COMPUTING INSPIRALS AND WAVEFORMS USING THE SELF-FORCE

Niels Warburton

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-What do we need to include in our models?

-Review methods and results

-Ongoing/recent work

-Future directions

Modeling goals

Waveform templates need to be accurate across the parameter space

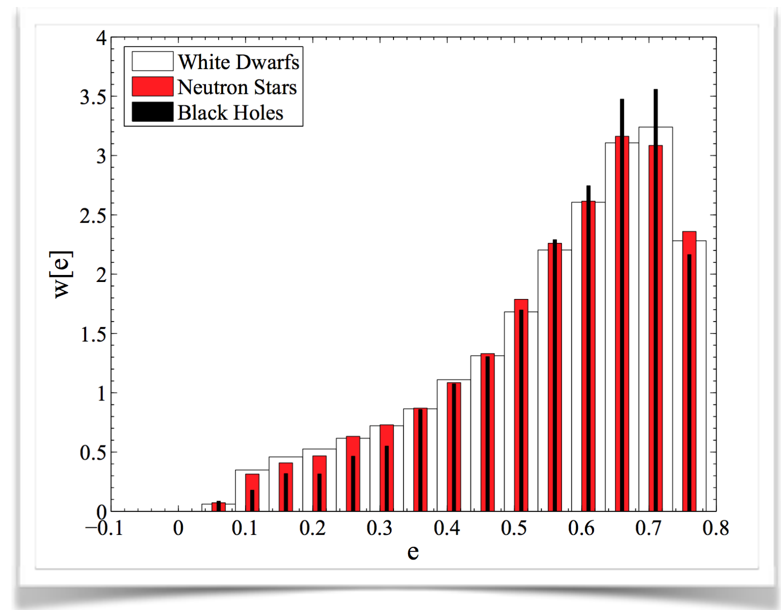
and generated rapidly

-Template must track waveform phase to better than 1 radian over 10s to 100s of thousands of cycles

-Need to cover 14 dimensional parameter space, so each template must be generated in a few seconds

-Primary and secondary spinning

-Motion of secondary can be highly eccentric and inclined



Hopman & Alexander ApJ 629 (2005) 362-372

Small mass-ratio, q, suggests modeling using black hole perturbation theory

Which forces do we need to include in our models and to what accuracy?

Equation of motion $\mu u^\beta \nabla_\beta u^\alpha = F^\alpha$

$$F^\alpha = \mu^2 \left(F^{(1)\alpha}_{\text{mono}} + \mu F^{(2)\alpha}_{\text{mono}} \right) + S \left(F^\alpha_{\text{spin-curvature}} + \mu F^{(1)\alpha}_{\text{dipole}} \right)$$

If the secondary is a Kerr black hole we can write

$$S = |s|\mu^2, \quad \text{where} \quad |s| \leq 1$$

The self-force can be split into orbit averaged quantities (fluxes) and pieces that oscillate on the orbital timescale

$$\begin{aligned} F^\alpha =& \mu^2 \left(\langle F^{(1)\alpha} \rangle + F^{(1)\alpha}_{\text{cons}} + F^{(1)\alpha}_{\text{diss}} + \mu \langle F^{(2)\alpha} \rangle \right) \\ &+ \mu^2 s \left(F^\alpha_{\text{spin-curvature}} + \mu \langle F^{(1)\alpha}_{\text{dipole}} \rangle \right) \end{aligned}$$

Which forces do we need to include in our models and to what accuracy?

Fluxes

Subleading fluxes and oscillatory forces

$$F^\alpha = \boxed{\mu^2 \langle F^{(1)\alpha} \rangle} \; \boxed{\begin{aligned} &+\mu^3 \left(\langle F^{(2)\alpha} \rangle + s \langle F^{(1)\alpha}_{\text{dipole}} \rangle \right) \\ &+\mu^2 \left(\boxed{F^{(1)\alpha}_{\text{cons}} + F^{(1)\alpha}_{\text{diss}}} + s F^\alpha_{\text{spin}} \right) \end{aligned}}$$

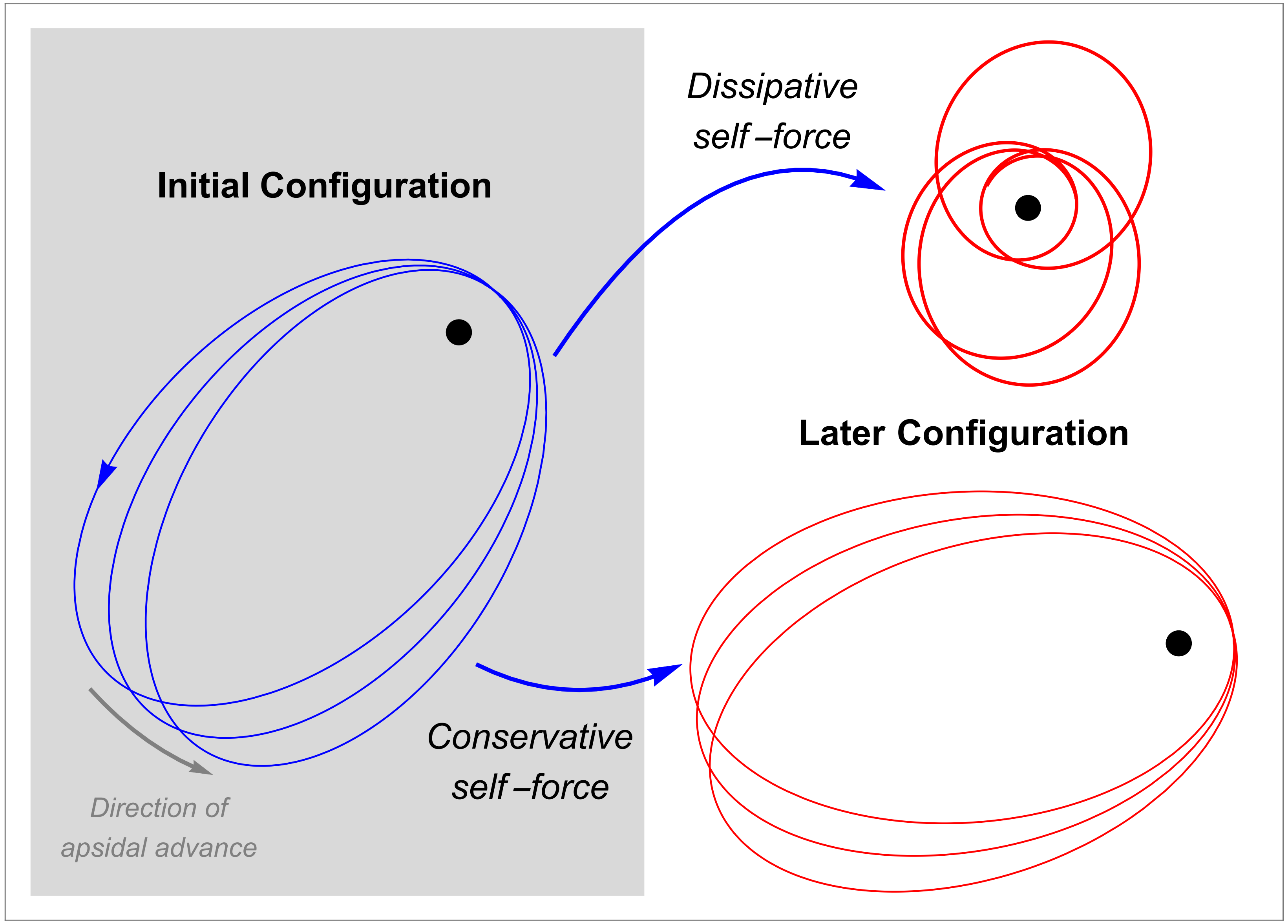
Kerr radial-polar resonance

Contribution to inspiral phase: $\mathcal{O}(q^{-1})$ $\quad$ $\mathcal{O}(q^{-1/2})$ $\quad$ $\mathcal{O}(1)$

Accuracy required in force, q=10^{-6}*:* 10^{-8} $\quad$ 10^{-5} $\quad$ 10^{-2}

How the different forces influence the inspiral phase can be made precise from a two-timescale analysis, e.g., Hinderer and Flanagan (2008). See also talk by Moxon.

How do these forces influence an inspiral?



Dissipative and conservative self-forces influence the inspiral differently

Which forces have been calculated?

		Schwarzschild	Kerr
$\langle F^{(1)\alpha} \rangle$	*1st order fluxes*	✓ *Tanaka et al.* *Cutler, Kennefick, Poisson*	✓ *Drasco and Hughes* *Fujita, Hikida, Tagoshi*
$F^{(1)\alpha}_{\mathrm{cons}} + F^{(1)\alpha}_{\mathrm{diss}}$	*1st order oscillatory*	✓ *Barack and Sago* *Akcay, NW, Barack* *Osburn et al.* *Merlin and Shah (circ)*	✓ *Dolan and Barack (circ)* *van de Meent (generic)*
$\langle F^{(2)\alpha} \rangle$	*2nd order fluxes*	✗ *but see talks by* *Pound, Wardell,* *Thompson, Yamada*	✗
$F^{\alpha}_{\mathrm{spin}}$	*spin-curvature force (conservative)*	✓ *Susuki and Maeda*	✓ *Hartl* *Ruangsri, Vigeland, Hughes*
$\langle F^{(1)\alpha}_{\mathrm{dipole}} \rangle$	*additional flux associated with spin of secondary*	✓ *Harms et al. (circ, aligned)*	✓ *Harms et al. (circ, aligned)*

A great deal of theoretical and preparatory numerical work underlies these calculations. Many of these effects have also been calculated in pN theory.

Given the various forces we can now compute, inspirals can be calculated via a number of different methods:

-Flux balance inspirals

-Kludge inspirals

-Geodesic self-force inspirals

-Self-consistent inspirals

Every method, except the final one, comes in two steps: computing the inspiral trajectory and then computing the associated waveform

Waveform generation

Snapshot waveform (Teukolsky)

$$\Psi_4(r \to \infty) \simeq \frac{1}{2}\left(\ddot{h}_+ - i\ddot{h}_\times\right)$$

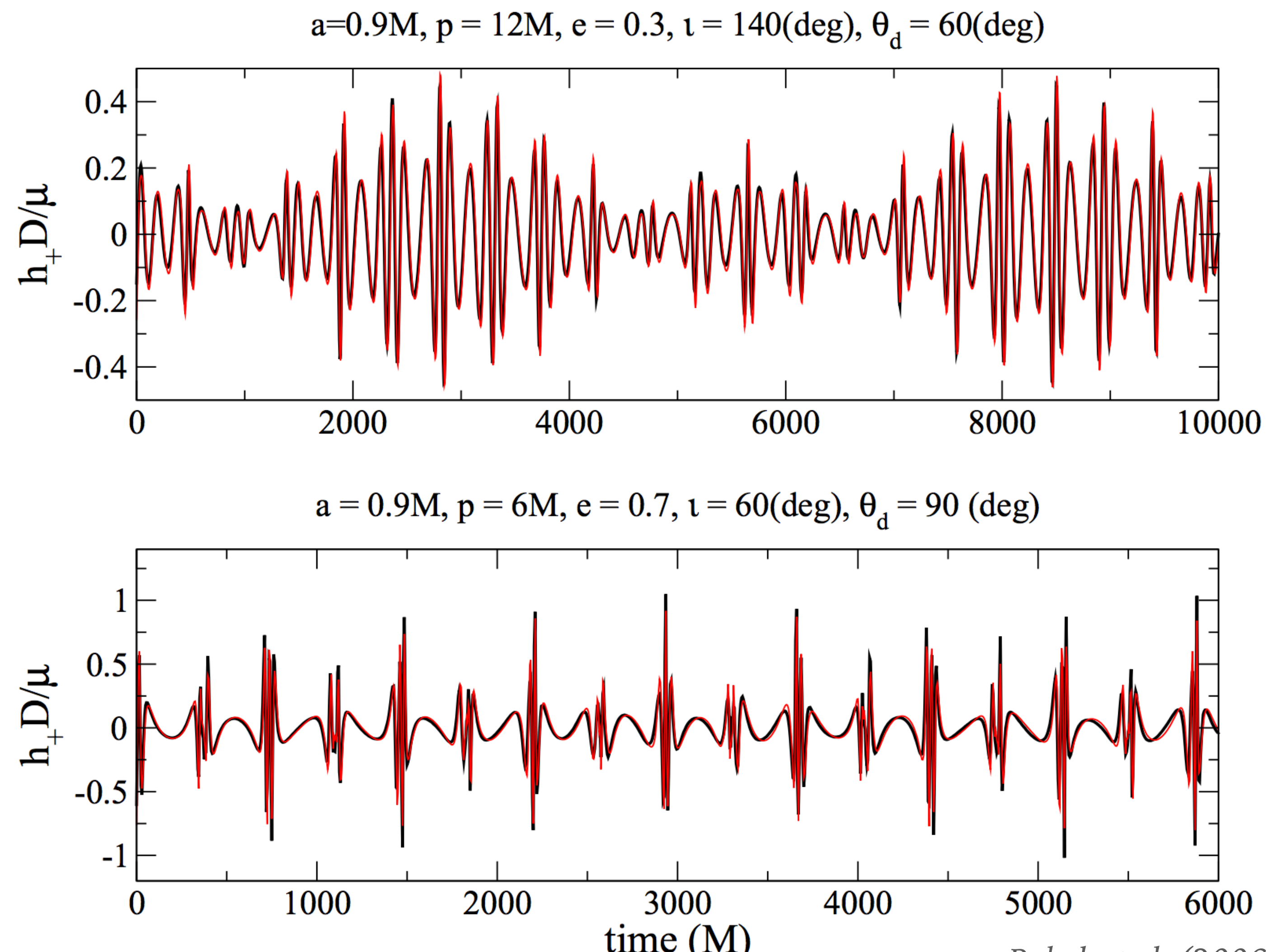
Can be computed accurately from frequency-domain codes

Quadrupole-octupole kludge

- map Boyer-Lindquist coordinates to flat space and use quadrupole and octupole formula
- Works surprising well, down as far as rmin >= 5M

