Transient instabilities of nearly extremal black holes

Aaron Zimmerman (CITA)

with Samuel Gralla and Peter Zimmerman

arXiv:1608.04739

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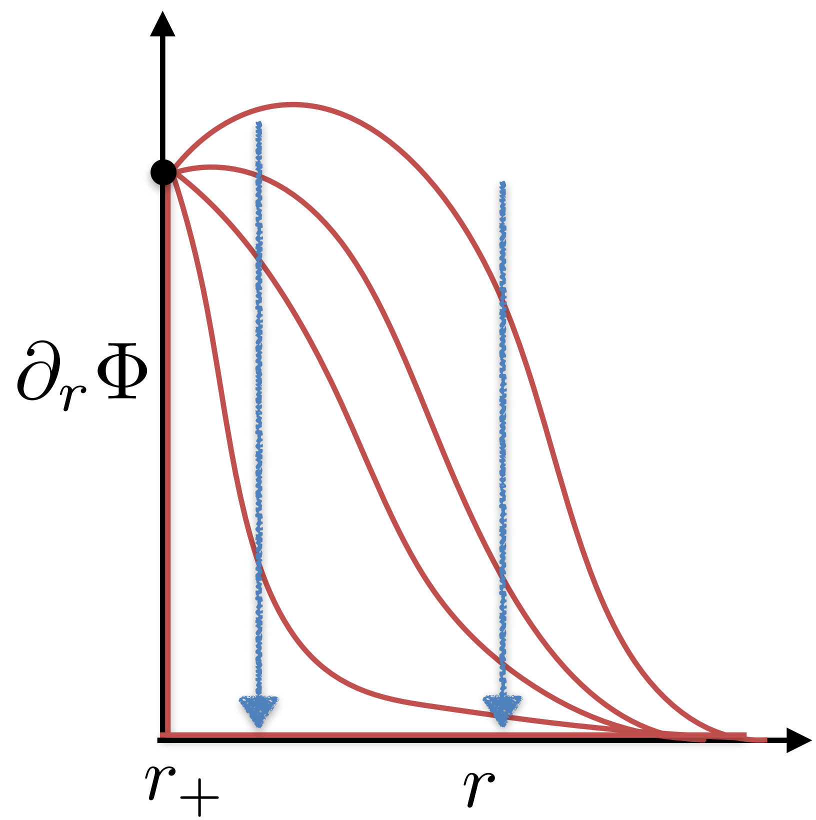
Instability of extremal horizons

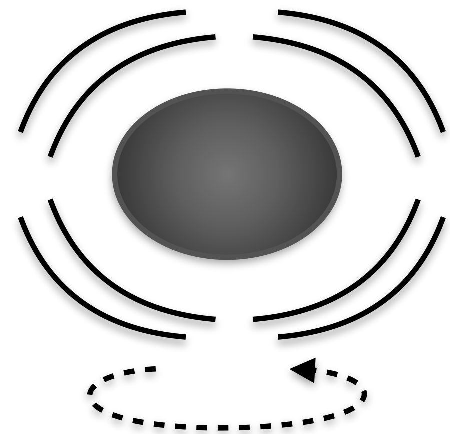
- No exponentially growing modes (Whiting 1989)
- Aretakis (2010): r derivs of scalars decay outside, not on horizon
- Two derivs grow unbounded
- Generalized: all extremal BHs, kinds of fields, beyond axisym
- What happens in near-extremal systems?



Aretakis (2010, 2012), Lucietti and Reall (2012), Casals et al. (2016)

Black hole perturbation theory

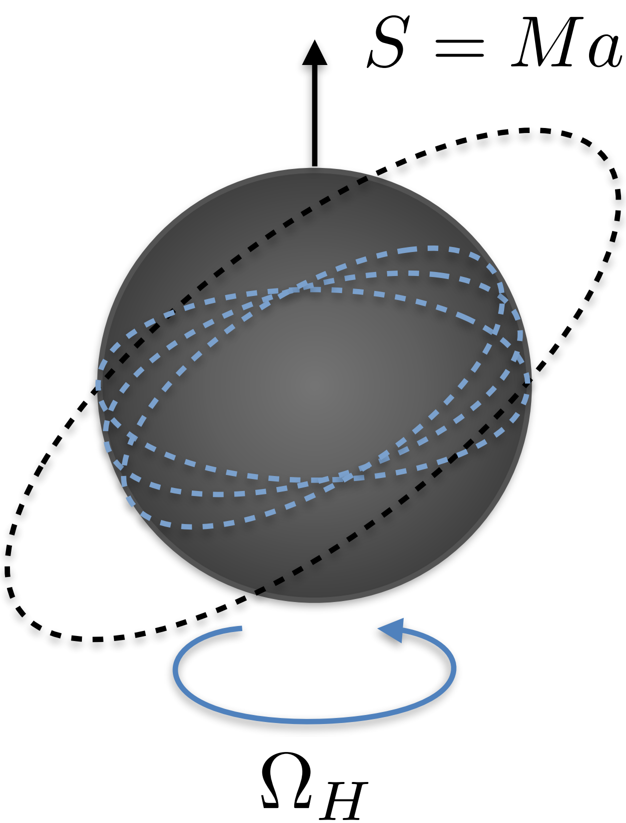
- Wave prop around BHs
- Perts to spacetime
- Orbits of test bodies
- Test for stability: modal, linear



$$\begin{matrix} \Phi \\ A_\mu \\ h_{\mu\nu} \end{matrix} \longrightarrow \psi \qquad \Box_s \psi = \mathcal{T}$$

$$\psi_{lm\omega} \sim e^{-i\omega t + im\phi} R_{lm\omega}(r) S_{lm\omega}(\theta)$$

Modes of nearly extremal BHs



- Rapid rotation: new expansion param

$$\epsilon = \sqrt{1-(a/M)^2}/2 \ll 1$$

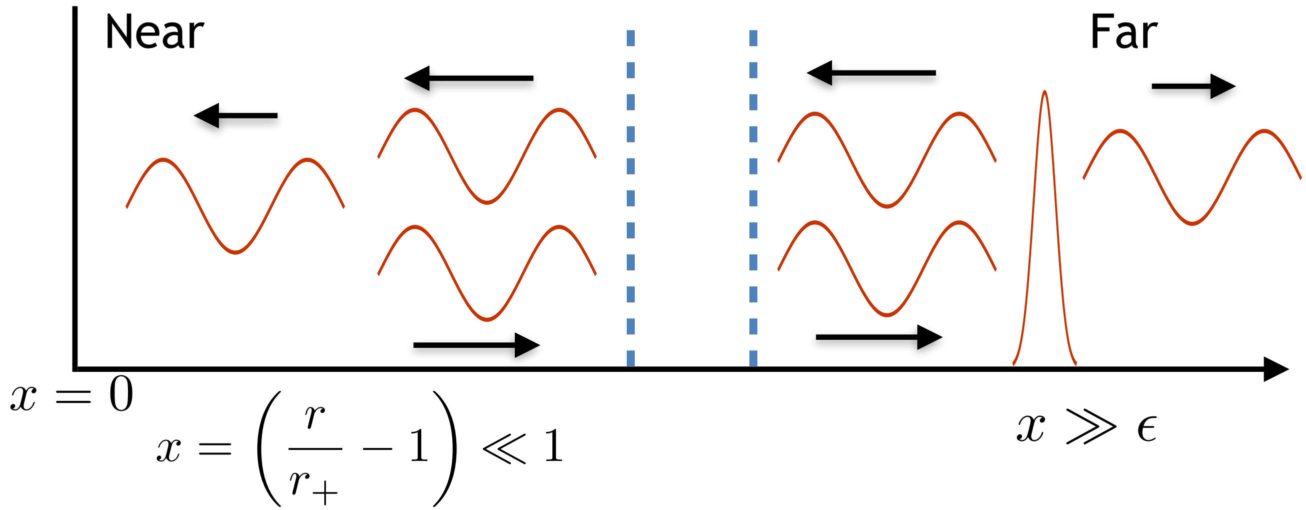
- Match near horizon to far region: $R_{lm\omega}$
- Analytically appx QNM freq, decay

$$\omega_R \approx m\Omega_H \qquad \gamma_n \approx \epsilon(n+1/2)$$

Slow decay!

