputs from black-hole perturbation theory
    3.1) Resummation of inspiral waveforms
    3.2) Merger waveforms
    3.3) Ringdown waveforms

II. *Inspiral-merger-ringdown models for aLIGO*

1) Nonprecessing models and calibration to NR
2) Precessing models
3) Parameter estimation of the first GW events
4) Unmodeled effects and recent developments

Introduction



GW170104

- **Synergy** of different approaches to general-relativistic 2-body problem has allowed construction of **accurate waveform models**

- As to the first GW observations, waveform models were crucial to:
  1. detect GW151226 [LVC1606.04855]
  2. establish 5-sigma **significance** of all detections [LVC1602.03839, LVC1606.04856, LVC1706.01812]
  3. measure **astrophysical properties of the sources** [LVC1602.03840, LVC1606.01210, LVC1606.01262, LVC1606.04856, LVC1706.01812]
  4. perform **tests of general relativity** [LVC1602.03841, LVC1606.04856, LVC1706.01812]

I. Effective-one-body (EOB) model

Introduction to the effective-one-body model

- Limitations of post-Newtonian (PN) description of binaries
  1. **PN does not account for merger-ringdown**, which is relevant for M>30MSun in aLIGO
  2. different PN templates are **distinguishable** in aLIGO if M>12MSun [Buonanno+09]

- Limitations of numerical-relativity (NR) description of binaries
  1. high computational cost: **~1000 CPU hours per millisecond** of BBH evolution close to merger
  2. parts of BBH parameter space are challenging (**high spins and high mass ratios**)

Introduction to the effective-one-body model

- Qualitative/quantitative predictions of EOB model of 1999 **before NR** simulation of BBH inspiral-merger-ringdown (IMR) in 2005
  1. plunge is smooth continuation of inspiral
  2. sharp transition at merger b/w inspiral and ringdown
  3. estimates of mass and spin of remnant BH
  4. full IMR waveform

- Ingredients:
  1. conservative Hamiltonian dynamics for binary motion for inspiral-plunge
  2. model of radiation reaction (dissipation) to complement 1.
  3. formulas for IMR waveform

- Original idea: use all available inputs from PN theory and resum them to extend their applicability to strong field + additional insights from BH perturbation theory

Conservative dynamics of the effective-one-body model

Conservative dynamics of the effective-one-body model

- Remember what happens in Newtonian 2-body problem

$$H_{\rm Newton} = \underbrace{\frac{\boldsymbol{p}_1^2}{2m_1} + \frac{\boldsymbol{p}_2^2}{2m_2} - \frac{G m_1 m_2}{|\boldsymbol{r}_1 - \boldsymbol{r}_1|}}_{\text{self-gravitating}} = \underbrace{\frac{\boldsymbol{p}^2}{2\mu} - \frac{G\mu M}{r}}_{\text{test mass in external gravitational field}} + \underbrace{\frac{\boldsymbol{P}_{\rm COM}^2}{2M}}_{\text{trivial center-of-mass motion}}$$

- Start with PN description of nonspinning BBH relative dynamics in COM

$$\hat{H}_{\rm PN} = \hat{H}_{\rm Newton} + \frac{1}{c^2}\hat{H}_{\rm 1PN} + \frac{1}{c^4}\hat{H}_{\rm 2PN} + \cdots$$

$$\hat{H}_{\rm Newton} = \frac{\boldsymbol{p}^2}{2} - \frac{1}{r}$$

$$\hat{H}_{\rm 1PN} = \frac{1}{8}(3\nu - 1)(\boldsymbol{p}^2)^2 - \frac{1}{2r}\left[(3+\nu)\boldsymbol{p}^2 + \nu\left(\frac{\boldsymbol{r}\cdot\boldsymbol{p}}{r}\right)^2\right] + \frac{1}{2\boldsymbol{r}^2}$$

$$\boldsymbol{r} \equiv (\boldsymbol{r}_1 - \boldsymbol{r}_2)/M \qquad \boldsymbol{p} \equiv \boldsymbol{p}_1/\mu = -\boldsymbol{p}_2/\mu$$

Conservative dynamics of the effective-one-body model

- Constants of motion: energy E and angular momentum L
- We use Hamilton-Jacobi equation

$$\hat{H}_{\mathrm{PN}}\left(\boldsymbol{r},\boldsymbol{p}\right)=\hat{H}_{\mathrm{PN}}\left(\boldsymbol{r},\frac{\partial S}{\partial \boldsymbol{r}}\right)=\frac{E}{\mu}$$

- Separation of variables: $S=-Et+L\phi+S_r(r;E,L)$

$$\left(\frac{dS_r}{dr}\right)^2=\mathcal{R}(r;E,L)$$

5th order polynomial in 1/r at 2PN

- Bound periodic motion: $I_r(E,L)\equiv\int_{r_{\mathrm{min}}}^{r_{\mathrm{max}}}\sqrt{\mathcal{R}(r;E,L)}dr$

$$N=L+I_r \qquad \alpha=G\mu M$$

$$E_{\mathrm{rel}}=E+Mc^2=Mc^2-\frac{1}{2}\frac{\mu\alpha^2}{N^2}\left[1+\mathcal{O}\left(\frac{1}{c^2}\right)\right]$$

Conservative dynamics of the effective-one-body model

- Spherically symmetric, static, stationary spacetime

$$ds^2_{\rm eff} = -A(R)c^2dt^2 + \frac{D(R)}{A(R)}dR^2 + R^2 d\Omega^2$$

$$A(R) = 1 + \frac{a_1}{c^2 R} + \frac{a_2}{c^4 R^2} + \cdots \quad D(R) = 1 + \frac{d_1}{c^2 R} + \frac{d_2}{c^4 R^2} + \cdots$$

- Geodesic motion of a test mass by extremizing $S_{\rm eff} = -m_0 c \int ds_{\rm eff}$

- Same Hamilton-Jacobi approach: $g^{\mu\nu}_{\rm eff} P_\mu P_\nu + m_0^2 c^2 = 0$

$$a_1 = -2GM_{\rm eff}, \quad \alpha_{\rm eff} = G m_0 M_{\rm eff}$$

$$E^{\rm eff}_{\rm rel} = m_0 c^2 - \frac{1}{2}\frac{m_0 \alpha^2_{\rm eff}}{N^2_{\rm eff}}\left[1 + \mathcal{O}\left(\frac{1}{c^2}\right)\right]$$

- **Assume there is a mapping** b/w PN binary (*real problem*) and test-mass motion in effective spacetime (*effective problem*)

Conservative dynamics of the effective-one-body model

- **Impose mappings**

$$\mu = m_0\,, \quad M = M_{\rm eff}\,,$$
$$L = L_{\rm eff}\,, \quad N = N_{\rm eff}$$
$$E_{\rm eff} = E\left[1 + \alpha_1\left(\frac{E}{\mu c^2}\right) + \alpha_2\left(\frac{E}{\mu c^2}\right)^2 + \cdots\right]$$

- Unknown a_i's, d_i's, alpha_i's can be fixed. **Energy mapping** is

$$E_{\rm rel} = Mc^2\sqrt{1 + 2\nu\left(\frac{E^{\rm eff}_{\rm rel}}{\mu c^2} - 1\right)}$$

- Since the mapping is coord-invariant, there is a canonical transformation

$$\boldsymbol{R} = \boldsymbol{R}(\boldsymbol{r}, \boldsymbol{p}), \boldsymbol{P} = \boldsymbol{P}(\boldsymbol{r}, \boldsymbol{p})$$

- Interpretation: the real problem is mapped to the effective problem via a canonical transformation + the energy mapping

Conservative dynamics of the effective-one-body model

- One works with the EOB dynamics obtained from

$$H_{\rm EOB}(\boldsymbol{R},\boldsymbol{P}) \equiv Mc^2\sqrt{1+2\nu\left(\frac{H_{\rm eff}(\boldsymbol{R},\boldsymbol{P})}{\mu c^2}-1\right)}$$

$$H_{\rm eff}(\boldsymbol{R},\boldsymbol{P}) = \mu c^2\sqrt{A(R)\left(1+\frac{P_R^2}{\mu^2 c^2}\frac{A(R)}{D(R)}+\frac{P_\phi^2}{\mu^2 c^2 R^2}\right)}$$

$$A(R) = 1-\frac{2GM}{c^2 R}+2\nu\left(\frac{GM}{c^2 R}\right)^3, \quad D(R) = 1-6\nu\left(\frac{GM}{c^2 R}\right)^2 \qquad \text{(at } \textbf{2PN}\text{)}$$

- At nonspinning **3PN** order to preserve the same energy mapping one has to introduce a non-geodesic term [Damour+00]

$$H_{\rm eff}(\boldsymbol{R},\boldsymbol{P}) = \mu c^2\sqrt{A(R)\left(1+\frac{P_R^2}{\mu^2 c^2}\frac{A(R)}{D(R)}+\frac{P_\phi^2}{\mu^2 c^2 R^2}+\left(\frac{GM}{c^2 R}\right)^2\frac{Q(P)}{\mu^4 c^4}\right)}$$

Conservative dynamics of the effective-one-body model

- **Spinning BBHs** in PN: leading order

$$\hat{H}^{\rm SO}_{\rm 1.5PN} = \frac{G}{c^2 r^3}\boldsymbol{L}\cdot(g_S \boldsymbol{S} + g_{S_*}\boldsymbol{S}_*)$$

$$\boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2, \quad \boldsymbol{S}_* = \frac{m_2}{m_1}\boldsymbol{S}_1 + \frac{m_2}{m_2}\boldsymbol{S}_2$$

$$g_S = 2 + O\left(\frac{1}{c^2}\right), \qquad g_{S_*} = \frac{3}{2} + O\left(\frac{1}{c^2}\right)$$

gyro-gravitomagnetic ratios

$$\hat{H}^{\rm SS}_{\rm 2PN} = \frac{G\nu}{2c^2} S_0^i S_0^j \partial_i \partial_j \frac{1}{r}$$

$$\boldsymbol{S}_0 = \left(1 + \frac{m_2}{m_1}\right)\boldsymbol{S}_1 + \left(1 + \frac{m_1}{m_2}\right)\boldsymbol{S}_2$$

- EOB for spinning BBHs as early as 2001 [Damour01, Damour,Jaranowski&Schaefer07,08,Nagar11,Balmelli&Damour15]: map to geodesic motion of **nonspinning test particle in a deformation of Kerr** (using a Kerr spin that is function of real spins)

$$H_{\rm eff} = \beta^i P_i + \alpha\sqrt{\mu^2 + \gamma^{ij}P_i P_j + Q}$$

$$\alpha = \frac{1}{\sqrt{-g^{00}_{\rm eff}}}, \quad \beta^i = \frac{g^{0i}_{\rm eff}}{g^{00}_{\rm eff}}, \quad \gamma^{ij} = g^{ij}_{\rm eff} - \frac{g^{0i}_{\rm eff} g^{0j}_{\rm eff}}{g^{00}_{\rm eff}}$$

$$\beta^i \approx \frac{2}{R^3}\epsilon^{ijk} S^j_{\rm Kerr} X^k, \qquad \boldsymbol{S}_{\rm Kerr} = \sigma_1 \boldsymbol{S}_1 + \sigma_2 \boldsymbol{S}_2$$

- Deformation regulated again by symmetric mass ratio. SS effects added “by hand”

Conservative dynamics of the effective-one-body model

- [Barausse&Buonanno09,10] Map real problem to geodesic motion of **spinning test particle in a deformation of Kerr**: exact at linear order in spin in the test-particle limit. PN spin effects included through 3.5PN SO, 2PN SS

$$H_{\rm eff} = \beta^i P_i + \alpha\sqrt{\mu^2 + \gamma^{ij} P_i P_j + Q} + H_S + H_{SS}$$

- Procedure:
  1. Apply canonical transformation to the PN ADM Hamiltonian to move to EOB coordinates
  2. Apply the energy mapping to the transformed PN Hamiltonian and expand in powers of 1/c
  3. Deform the Hamiltonian of a test particle in Kerr and expand it in powers of 1/c
  4. Comparing 2. and 3., work out the mapping between the spin variables in the real and effective problems

Conservative dynamics of the effective-one-body model

- Gyro-gravitomagnetic ratios are identified in the SO part of the effective problem. Mapping leaves undetermined coefficients a_i's, b_i's

$$
\begin{aligned}
g_S^{\text{eff}} &= 2 + \frac{1}{c^2} g_S^{(2)}\left(\mathbf{P}^2, P_R^2, \frac{GM}{R}; a_0\right) \\
&\quad + \frac{1}{c^4} g_S^{(4)}\left(\mathbf{P}^2, P_R^2, \frac{GM}{R}; a_0, a_1, a_2, a_3\right) + \ldots \\
g_{S^*}^{\text{eff}} &= \frac{3}{2} + \frac{1}{c^2} g_{S^*}^{(2)}\left(\mathbf{P}^2, P_R^2, \frac{GM}{R}; b_0\right) \\
&\quad + \frac{1}{c^4} g_{S^*}^{(4)}\left(\mathbf{P}^2, P_R^2, \frac{GM}{R}; b_0, b_1, b_2, b_3\right) + \ldots
\end{aligned}
$$

$$u = \frac{GM}{c^2 R}$$

- These ratios are different for different spinning EOB Hamiltonians

Inputs from conservative gravitational self-force

Inputs from conservative gravitational self-force

- Advantages of synergy b/w GSF and EOB:
  1. GSF data are numerically very accurate
  2. In GSF calculations, unlike in NR, it is straightforward to disentangle conservative effects from dissipative ones
  3. GSF data are available in the strong-field/extreme-mass-ratio regime, currently, inaccessible to either PN or NR

- Conservative GSF results can inform the EOB potentials [Damour09]

PN-like expansion

$$A(u;\nu) = 1 - 2u + 2\nu\, u^3 + \nu\, a_4\, u^4 + \mathcal{O}(u^5)\,,$$

$$\bar{D}(u;\nu) = 1 + 6\nu\, u^2 + 2(26 - 3\nu)\,\nu\, u^3 + \mathcal{O}(u^4)\,,$$

$$Q(u, P_R;\nu) = 2(4-3\nu)\nu u^2 P_R^4/\mu^2 + \mathcal{O}(p_r^6)$$

$$u = \frac{GM}{c^2 R}$$

GSF-like expansion

$$A(u;\nu) = 1 - 2u + \nu\, a(u) + \nu^2\, a_2(u) + \mathcal{O}(\nu^3)\,,$$

$$\bar{D}(u;\nu) = 1 + \nu\, \bar{d}(u) + \nu^2\, \bar{d}_2(u) + \mathcal{O}(\nu^3)\,,$$

$$Q(u, P_R;\nu) = \nu[q(u) P_R^4 + \bar{q}(u) P_R^6] + \mathcal{O}(\nu^2) + \mathcal{O}(p_r^6)$$

ISCO shift

- **Schwarzschild ISCO shift** in EOB [Damour09]

$$A(u_{\rm ISCO})A'(u_{\rm ISCO}) + 2u_{\rm ISCO}A'(u_{\rm ISCO})^2 - u_{\rm ISCO}A(u_{\rm ISCO})A''(u_{\rm ISCO}) = 0$$

$$x_{\rm ISCO} = (GM\Omega_{\rm ISCO})^{2/3} = u_{\rm ISCO}\left[\frac{-A'(u_{\rm ISCO})/2}{1 + 2\nu(H_{\rm eff}(u_{\rm ISCO})|_{P_R=0}/\mu - 1)}\right]^{1/3}$$

$$x_{\rm ISCO} = \frac{1}{6}\left\{1 + \nu\left[a\left(\frac{1}{6}\right) + \frac{1}{6}a'\left(\frac{1}{6}\right) + \frac{1}{18}a''\left(\frac{1}{6}\right)\right] + \mathcal{O}(\nu^2)\right\}$$

  and in GSF [Barack&Sago09,Akcay+12]

$$x_{\rm ISCO} = \frac{1}{6}\left[1 + 0.8342(4)\nu + \mathcal{O}(\nu^2)\right]$$

- GSF Schwarzschild ISCO shift is currently included in latest NR-calibrated EOB models: it puts a constraint on certain calibration parameters that enter the A potential

- Kerr ISCO shift in GSF is available [Isoyama+14, vandeMeent16] but has not yet been included in EOB (numerical constraints on calibration parameters in non-closed form)

Periastron advance

- **Schwarzschild periastron advance**

$$\left(\frac{\omega_r}{\Omega}\right)^2 = 1 - 6x + \nu\,\rho(x) + \mathcal{O}(\nu^2)\,,$$

in EOB [Damour09]

$$\rho(x) \equiv 4x\left(1 - \frac{1-2x}{\sqrt{1-3x}}\right) + (1-6x)\,\bar{d}(x) + a(x) + x\,a'(x) + \frac{1}{2}\,x(1-2x)\,a''(x)$$

and in GSF [Barack,Damour&Sago10] (fit)

$$\rho(x) = \frac{14x^2(1+12.9906x)}{1+4.57724x-10.3124x^2}$$

- Comparisons also with NR and PN for periastron advance with nonspinning [LeTiec+09] and extension to spinning BBHs [Hinderer+13,LeTiec+13,vandeMeent16] are fruitful cross-validations

First law and redshift

- Using **first law** of mechanics for nonspinning BBHs [LeTiec+12]

$$\delta M - \Omega\,\delta J = z_1\,\delta m_1 + z_2\,\delta m_2$$

  one gets [LeTiec+12] GSF correction to *circular-orbit* binding energy as function of GSF correction to redshift [Detweiler08, Sago+08, Shah+11]

$$\hat{E}(x) = \frac{1-2x}{\sqrt{1-3x}} - 1 + \nu\left[\frac{1}{2}\,z_{\rm SF}(x) - \frac{x}{3}\,z'_{\rm SF}(x) + \sqrt{1-3x} - 1 + \frac{x}{6}\,\frac{7-24x}{(1-3x)^{3/2}}\right] + \mathcal{O}(\nu^2)\,;$$

- The *circular-orbit* binding energy in EOB is [Barausse+12]

$$\hat{E}_{\rm EOB}(x) = \frac{1-2x}{\sqrt{1-3x}} - 1 + \nu\left\{\frac{1-4x}{(1-3x)^{3/2}}\,\frac{A_{\rm SF}(x)}{2} - \frac{x}{\sqrt{1-3x}}\,\frac{A'_{\rm SF}(x)}{3} - \left(\frac{1-2x}{\sqrt{1-3x}} - 1\right)\times \left[\frac{x}{3}\,\frac{1-6x}{(1-3x)^{3/2}} + \frac{1}{2}\left(\frac{1-2x}{\sqrt{1-3x}} - 1\right)\right]\right\} + \mathcal{O}(\nu^2)\,.$$

- **Fully determine linear-in-nu term in A potential** down to ISCO [Barausse+12] and LR [Akcay+12]

$$A_{\rm SF}(x) = \sqrt{1-3x}\;z_{\rm SF}(x) - x\left(1 + \frac{1-4x}{\sqrt{1-3x}}\right)$$

First law and redshift

- Knowledge of linear-in-nu term in A + periastron advance give information about **linear-in-nu term in D** [Barausse+12, Bini+15]

- Extension of the first law of nonspinning BBH mechanics to include eccentricity [LeTiec+15] can be exploited to **fully determine linear-in-nu term in Q potential** [Akcay&vandeMeent15]

- Extension of the first law of BBH mechanics to include spins [Blanchet +12] can be exploited to **fully determine linear-in-nu term in one EOB gyro-gravitomagnetic coefficient** on **circular orbits**: use redshift of equatorial spinless test mass in Kerr to get $g^{\mathrm{eff}}_{S(1\mathrm{GSF})}$ [Bini+15,Kavanagh+16]

Gyroscopic precession

- Spin-orbit precession of gyroscopes in Schwarzschild to get **fully determine linear-in-nu term in one EOB gyro-gravitomagnetic coefficient** $g^{\mathrm{eff}}_{S_*(1\mathrm{GSF})}$
  1. on **circular orbits** [Bini&Damour+14]
  2. on **eccentric orbits** [Kavanagh+17]