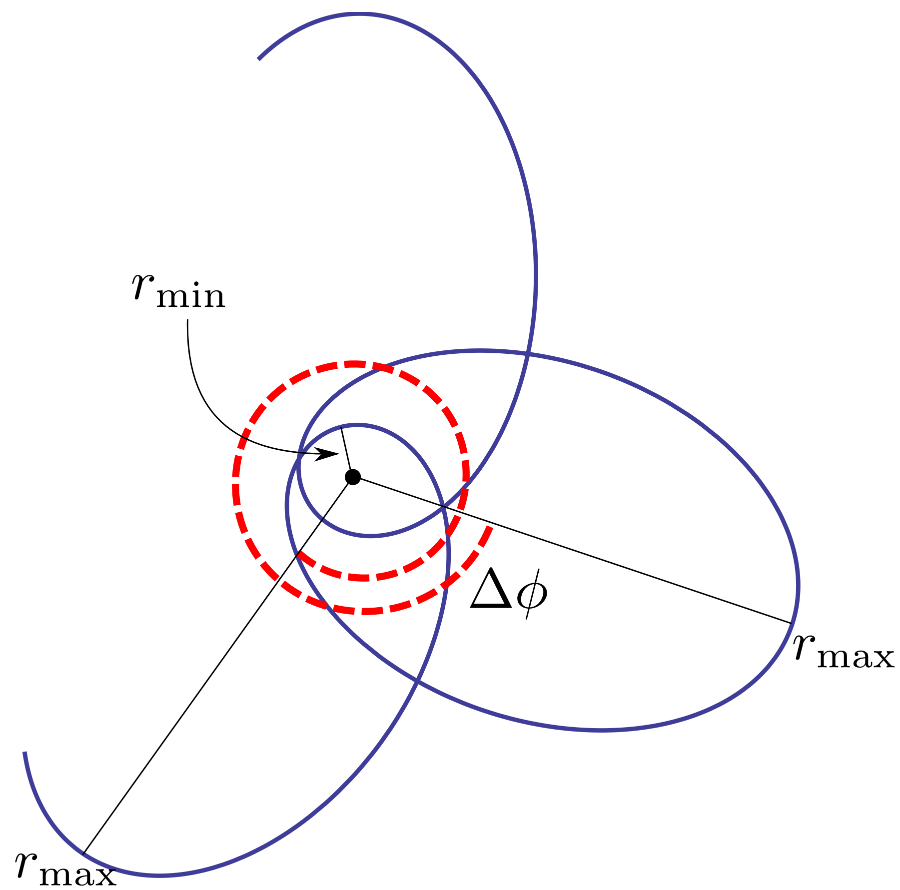
Time-domain simulations

Use trajectory as source for TD code. Slow to compute but important validation



Babak et al. (2006)

Description of geodesic



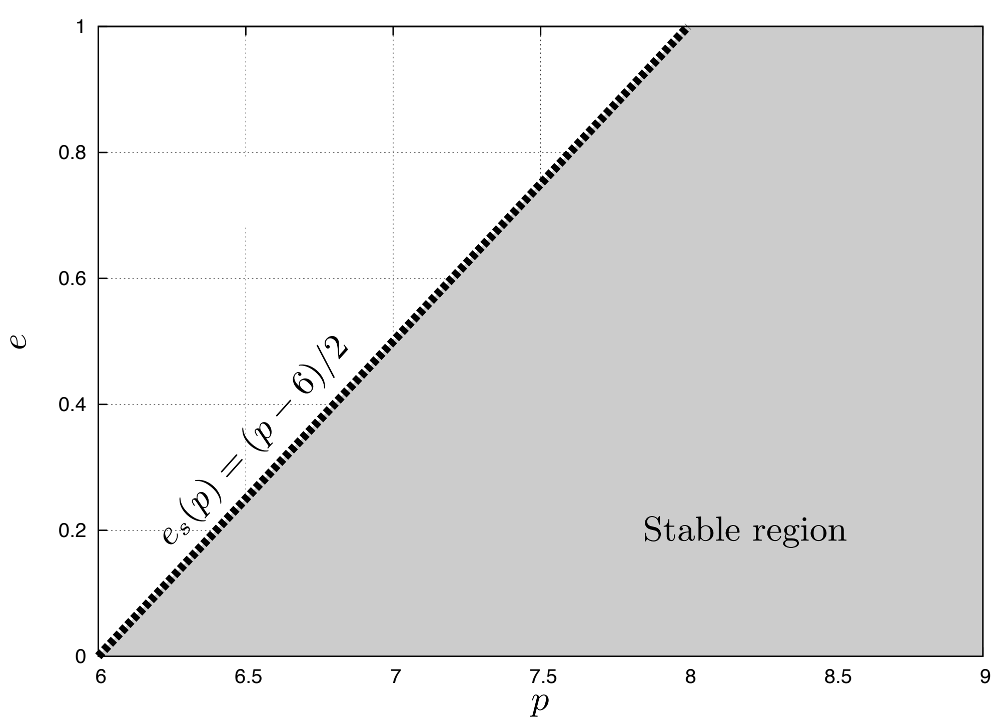
Up to orientation, bound geodesic orbits in Schw. spacetime are uniquely specified by $\mathcal{E}$ *and* $\mathcal{L}$

$$p \equiv \frac{2 r_{\rm max} r_{\rm min}}{M(r_{\rm max} + r_{\rm min})} \qquad e \equiv \frac{r_{\rm max} - r_{\rm min}}{r_{\rm max} + r_{\rm min}}$$

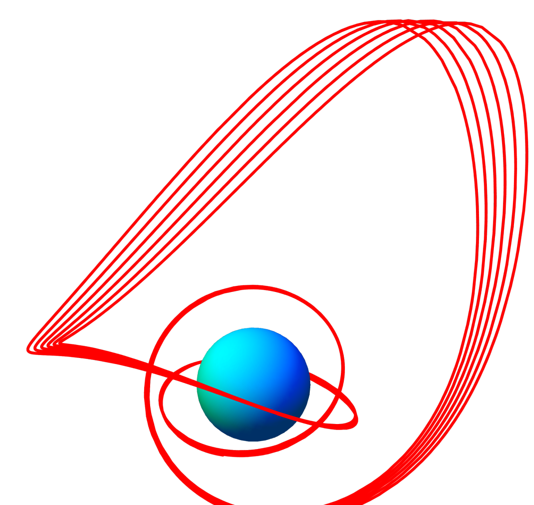
We also introduce the relativistic anomaly parameter, χ*, such that*

$$r(t) = \frac{pM}{1 + e\cos[\chi(t) - \chi_0]}$$

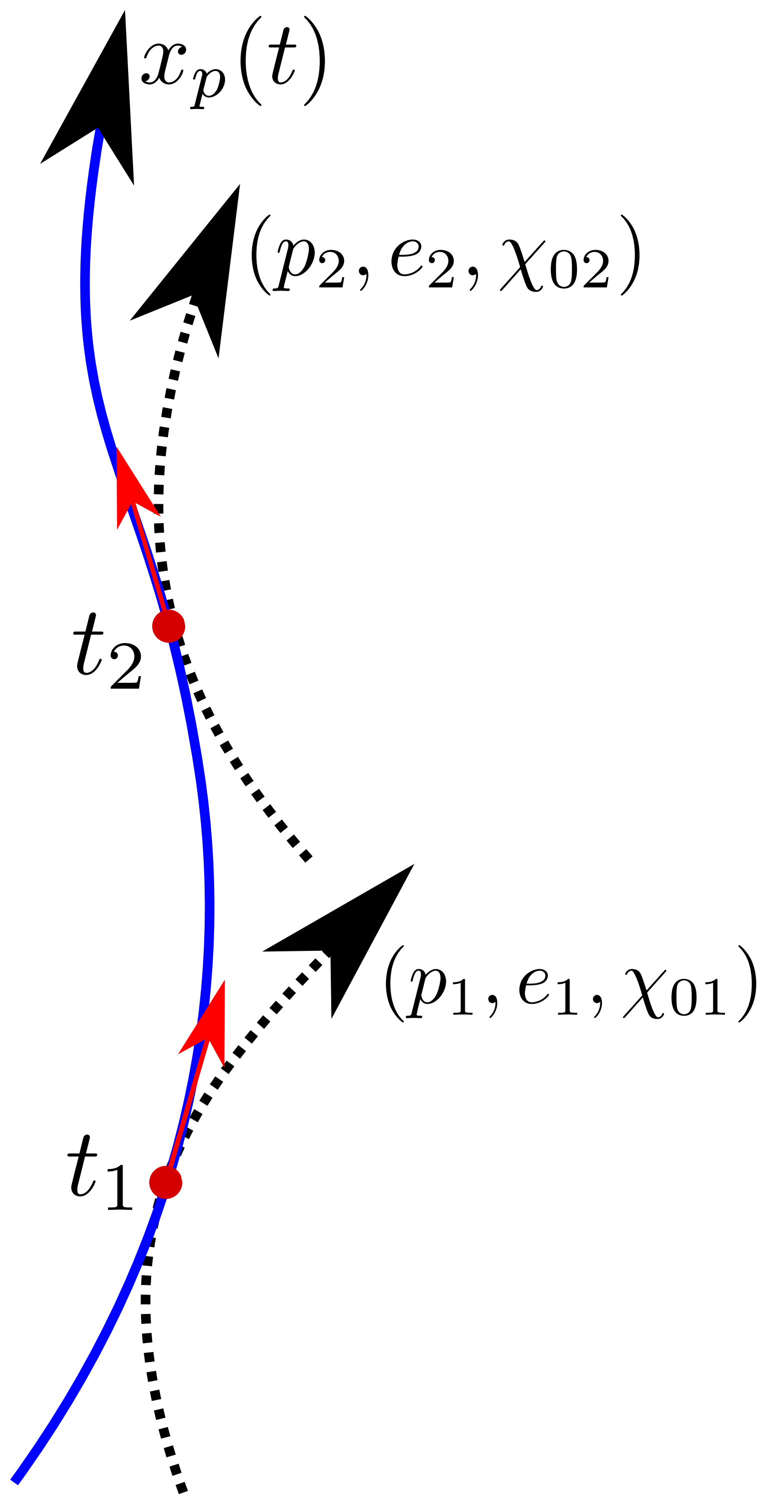
where χ_0 *is the periastron phase*



Extension to Kerr adds the Carter constant, $\mathcal{Q}$*, or alternatively an inclination angle* $\theta_{\rm min}$



Description of inspiral trajectory



Equation of motion $\mu u^\beta \nabla_\beta u^\alpha = F^\alpha$

Trajectory described by $x_p^\alpha(t) \quad u^\alpha(t)$

At each time, particle has a position and velocity which uniquely matches a geodesic

$$\{p, e, \chi_0\} \to \{p(t), e(t), \chi_0(t)\}$$

Relativistic osculating elements
Schwarz: Pound & Poisson (2007)
Kerr: Gair et al. (2010)

$$\dot{p} = \mathcal{F}_p[p, e, \chi - \chi_0, F^{\text{diss}}_{\text{self}}(t)]$$
$$\dot{e} = \mathcal{F}_e[\cdots]$$
$$\dot{\chi_0} = \mathcal{F}_{\chi_0}[p, e, \chi - \chi_0, F^{\text{cons}}_{\text{self}}(t)]$$

No small force approximations made, just a recasting of the equation of motion

Flux balance inspirals

$$\frac{dr}{dt} = \left(\frac{d\mathcal{E}}{dr}\right)^{-1} \frac{d\mathcal{E}}{dt}$$
$$= \left(\frac{d\mathcal{E}}{dr}\right)^{-1} \dot{E}(r)$$

So long as the orbital evolution is adiabatic we can balance the change in the orbital energy with the radiated energy flux

Can be computed in nice formulations (Regge-Wheeler, Teukolsky)
No local calculation of the self-force necessary

Equatorial orbits can be computed in a similar fashion by balancing change in E and L with associated fluxes

Adiabaticity condition breaks down near the separatrix, e.g., analysis by Cutler, Kennefick and Poisson in Schwarz. showed

$$\mu/M \ll (p-6-2e)^2$$

Mino showed so long as the inspiral is adiabatic the change in the Carter constant can also be derived

Kludge inspirals

Conceived to meet the data analysis task. Speed was key aim and over the years the accuracy has been improving

Two main flavors, analytic kludges (AK) and numerical kludges (NK):

Analytic kludge

1. Small object moves along a Keplerian orbit
2. Amend motion to incorporate periastron and Lense-Thirring precession and radiation reaction
3. Waveform from quadrupole formula

Barack and Cutler (2003)

Numerical kludge

1. Calculate inspiral trajectory in (E,L,Q) space (using pN and Teuk. fluxes)
2. Numerically integrate the Kerr geodesic equations along the inspiral trajectory to obtain the Boyer-Lindquist coordinate of the inspiral
3. Waveform from quadrupole-octupole formula

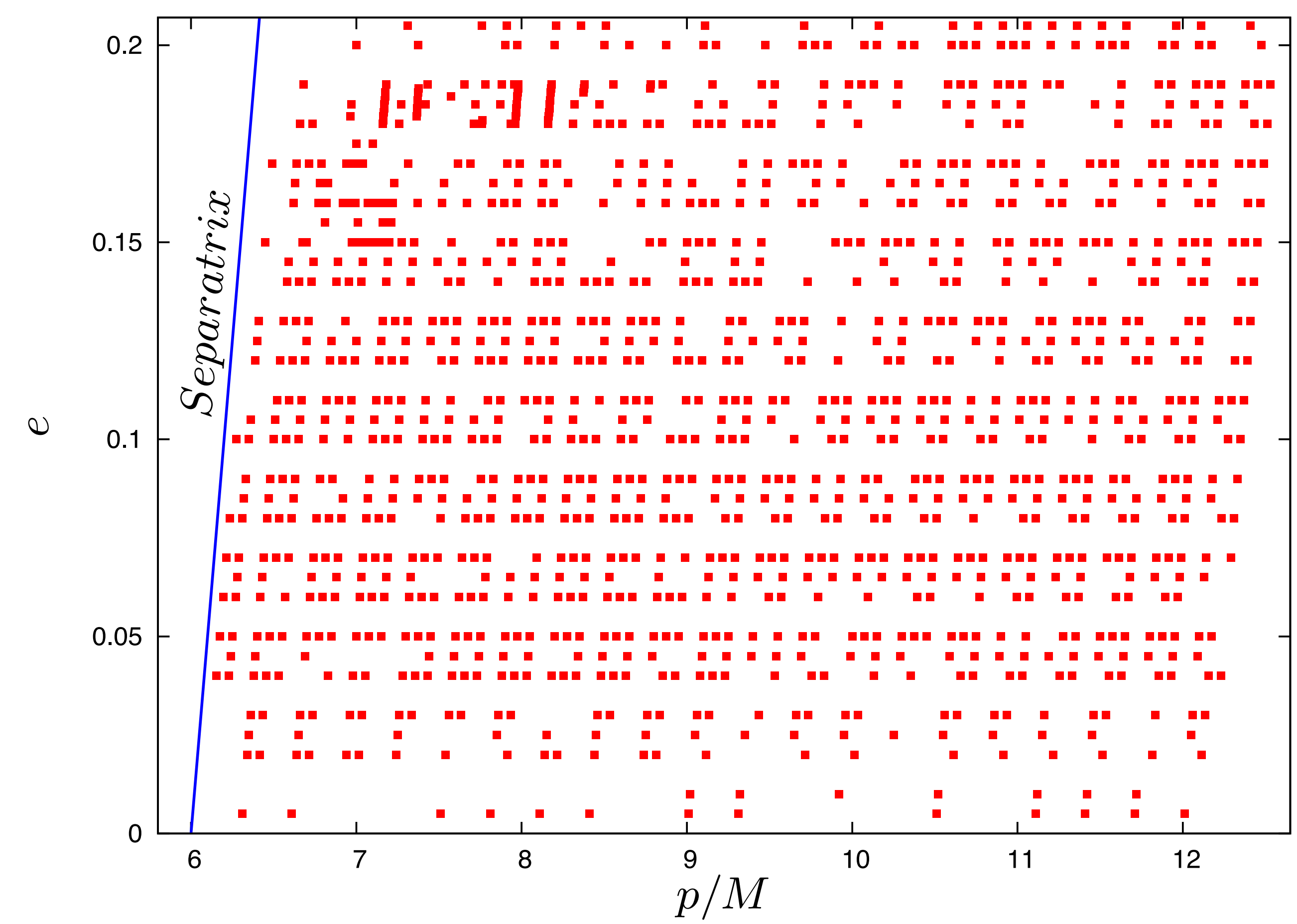
5-15 times slower than AK

Babak et al. (2006)