Teukolsky and Press (1974), Detweiler (1980), Hod (2008), Yang et al. (2012, 2013a, 2013b)

Wavefunctions of nearly extremal BHs



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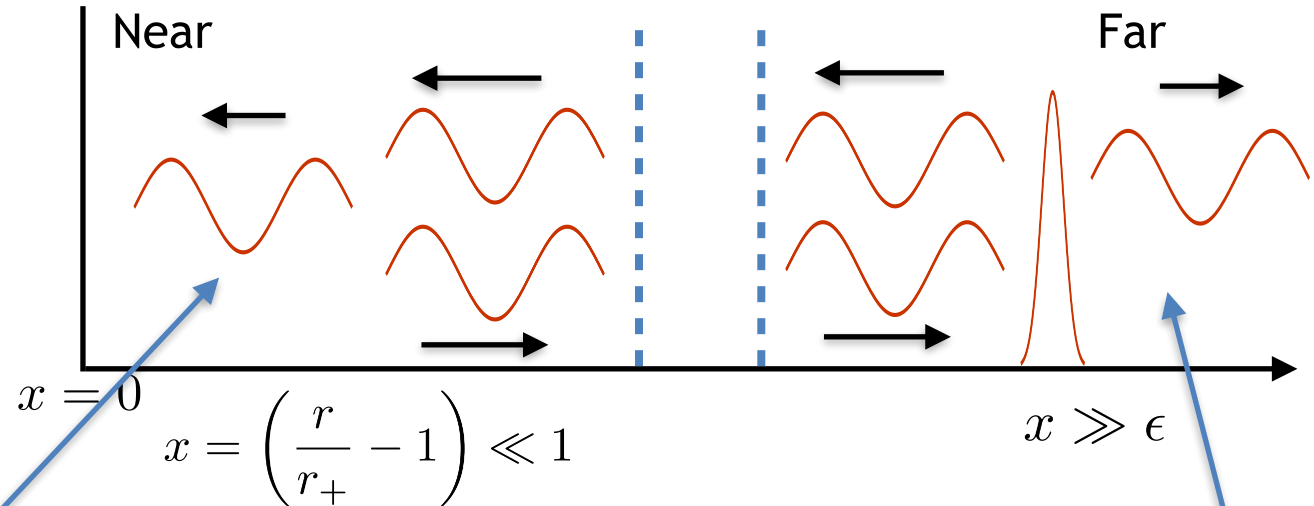
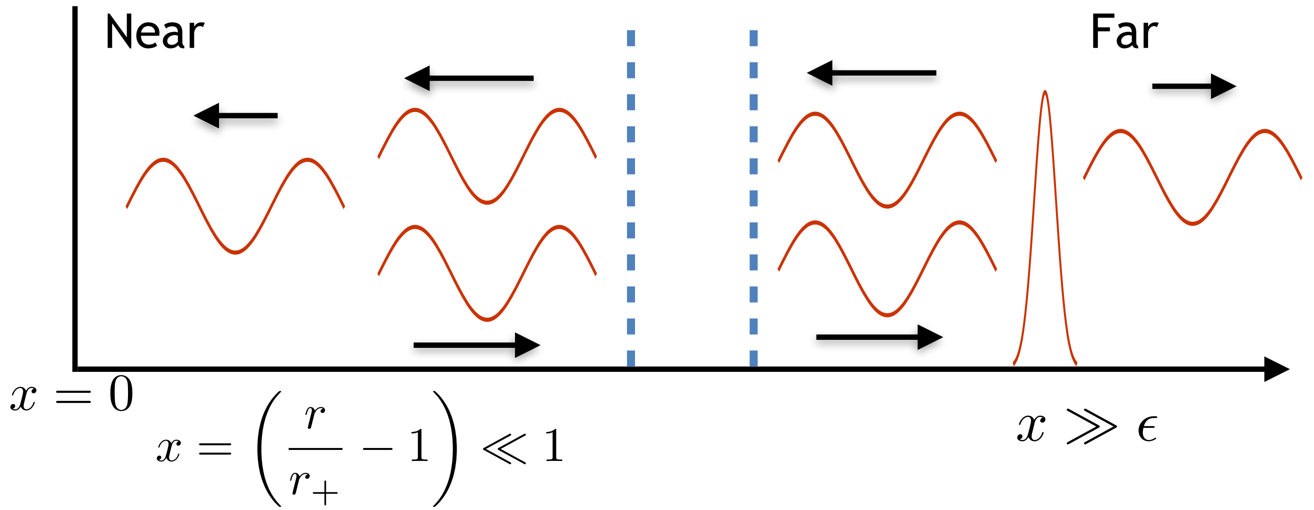


$R \sim {}_2F_1(\alpha_+, \alpha_-, \gamma, 4x/\epsilon)$

$R \sim x^{-1/2 - s \pm i\delta + 2i\omega r_+} {}_1F_1(\beta, 1 \pm 2i\delta, 2i\omega r_+ x)$

Teukolsky and Press (1974), Detweiler (1980), Hod (2008), Yang et al. (2012, 2013a, 2013b)

Wavefunctions of nearly extremal BHs



Matching with BCs gives

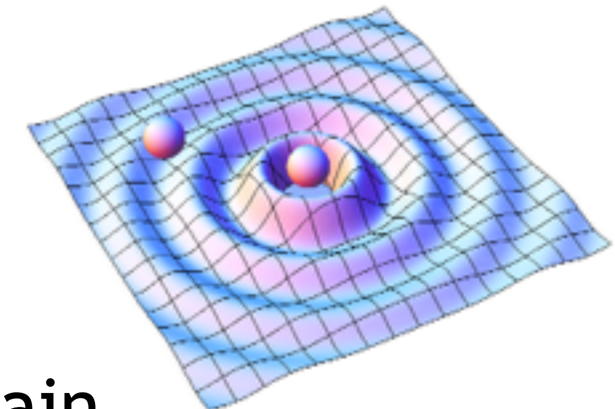
$$\omega_{lmn} = \frac{m}{2} - \epsilon\left[\delta + i\left(n + \frac{1}{2}\right)\right]$$

$$\delta^2 = 4(\omega r_+)^2 - (s + 1/2)^2 - {}_s\lambda_{lm\omega}$$

Teukolsky and Press (1974), Detweiler (1980), Hod (2008), Yang et al. (2012, 2013a, 2013b)

Quasinormal mode response

- Have source-free solutions



$$\psi_{lm\omega} \sim e^{-i\omega v + im\tilde{\phi}} R_{lm\omega}(r) S_{lm\omega}(\theta)$$

- Build response func in time domain

$$G(x^\mu, x^{\mu\prime}) = \frac{1}{2\pi} \sum_{l,m} \int d\omega \, e^{-i\omega v} \tilde{g}_{lm\omega}(r, r') \Omega(\theta, \theta', \tilde{\phi}, \tilde{\phi}')$$

$$\tilde{g}_{lm\omega} = \frac{w(r') R^{\rm in}_{lm\omega}(r') R^{\rm up}_{lm\omega}(r)}{2i\omega A_{\rm in}(\omega)}$$

Quasinormal mode response

- Inverse Laplace transform

$$G(x^\mu, x^{\mu\prime}) \sim \sum_{l,m} \int d\omega\, e^{-i\omega v} \frac{w(r') R^{\mathrm{in}}_{lm\omega}(r') R^{\mathrm{up}}_{lm\omega}(r)}{2i\omega A_{\mathrm{in}}}$$

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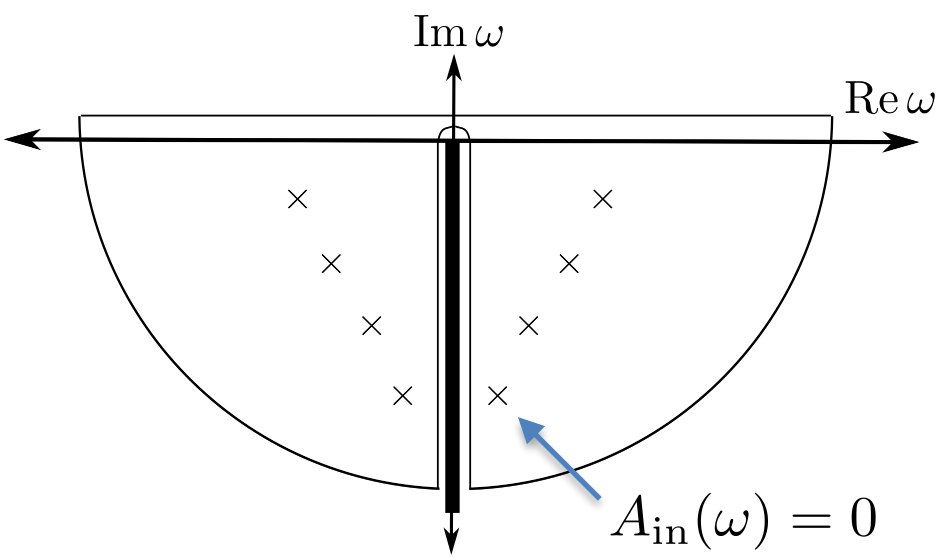