Waveforms and inputs
from black-hole perturbation theory

Resummation of waveform formulas for inspiral

- Goal: **resum** different PN effects in the waveforms by adopting a factorized form [Damour&Nagar07,09, Pan+11]

$$h_{\ell m}^{\mathrm{fact}} = h_{\ell m}^{\mathrm{Newt}} S_{\ell+m} T_{\ell m} (\rho_{\ell m})^{\ell} e^{i\delta_{\ell m}}$$

- For (2,2) mode: (quadrupole waveform) X (relativistic energy of the source) X (tail due to backscattering of waves off the background curvature) X (residual amplitude) X (residual phase)

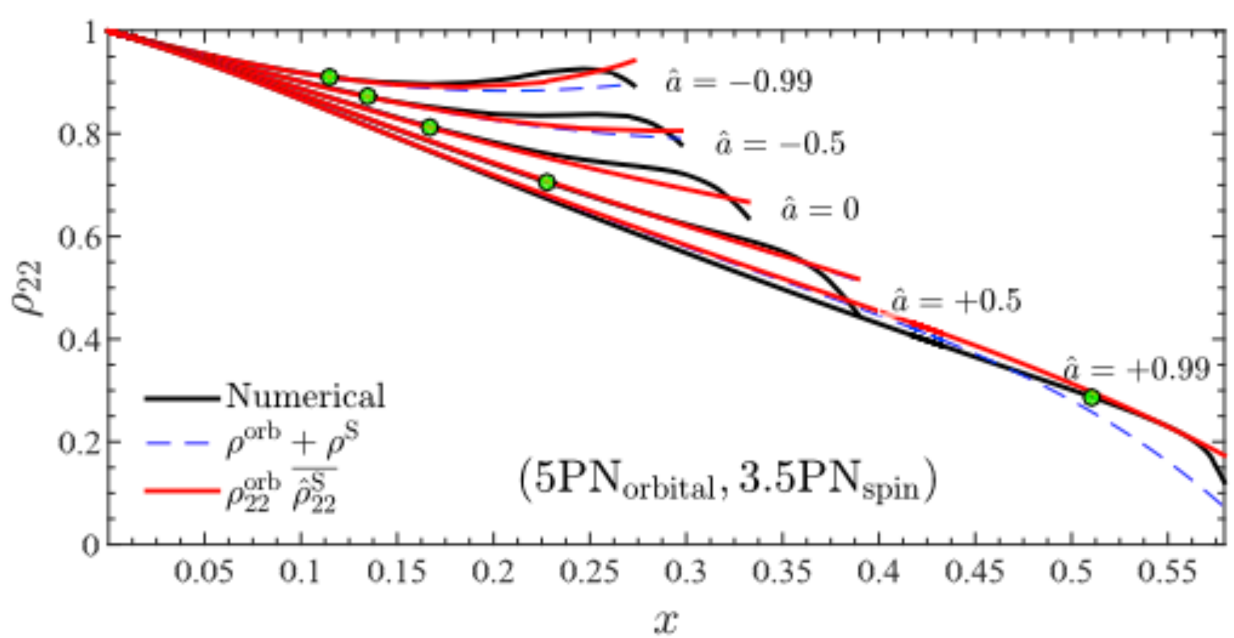
- *S-factor*: in the test-mass limit, each mode obeys a (frequency-domain) wave equation of the Regge-Wheeler-Zerilli type whose source term is a linear combination of terms linear in the stress-energy tensor of a test-particle of mass μ moving around a black hole of mass M

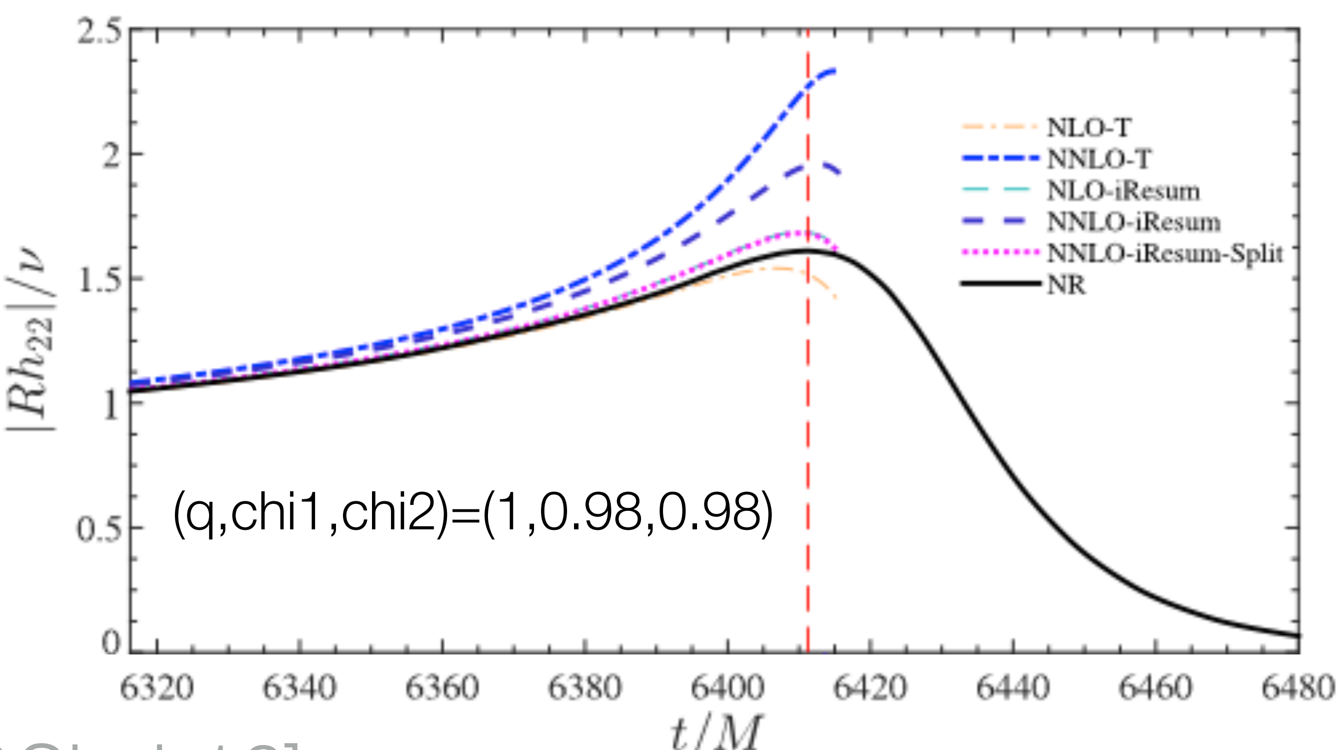
- *T-factor*: asymptotic modes are related to their corresponding near-zone expression by tail factor; in the comparable mass case, this tail factor is resummation of an infinite number of leading logarithms that appear when computing asymptotic modes in MPM formalism

Resummation of waveform formulas for inspiral

- Factorized formulas have been compared to GW fluxes/waveforms computed via BH perturbation theory (**frequency Regge-Wheeler-Zerilli/Teukolsky**) for spinless test masses on equatorial circular orbits in Schwarzschild/Kerr: further **resummations of the residuals** (rho, delta) have been devised [Damour&Nagar09, Pan+09, AT+13, Nagar&Shah16]

- Factorized formulas exhibit better behavior also when compared to NR



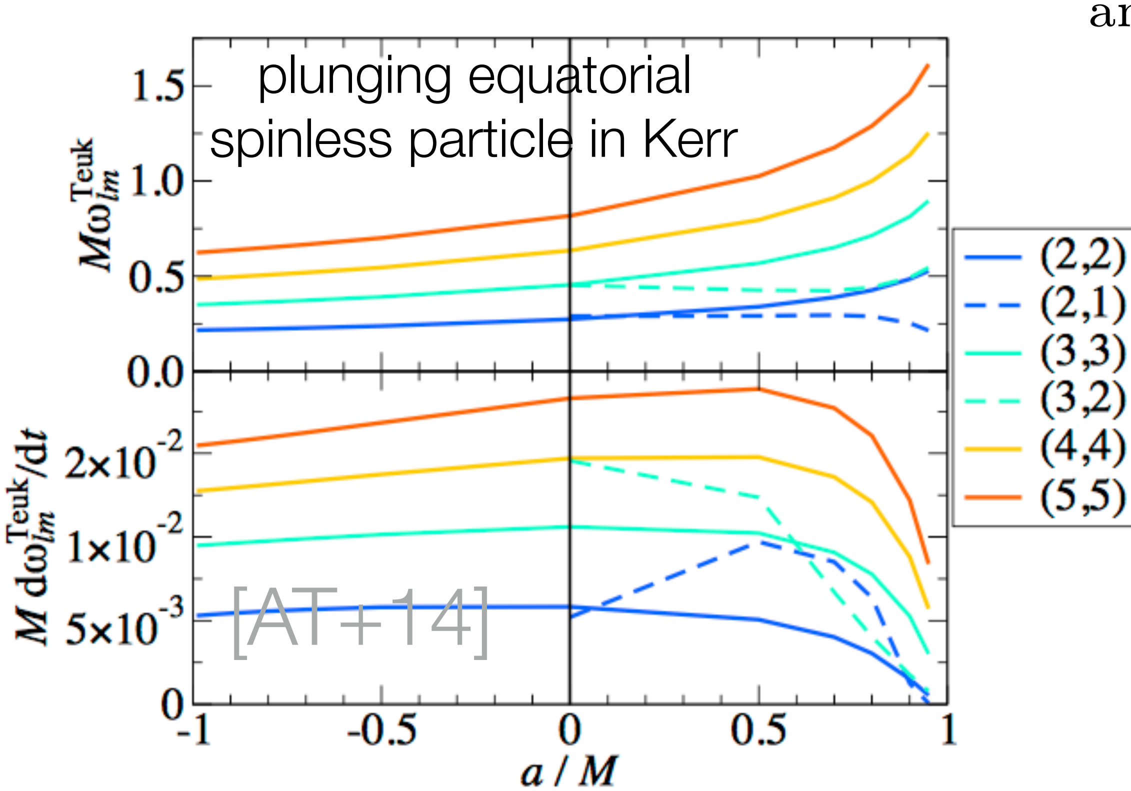


[Nagar&Shah16]

Merger waveforms

- EOB defines the merger time by the **light-ring crossing**, a special point in the EOB dynamics
- **At merger**, inspiral (quasicircular) formulas are **corrected phenomenologically** requiring that: (i) amplitude, (ii) 2nd time-derivative of amplitude, (iii) GW frequency, and (iv) time-derivative of GW frequency agree with fits to NR + **time-domain Teukolsky** waveforms [done in all state-of-the-art EOB models]

$$h_{\ell m}^{\mathrm{insp-plunge}} = h_{\ell m}^{\mathrm{fact}} \times \underbrace{A_{\ell m}(R, P_R; c_1, c_2, c_3)}_{\text{amplitude correction}} \underbrace{e^{i\psi_{\ell m}(R, P_R; c_4, c_5)}}_{\text{phase correction}}$$



- Time-domain Teukolsky merger waveforms from precessing plunging orbits could inform precessing EOB

[Nagar+07, Bernuzzi+11, Han+11, Barausse+13, AT+14, Nagar+14]

Ringdown waveforms

- After merger, signal can be modeled by **linear combination of QNMs** of the remnant BH [Buonanno&Damour00] that is smoothly attached to inspiral-plunge waveform

- Fitting formulas to NR predict **final mass and spin of remnant BH** as functions of progenitor BBH [Hofmann+16,Keitel+16,Healy+17]

- More recently, purely **phenomenological fits** to NR ringdowns are used for improved stability of attachment to inspiral-plunge waveform [Baker +08,Damour&Nagar14,Nagar&DelPozzo16,Bohe,Shao,AT+16]

- Time-domain Teukolsky ringdowns from plunging particles in Kerr can inform modeling of ringdowns for comparable-mass binaries [AT+14, Babak,AT+16]

Modeling EMRIs with EOB [Yunes+09,10]

- Studies limited to **equatorial**, **quasicircular** orbits of a small body around supermassive BH, both possibly spinning

- Accurate **fits to frequency-domain Teukolsky fluxes** starting from the EOB factorized waveform formulas. Include also fits of BH absorption. Fits extended down to LR [AT+14]

- Evolve spinning EOB Hamiltonian of [Barausse&Buonanno09] using fitted flux within an adiabatic approximation to speed up computation, competitive with kludges

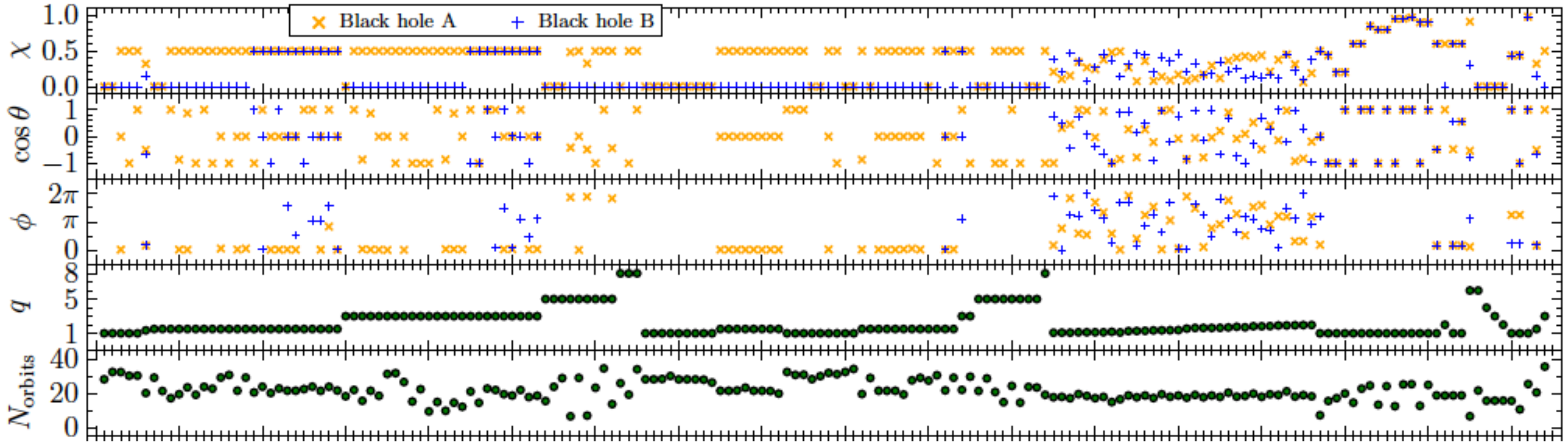
- Matches to Teukolsky-based waveforms >97% over a period between 4 and 9 months, depending on the system, better than kludges [Gair&Glampedakis06] in classical LISA

- Limitations: EOB H only contained PN linear-in-nu corrections, no eccentricity, no inclinations

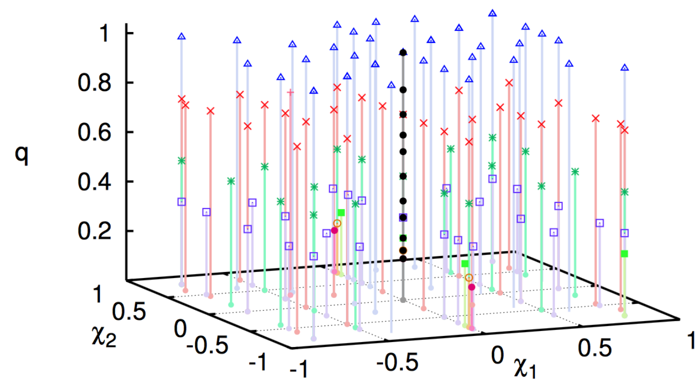
II. Inspiral-merger-ringdown models for aLIGO

Nonprecessing models and calibration to numerical relativity

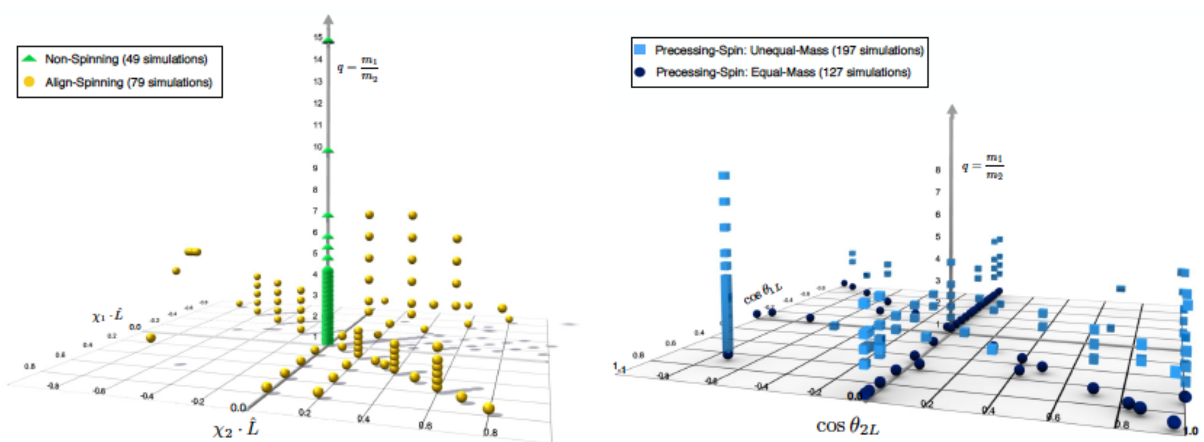
Numerical-relativity catalogs of BBHs



[Mroue+13, Chu+15]



[Healy+17]



[Jani+16]

… and many more NR waveforms from many groups [SXS, GATech, RIT, Cardiff-UIB, NCSA, etc.] are being computed, also in response to observations

Direct use of numerical relativity

Besides guiding construction of models (waveforms, remnant properties), there are other avenues to use NR for data analysis:

- **Direct comparison of existing NR catalogs to observations** [LVC1602.03843, LVC1606.01262]

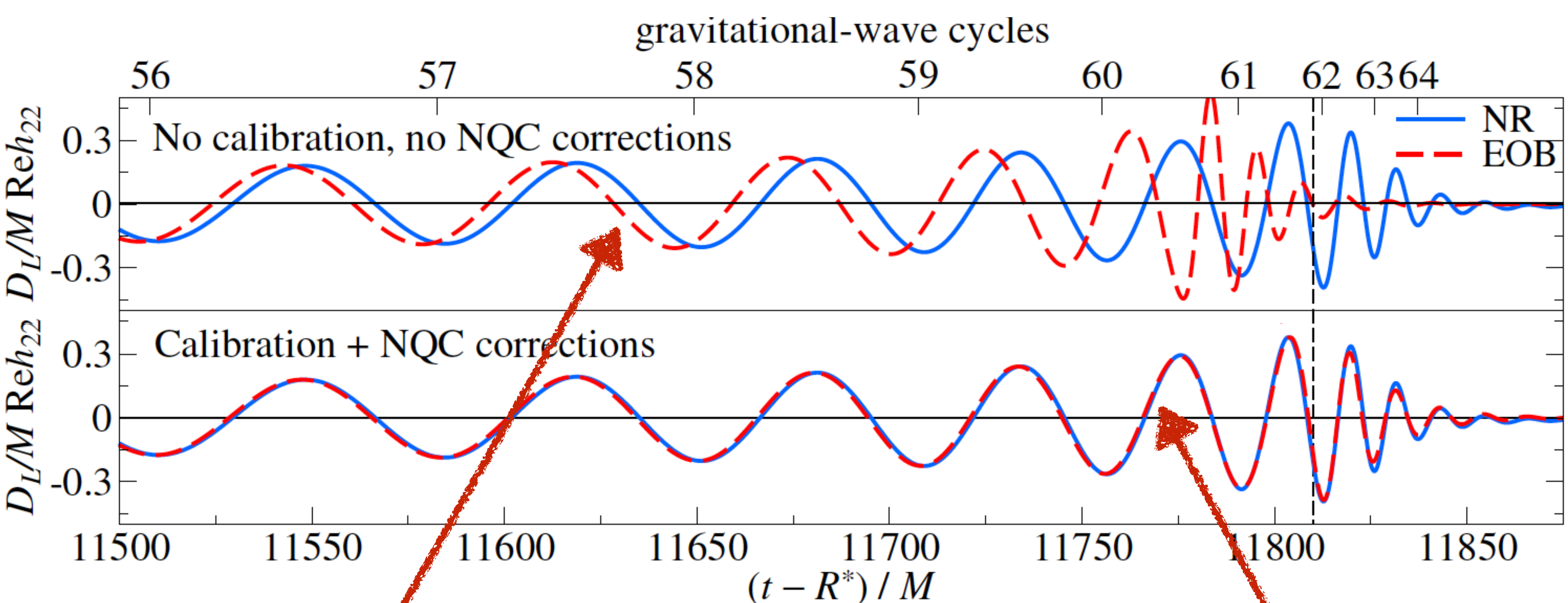
- **NR follow-ups to observations** [LVC detection papers, Lovelace+16]:
  1. comparisons to unmodeled reconstructions
  2. validate models