Geodesic self-force inspirals (2012)

To evolve equations of motion need self-force for arbitrary values of p and e

$$\dot{p} = \mathcal{F}_p[p, e, \chi - \chi_0, F^{\rm diss}_{\rm self}(t)]$$
$$\dot{e} = \mathcal{F}_e[\cdots]$$
$$\dot{\chi}_0 = \mathcal{F}_{\chi_0}[p, e, \chi - \chi_0, F^{\rm cons}_{\rm self}(t)]$$



NW, Akcay, Barack, Gair, Sago

In 2012 we used a global interpolation model expanding self-force in Fourier coefficients

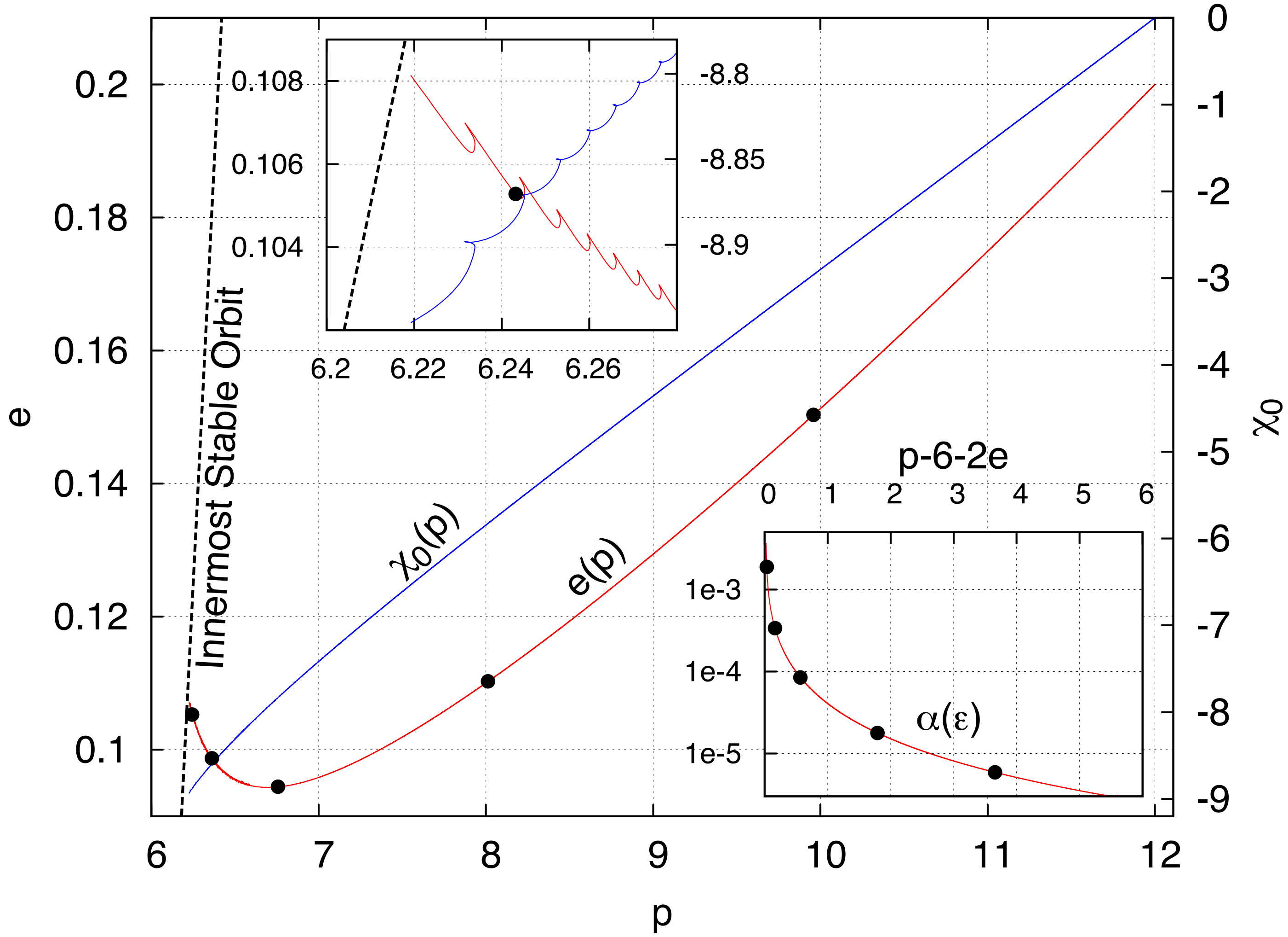
$$F^r_{\rm cons} = (\mu/M)^2 \sum_{n=0}^{\bar{n}} A_n(p, e)\cos(nv)$$

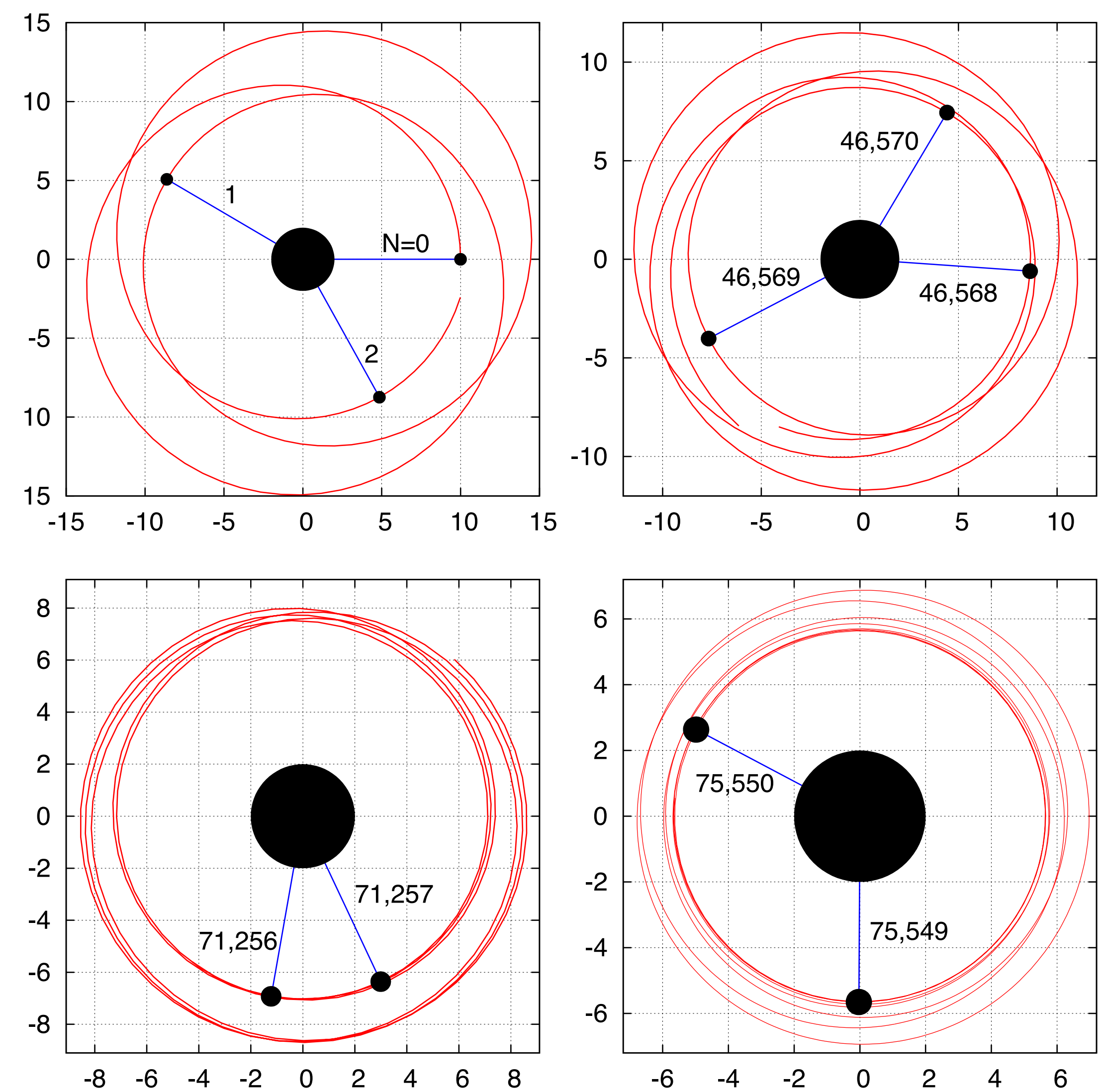
$$A_n(p, e) = p^{-2} \sum_{j=n}^{\bar{j}_a} \sum_{k=0}^{\bar{k}_a} a^n_{jk} e^j p^{-k}$$

Fit the model with over 1000 geodesic SF values computed using frequency-domain Lorenz-gauge code (Akcay, NW, Barack). Model was validated against time-domain results from Barack and Sago

$$\delta F \equiv \frac{F(\text{model}) - F(\text{data})}{F(\text{data})} < 10^{-3}$$

Geodesic self-force inspirals (2012 results)





mass ratio 10^{-5} example (NW et al.)

Oscillations in (p,e) on the orbital timescale

Computed inspiral for a particular setup, found chi0 subtracted (self-force acts against GR periastron advance) ~9 radians over inspiral

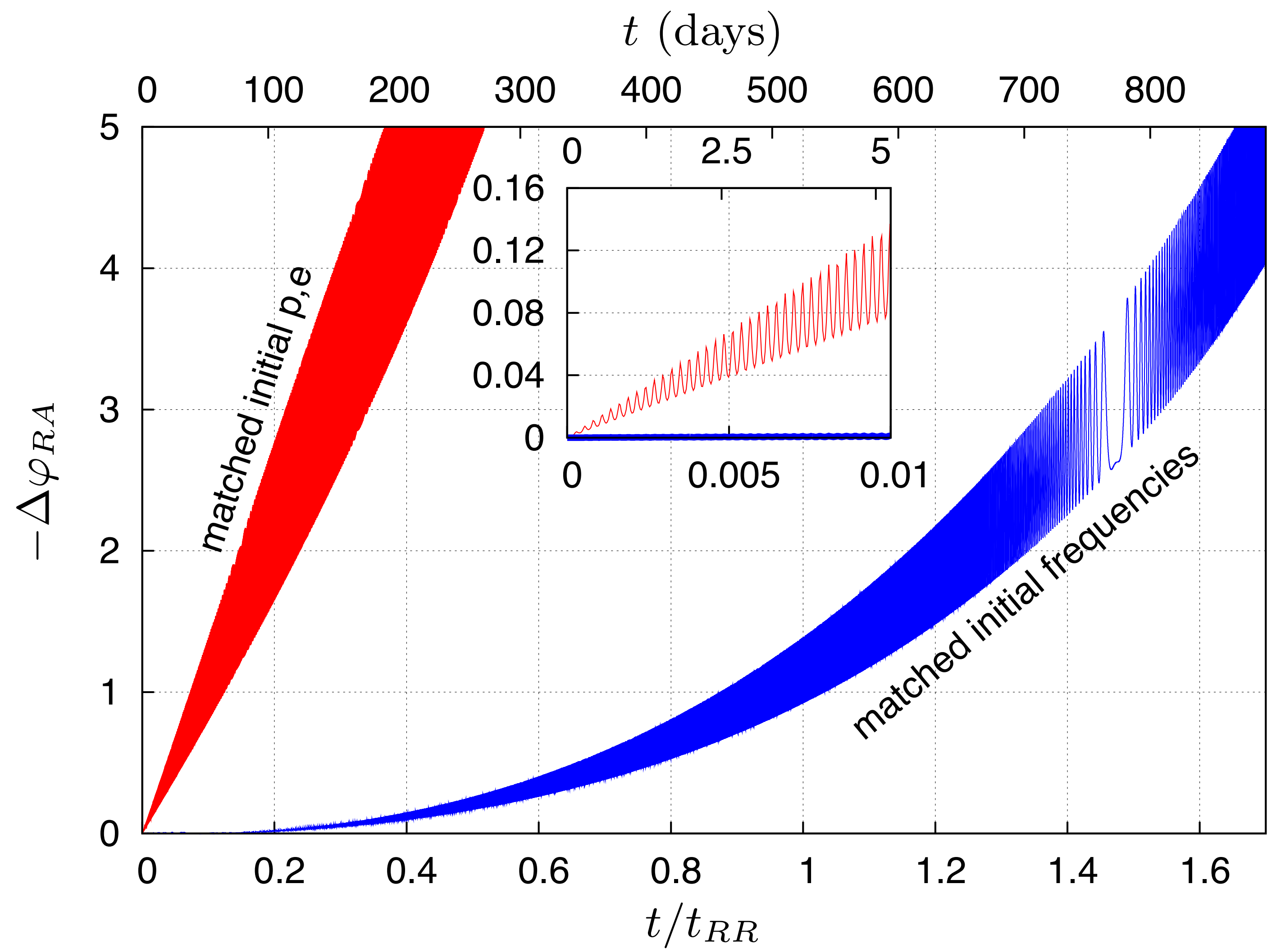
How to compare two inspirals?