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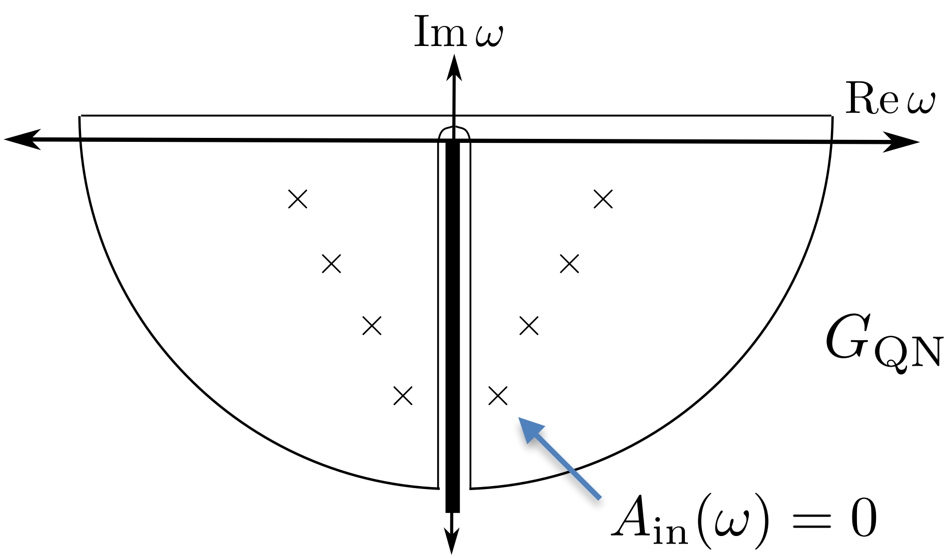
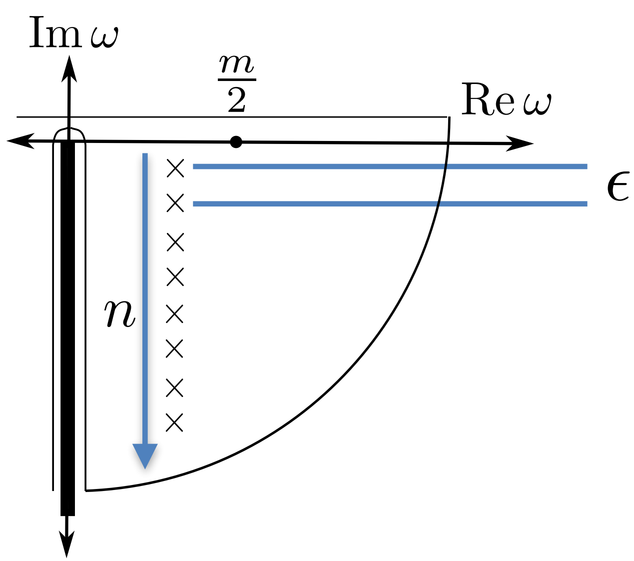


$$G_{\mathrm{QNM}} \sim \sum_{\omega_{lmn}} e^{-i\omega V} \frac{R^{\mathrm{in}}_{lm\omega}(r') R^{\mathrm{up}}_{lm\omega}(r)}{2i\omega \partial_\omega A_{\mathrm{in}}}$$

Near-horizon response of near-extremal holes



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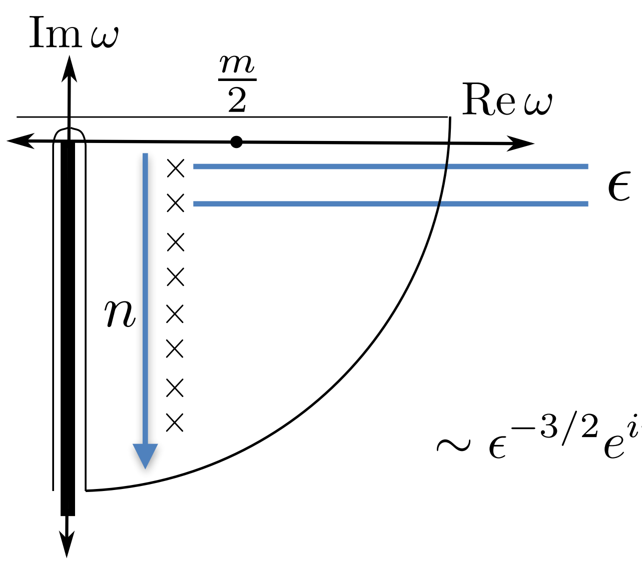

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$$\sim \epsilon^{-3/2} e^{im\Omega_H V} e^{-\epsilon V/2} \sum_n \frac{e^{-n\epsilon V}\,{}_2F_1(\alpha_+, \alpha_-, \alpha_+ - n; 4x/\epsilon)}{n!\Gamma(\alpha_+ - n)}$$

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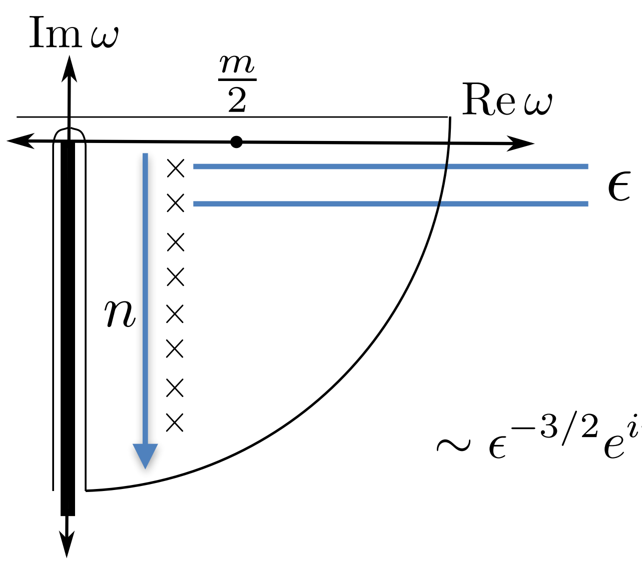

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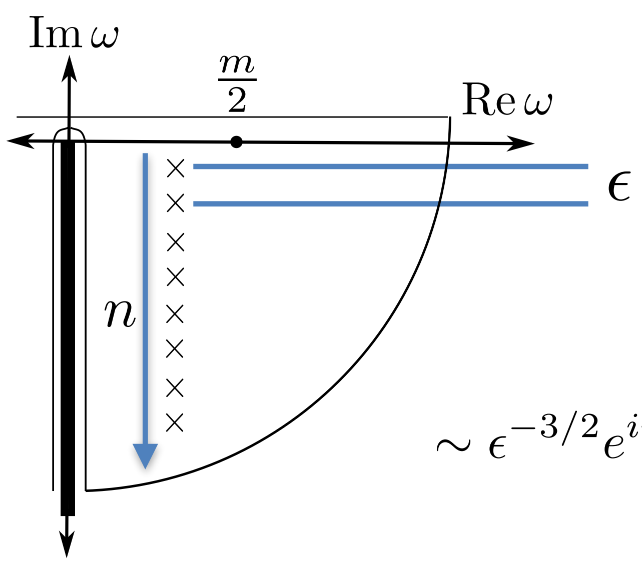
$$|G_{\text{QNM}}| \sim \epsilon^{-1/2-s} e^{-\epsilon v} \left(1 + e^{-\epsilon v}\right)^{-1/2+s} \left[1 + \frac{4x}{\epsilon}\left(1 - e^{-\epsilon v}\right)\right]^{-1/2-s}$$