- **Surrogate waveform models** [Blackman+15,17]
  1. restricted parameter space (high mass, q<=2, spins<=0.8, generic spin orientations)
  2. many NR simulations to construct basis
  3. **interpolation** across NR runs
  4. they do not extrapolate to low mass: need models or long NR

Tuning EOB to numerical relativity

$$A = \overbrace{1-2u}^{\text{Schwarzschild}} + 2\nu u^3 + \left(\frac{94}{3} - \frac{42}{32}\pi^2\right)\nu u^4 + a_5 u^5 + \cdots \quad (u = GM/Rc^2)$$

example of tuning parameter



no tuning

tuned

How good is a model?

$$\langle h_{\rm NR}, h_{\rm model}\rangle = 4\,{\rm Re}\int_{f_{\rm low}}^{f_{\rm high}} \frac{\tilde{h}_{\rm NR}(f)\tilde{h}^*_{\rm model}(f)}{S_n(f)}df$$

$$\mathcal{O}(h_{\rm NR}, h_{\rm model}) = \frac{\langle h_{\rm NR}, h_{\rm model}\rangle}{\sqrt{\langle h_{\rm NR}, h_{\rm NR}\rangle\langle h_{\rm model}, h_{\rm model}\rangle}}$$

- **Banks of templates** used to search the data for GWs tolerate 97% overlaps ~ 10% loss in event rate

- **Parameter estimation**: (sufficient) accuracy requirement [Lindblom+08]

$$\mathcal{O}(h_{\rm NR}, h_{\rm model}) > 1 - \frac{1}{2\,{\rm SNR}^2}$$

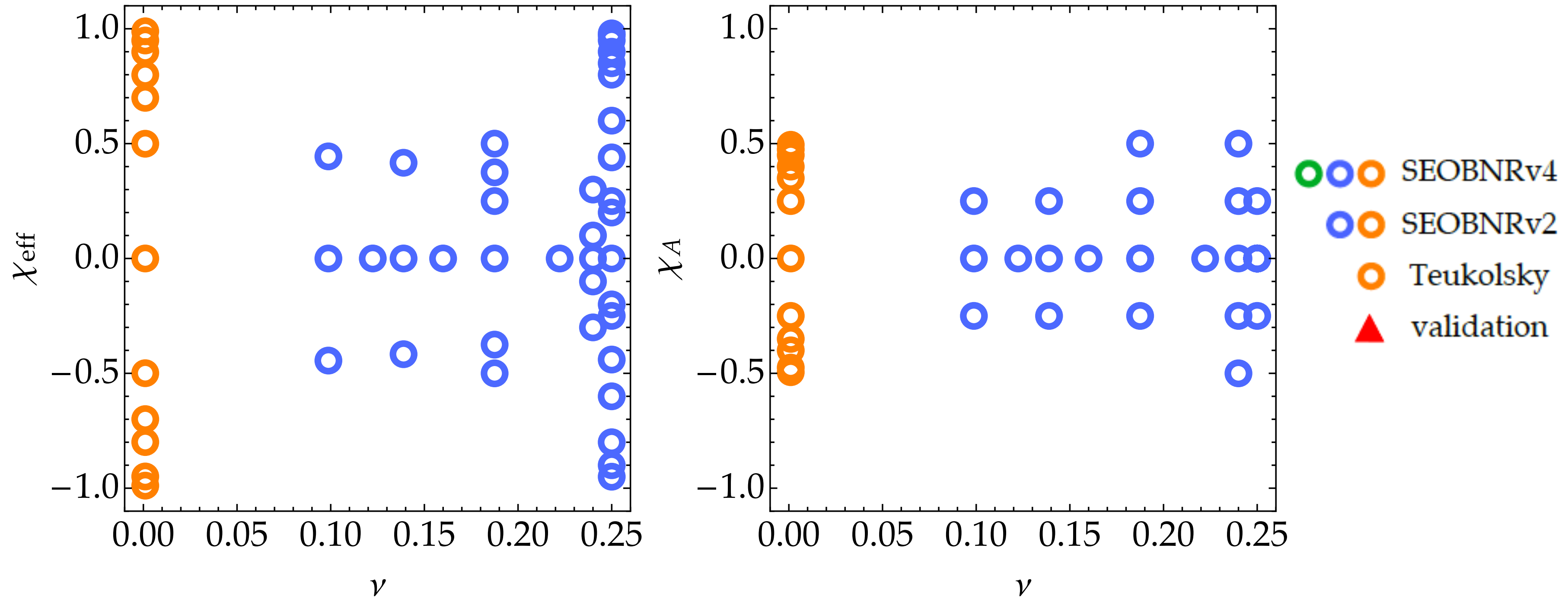
Effective-one-body model of nonprecessing BBHs for O1

$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2}$$

$$\chi_{\rm eff} = \left(\frac{\boldsymbol{S}_1}{m_1} + \frac{\boldsymbol{S}_2}{m_2}\right)\cdot\hat{\boldsymbol{L}}$$

$$\chi_A = \left(\frac{\boldsymbol{S}_1}{m_1^2} - \frac{\boldsymbol{S}_2}{m_2^2}\right)\cdot\hat{\boldsymbol{L}}$$

only (2,2) mode



- **SEOBNRv2** calibrated to better than 99% overlap with NR for design aLIGO [AT+14]
- Used in its **reduced-order-model** version [Pürrer14,15] in O1 for filtering and parameter estimation

Effective-one-body model of nonprecessing BBHs for O2

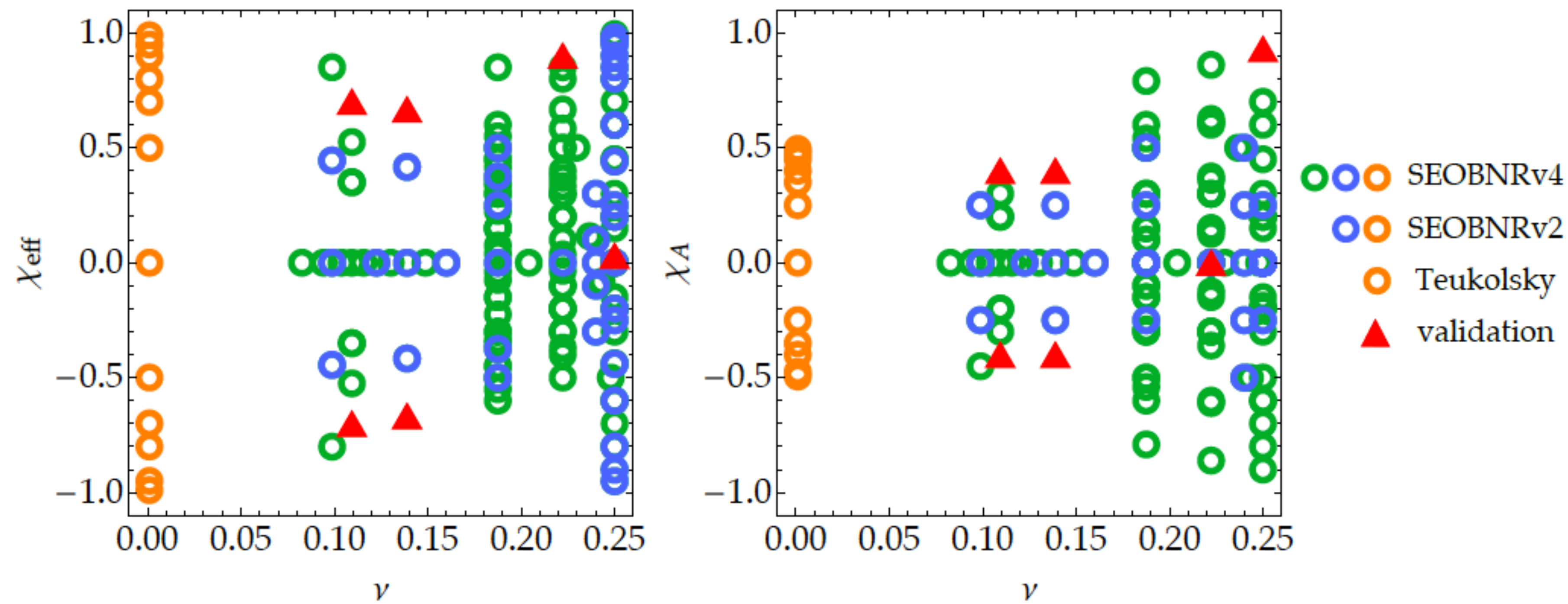
- **SEOBNRv4** [Bohe,Shao,AT+16]

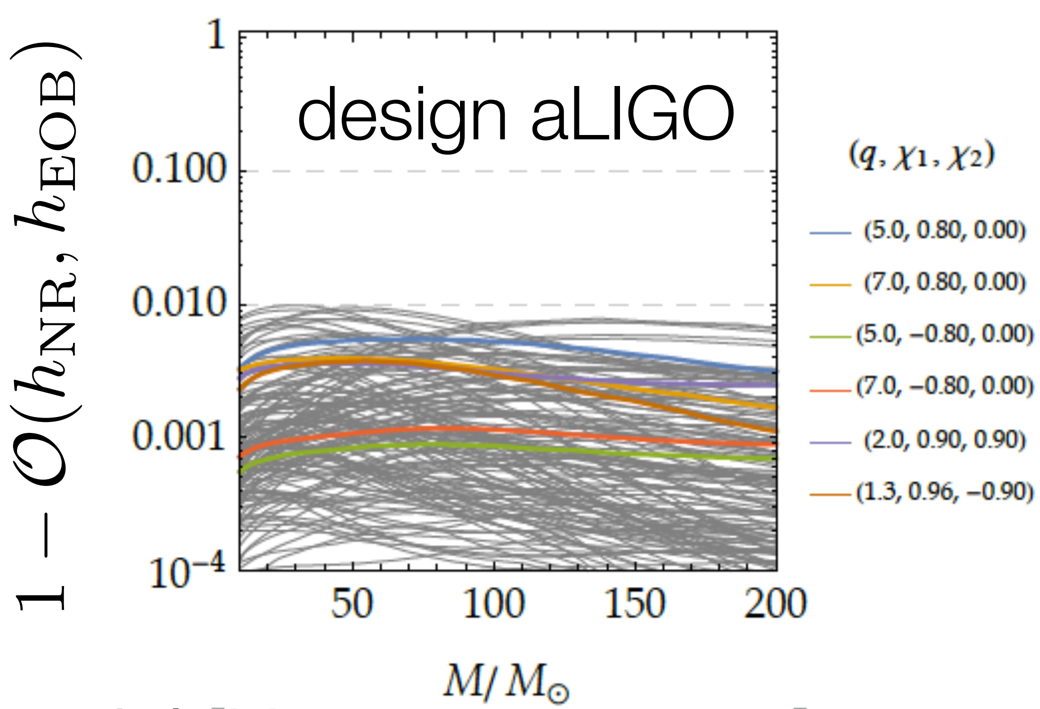
$$\nu = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

$$\chi_{\rm eff} = \left(\frac{\boldsymbol{S}_1}{m_1} + \frac{\boldsymbol{S}_2}{m_2}\right) \cdot \hat{\boldsymbol{L}}$$

$$\chi_A = \left(\frac{\boldsymbol{S}_1}{m_1^2} - \frac{\boldsymbol{S}_2}{m_2^2}\right) \cdot \hat{\boldsymbol{L}}$$

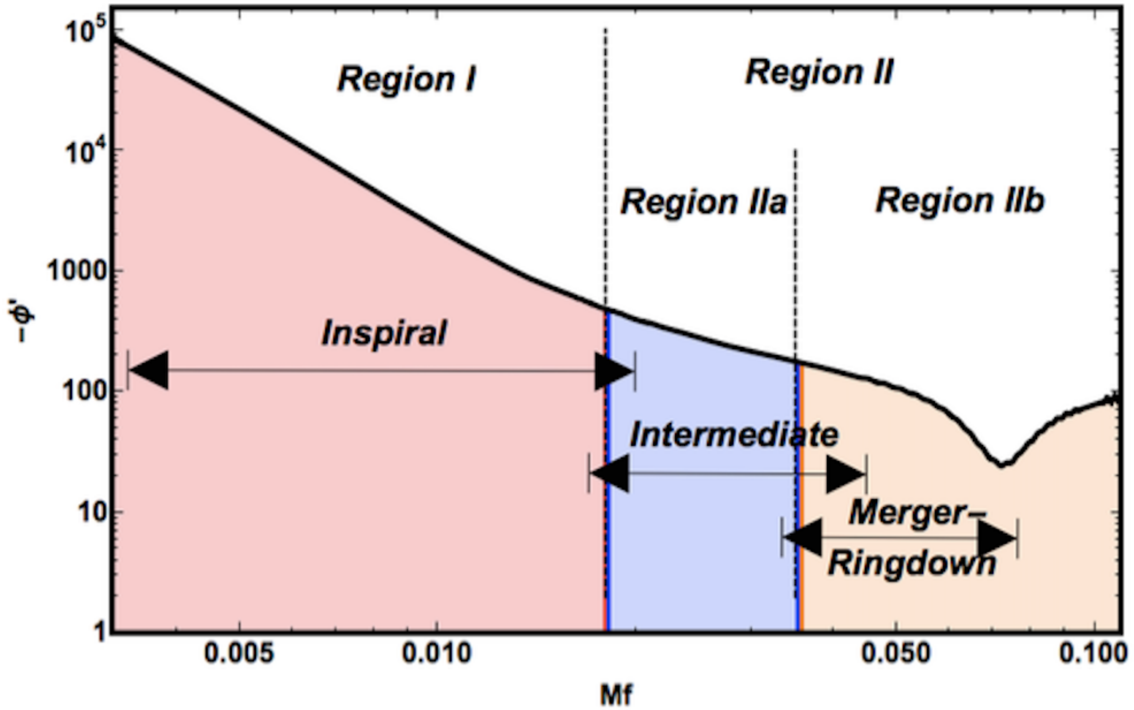
only (2,2) mode





- Latest **IHES EOB** model [Nagar+15,16,17] uses different spinning Hamiltonian and has comparable performance to NR

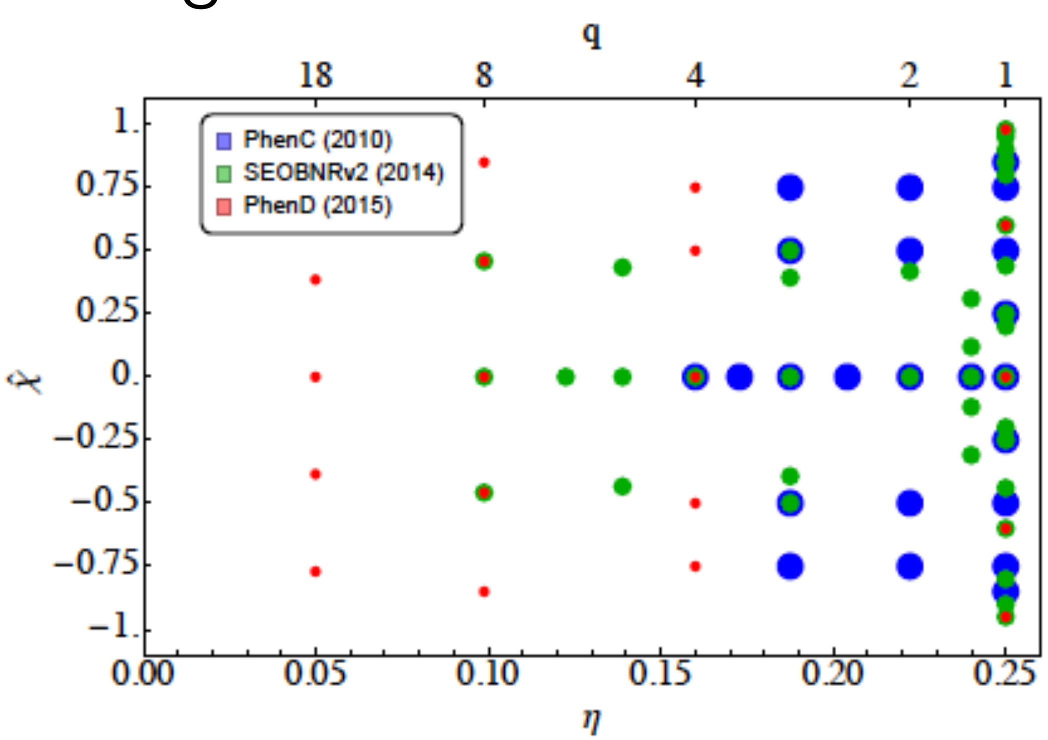
Phenomenological model of nonprecessing BBHs

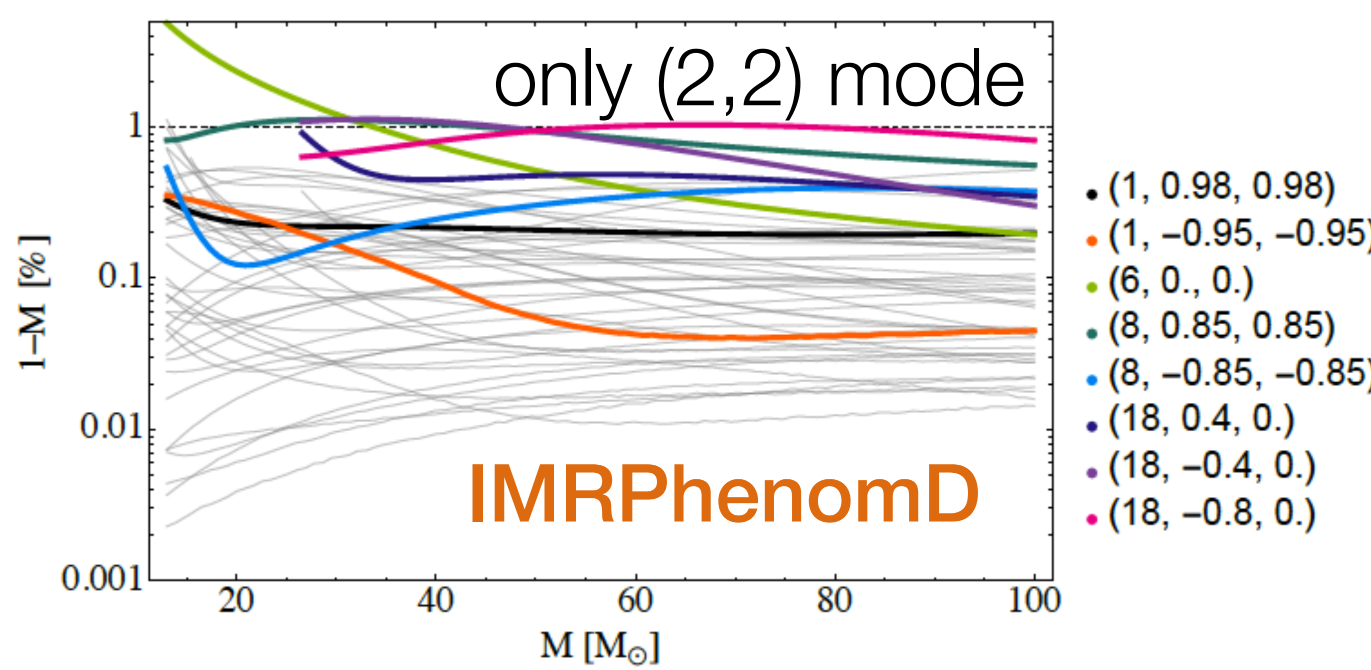


(fits for Fourier phase)

$$
\begin{aligned}
\phi_{\rm Ins} =& \phi_{\rm TF2}(Mf;\Xi) \\
&+\frac{1}{\eta}\left(\sigma_0+\sigma_1 f+\frac{3}{4}\sigma_2 f^{4/3}+\frac{3}{5}\sigma_3 f^{5/3}+\frac{1}{2}\sigma_4 f^2\right) \\
\phi_{\rm Int} =& \frac{1}{\eta}\left(\beta_0+\beta_1 f+\beta_2\,{\rm Log}(f)-\frac{\beta_3}{3}f^{-3}\right) \\
\phi_{\rm MR} =& \frac{1}{\eta}\Big\{\alpha_0+\alpha_1 f-\alpha_2 f^{-1}+\frac{4}{3}\alpha_3 f^{3/4} \\
&+\alpha_4\tan^{-1}\left(\frac{f-\alpha_5 f_{\rm RD}}{f_{\rm damp}}\right)\Big\}\,.
\end{aligned}
$$