Interesting to compare flux balance inspirals with inspirals that include the conservative self-force

How quickly does the flux balance inspiral dephrase from the more accurate self-force inspiral?

Conservative self-force changes the orbital frequencies e.g., the rate of periastron advance changes

When comparing inspirals must match inspiral frequencies not initial (p,e) values

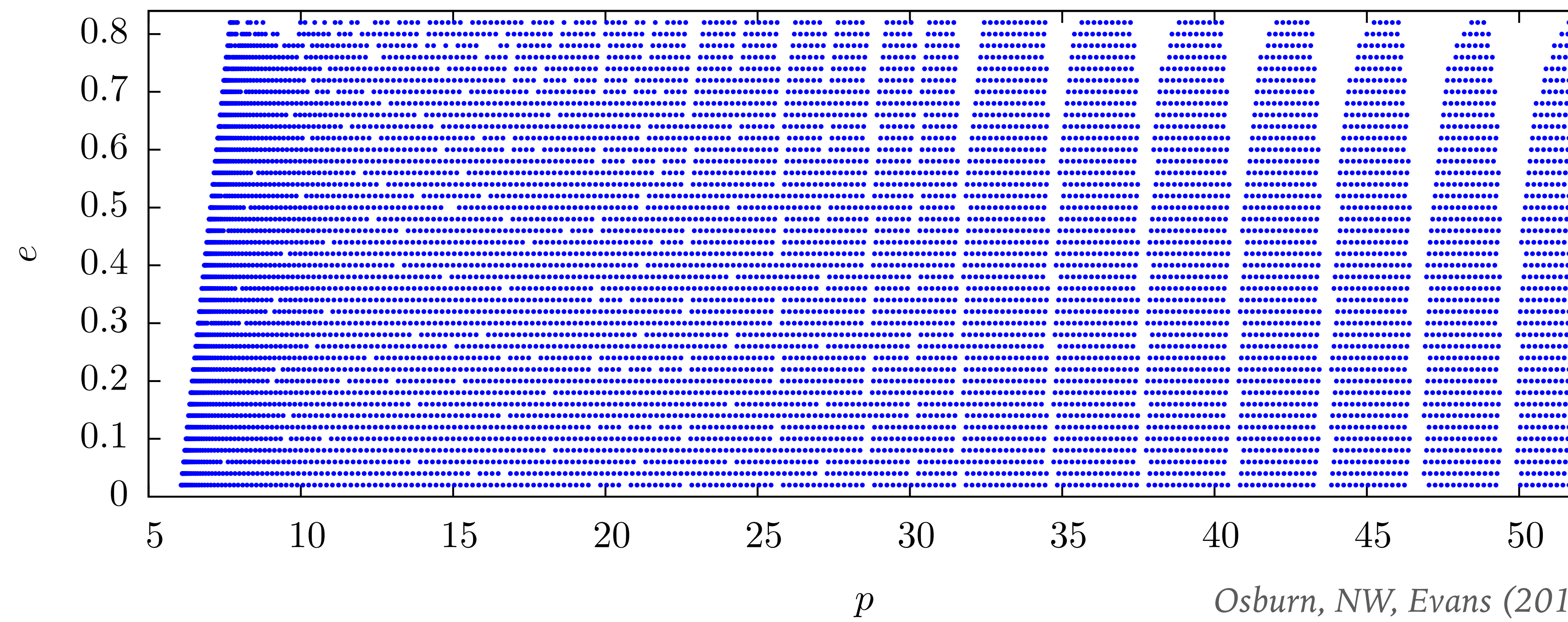


$$t_{RR} = (M/\mu)T_c, \qquad T_c = 2\pi/6^{3/2}M$$

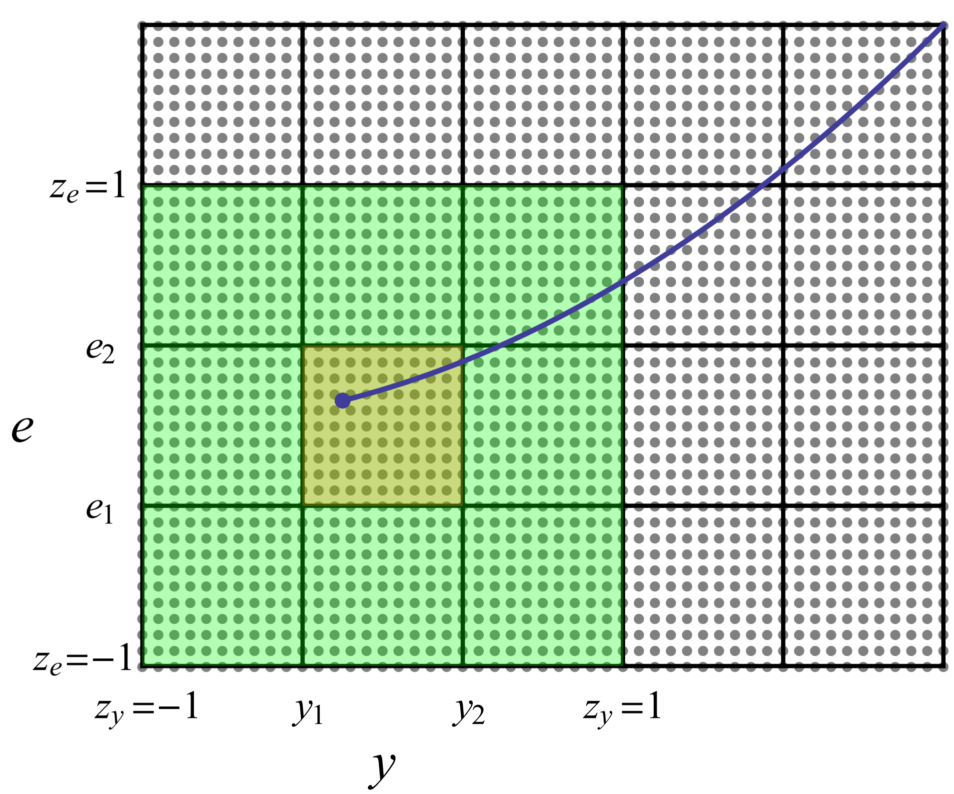
Dephase about 1 radian over a radiation reaction timescale. Suggests you can use flux balance waveforms for matched filtered searches but might introduce a parameter bias

Geodesic self-force inspirals (2015)

- *Early work was only for small eccentricity*
- *Accuracy in self-force was not sufficient*
- *Use a hybrid scheme with local interpolation model*



Osburn, NW, Evans (2015)



Compute fluxes to high precision using RW code

$$\langle F^{(1)\alpha} \rangle$$

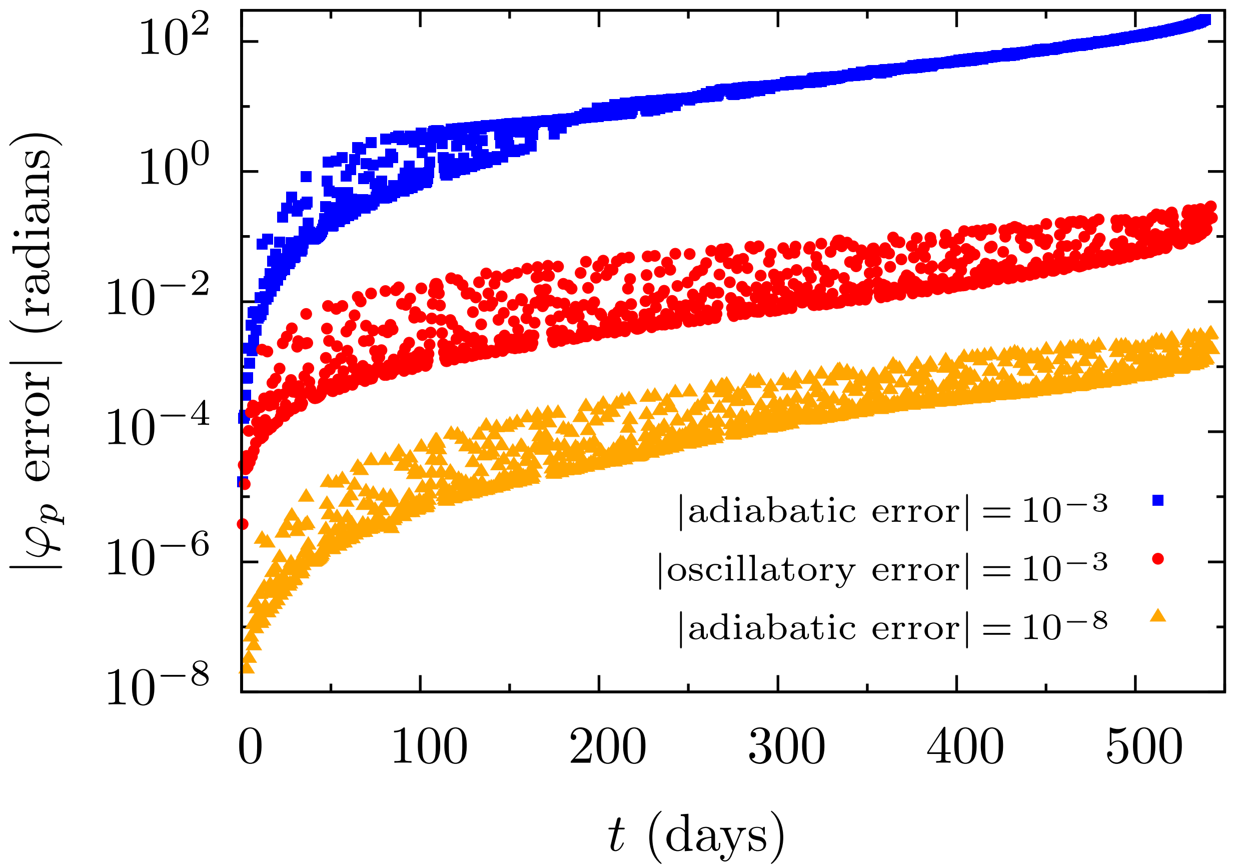
Compute oscillatory pieces using Lorenz-gauge code

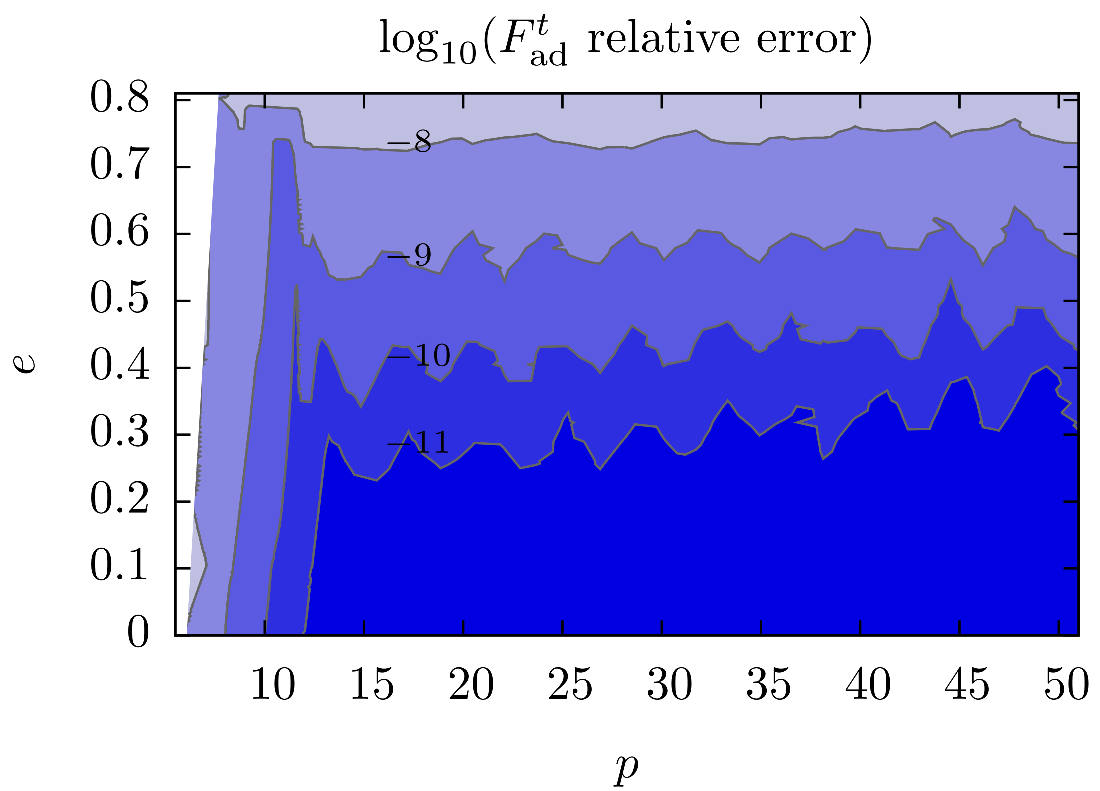
$$F^{(1)\alpha}_{\text{cons}} + F^{(1)\alpha}_{\text{diss}}$$

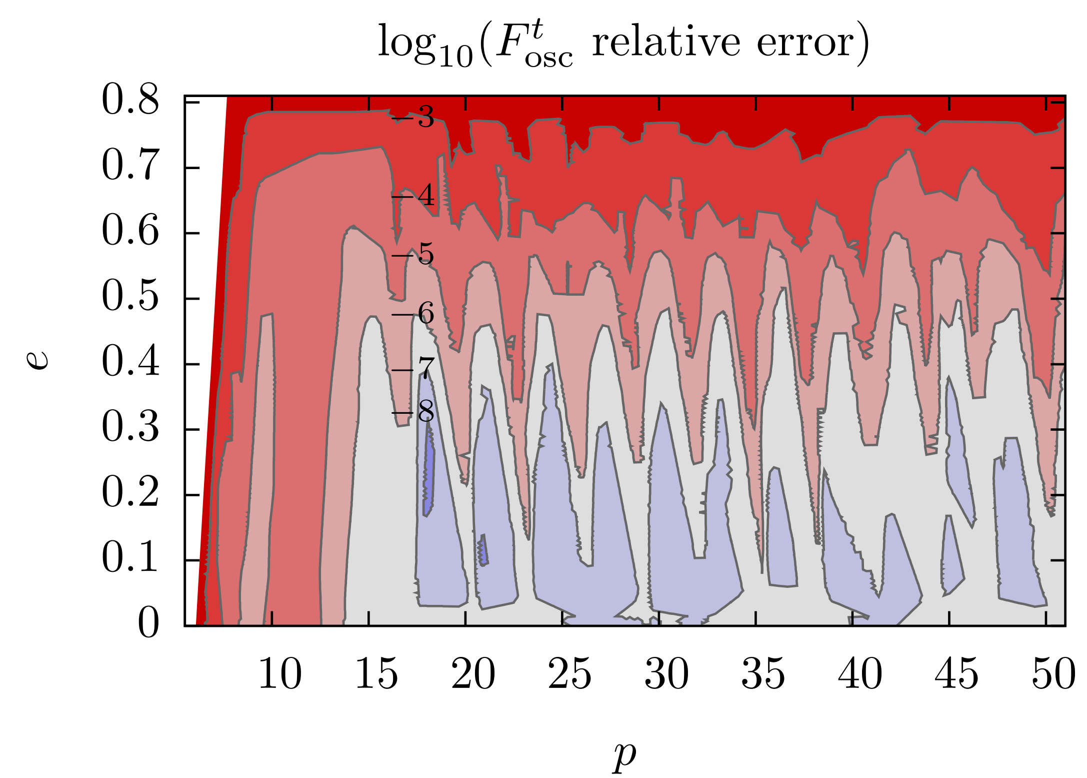
Local interpolation rather than global

Geodesic self-force inspirals (2015)