Near-horizon response of near-extremal holes



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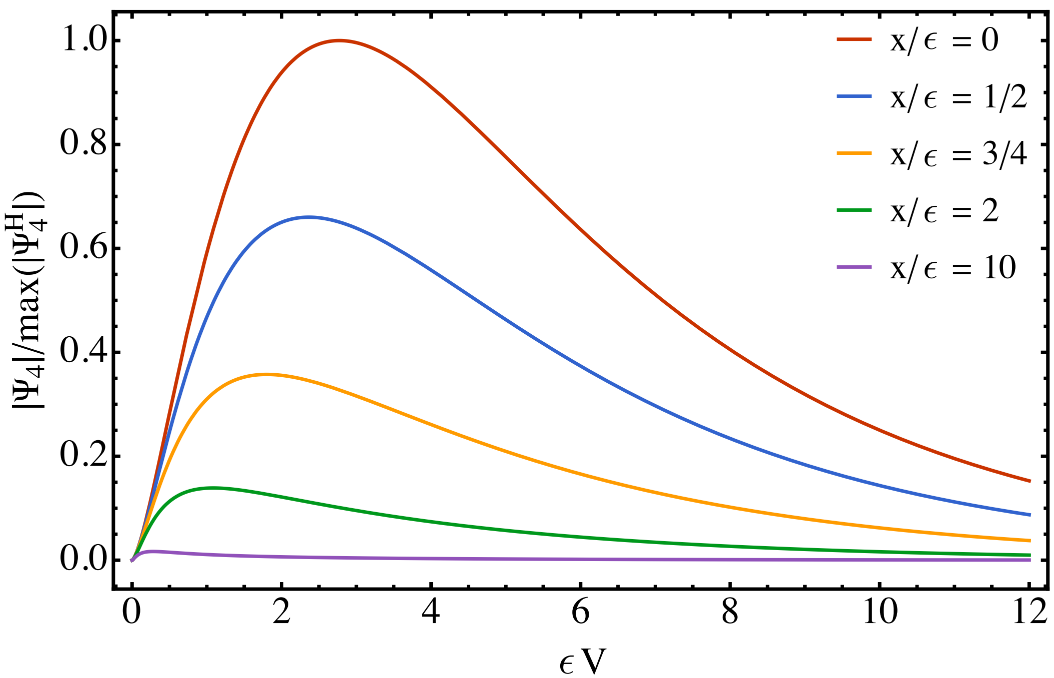
$$|G_{\mathrm{QNM}}| \sim \begin{cases} V^{3/2}\left(1 + \dfrac{Vx}{4}\right)^{-3/2} & V \ll 1/\epsilon \\ \epsilon^{-3/2} e^{-\epsilon V/2}\left(1 + \dfrac{4x}{\epsilon}\right)^{-3/2} & V \gg 1/\epsilon \end{cases}$$

Near-horizon response

- Collective response: transient QNM growth
- For grav and EM perts, early growth to large amplitudes

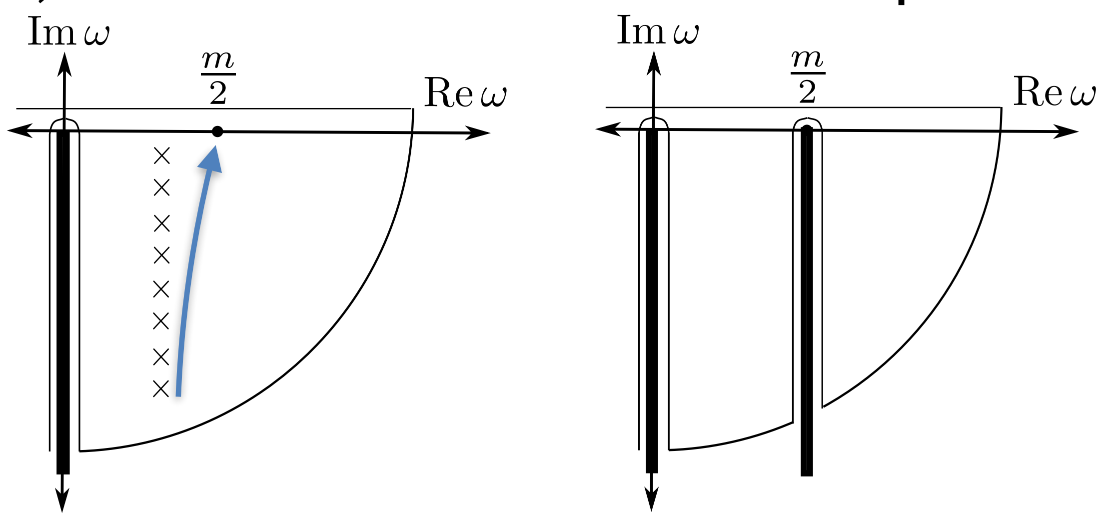
  Grav $\sim \epsilon^{-3/2}$ EM $\sim \epsilon^{-1/2}$

- Scalar fields only decay, but r derivs grow transiently



Instability through modes

- Smooth transition to horizon instability
- As $\epsilon \to 0$, modes collect into a branch point



- Casals, Gralla, P. Zimmerman (2016): Branch point gives power law response
- Radial der: unbounded growth for scalars (m = l)

$$|\partial_x^d G_{\text{late}}(x=0)| \sim v^{d+s-1/2}$$

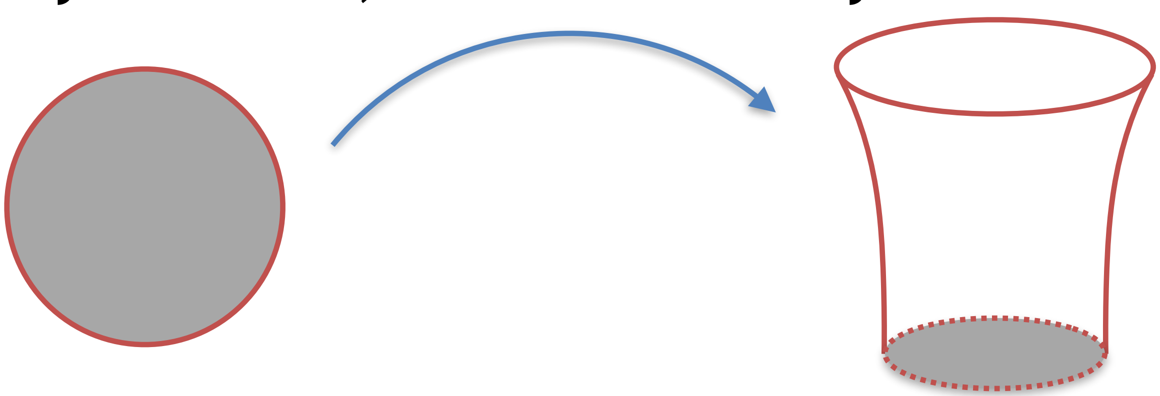
Physical picture: Near horizon extremal Kerr

- Extremal black holes hide a lot at the horizon
- Make a coord transform

$$\bar{v} = \frac{2\epsilon v}{M} \qquad \bar{x} = \frac{x}{4\epsilon} \qquad \bar{\phi} = \tilde{\phi} - \Omega_H v \qquad \epsilon \to 0$$

$$ds^2 = -\bar{x}(\bar{x}+2)d\bar{v}^2 + 2d\bar{v}d\bar{x} + 4(d\bar{\phi} + (\bar{x}+1)d\bar{v})^2$$

- Result: NHEK spacetime, not asymp flat
- More symmetries, Kerr/CFT duality

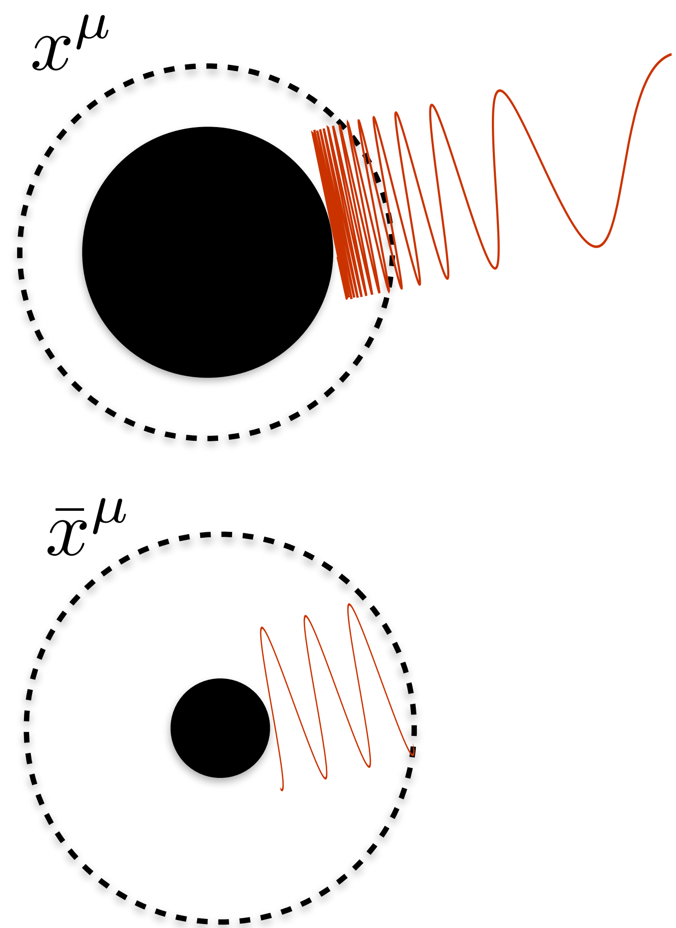


Bardeen, Press, Teukolsky (1972);Bardeen, Horowitz (1999); Guica et al. (2009); many others…