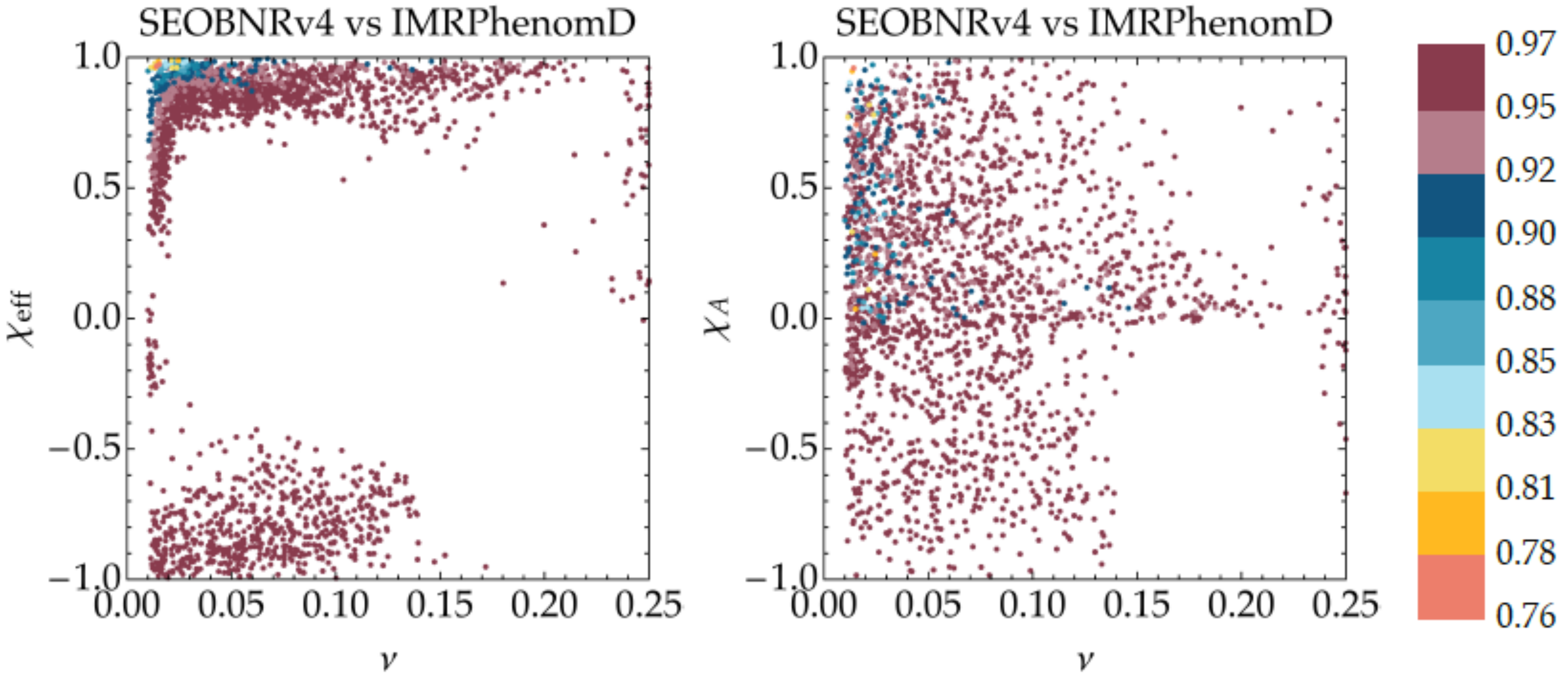
- Directly fit **hybrids** of uncalibrated EOB and NR in the frequency domain using different ansaetze in different regimes [Husa+15, Khan+15]





Comparing nonprecessing IMR BBH models (only (2,2))

only **2.1%** out of 100,000 random configurations
have effectualness <0.97



[Bohe,Shao,AT+16]

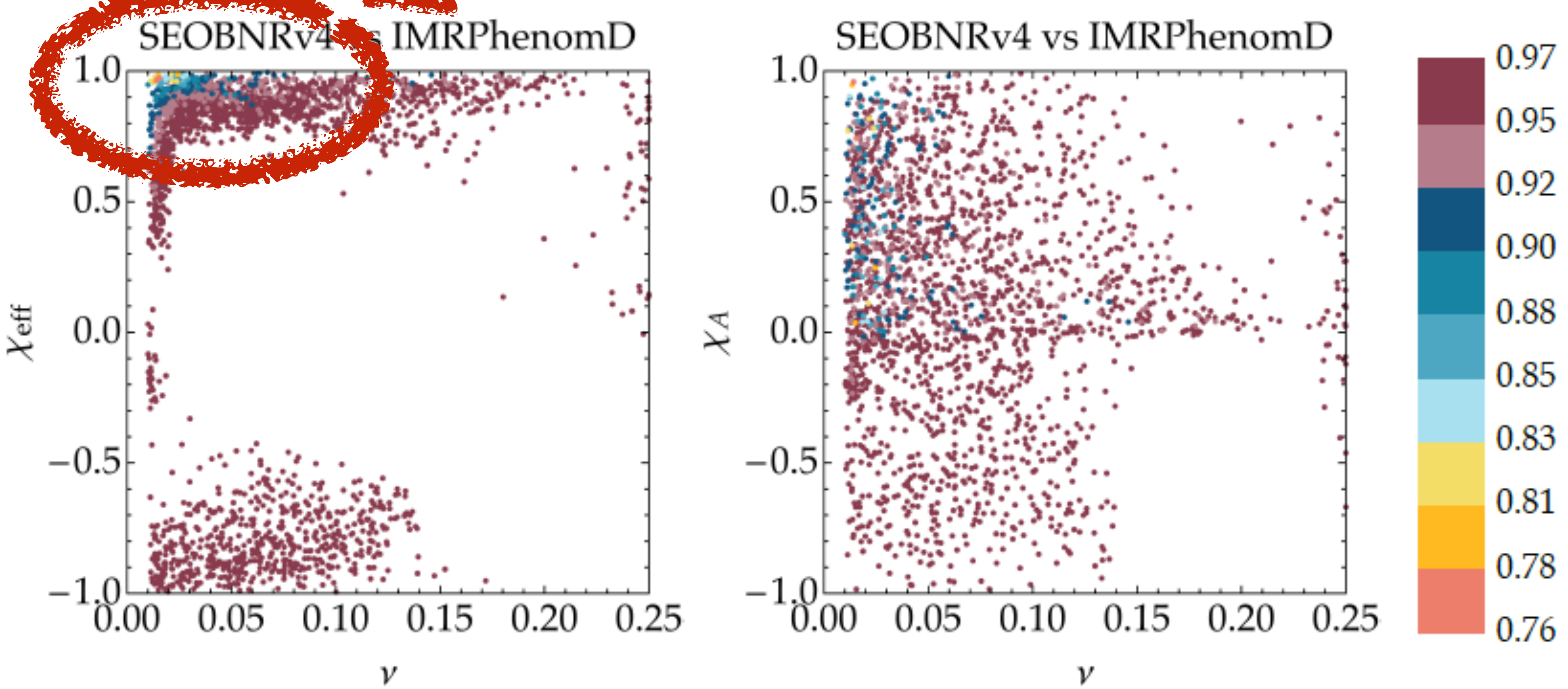
$$1 - \mathcal{O}(h_1, h_2)$$

maximized over
masses and spins
(in template bank)

(O1 aLIGO)

Comparing nonprecessing IMR BBH models (only (2,2))

new, long numerical-relativity simulations are needed here



[Bohe,Shao,AT+16]

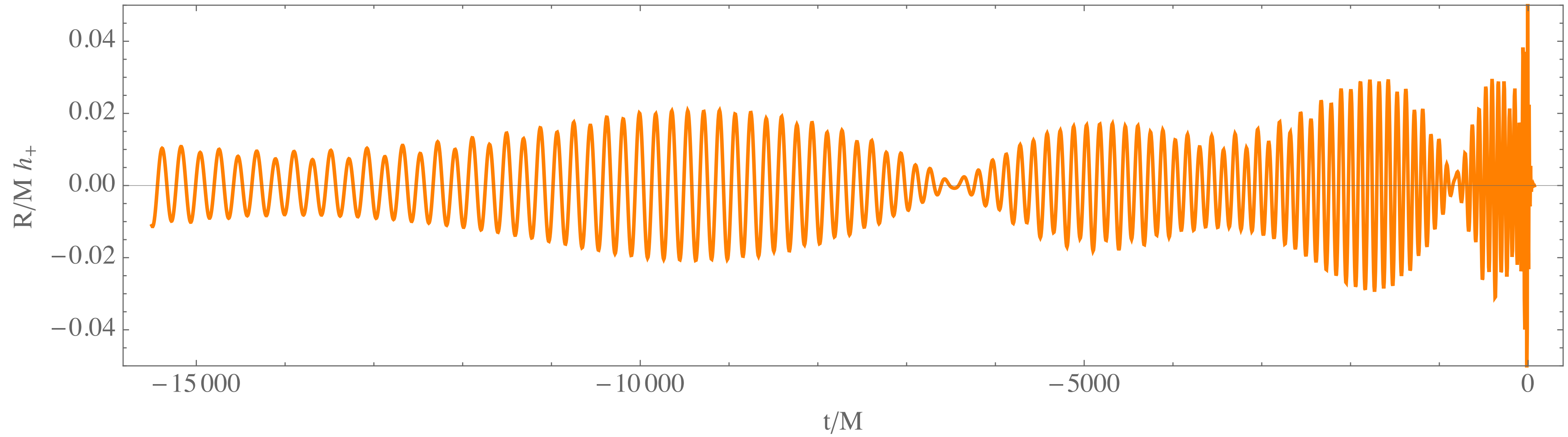
$$1 - \mathcal{O}(h_1, h_2)$$

maximized over masses and spins (in template bank)

(O1 aLIGO)

Precessing models

Precessing IMR BBH models



- When BH spins are not parallel to angular momentum of the binary, the **orbital plane precesses**

- **Precessing frame** [Buonanno+03, Schmidt+11, O'Shaughnessy+11, Boyle+11]
  1. In precessing frame, use **calibrated nonprecessing model**
  2. Inertial-frame modes from **rotation of precessing-frame modes according to motion of orbital angular momentum**

- Both EOB [Pan+13, Babak, AT+16] and phenomenological [Hannam+13] models available. These are not calibrated to precessing simulations

- Ongoing analytical work with inspiral-only PN waveforms [Chatziioannou+17]

Parameter estimation of the first GW events

Parameter estimation in one slide

- Precessing BBH templates depend on **15 parameters**: BH masses, BH spin vectors, luminosity distance, right ascension, declination, direction of emission in the source frame (2 angles), time of merger, phase at merger

- **Bayesian inference**

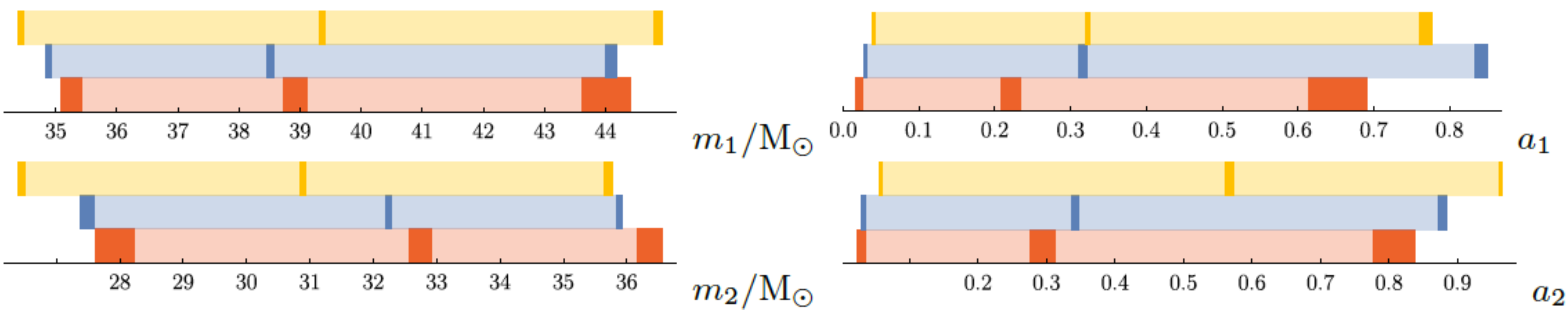
$$\mathrm{likelihood}(\boldsymbol{\xi}|\mathrm{data}) \propto e^{-\langle \mathrm{data}-h_M(\boldsymbol{\xi})|\mathrm{data}-h_M(\boldsymbol{\xi})\rangle/2}$$

$$p(\boldsymbol{\xi}|\mathrm{data}) \propto \mathrm{prior}(\boldsymbol{\xi}) \times \mathrm{likelihood}(\boldsymbol{\xi}|\mathrm{data})$$

$$p_i(\xi_i) = \int p(\boldsymbol{\xi}|\mathrm{data})\,\mathrm{d}\xi_1 \cdots \widehat{\mathrm{d}\xi_i} \cdots \mathrm{d}\xi_{15}$$

IMR precessing models vs GW150914

- Nonprecessing EOBNR, precessing EOBNR, and precessing Phenom measure consistent parameters for GW150914
  1. SNR
  2. comparable mass
  3. face off/on
  4. short signal

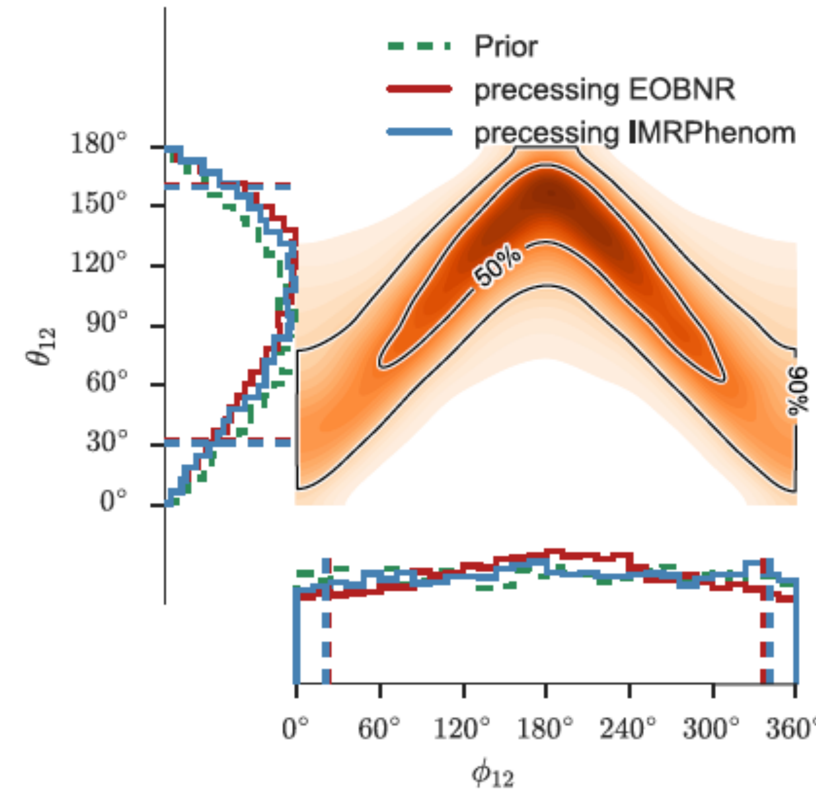


[LVC1606.01262]