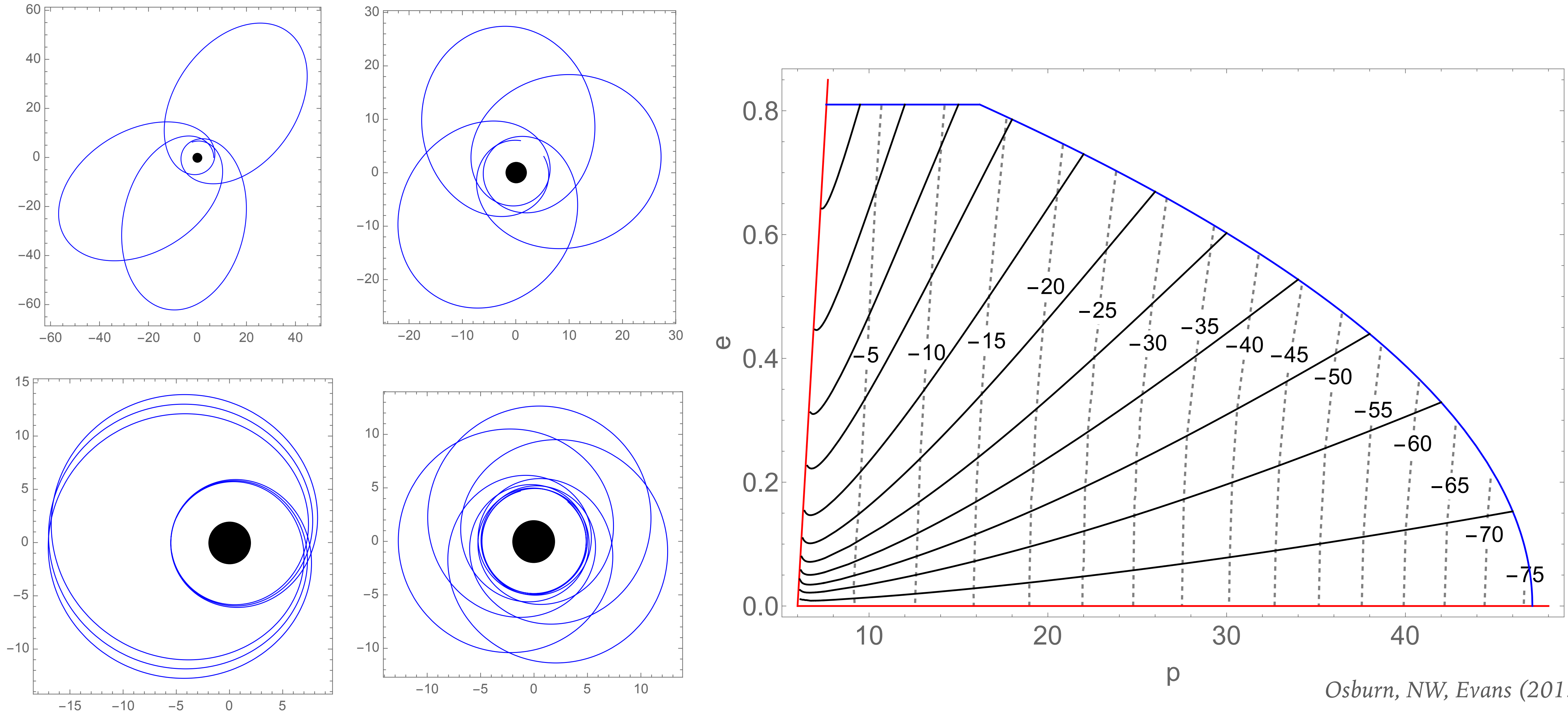
- *Met accuracy goals for adiabatic and oscillatory self-force across parameter space*
- *Verified influence of error on orbital phase by injecting noise into self-force*







Geodesic self-force inspirals (2015)



Osburn, NW, Evans (2015)

Can now accurately compute inspirals across whole parameter space including first-order self-force

Model can easily incorporate other forces (2nd order, spin) but… geodesic SF model is making an O(1) error

Geodesic self-force inspirals: error from osculating assumption

Fluxes | *Subleading fluxes and oscillatory forces*

$$F^\alpha = \boxed{\mu^2\langle F^{(1)\alpha}\rangle} \quad \boxed{\begin{aligned} &+\mu^3\left(\langle F^{(2)\alpha}\rangle + s\langle F^{(1)\alpha}_{\mathrm{dipole}}\rangle\right) \\ &+\mu^2\left(F^{(1)\alpha}_{\mathrm{cons}} + F^{(1)\alpha}_{\mathrm{diss}} + sF^{\alpha}_{\mathrm{spin}}\right)\end{aligned}}$$

Contribution to inspiral phase: $\mathcal{O}(\mu^{-1})$ | $\mathcal{O}(1)$

Geodesic self-force inspirals: error from osculating assumption

$$F^\alpha = \underbrace{\mu^2 \langle \underbrace{\boxed{F^{(1)\alpha}_{\text{geo}}}}_{\mathcal{O}(\mu^{-1})} + \mu \underbrace{\boxed{F^{(1)\alpha}_{\text{insp}}}}_{\mathcal{O}(1)} \rangle}_{\textit{Fluxes}} \underbrace{\boxed{\begin{aligned} &+\mu^3 \left(\langle F^{(2)\alpha} \rangle + s \langle F^{(1)\alpha}_{\text{dipole}} \rangle \right) \\ &+\mu^2 \left(F^{(1)\alpha}_{\text{cons}} + F^{(1)\alpha}_{\text{diss}} + s F^\alpha_{\text{spin}} \right) \end{aligned}}}_{\mathcal{O}(1)}$$

Subleading fluxes and oscillatory forces

Contribution to inspiral phase: $\mathcal{O}(\mu^{-1})$ $\mathcal{O}(1)$

Geodesic self-force approximation introduces an error at the order we are trying incorporate in our model!

Fortunately, initial indications are the coefficient of this term is very small (more on this later)

In principle the subleading terms split into geo and insp contributions but insp contributions are suppressed to O(mu) so we need not worry about them

Self-consistent inspirals

Evolve field equations and equations of motion together

Turn to time-domain or Green function methods

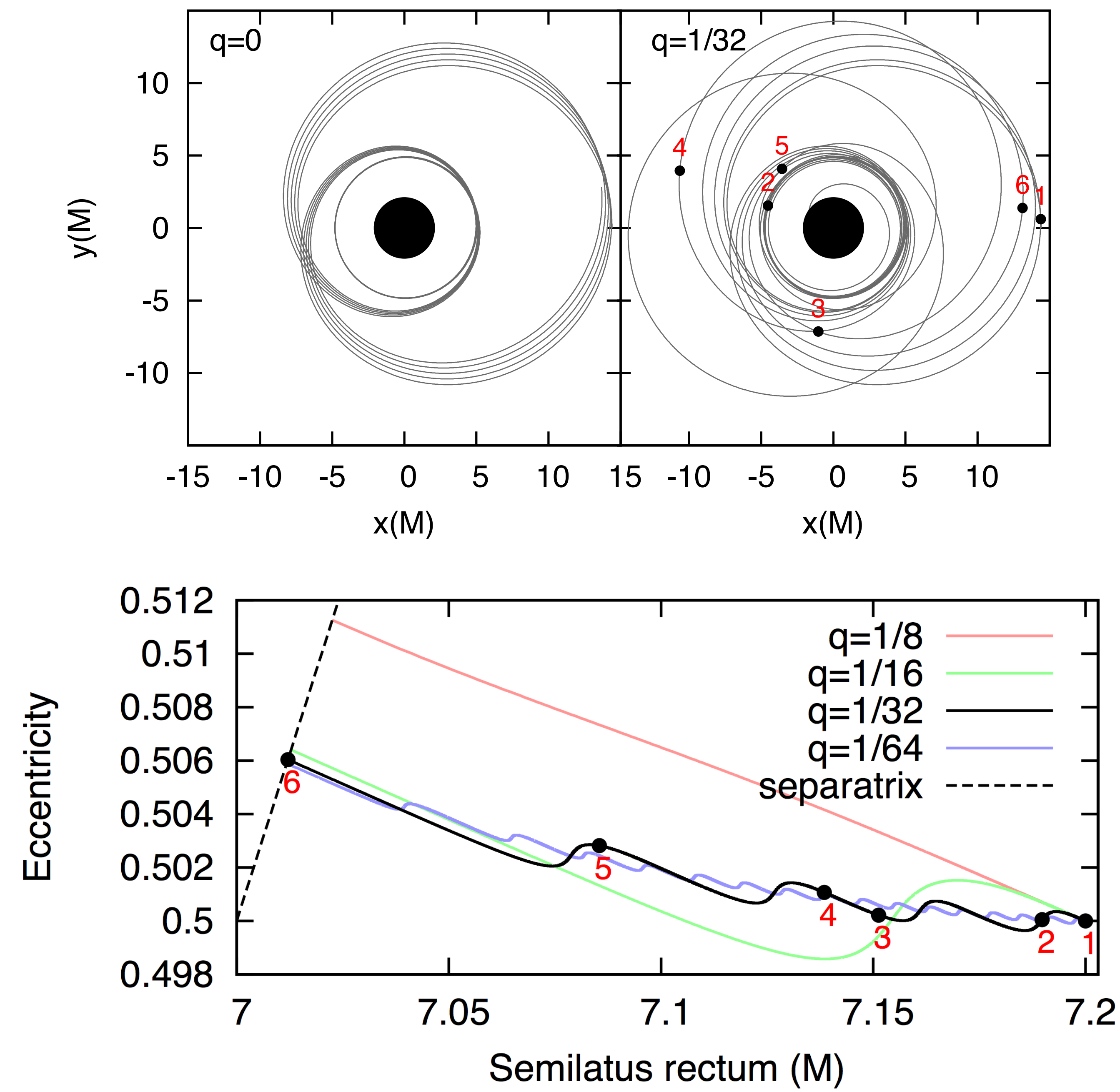
Time-domain simulations give waveform as well (GF inspirals would need to use a waveform generation)

No adiabaticity assumptions made

Typically much slower to compute but are gold standard as no approximation (beyond BH pert theory) used

So far achieved in scalar-field case

Scalar-field example



Diener, Vega, Wardell, Detweiler (2012)

Which inspirals have been computed?

	Schwarzschild	Kerr
Flux balance	✓ Tanaka et al. Cutler, Kennefick, Poisson	✓ Circular, equatorial and spherical done but not generic
Kludge	✓	✓ Barack and Cutler Babak et al. Chua, Moore, Gair
Geodesic SF	✓ NW et al. Osburn, NW, Evans	✗
Self-consistent	✗ but see scalar-field work by Diener et al.	✗

✗ No 2nd-order fluxes, or dissipative spin effects included in any of these

Ongoing/recent research

-Improved kludge models

-Inspirals with a spinning secondary

-Comparison of self-consistent and geodesic SF inspirals

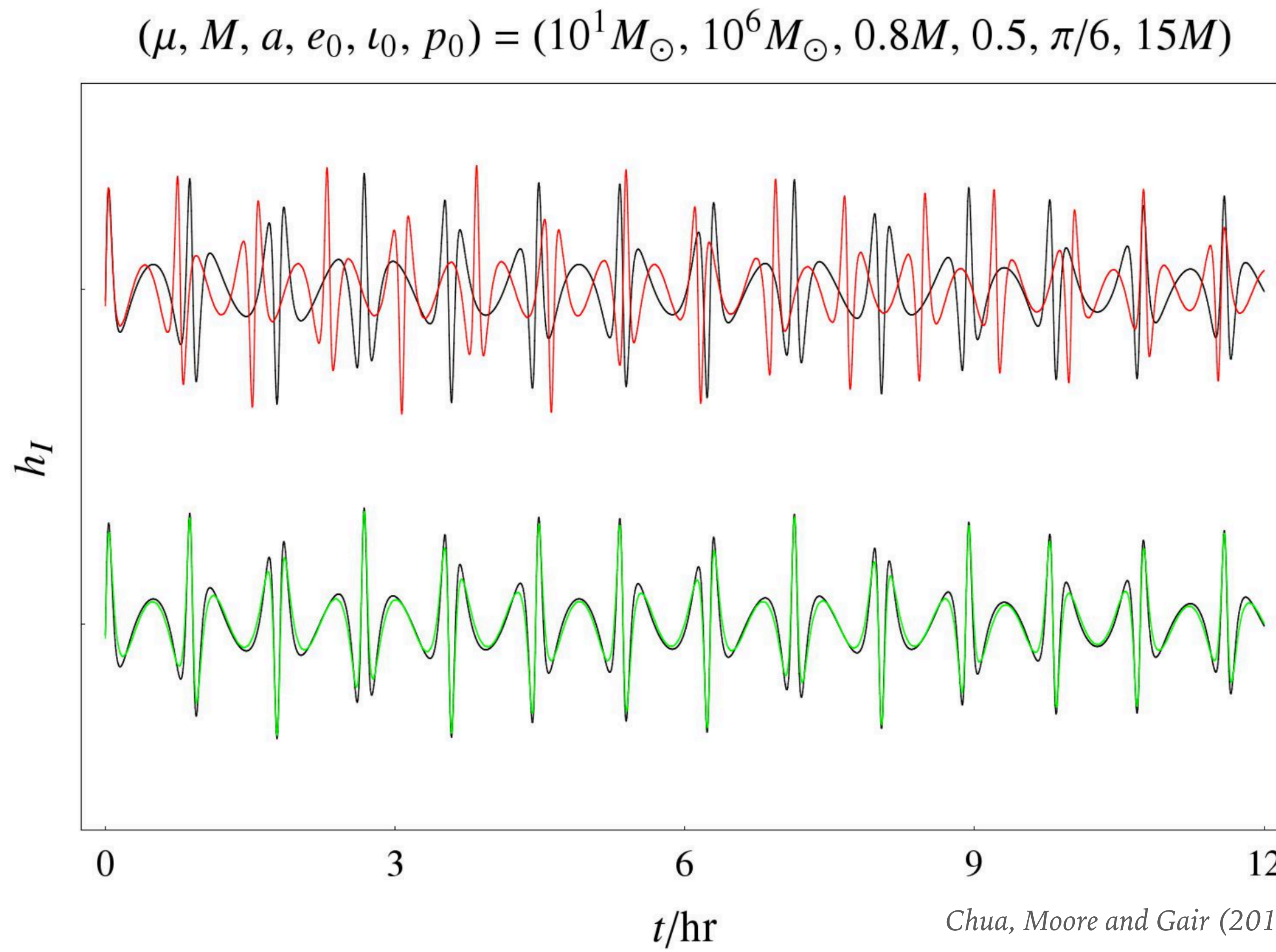
Improved kludge models

Augmented Analytic Kludge (AAK) Chua, Moore, Gair

AAK maps the parameters of the AK model to match the frequencies of NK waveforms

About as fast to evaluate as AK models but with similar accuracy to the NK models (its a hybrid)