Physical picture: Nearly extremal Kerr

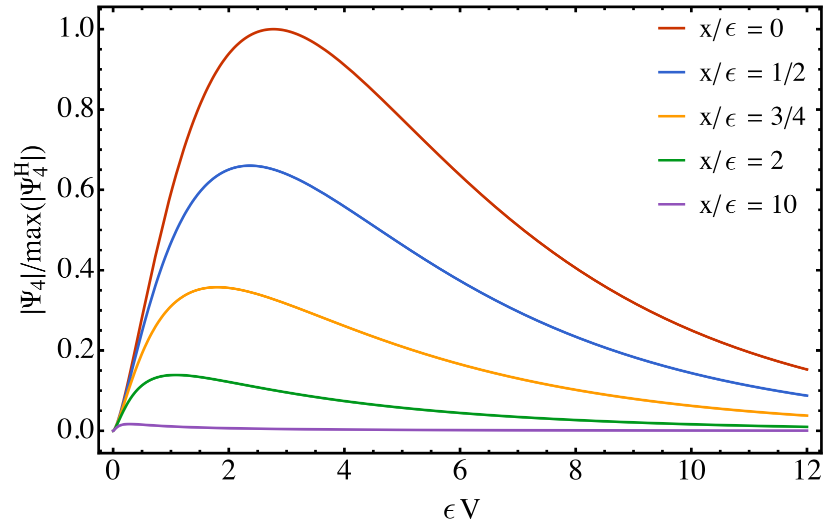
- Imprint of NHEK remains when $\epsilon \ll 1$
- Near horizon region has natural coords $\bar{x}^\mu$
- Transform $x^\mu \to \bar{x}^\mu$ large
- Finite perts in $\bar{x}^\mu$ have steep gradients in x^μ
- GF: regions are connected

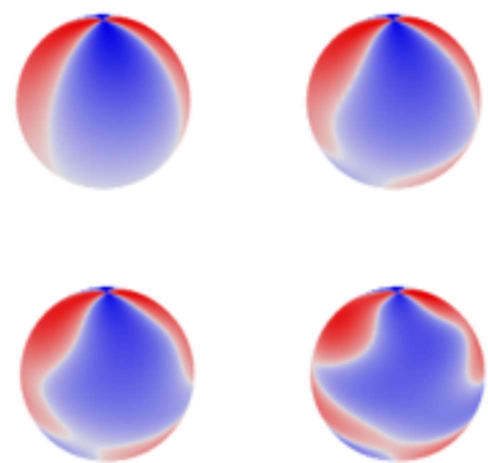
$$G_{\rm QNM} = \sum_{lm} \epsilon^{-3/2} \mathcal{G}_{lm}(\bar{x}^\mu, x^{\mu\prime})$$



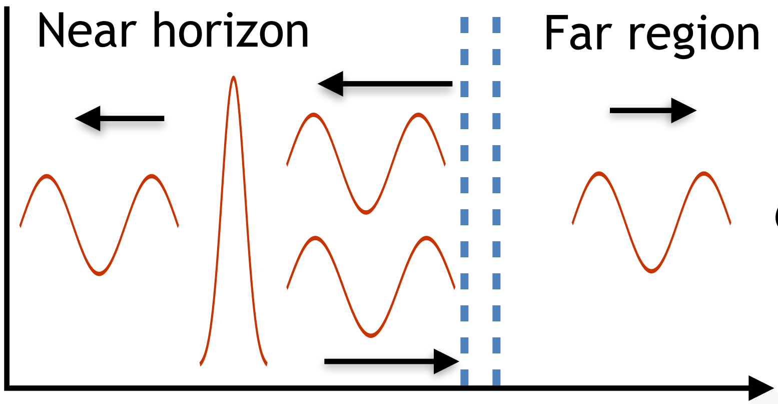
Possible consequences

- Infalling probes generically see large fields: BHs as particle accelerators
- EM fields grow near BH, drive unique dynamics?
- Grav perts enhanced, stronger backreaction?
- Enhance grav turbulence?





Near response to near perts



$$G_{\mathrm{QNM}} \sim \sum_{\omega_{lmn}} e^{-i(\omega - m\Omega_H)V} \frac{R^{\mathrm{in}}_{lm\omega}(r')R^{\mathrm{up}}_{lm\omega}(r)}{2i\omega\partial_\omega A_{\mathrm{in}}}$$

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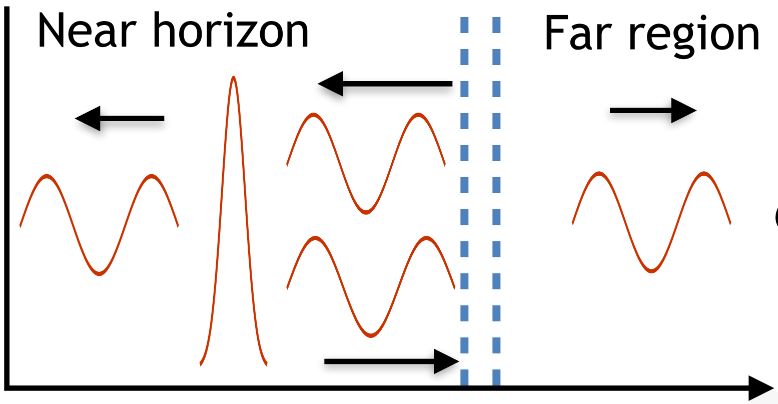

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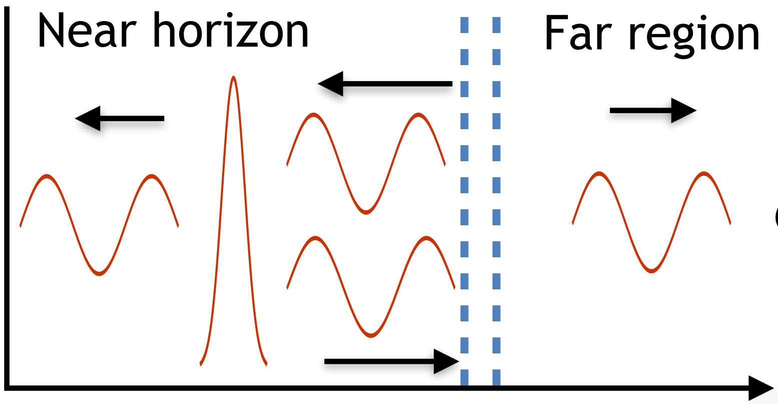
$$|G_{\rm QNM}| \sim [\bar{x}'(1+\bar{x}')]^s e^{-\bar{V}/4}$$
$$\times \sum_{n=0}^{\infty} \frac{\Gamma(2i\delta - n)}{\Gamma(\alpha_+ - n)^2 n!}(-e^{-n\bar{V}/2})\, {}_2F_1(\alpha_+, \alpha_-, \alpha_+ - n, -\bar{x}')$$

Near response to near perts



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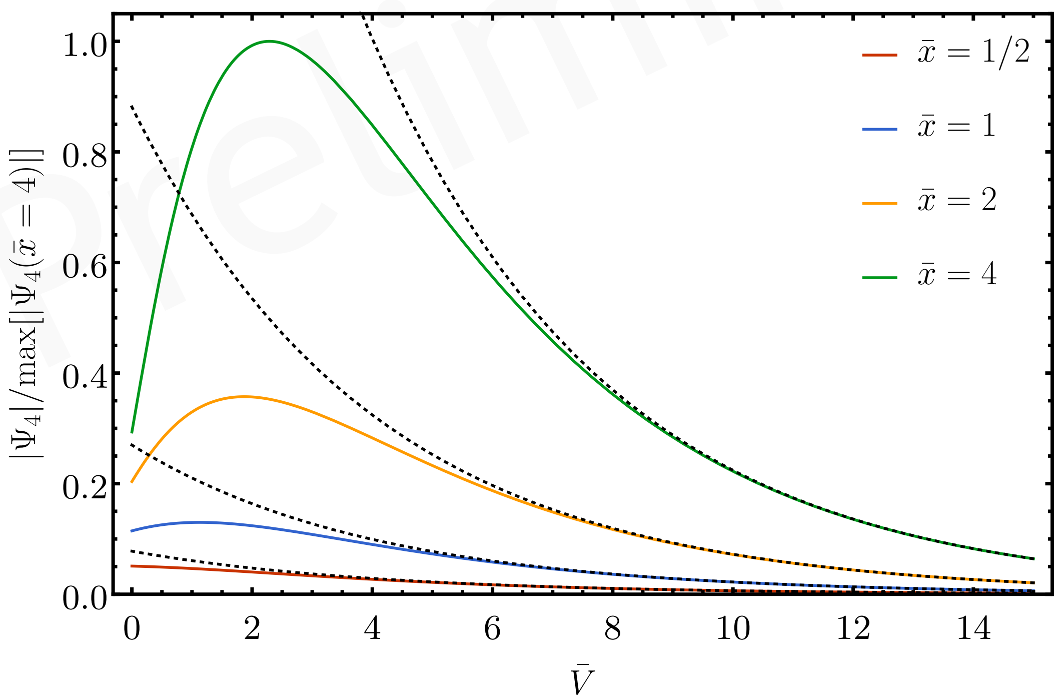
$$\sim \left(\frac{\bar{x}'}{1+\bar{x}'}\right)^s e^{-s\bar{V}/4} z^{1-\alpha_+}\,{}_2F_1(1-\alpha_+, 1-\alpha_+, 1-2i\delta, z)$$

$$z = \frac{e^{-\bar{V}/2}}{1+\bar{x}'(1-e^{-\bar{V}/2})}$$

Near response to near perts

- Response is regular in near-horizon coordinates
- Growth rates initially to the power 2s
- Again scalars decay but EM, grav fields grow
- s < 0 and axisymmetric cases still TBD