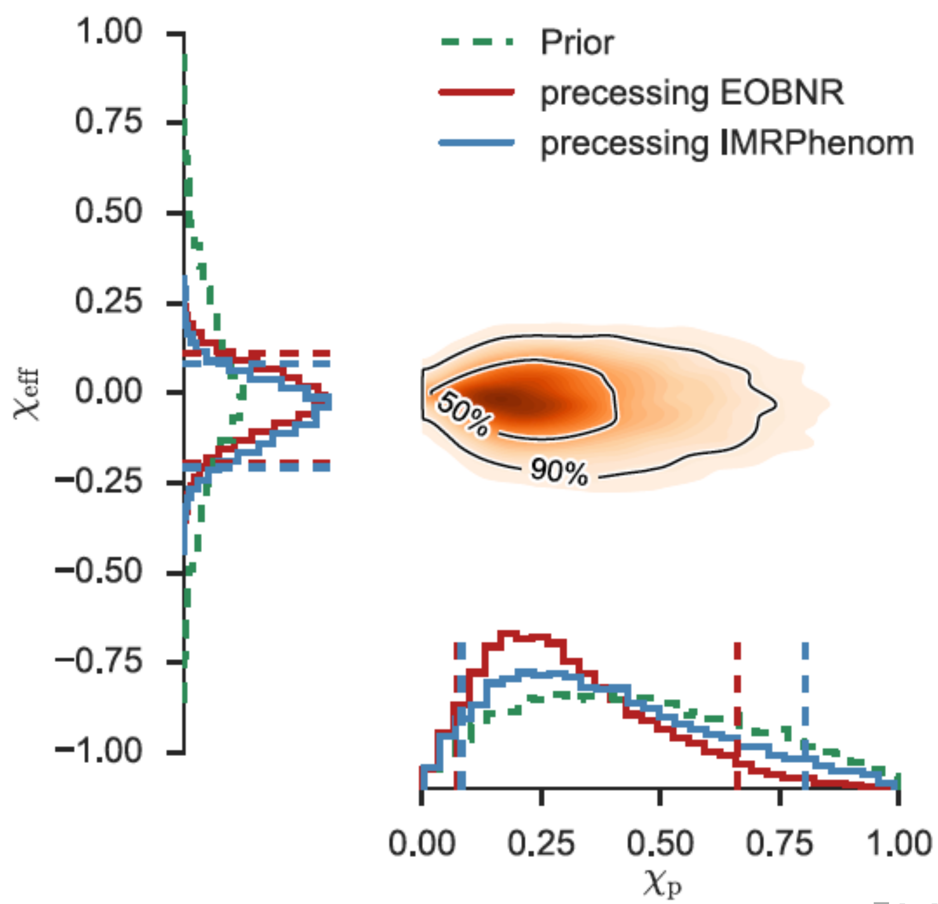
IMR precessing models vs GW150914

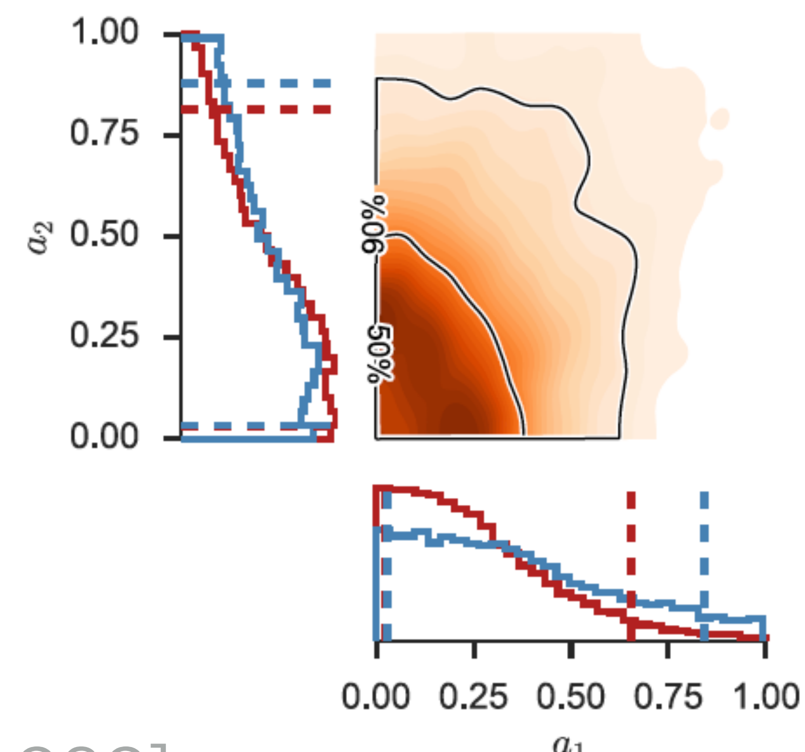
$$\chi_{\rm eff} = \frac{c}{G}\left(\frac{\boldsymbol{S}_1}{m_1} + \frac{\boldsymbol{S}_2}{m_2}\right) \cdot \frac{\hat{\boldsymbol{L}}_{\rm N}}{M},$$

$$\chi_{\rm p} = \frac{c}{B_1 G m_1^2} \max(B_1 S_{1\perp}, B_2 S_{2\perp}),$$



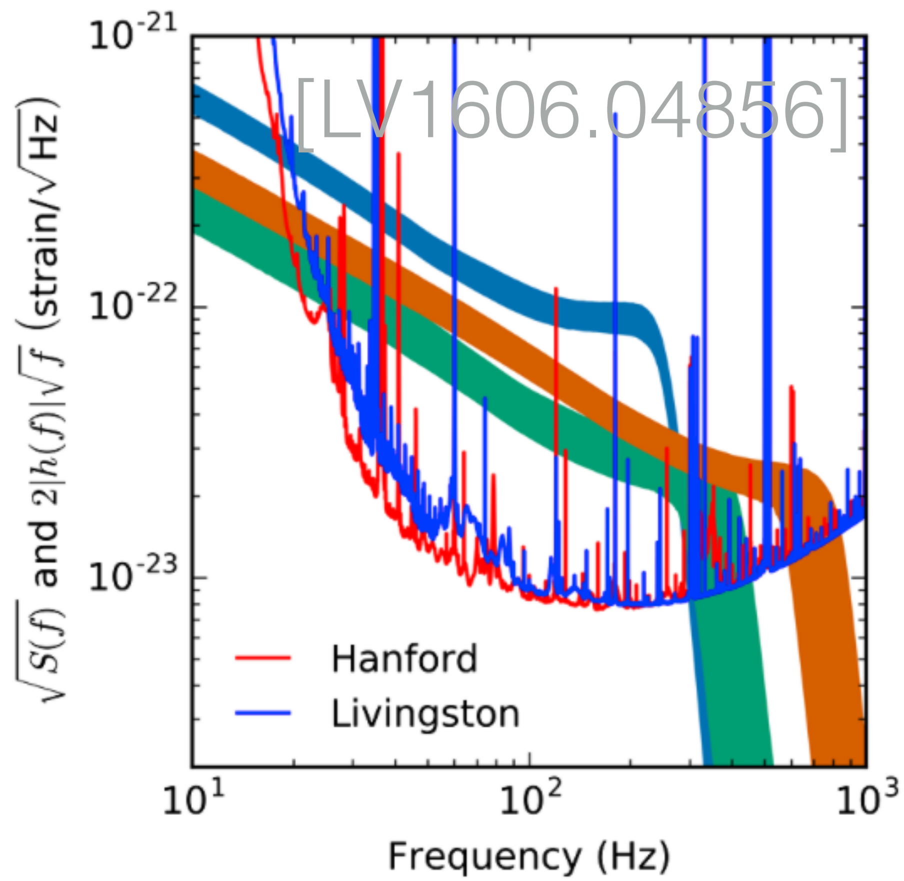
measured at 20Hz

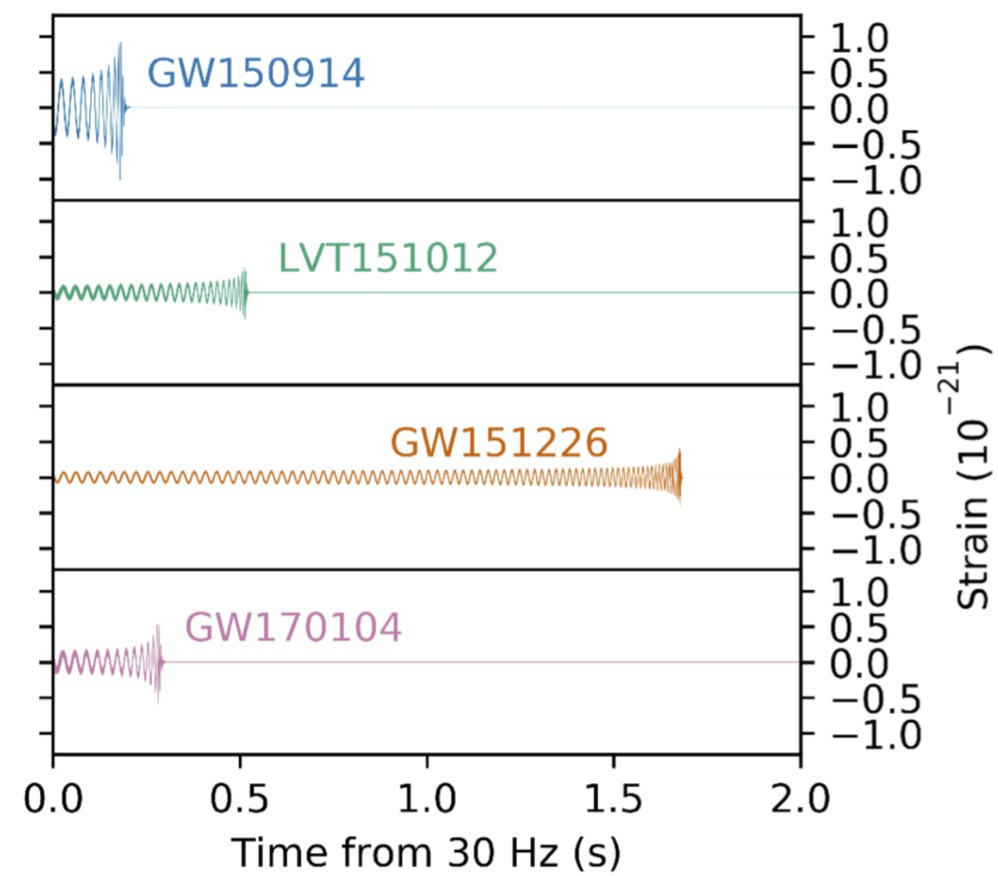


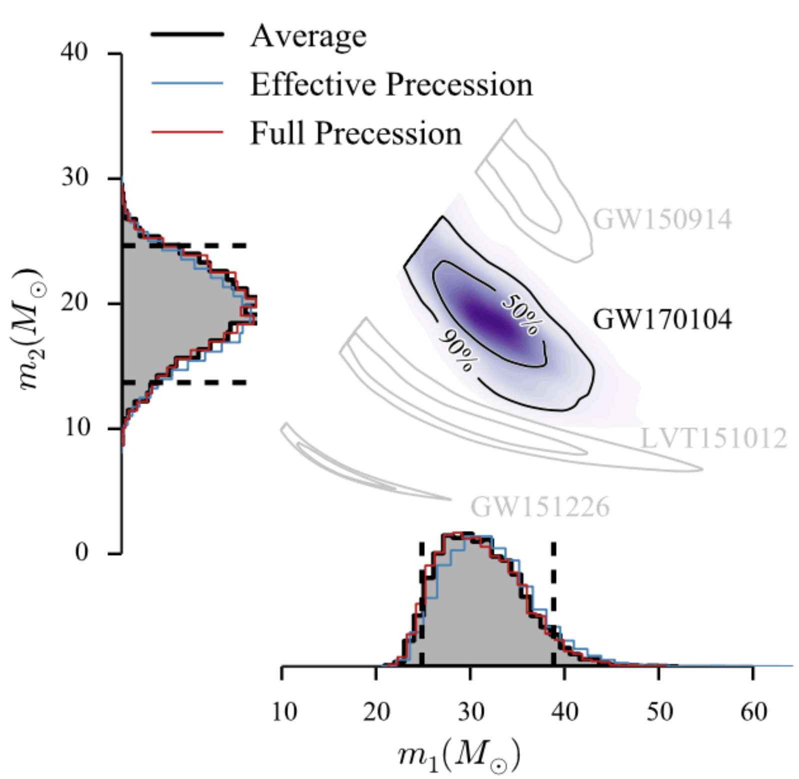


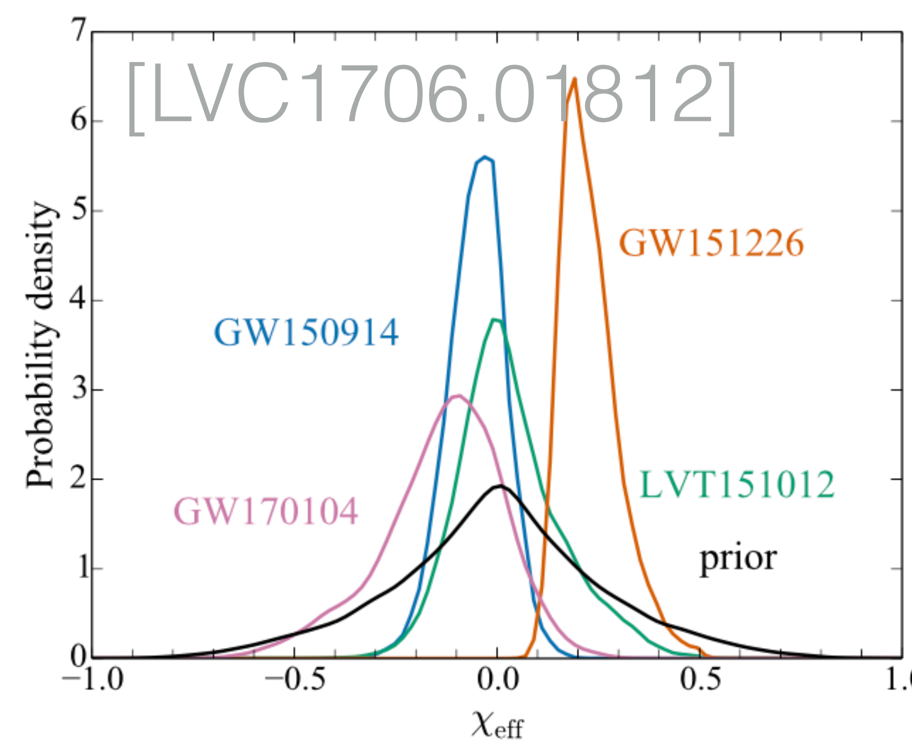
[LVC1606.01262]

BBH observations so far









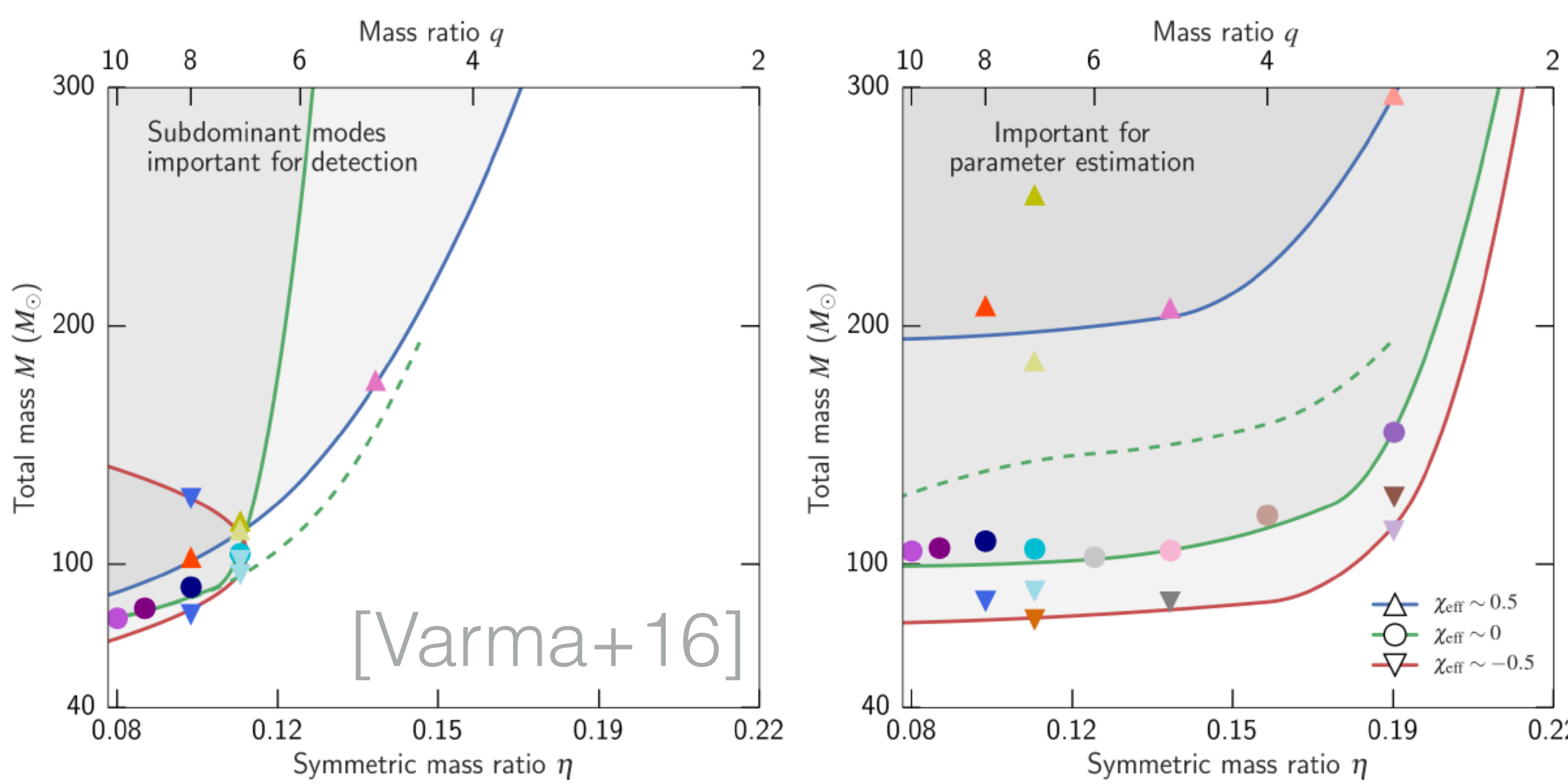
Unmodeled effects and recent developments

Higher-order modes

- **Inspiral-merger-ringdown** higher-order modes for nonspinning BBHs available [Pan+11], but for **spinning, nonprecessing** BBHs not available

- For nonspinning searches, no impact for 3MSun ≤ m1, m2 ≤ 200MSun and M < 360MSun [Capano+13]

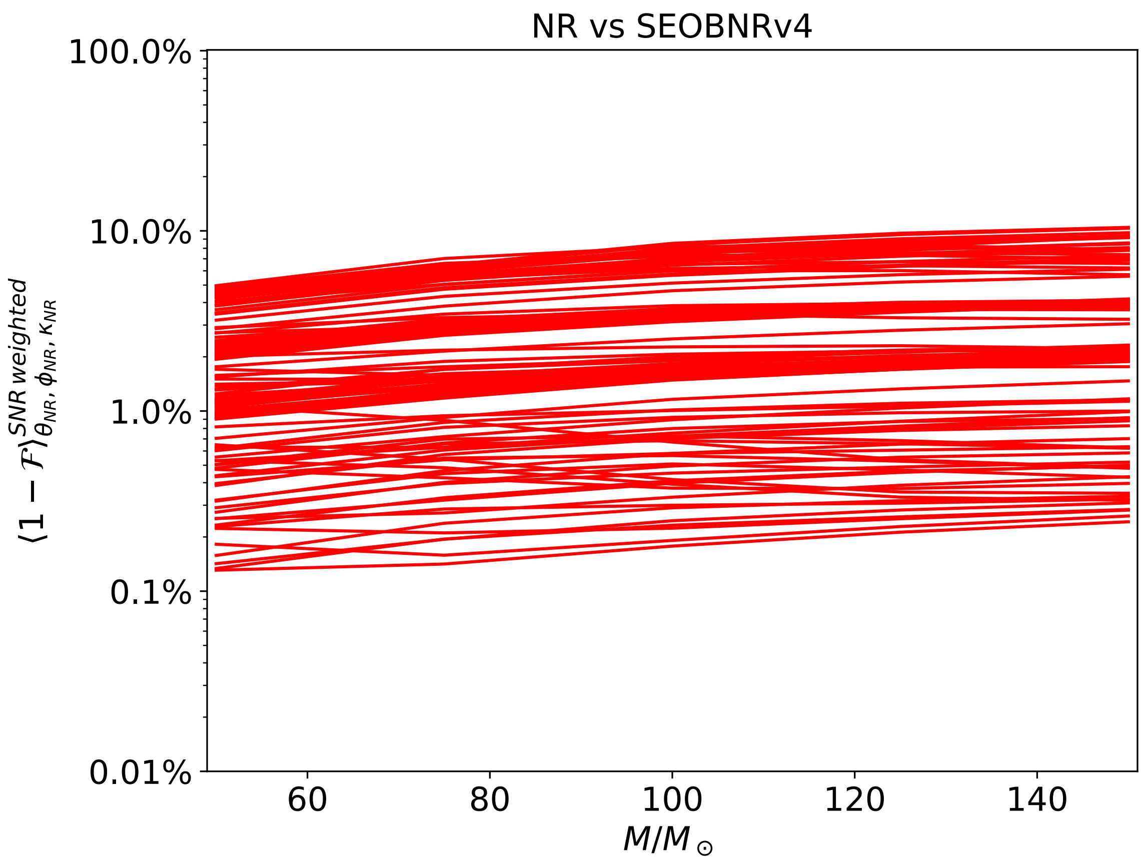
- Higher-modes systematics > statistical errors for q>4 and M>100Msun at SNR>8 (orientation avg) [Calderon-Bustillo+15,16, Varma+16]