AAK model publicly available: github.com/alvincjk/EMRI_Kludge_Suite



Chua, Moore and Gair (2017)

Plunge criterion	Population model	Number of events in mass range									
		$M_{10} < 5$		$5 < M_{10} < 5.5$		$5.5 < M_{10} < 6$		$M_{10} > 6$		Total	
		AAK	AK	AAK	AK	AAK	AK	AAK	AK	AAK	AK
Schwarzschild	M1	20	0	240	10	110	10	10	0	380	20
	M2	30	0	190	10	70	10	0	0	290	10
	M3	20	0	310	10	510	40	40	10	880	50
	M4	70	0	280	20	80	20	0	0	440	40
	M5	0	0	10	0	20	0	0	0	30	0
	M6	20	0	270	10	210	10	20	0	520	30
	M7	230	0	2190	60	1040	100	60	10	3530	180
	M8	0	0	30	0	10	0	0	0	50	0
	M9	20	0	210	0	110	10	10	0	350	20
	M10	30	0	240	10	100	10	10	0	370	10
	M11	0	0	0	0	1	0	0	0	1	0
	M12	230	10	2420	70	1730	130	180	30	4560	230

Spinning secondary around Schwarz. black hole

Quasi-circular inspirals: Burko and Khanna

Eccentric inspirals: NW, Osburn, Evans (in prep)

Include the spin-curvature force

Spin-curvature MPD force:

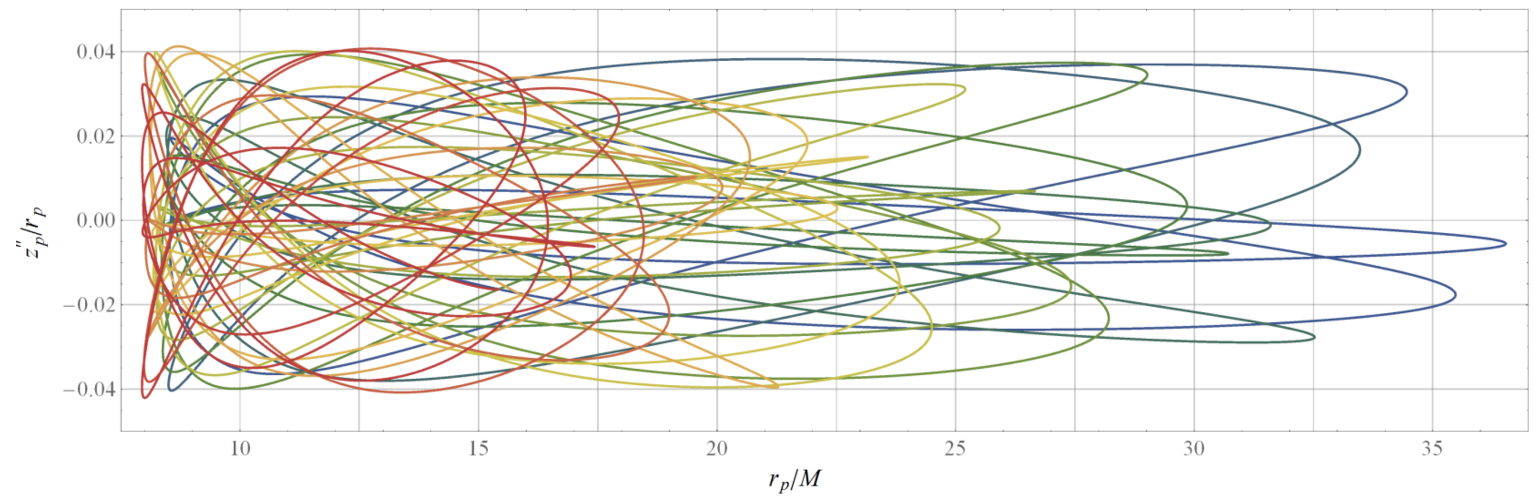
$$F^t_{\text{spin}} = \frac{3Mu^r \sin\theta_p (S^\varphi u^\theta - S^\theta u^\varphi)}{r_p f_p},$$

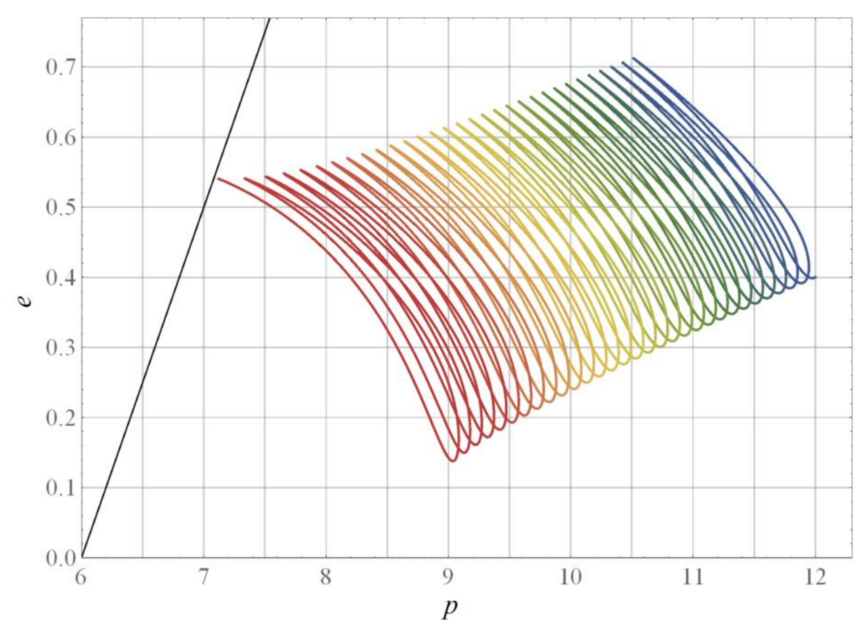
$$F^r_{\text{spin}} = \frac{3M f_p u^t \sin\theta_p (S^\varphi u^\theta - S^\theta u^\varphi)}{r_p},$$

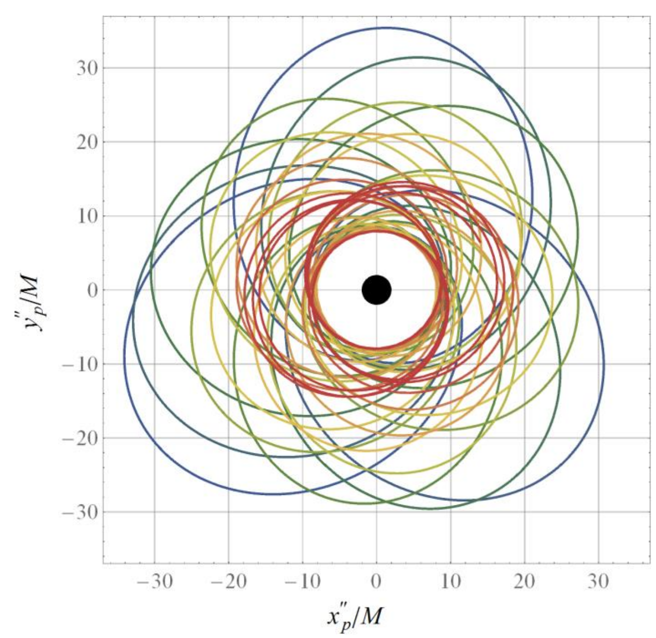
$$F^\theta_{\text{spin}} = \frac{3Mu^\varphi \sin\theta_p (S^t u^r - S^r u^t)}{r_p^3},$$

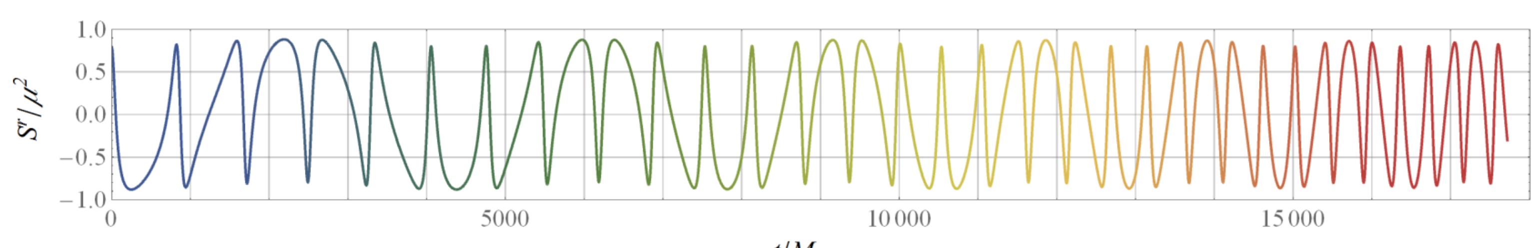
$$F^\varphi_{\text{spin}} = -\frac{3Mu^\theta (S^t u^r - S^r u^t)}{r_p^3 \sin\theta_p}.$$

Extend osculating element equations to non-equatorial motion when the spin is not aligned with the orbital ang. mom.









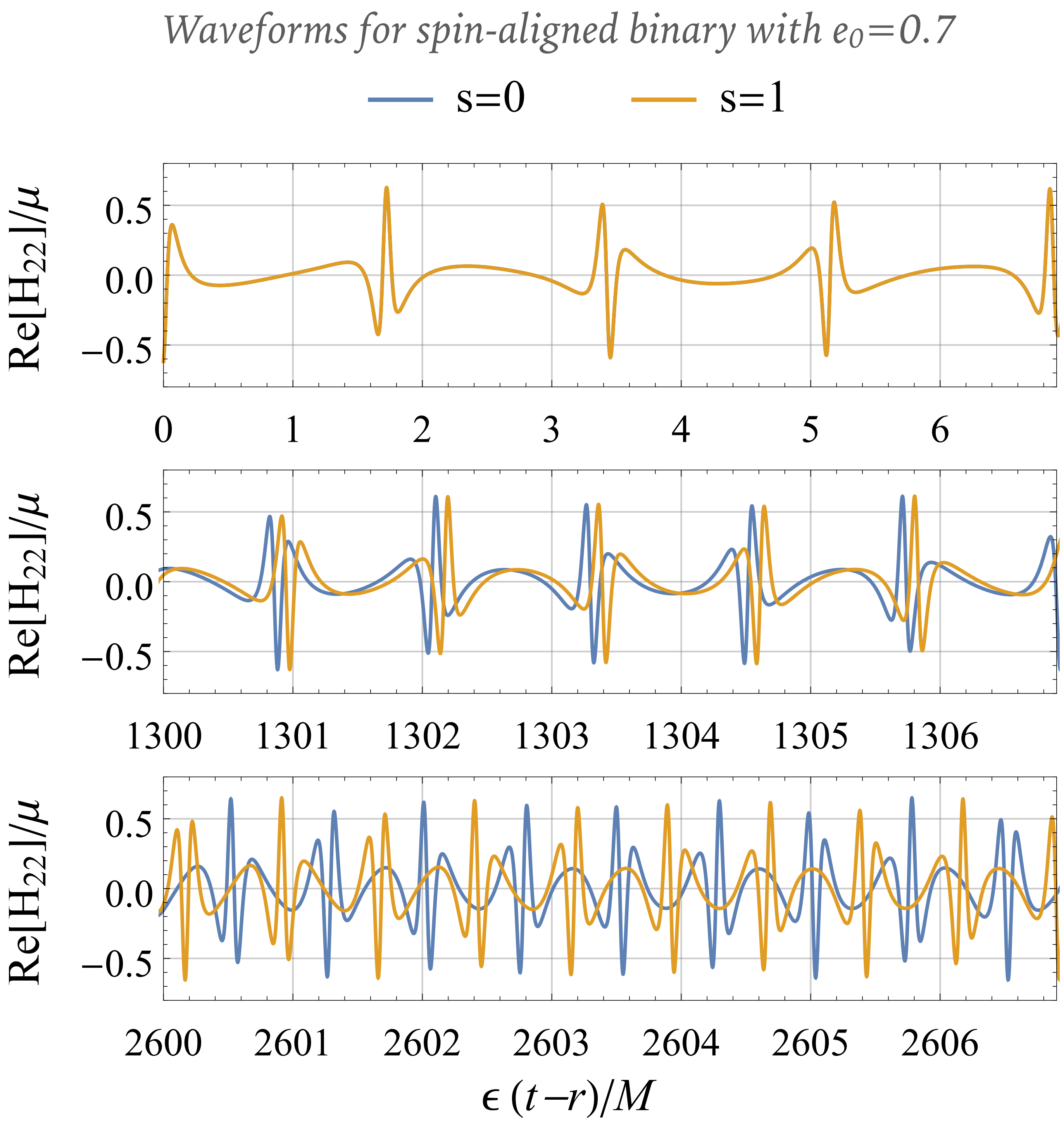
Spinning secondary around Schwarz. black hole

Quasi-circular inspirals: Burko and Khanna

Eccentric inspirals: NW, Osburn, Evans (in prep)