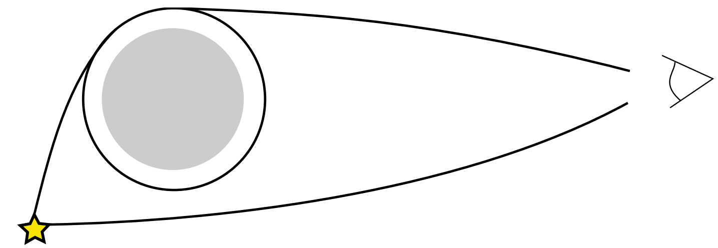
Summary

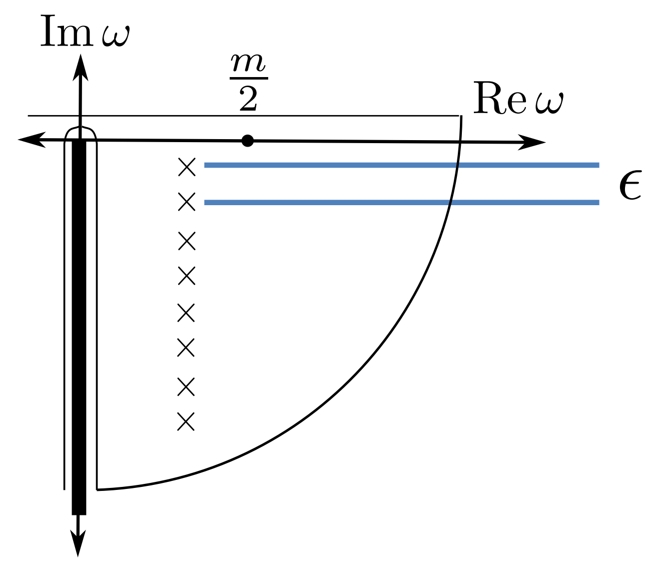
- Perts of nearly extremal BHs experience transient growth near horizon
- QNM perspective: consequence of collective oscillation of many modes
- Physical picture: result of (almost) singular map between near-horizon region and asymptotic observers
- Outlook: many potential consequences to explore
- Near-near case under active investigation

EXTRAS

Scalar response: Far observer

$$G_{\mathrm{QNM}} \sim \sum_n \frac{e^{-i\omega_n(t - r_* - r'_*)}}{d\mathcal{W}/d\omega|_{\omega_n}}$$





Glampedakis, Andersson (2001)
Yang, AZ et al (2013)

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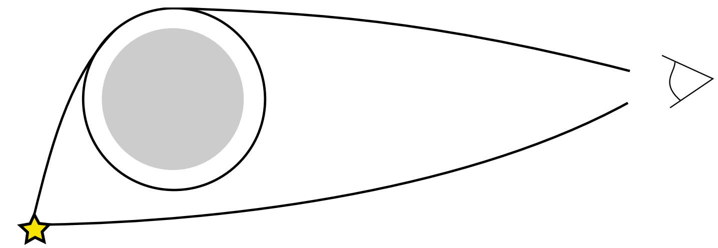

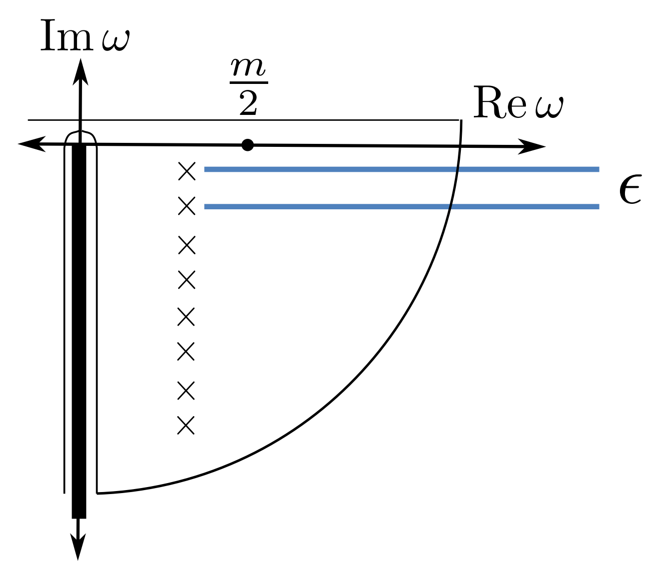

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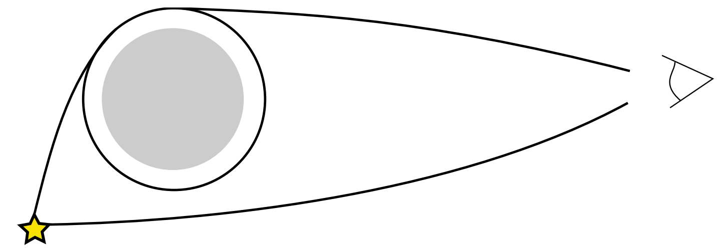

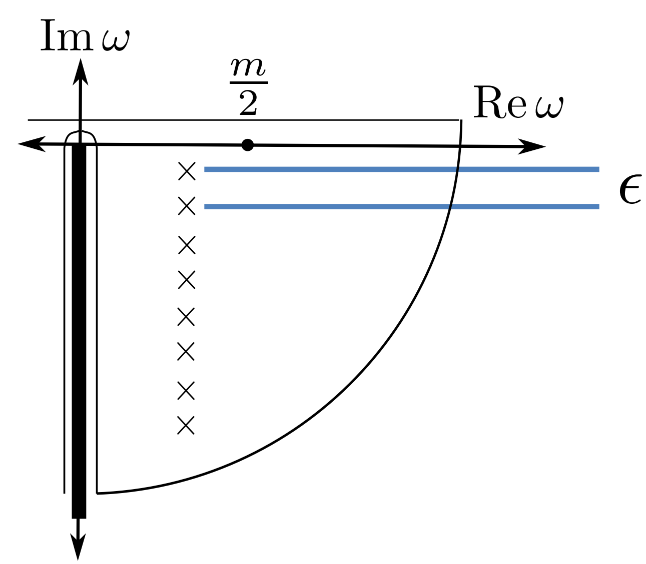

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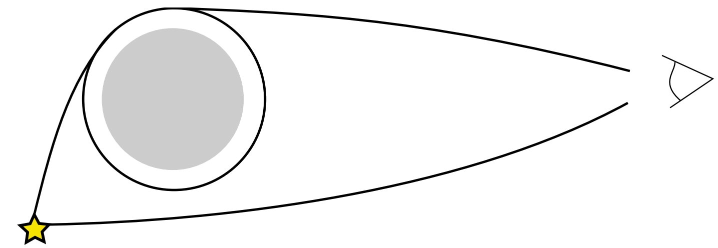

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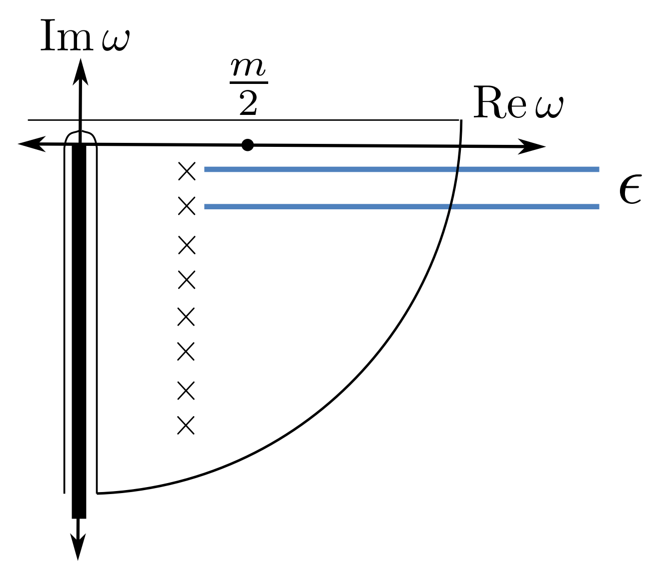