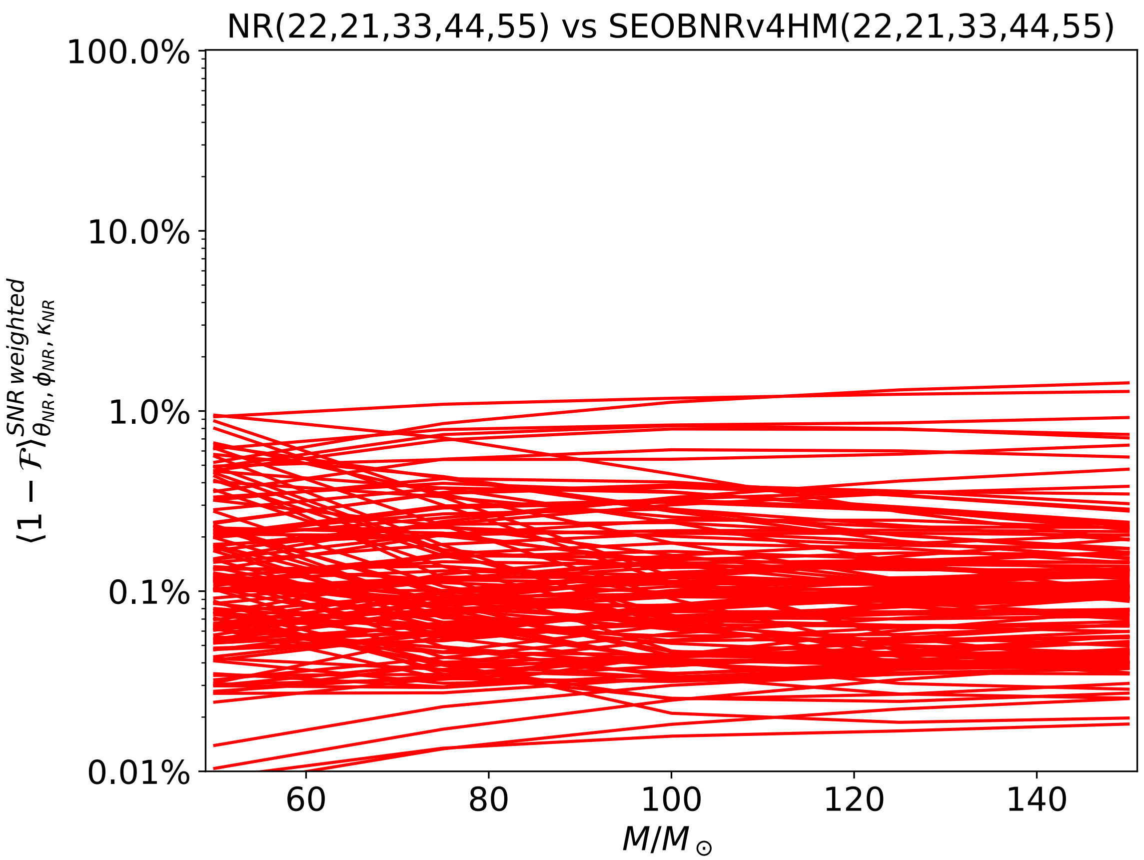


Spin-aligned EOB model with higher-order modes

- Development of merger-ringdown waveforms for higher-order modes **(2,1),(3,3),(4,4),(5,5)** for spinning, nonprecessing BBHs is underway [Cotesta+(in prep)]

- Comparison and tuning to ~150 SXS NR simulations + time-domain Teukolsky waveforms





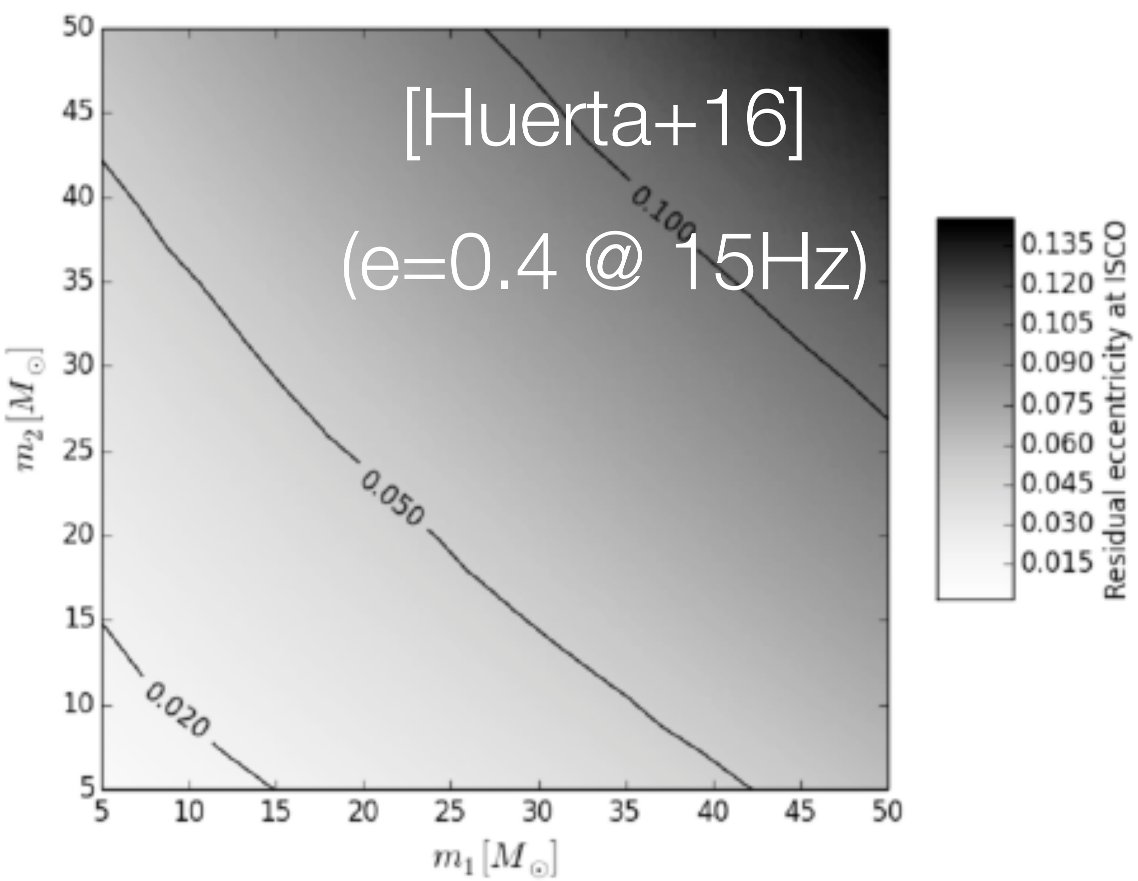
SNR-weighted sky- and polarization-averaged mismatch (design aLIGO)

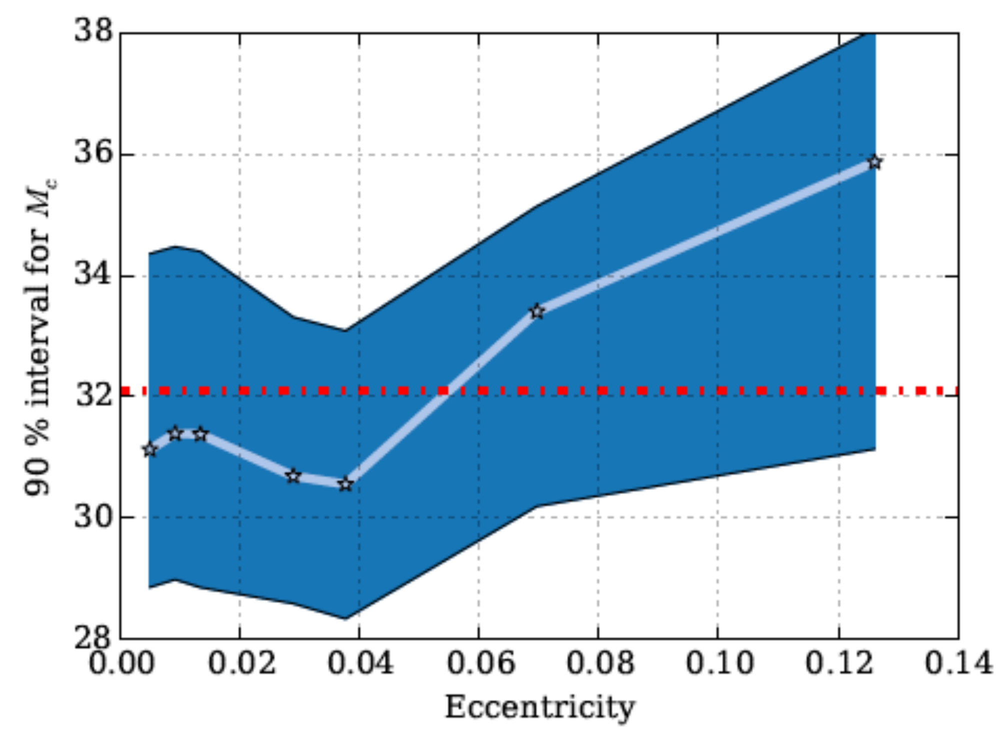
Eccentric binary black holes of comparable masses

- **Dynamical environment** scenarios for the binary can create BBHs that enter aLIGO band with e>0.1 [Antonini+15]

- Searches for BNS using quasicircular templates ok for e<=0.02 (M=2.6Msun) [Huerta+13]. BBH case studied in [Huerta+16]

- Small residual eccentricity can **bias** parameter estimation [Favata14, LVC1611.07531]

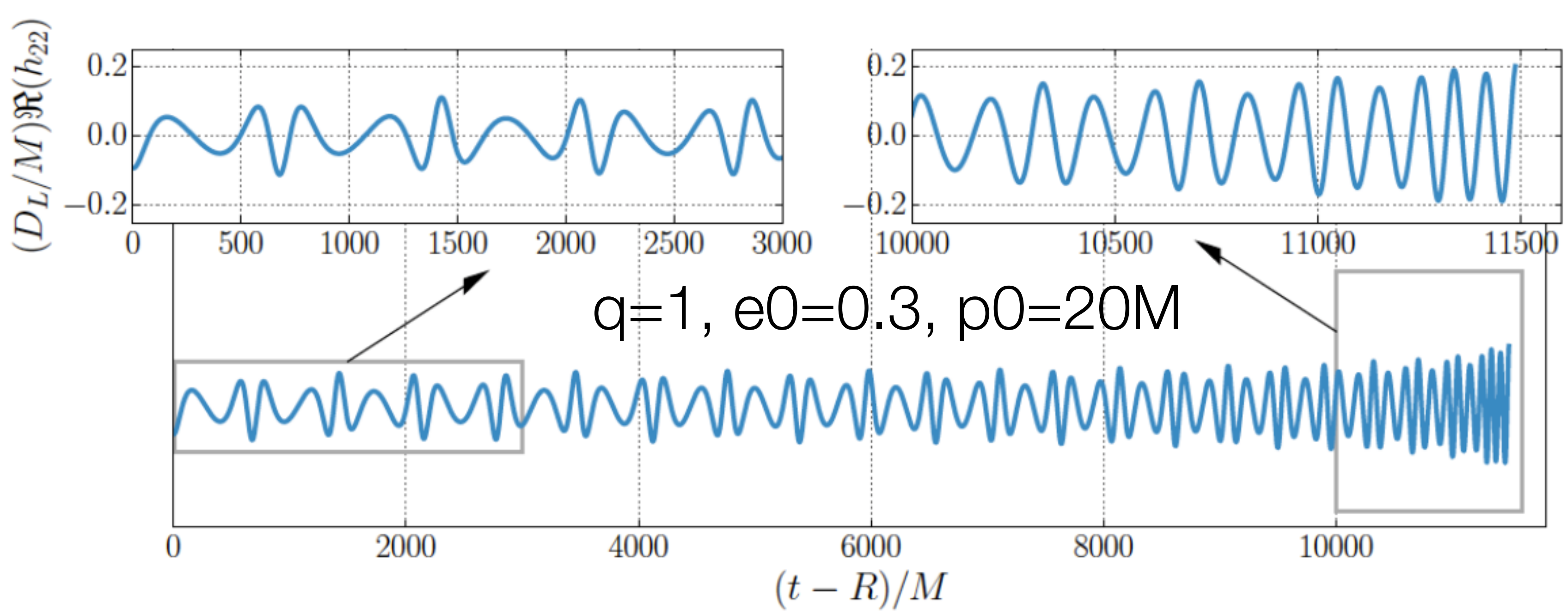




[LVC1611.07531]

EOB model for nonspinning BBHs with eccentricity

- Adopt more convenient orbital variables (semilatus rectum p, eccentricity e, 2 angles)
- Compute waveform modes sourced by EOB eccentric dynamics (only up to 1.5PN for now). Recover PN formulas in weak-field limit
- Complete IMR signal with merger-ringdown of circular model



[Hinderer&Babak(in prep)]

Conclusions

Conclusions

- Where we stand
  1. unprecedented wealth of information about GR 2-body problem from PN, NR, BHPT, GSF, but limited use of GSF in models used for data analysis
  2. very accurate (2,2)-mode spin-aligned models for q<=6
  3. reasonably good precessing models for moderate spins (<=0.5) and q<=4 (ℓ=2)

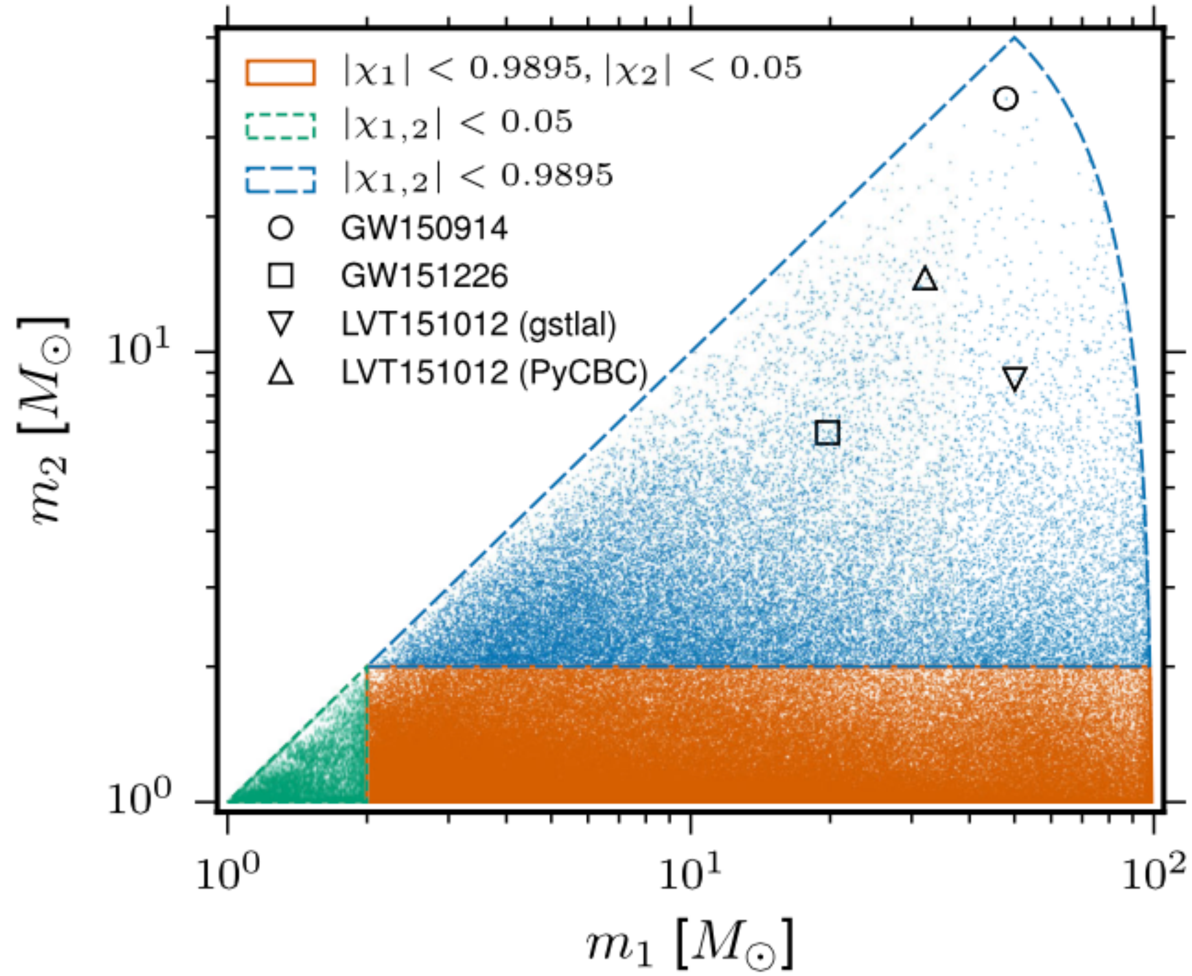
- Open problems
  1. **improve integration** of information from different regimes into waveform models that are used for data analysis
  2. **(large q, large spins, "low" M)** domain not constrained by NR: need for new simulations or inputs from GSF
  3. major physical effects that are important and still require proper modeling are **higher harmonics** and **eccentricity**

Additional slides

Template banks

- We don't know a priori the parameters of sources: build **banks of plausible signals** (templates) taking correlations into account
- **Filter** data through each template to see which fits best
- ~**200,000 spin-aligned EOB templates** were used in O1
- Nonprecessing bank sensitive to precessing signals around GW150914

[LVC1606.04856]