Include the spin-curvature force

Waveforms for spin-aligned binary with e_0=0.7



Inspiral waveforms computed by osculating between Teukolsky snapshot waveforms

Shows the dephasing of the s=0 and s=1 waveforms (initially matched in frequencies)

Currently computing dephasing as a function of initial (p,e) values - results soon

For more details see talk by Osburn

Comparison of self-consistent and geodesic self-force inspirals

Comparison for scalar-field case between geodesic SF inspiral and 3+1 time-domain self-consistent simulation

Fluxes

Subleading fluxes and oscillatory forces

$$F^\alpha = \mu^2 \langle \boxed{F^{(1)\alpha}_{\text{geo}}} + \mu \boxed{F^{(1)\alpha}_{\text{insp}}} \rangle \quad \boxed{\begin{aligned} &+\mu^3 \left(\langle F^{(2)\alpha} \rangle + s \langle F^{(1)\alpha}_{\text{dipole}} \rangle \right) \\ &+\mu^2 \left(F^{(1)\alpha}_{\text{cons}} + F^{(1)\alpha}_{\text{diss}} + s F^{\alpha}_{\text{spin}} \right) \end{aligned}}$$

Contribution to inspiral phase: $\mathcal{O}(q^{-1})$ $\mathcal{O}(1)$

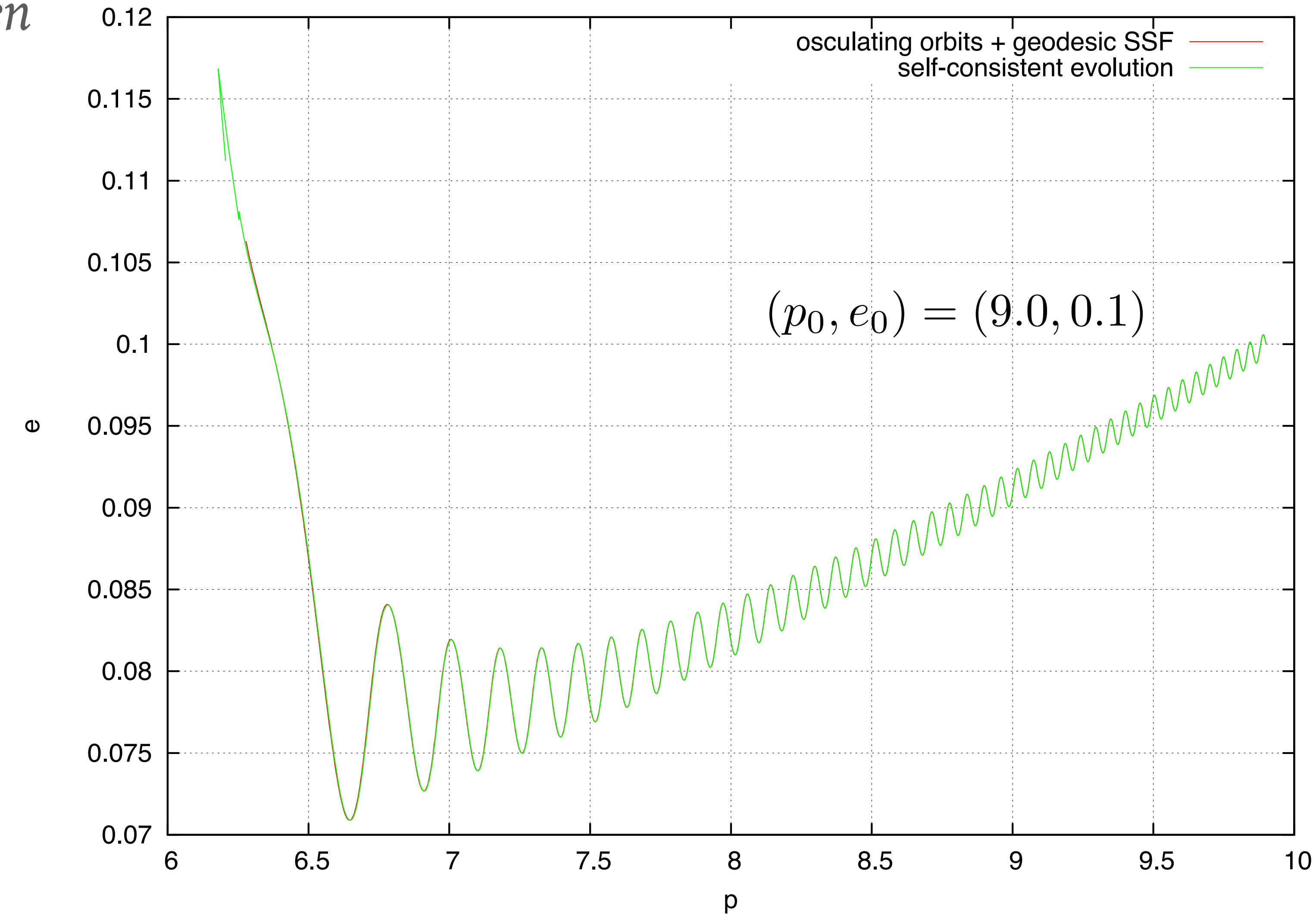
Want to confirm scaling of insp error term and find coefficient

Comparison of self-consistent and geodesic self-force inspirals

Comparison for scalar-field case between geodesic SF inspiral and 3+1 time-domain self-consistent simulation

Difference in phase evolution smaller than numerical noise

Great to see such good agreement between two completely distinct codes



Diener, NW, Wardell

Difference between inspirals is very small - calls for a very accurate time-domain code *See talk by Diener*

Inspiral is not a geodesic $\mu u^\beta \nabla_\beta u^\alpha = F^\alpha$

Need to account for acceleration in regularization procedure *See talk by Heffernan*

Future directions

- *2nd order fluxes (see talks tomorrow), spin flux (work in progress)*
- *Comparison of inspirals from self-force in different gauges*
- *Further improvements to kludge models*
- *EOBSF? (see Taracchini's talk)*
- *Green function inspirals (see work by Galley and Wardell)*
- *Evolve through resonances (see van de Meent and Nasipak's talks)*
- *Numerical Relativity? (see Schutz's discussion session on Thursday)*
- *Faster geodesic self-force models*
- *Augmented flux models with high-order pN*
- *Self-consistent inspirals in gravity*

Faster inspirals with conservative corrections

Current geodesic SF inspirals are slow to compute as need to resolve small oscillations on orbital timescale

$$\dot{p} = \mathcal{F}_p[p, e, \chi - \chi_0, F^\alpha]$$
$$\dot{e} = \mathcal{F}_e[\cdots]$$
$$\dot{\chi}_0 = \mathcal{F}_{\chi_0}[\cdots]$$

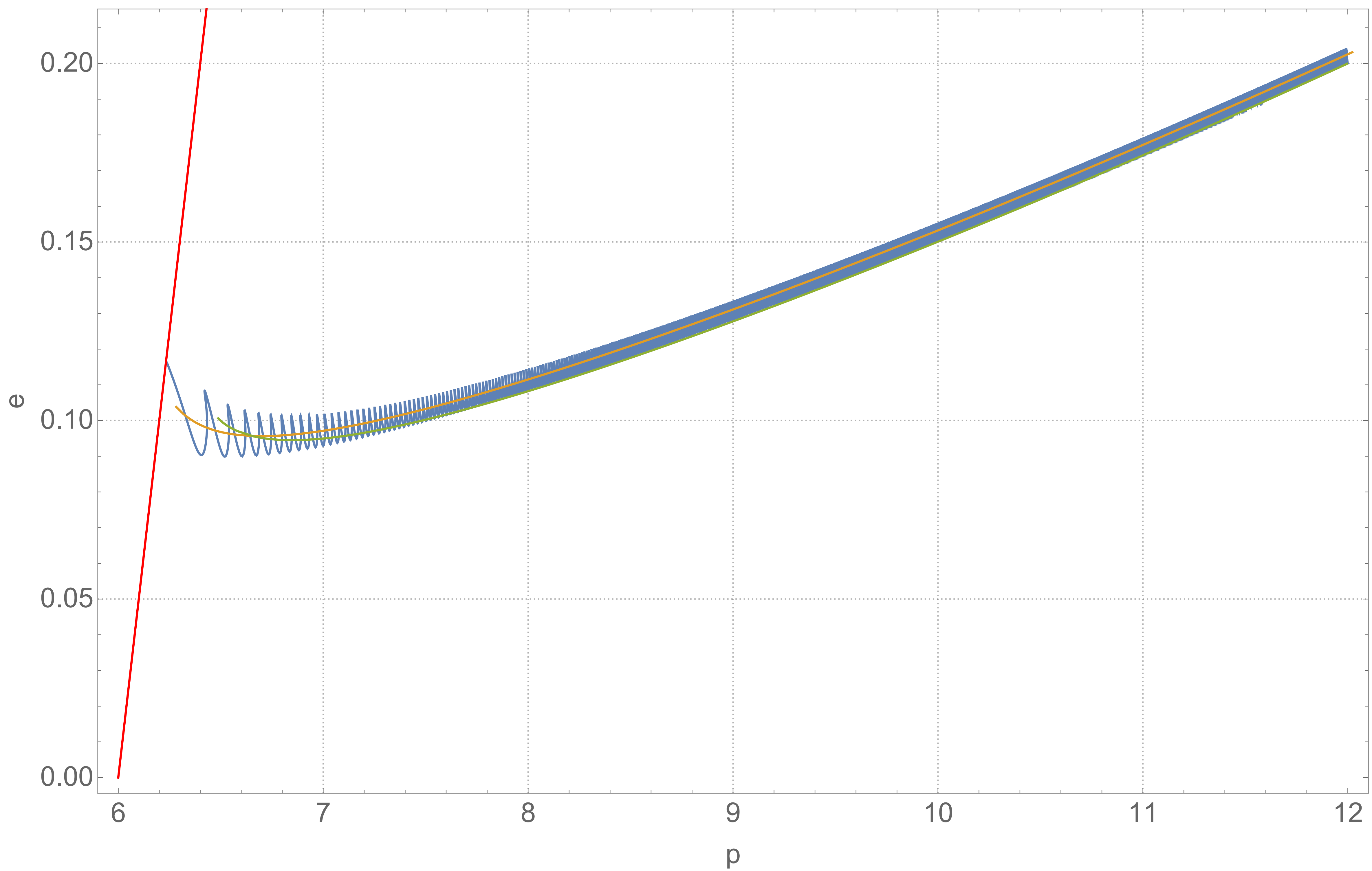
RHS varies on orbital timescale. Try replacing it with averaged quantities

$$\dot{p} = \langle \mathcal{F}_p[p, e, \chi - \chi_0, F^\alpha] \rangle_\chi$$
$$\dot{e} = \langle \mathcal{F}_e[\cdots] \rangle_\chi$$
$$\dot{\chi}_0 = \langle \mathcal{F}_{\chi_0}[\cdots] \rangle_\chi$$

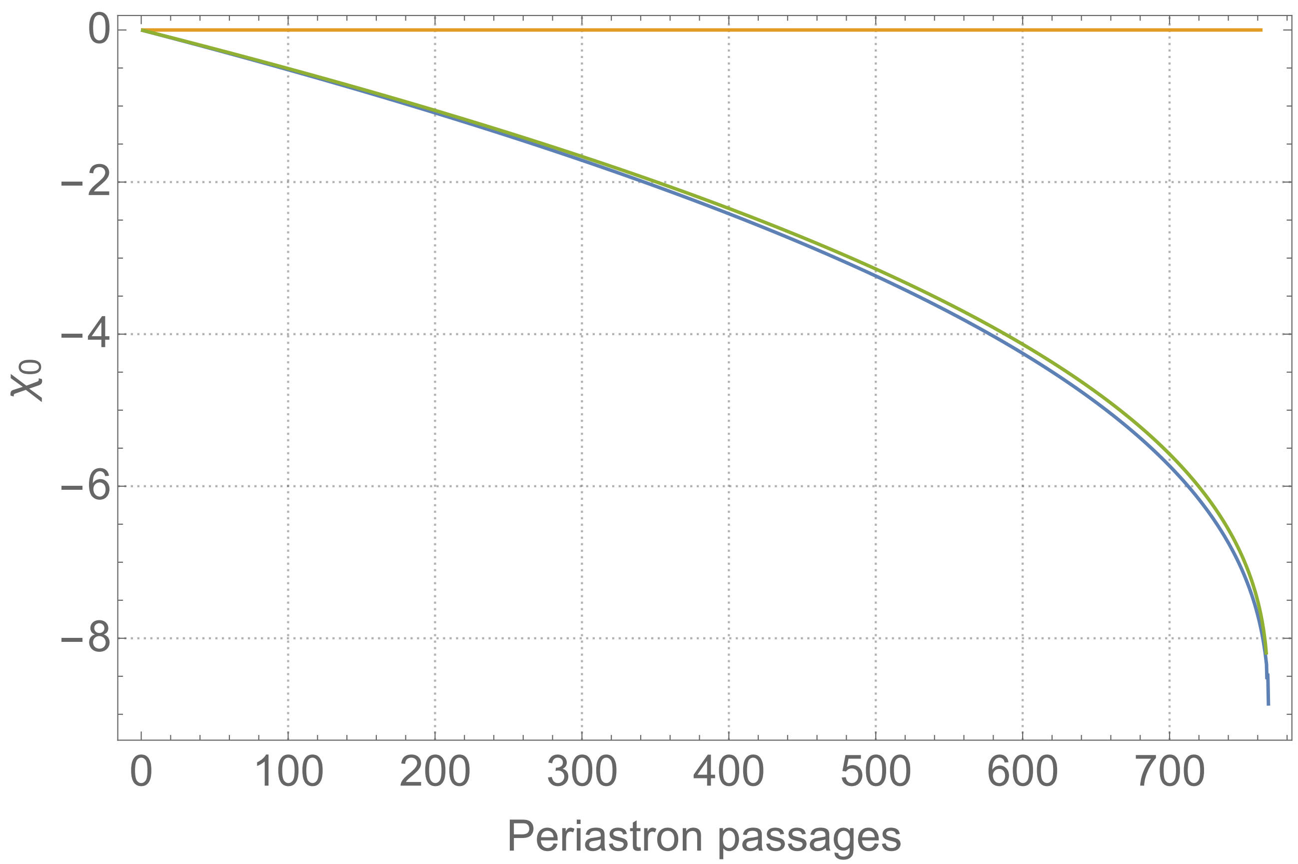
Find averaging centered around periastron passages works well. Averaging can be performed in an offline step