- Sum gives a surprise

$$G_{\mathrm{QNM}} \sim e^{-im\Omega_H T} \frac{\epsilon\, e^{-\epsilon T/2}}{1 - e^{-\epsilon T}}$$

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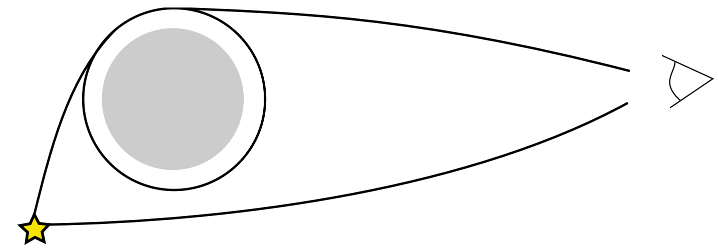

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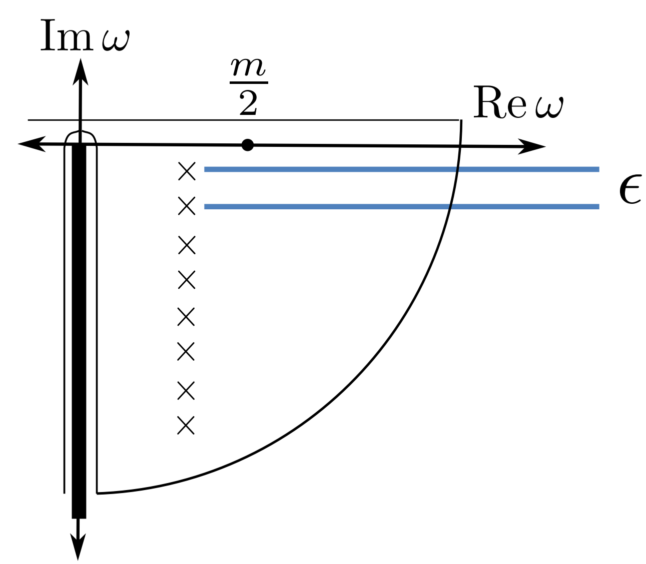

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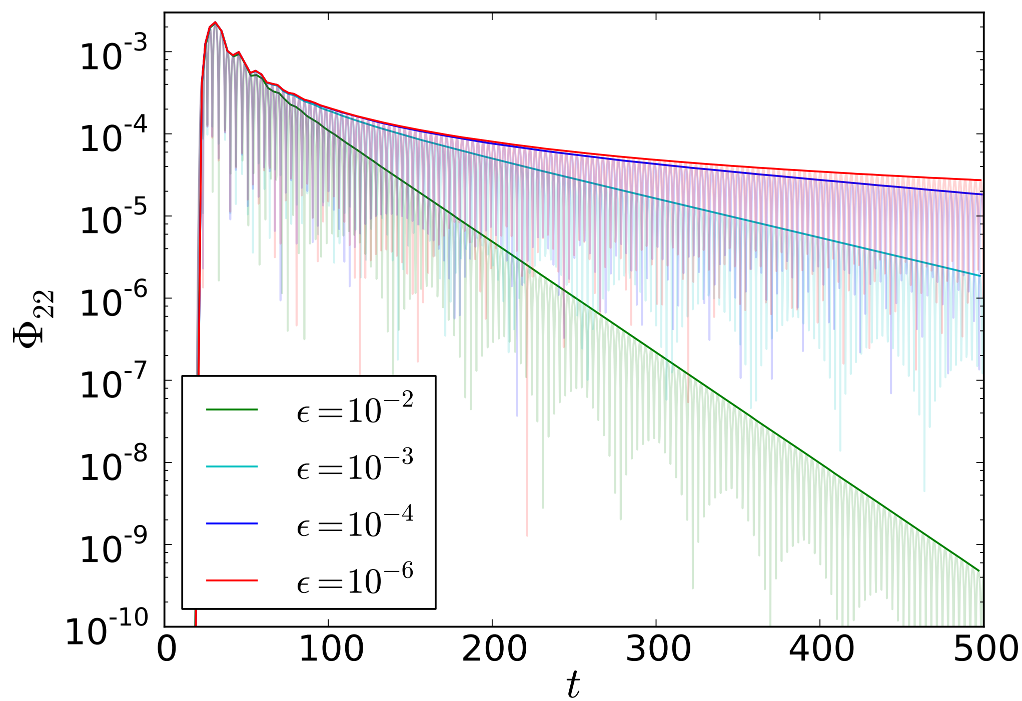
$$\sim \begin{cases} e^{-im\Omega_H T}\, T^{-1} & T \ll 1/\epsilon \\ \epsilon e^{-im\Omega_H T - \epsilon T/2} & T \gg 1/\epsilon \end{cases}$$

Glampedakis, Andersson (2001)
Yang, AZ et al (2013)

Power law ringdown

- This collective excitation provides a unique ringdown
- Initially a power-law decay
- Slowest mode takes over at end



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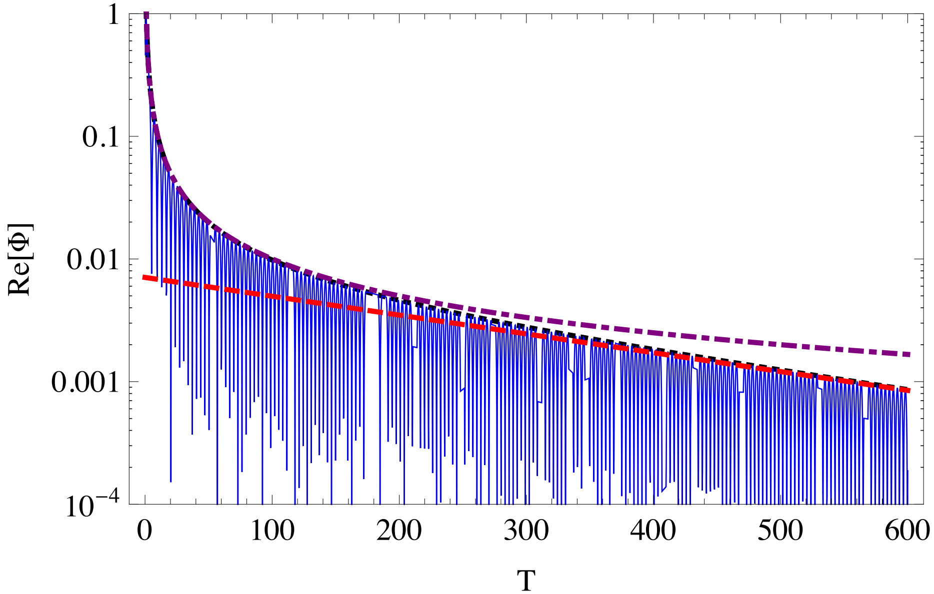