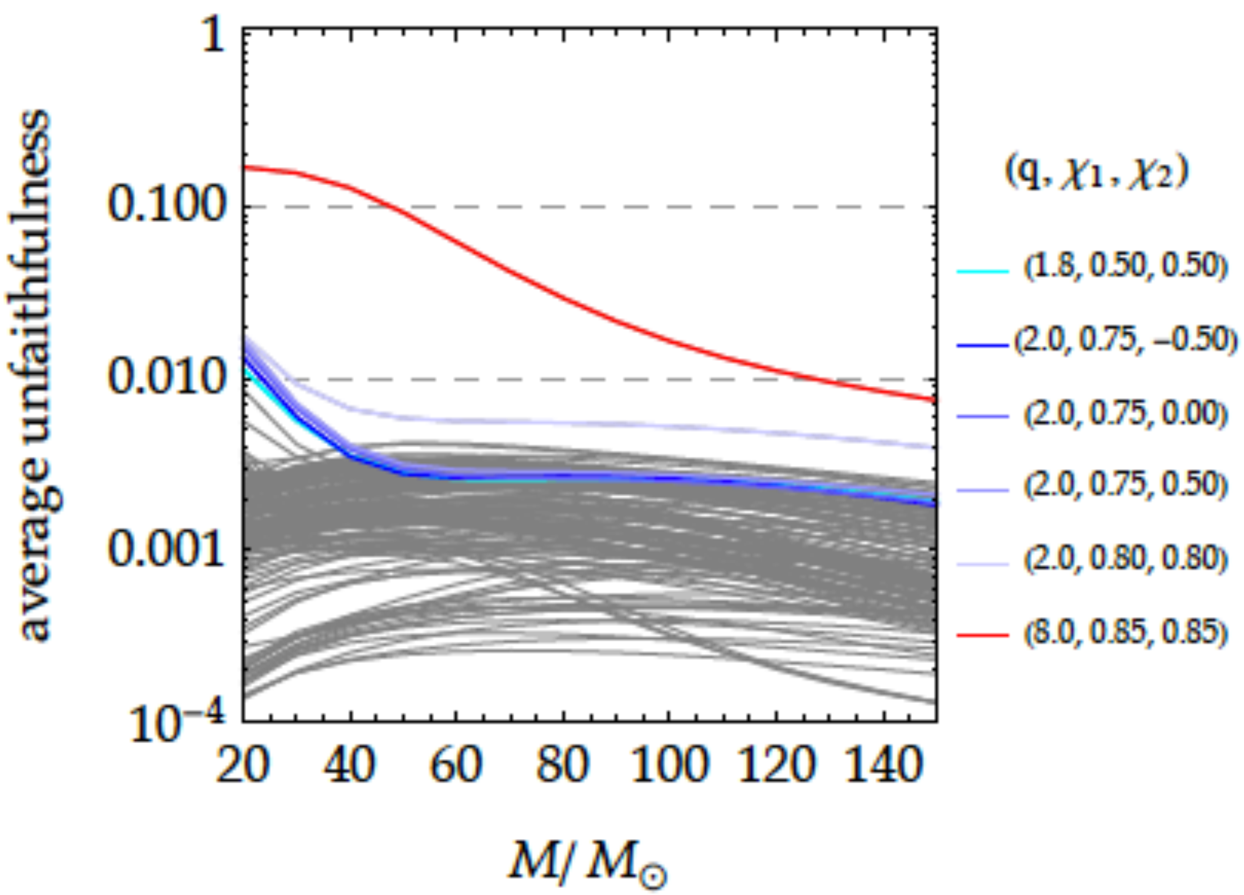


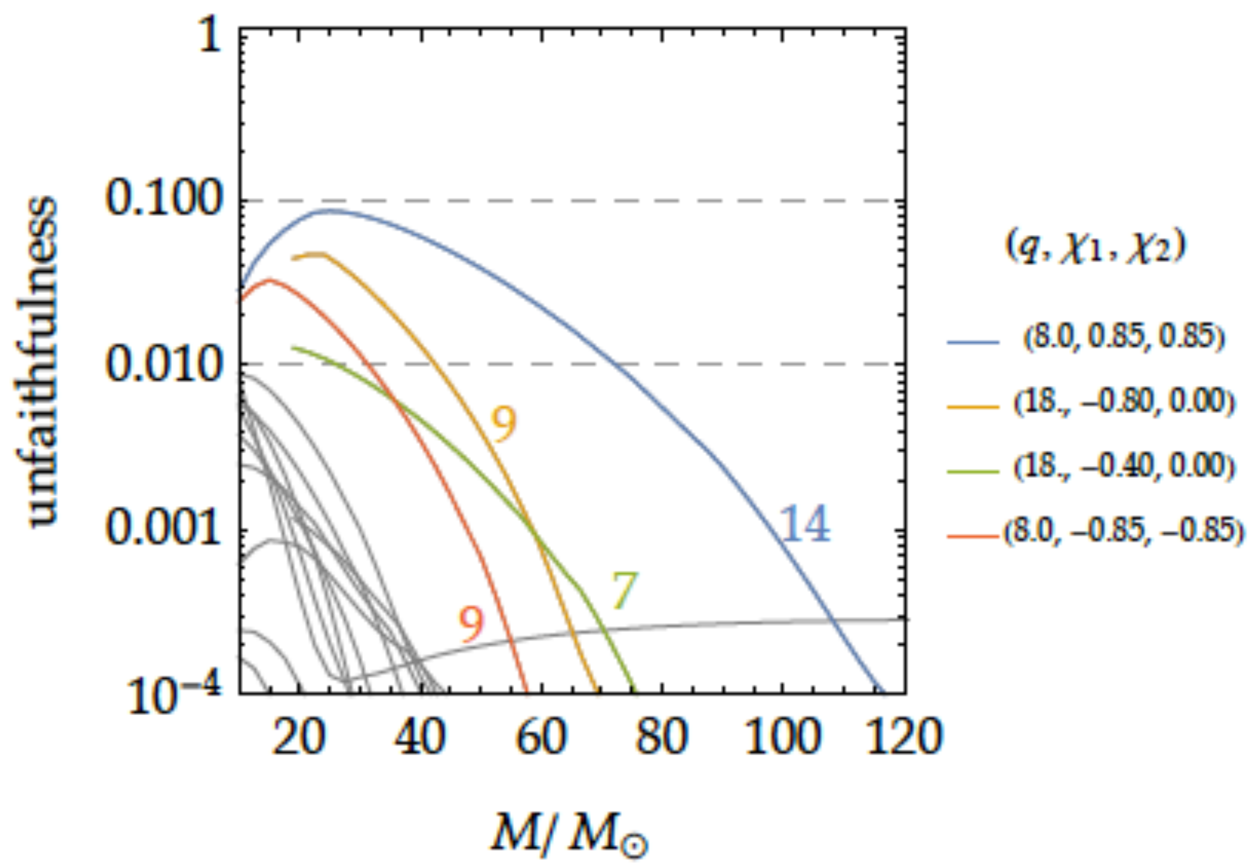
[Harry+2009, Brown+2012, Harry+2014, Ajith+2014, Privitera+2014, Capano+2016]

Extrapolation to low frequencies

are NR waveforms long enough to constrain down to 25Hz for moderate M?

are hybrids reliable?

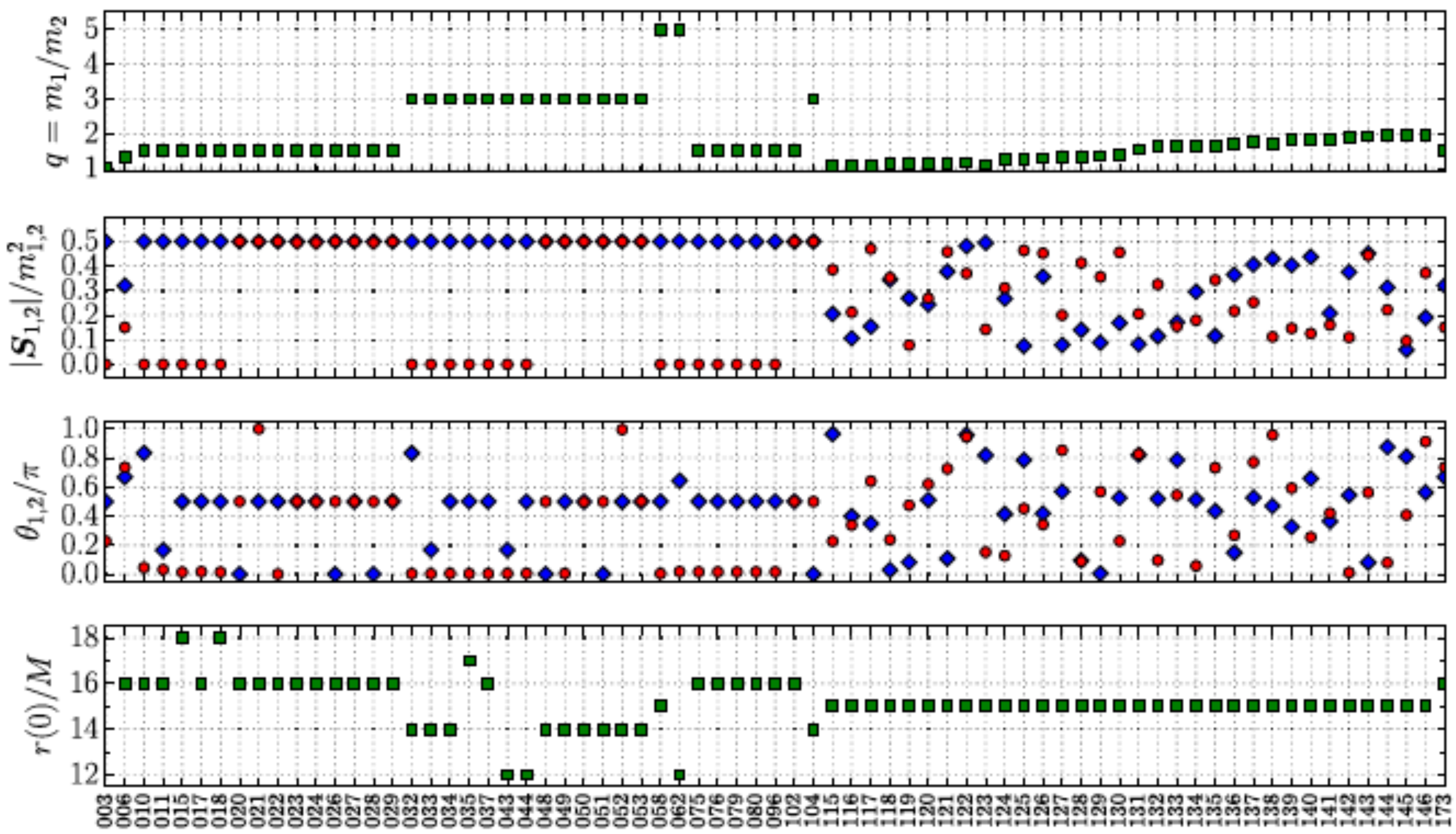




[Bohe,Shao,AT+16]

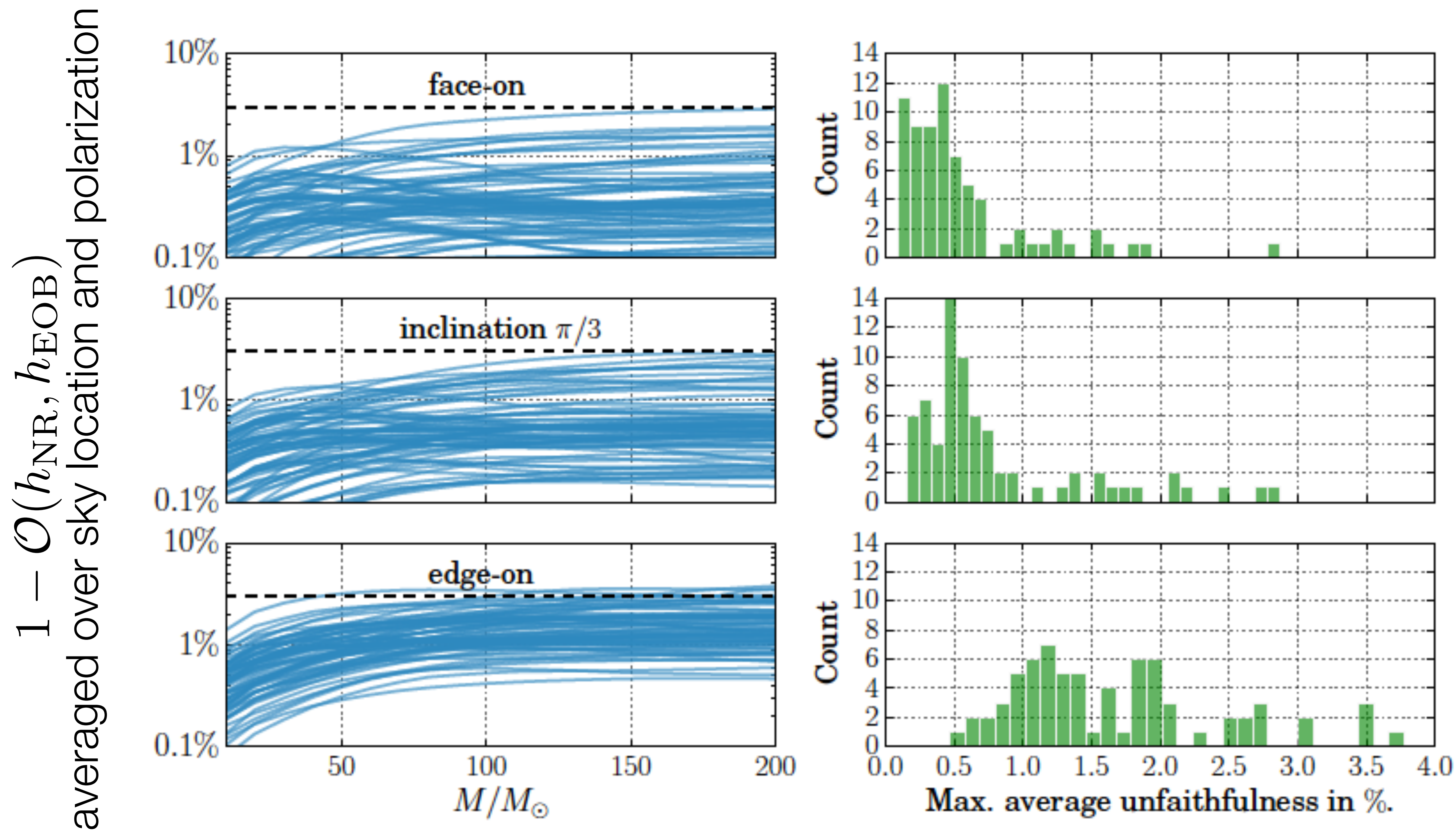
Effective-one-body model for precessing BBHs (ℓ=2)

- 70 NR waveforms from SXS public catalog used to test model



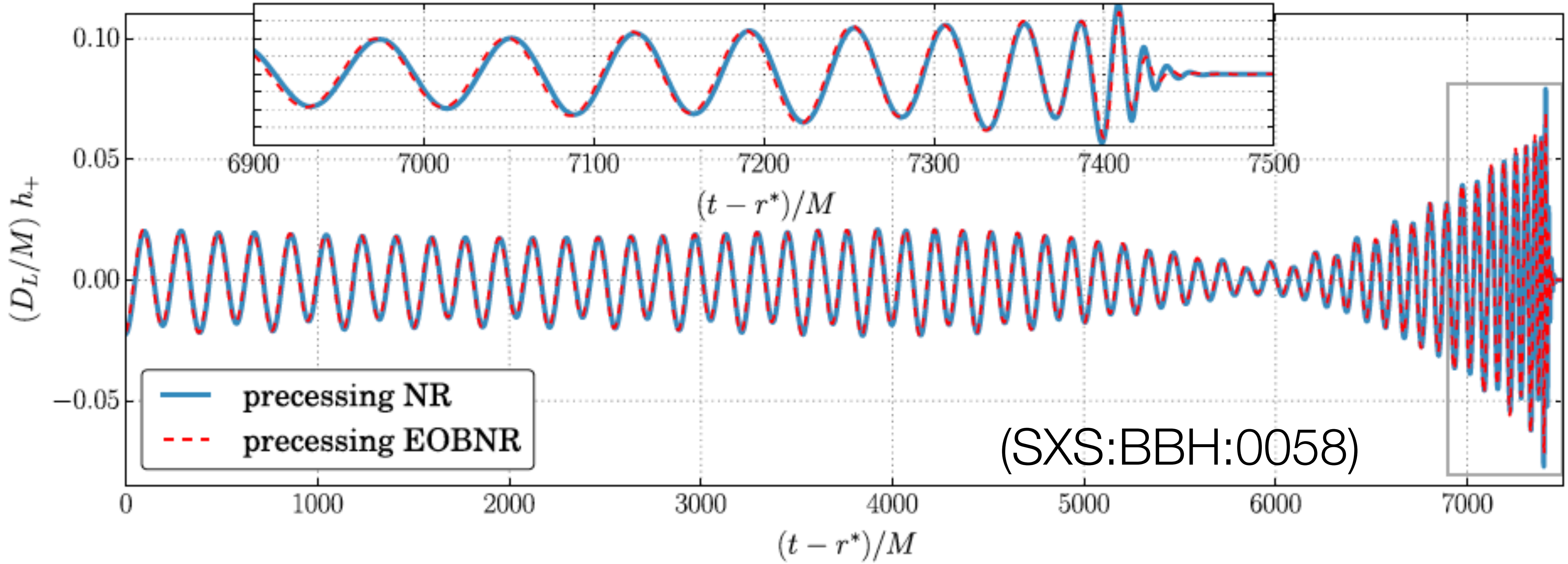
[Babak, AT+16]

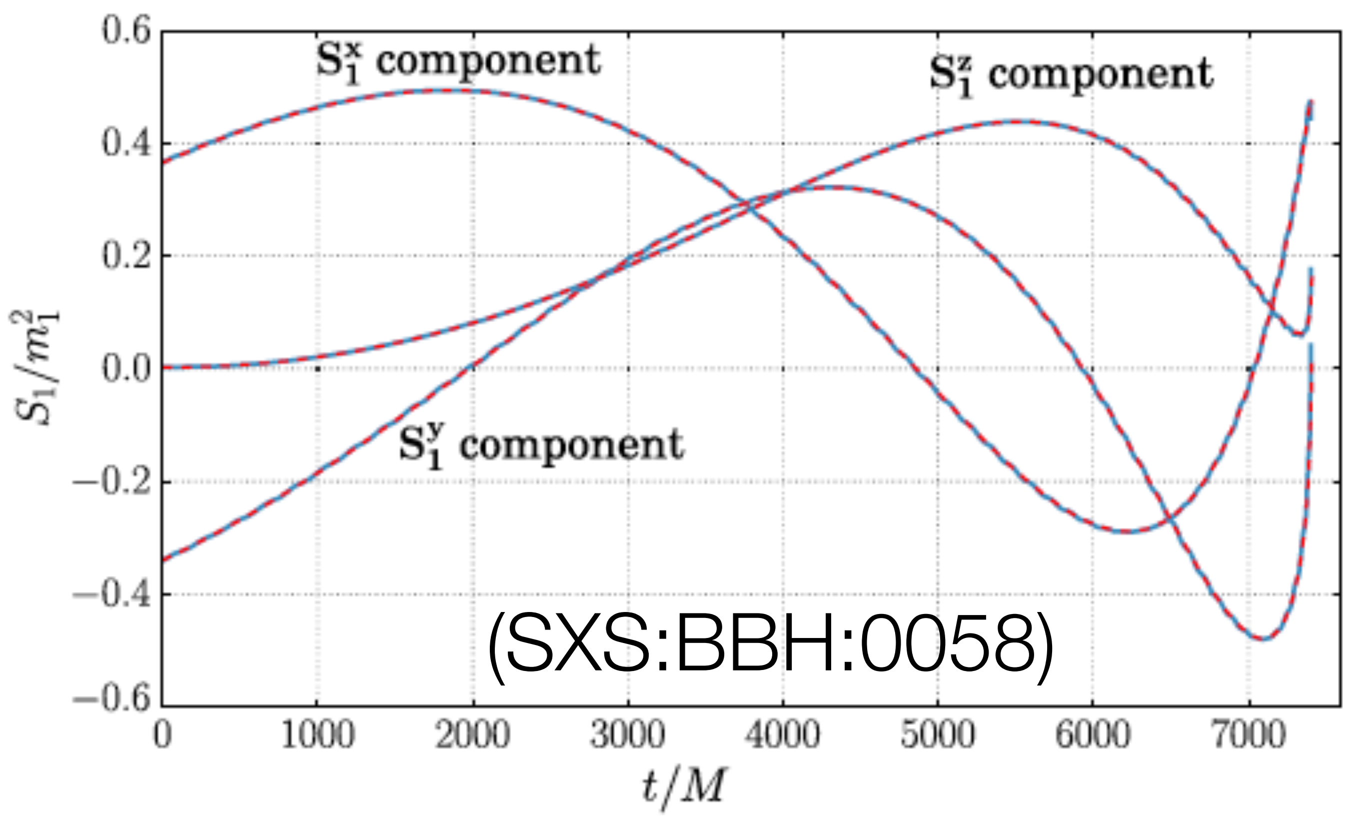
Effective-one-body model for precessing BBHs (ℓ=2)



[Babak, AT+16]

Effective-one-body model for precessing BBHs (ℓ=2)