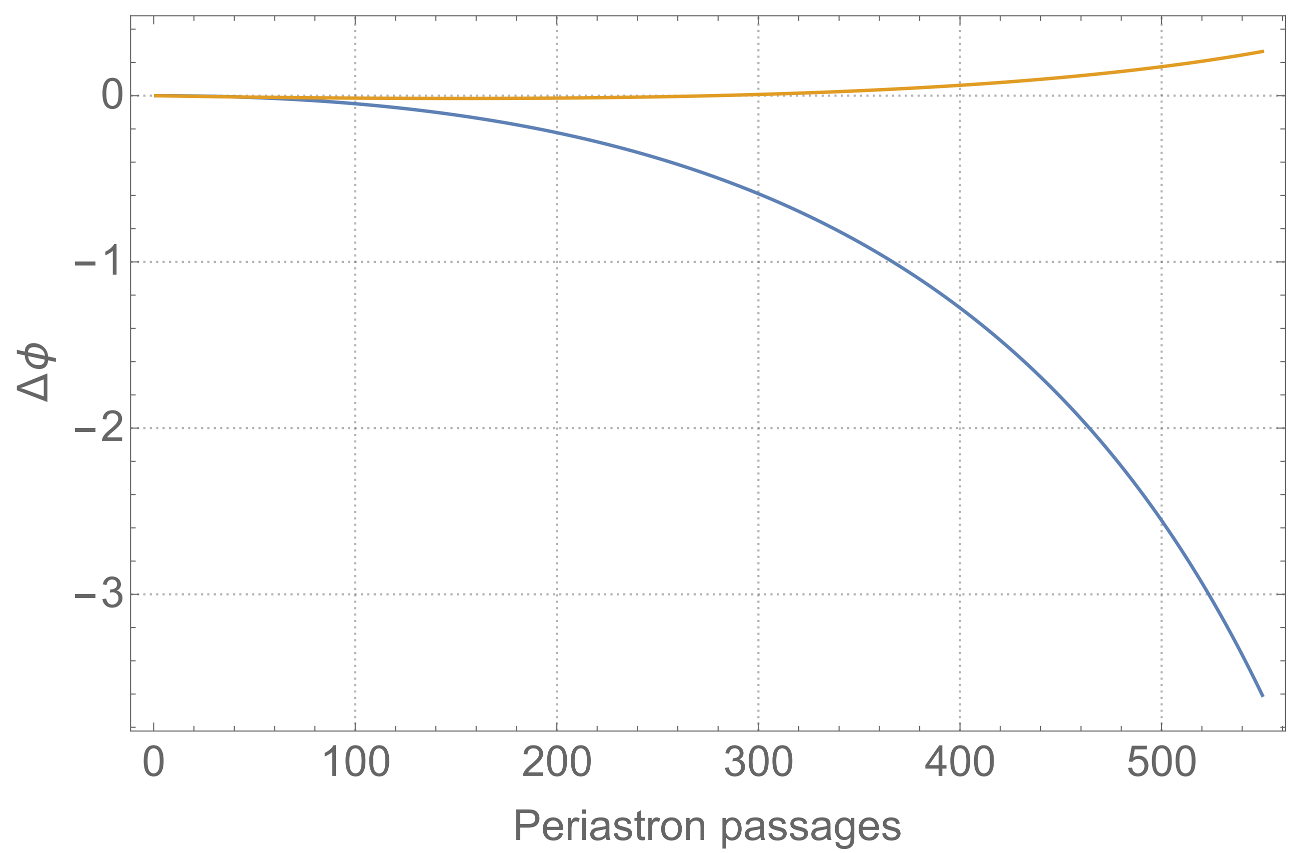
Resulting system should be as fast to solve for as flux balance case - could be used to improve kludges

Faster inspirals with conservative corrections



Faster inspirals with conservative corrections





Averaged version tracks evolution of chi0 well and remains in phase with the full inspiral better than radiative approximation

Which forces do we need to include in our models and to what accuracy?

Fluxes

Subleading fluxes and oscillatory forces

$$F^\alpha = \boxed{\mu^2 \langle F^{(1)\alpha} \rangle} \quad \boxed{\begin{aligned} &+\mu^3 \left(\langle F^{(2)\alpha} \rangle + s \langle F^{(1)\alpha}_{\text{dipole}} \rangle \right) \\ &+\mu^2 \left(\boxed{F^{(1)\alpha}_{\text{cons}} + F^{(1)\alpha}_{\text{diss}}} + s F^\alpha_{\text{spin}} \right) \end{aligned}}$$

Kerr radial-polar resonance

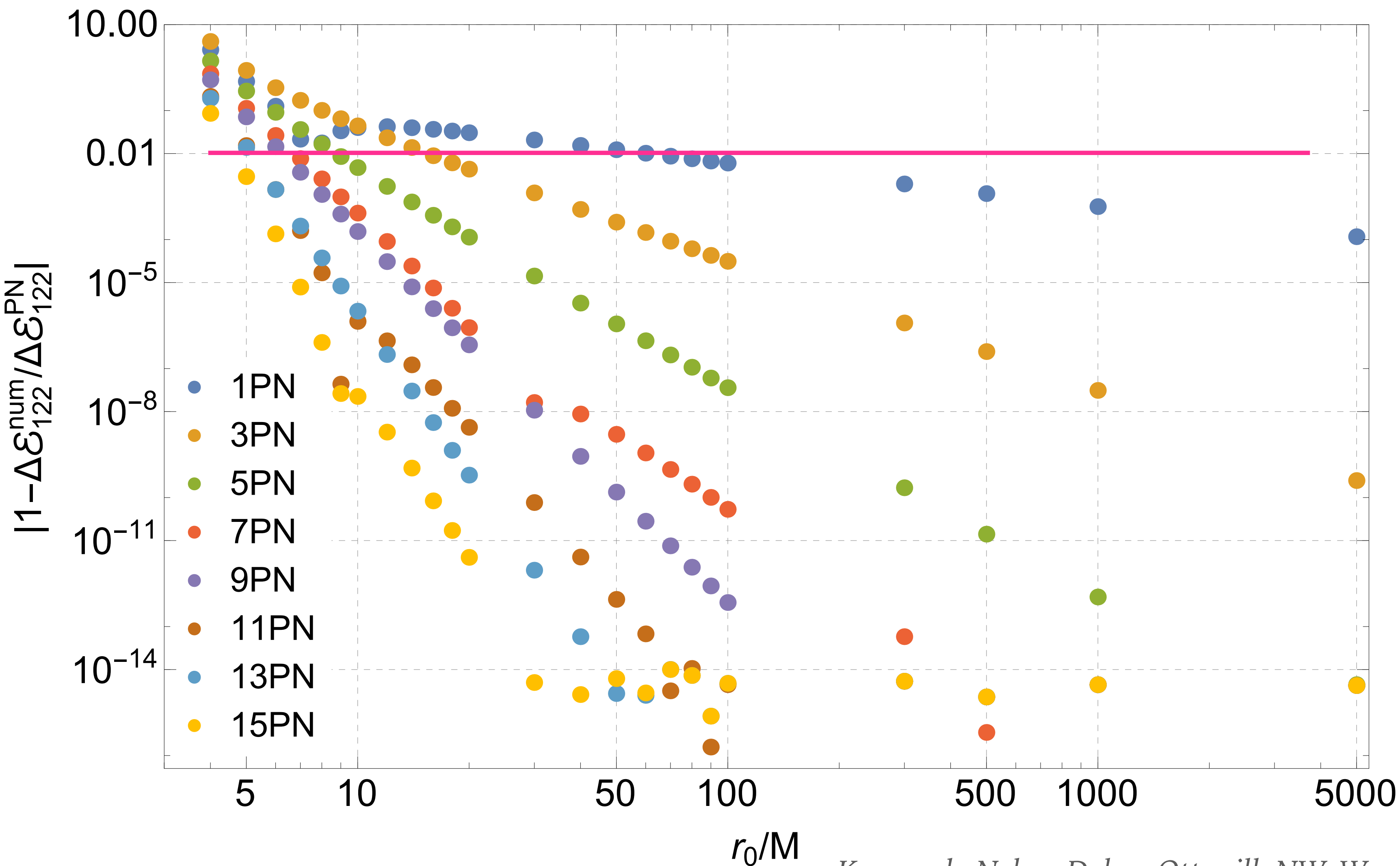
Contribution to inspiral phase:	$\mathcal{O}(\mu^{-1})$	$\mathcal{O}(\mu^{-1/2})$	$\mathcal{O}(1)$
Accuracy required in force, q=10^{-6}:	$\boxed{10^{-8}}$	10^{-5}	$\boxed{10^{-2}}$

High precision numerical results required in strong-field

Can pN reach this precision in the strong-field?

Augmenting inspiral models using high-order pN (MST)

Using MST methods we can (at first-order) reach very high pN

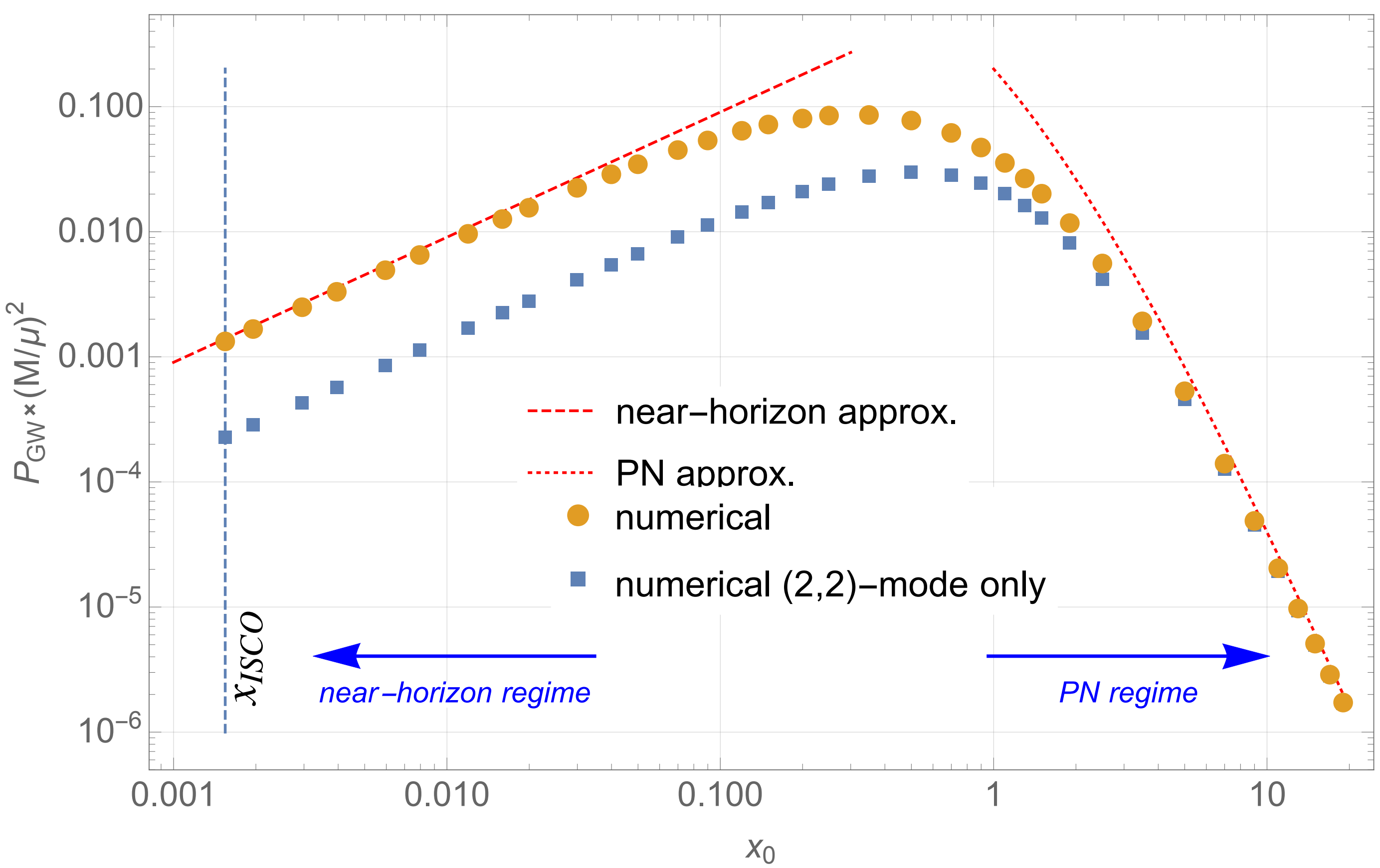


Kavanagh, Nolan, Dolan, Ottewill, NW, Wardell (2015)

Suggests ~10 pN sufficient to reach 10^{-2} accuracy for all stable orbits

Augmenting inspiral models using high-order pN (MST)

- *Need to compute force using MST methods (done by Kavanagh for radiation-gauge force in Schwarz. spacetime). We think we can do this with the Lorenz-gauge force also*
- *pN series grows when orbit is eccentric and inclined, but can might still cover a fair piece of the parameter space*
- *For retrograde orbits the ISCO moves out to 9M (extreme Kerr)*
- *For prograde orbits the ISCO moves in to 1M (extreme Kerr), maybe argument with near-horizon, near-extremal expansion (see Zimmerman's talk)*



Self-consistent inspirals with gravity

Why not done? Low multiple modes (l=0,1) are numerically unstable in Lorenz gauge

Fluxes

Subleading fluxes and oscillatory forces

$$F^\alpha = \mu^2 \langle \boxed{F^{(1)\alpha}_{\rm geo}} + \mu \boxed{F^{(1)\alpha}_{\rm insp}} \rangle \quad \boxed{\begin{aligned} &+\mu^3 \left(\langle F^{(2)\alpha} \rangle + s \langle F^{(1)\alpha}_{\rm dipole} \rangle \right) \\ &+\mu^2 \left(F^{(1)\alpha}_{\rm cons} + F^{(1)\alpha}_{\rm diss} + s F^{\alpha}_{\rm spin} \right) \end{aligned}}$$

Contribution to inspiral phase: $\mathcal{O}(q^{-1})$ $\mathcal{O}(1)$

Do we need to compute the low-modes self-consistently? No, they do not radiate so must contribute at subleading order. Thus difference between geodesic and self-consistent not important for these modes.

Recap and conclusions

- *Can accurately compute inspirals with first-order force*
- *Comparison between geodesic SF and self-consistent inspiral ongoing*
- *Need 2nd-order fluxes and contribution from spin of secondary*
- *These inspirals are slow to compute: improve Kludges, EOBSF?*
- *Good to have multiple approaches, geodesic SF, self-consistent, kludge, EOBSF…*