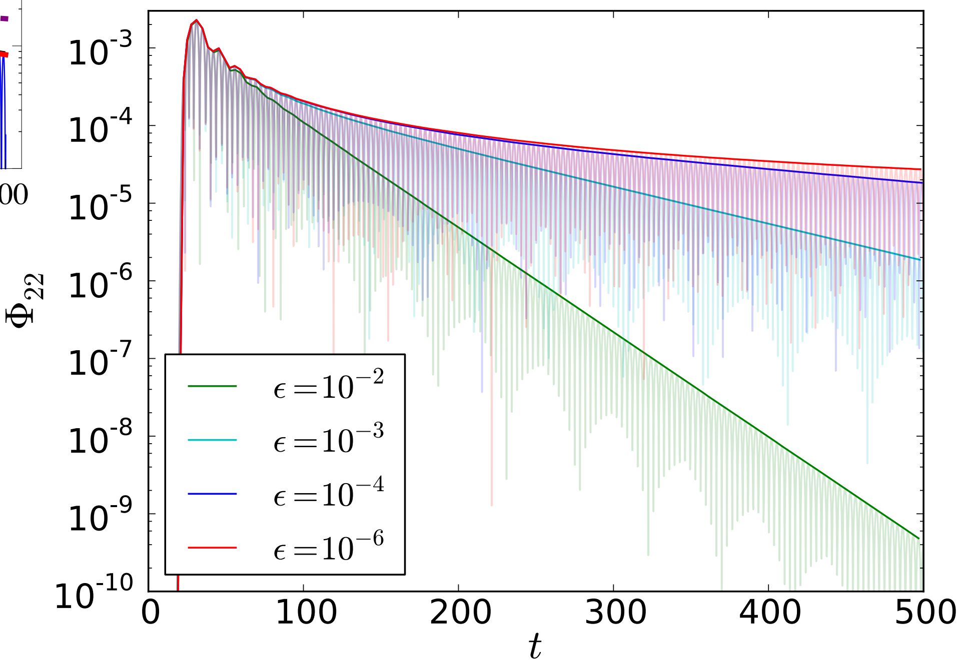

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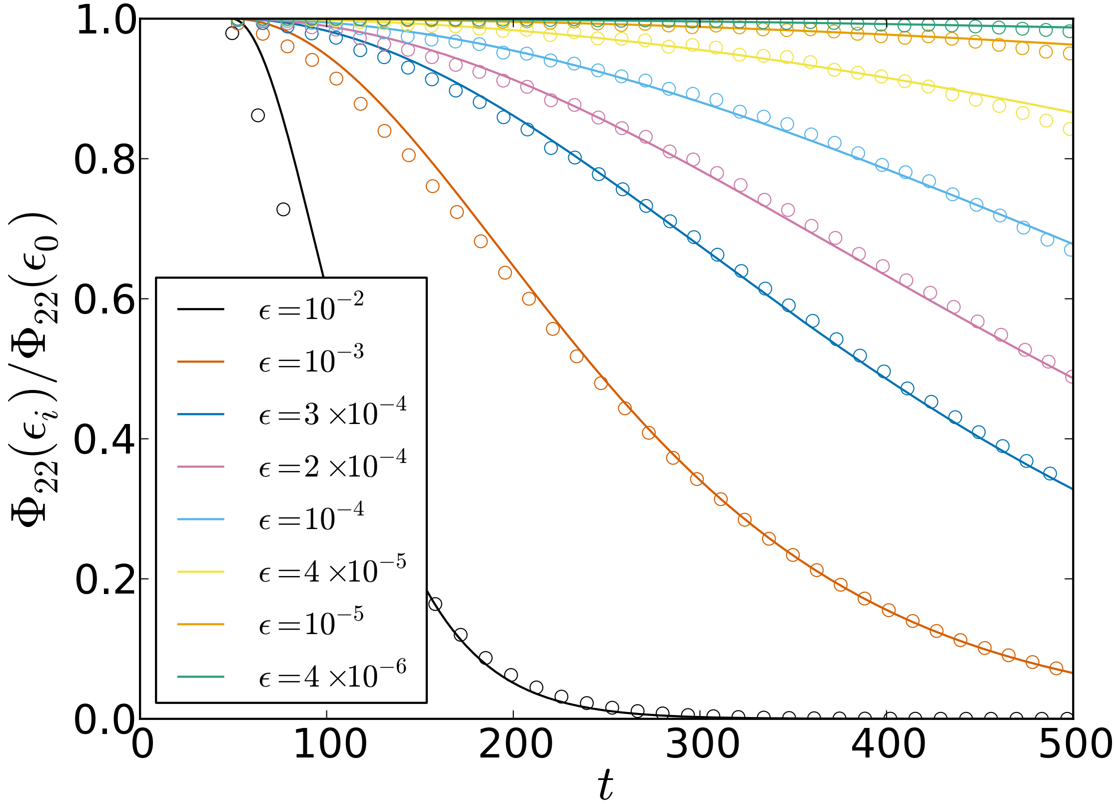
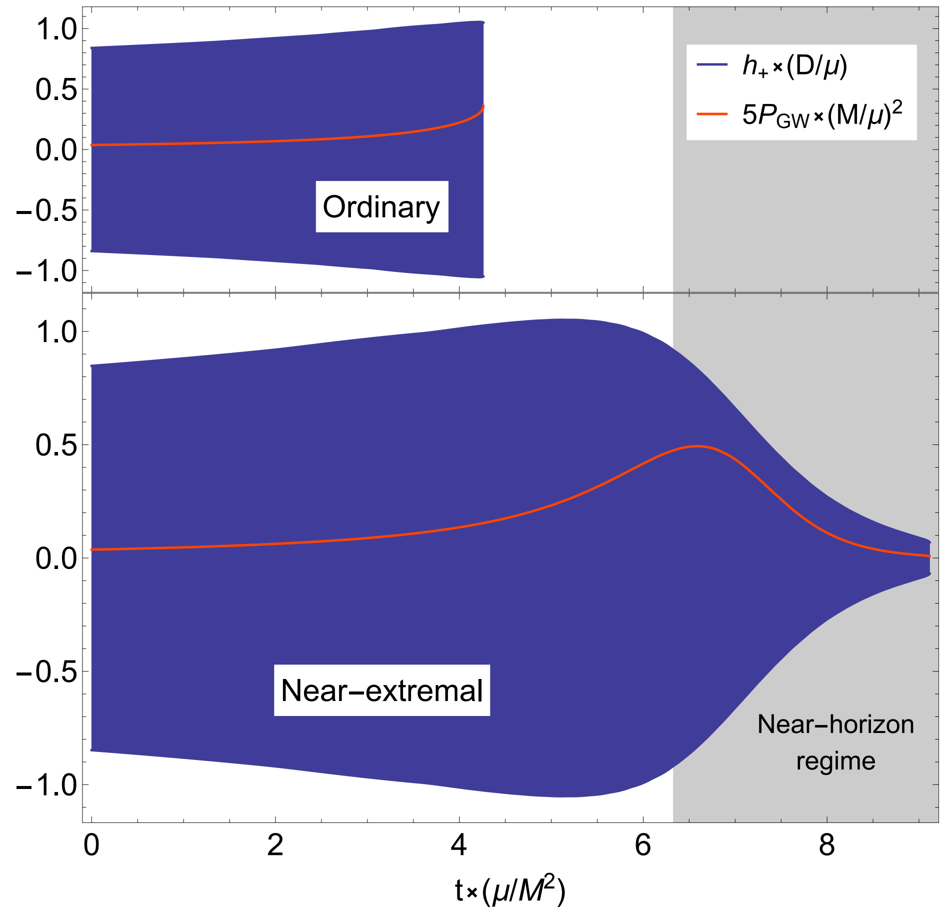


Yang, AZ et al (2013)

Strange chirps, power law from plunge

- Related effects seen in the inspiral and plunge of test particles
- Farway observers: particle locks onto the horizon, redshifts away



Gralla et al (2016)

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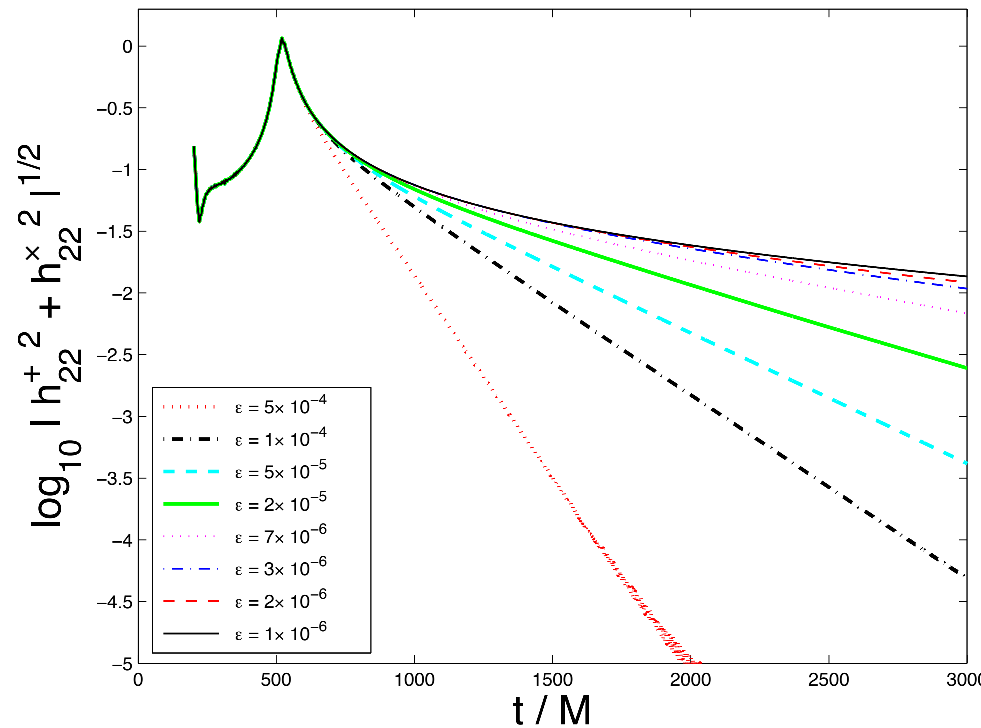
- Related effects seen in the inspiral and plunge of test particles
- Farway observers: particle locks onto the horizon, redshifts away



Burko, Khanna (2016)