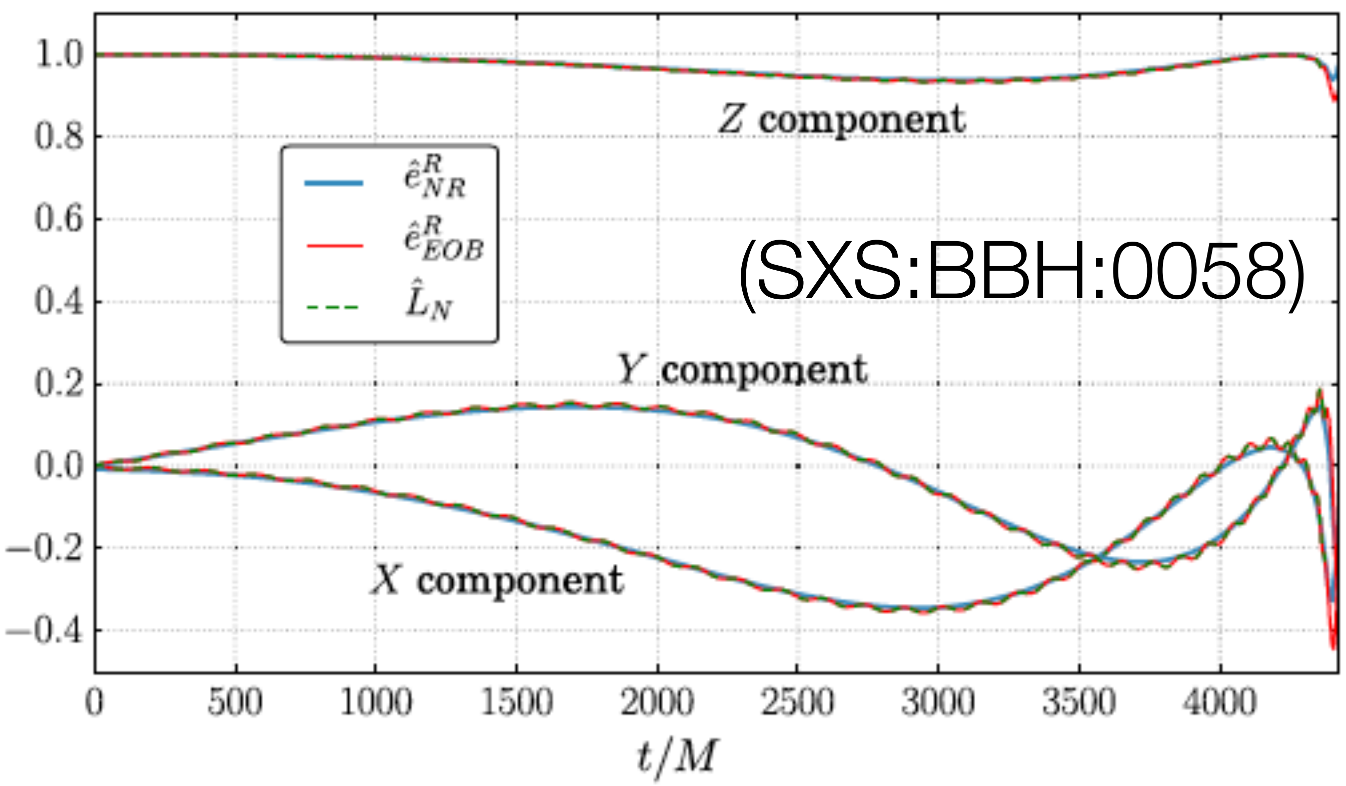


q=5, a1=0.5, a2=0
S1 in-plane at t=0

[Babak, AT+16]

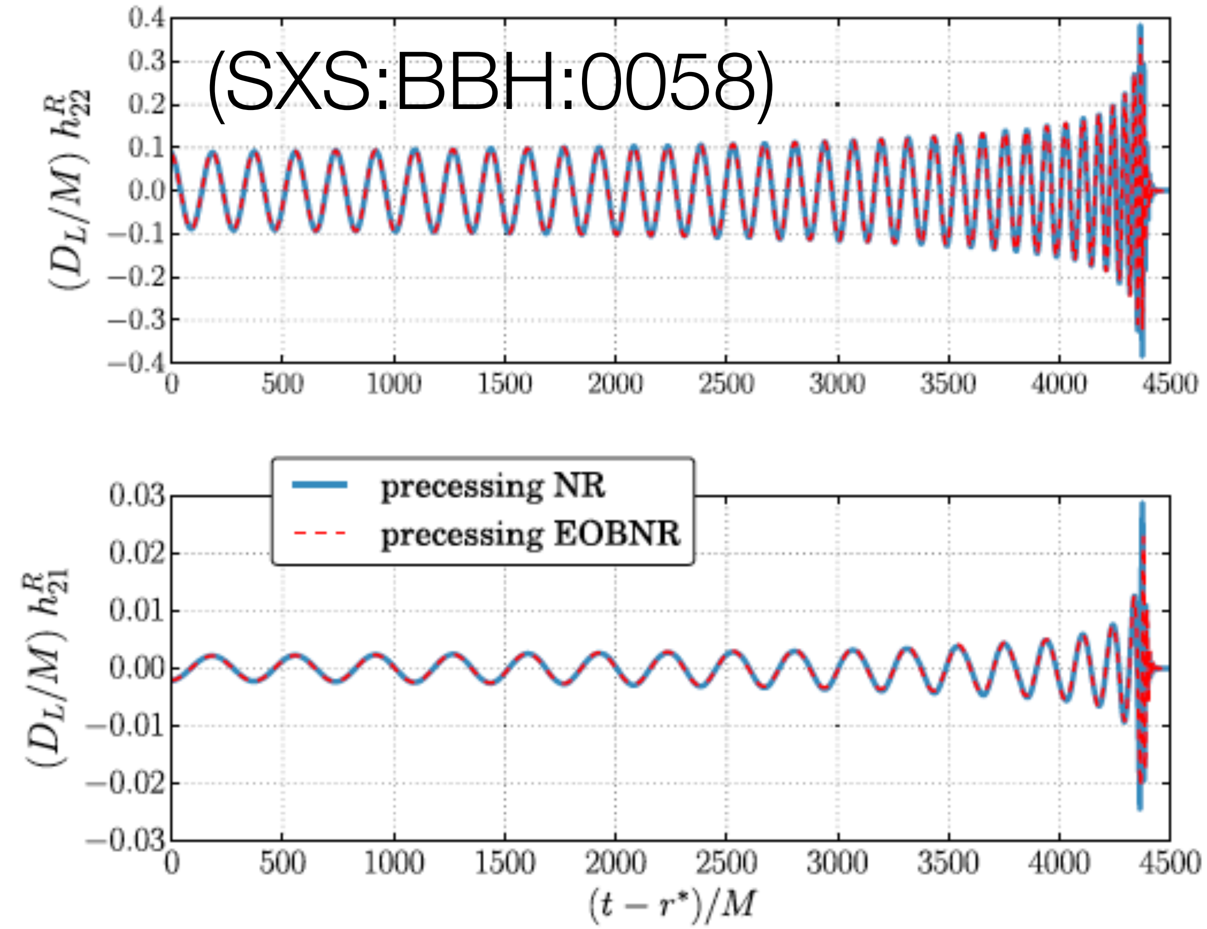
Effective-one-body model for precessing BBHs (ℓ=2)

testing the **rotation**
via maximum-radiation
direction



motion of EOB angular momentum
closely tracks
NR direction of max radiation

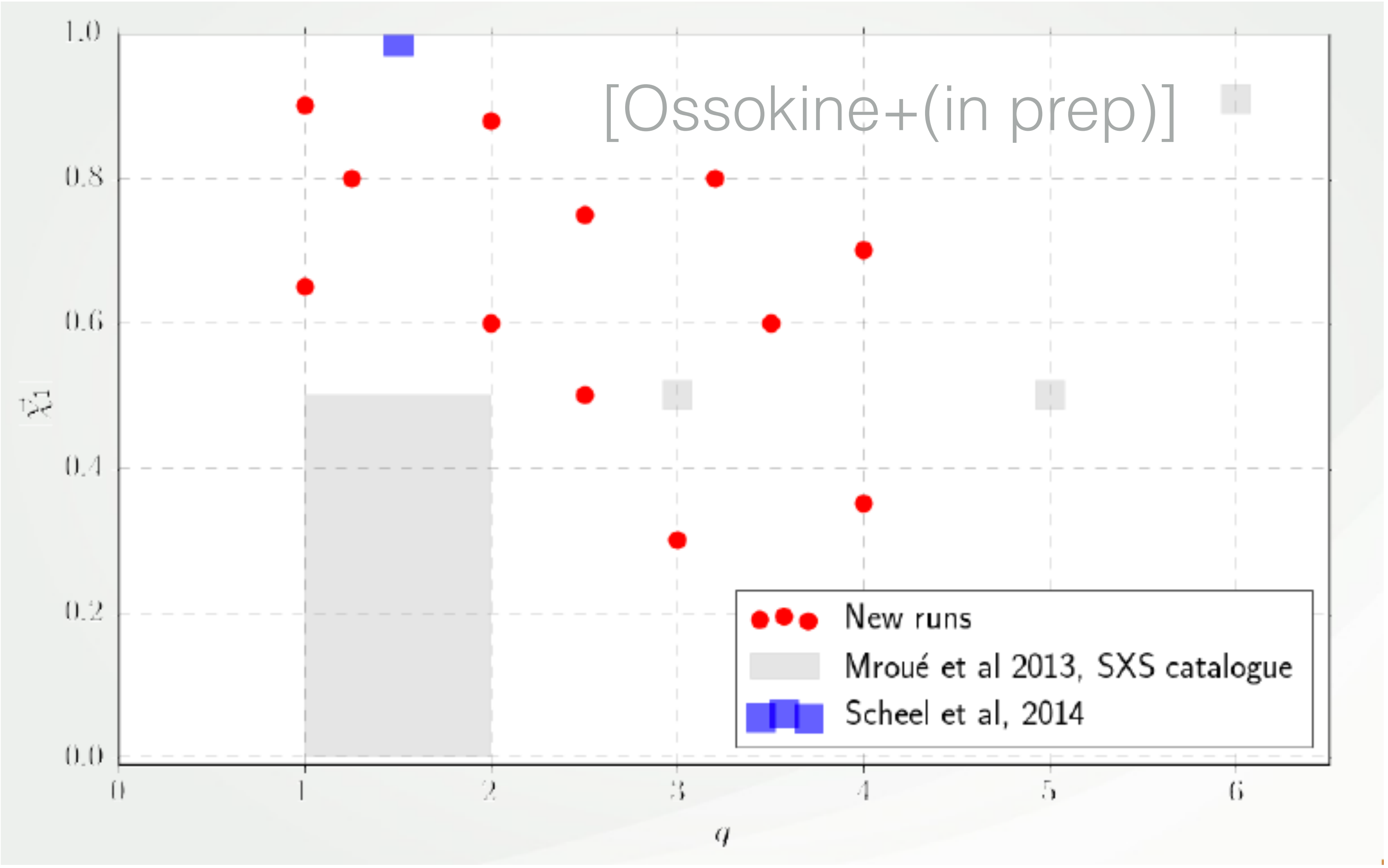
testing the waveforms
in the **precessing frame**



(2,2) good, (2,1)
to improve, especially RD

Effective-one-body model for precessing BBHs

- New SXS NR waveforms [Ossokine+(in prep)] used to
  1. improve model [AEI(in prep)]
  2. assess PE systematics [AEI(in prep)]



Phenomenological model of precessing BBHs (ℓ=2)

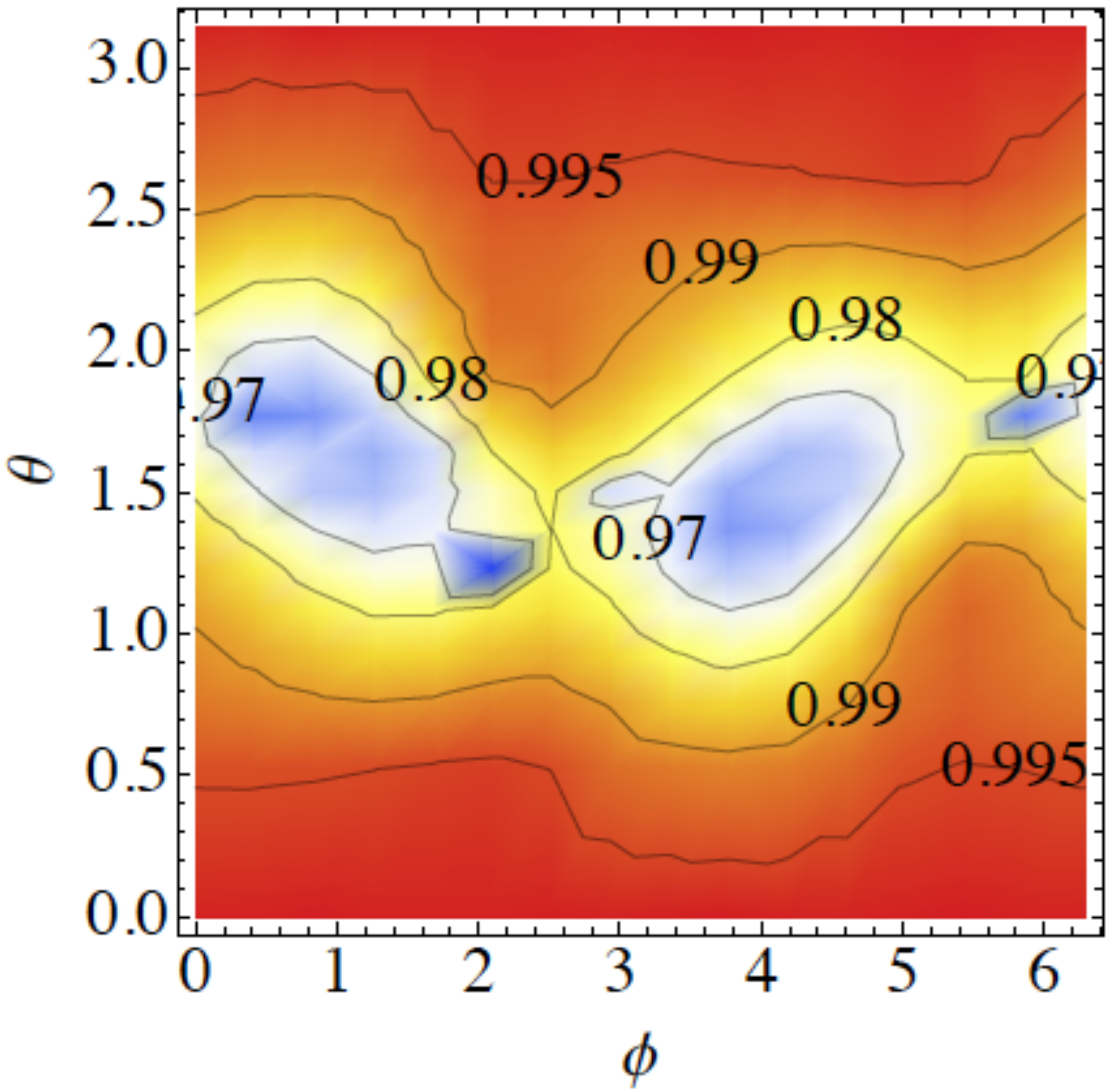
- Start from PN and find **single effective spin** (+ phase) that dominates precessional effects [Schmidt+14]

  1. Closed-form frequency domain formulas for precession of angular momentum

  2. Rotate nonprecessing PhenomD directly in frequency domain

- **IMRPhenomPv2**: comparisons to many NR runs during LIGO software review

PN+NR q=3, a1=0.75 in-plane



[Hannam+13]

Differences between precessing IMR models

precessing Phenom

- Dof: S1z, S2z, chip, phase
- Purely nonprecessing model in the precessing frame
- Ringdown built in the precessing frame
- In the precessing frame only (2,2) mode included
- SPA for modes rotation
- Euler angles for modes rotation derived in analytic form under approximations
- Initial in-plane spin components enter final-spin formula

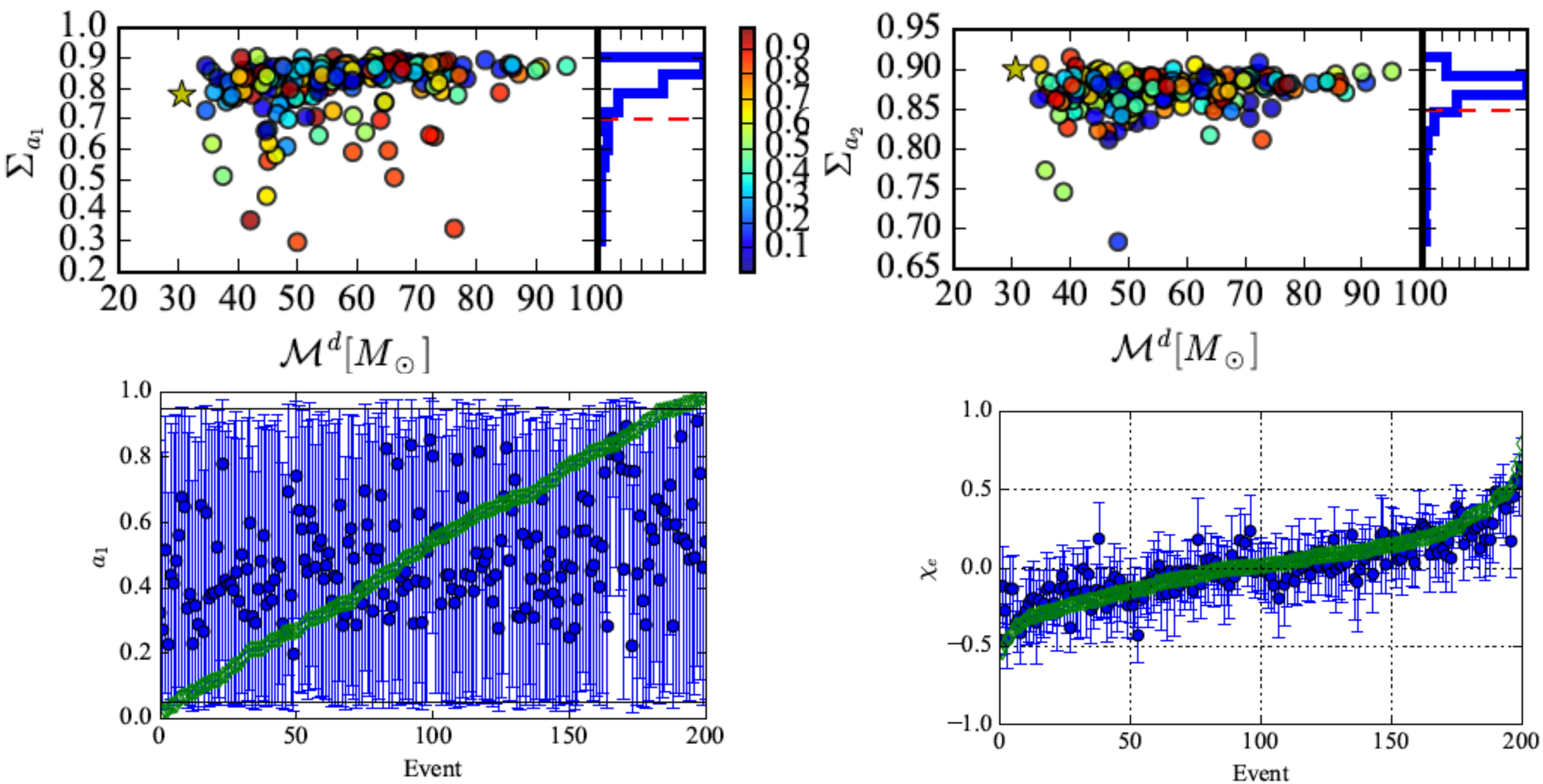
precessing EOBNR

- Dof: S1x, S1y, S1z, S2x, S2y, S2z
- Fully precessing conservative orbital dynamics
- Ringdown built in final-spin frame
- In the precessing frame uncalibrated (2,1) mode included
- Exact time-domain modes rotation
- Euler angles for modes rotation from motion of LN
- Spin-aligned formula for remnant spin evaluated at merger

Expected uncertainties for heavy BBHs [Vitale+16]

- 200 precessing BBHs w/ m1,m2 uniform in [30,50]MSun, a1,a2 uniform in [0,0.98], isotropic sky location, uniform inclination, uniform in comoving volume, threshold network SNR=12

- Model: IMRPhenomPv2. Detectors: HLV at design sensitivity



Expected uncertainties for heavy BBHs [Vitale+16]

- a1<0.2: can rule out ~maximal a1 90% of the times
- a1>0.8: can rule out ~zero a1 75% of the times
- chieff better measured (90% C.I. of typical width ~0.35)
- Aligned-spins yield smaller uncertainties (90% C.I. of width ~0.2 on a1)
- For unequal-mass BBHs: the more edge-on, the easier the measurement of a1. For equal-mass BBHs: no dependence on inclination
- Tilts are poorly measured
- **Uncertainties of GW150914 are typical of similar BBHs**

Unmodeled precessional effects

- **Precessional effects** not fully modeled
  1. mode asymmetry in precessing frame [O'Shaughnessy+13, Pekowsky +14, Boyle+14]
  2. radiation axis keeps precessing during ringdown [O'Shaughnessy+13]
  3. no calibration to precessing NR ever done