PROGRESS AT THE INTERFACE BETWEEN EFFECTIVE ONE BODY THEORY AND THE SMALL MASS RATIO APPROXIMATION

Andrea Antonelli,

Max Planck Institute for Gravitational Physics (Albert Einstein Institute,AEI), Potsdam-Golm.

In collaboration with:

M. van de Meent, A. Buonanno, J. Steinhoff, J.Vines

21st Capra Meeting on Radiation Reaction in General Relativity@ AEI

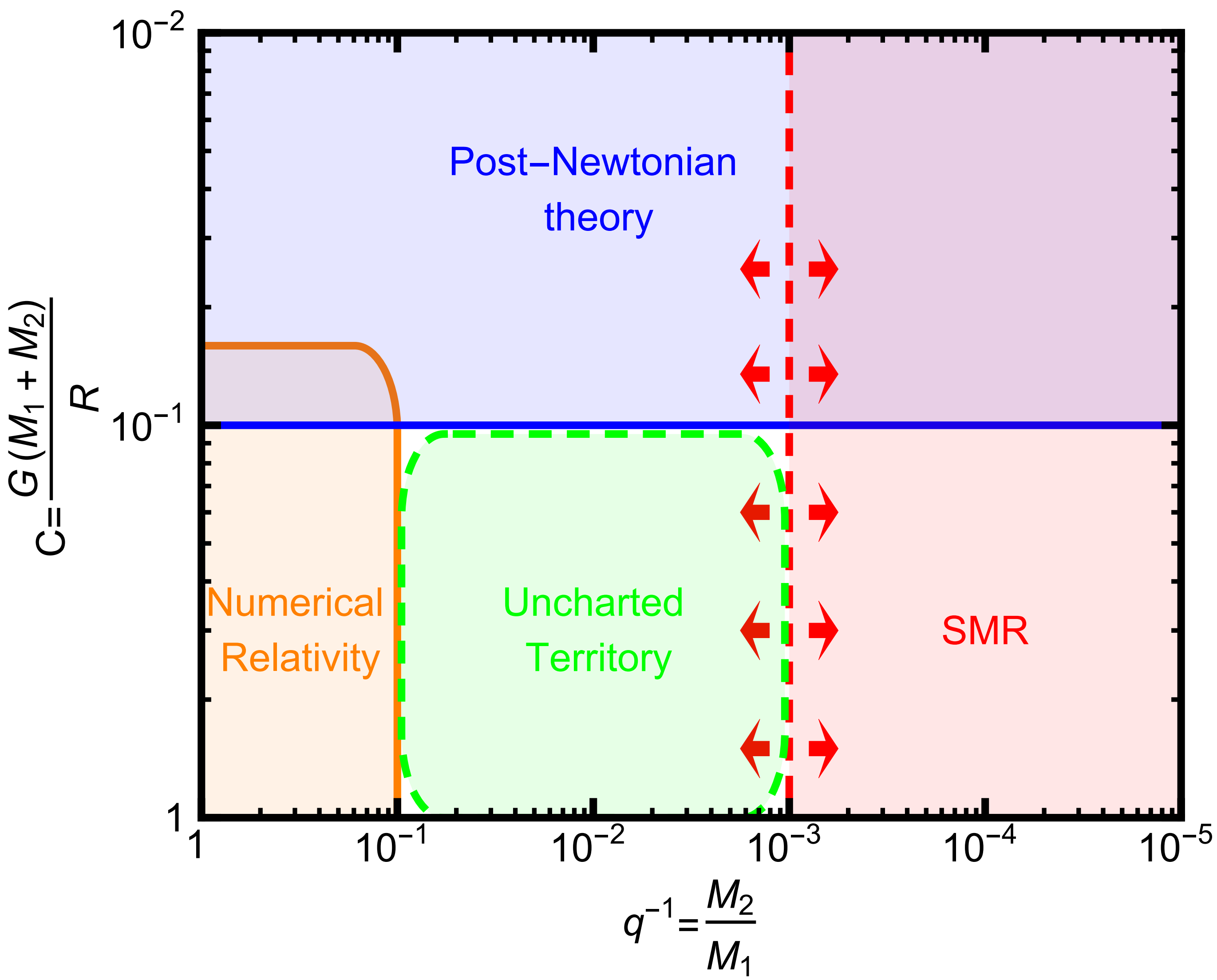
26/06/2018

MAX-PLANCK-GESELLSCHAFT

MOTIVATIONS

- Various approximations to the two body problem [Post-Newtonian (PN), Post-Minkowskian (PM) and Small Mass Ratio (SMR)] have different domains of validity in the "compactness - mass ratio" parameter space.

- The Effective One Body (EOB) theory can extend these domains of validity.



- SMR terms linear in the symmetric mass ratio $\nu = \frac{q}{(q+1)^2}$, but at very high PN orders have been included in the EOB Hamiltonians. [Bini, Damour, Geralico, Kavanagh, …]

- We want a Hamiltonian which is not PN truncated and that contains information at linear order in ν. [Akcay, Barausse, Buonanno, Damour, Le Tiec, van de Meent,…]

CURRENT EOB HAMILTONIAN FOR NON-SPINNING BLACK HOLES

- The EOB theory is based on an energy map linking the real two-body problem to an effective one:

$$H_{EOB} = M\sqrt{1+2\nu\left(\frac{H_{Eff}}{\mu}-1\right)}$$

$$G = c = 1$$

$$\frac{H_{Eff}}{\mu} = \sqrt{A(u,\nu)[1+p_\phi^2 u^2 + A(u,\nu)D(u,\nu)^{-1}p_r^2 + Q(u,p_r,\nu)]}$$

$M = M_1 + M_2$ Total mass p_ϕ reduced angular momentum $u = 1/R$ reduced inverse radius

$\mu = M\nu$ Reduced mass p_r reduced radial momentum

- At 2PN order, the effective body moves on a geodesic of a deformed Schwarzschild spacetime [Buonanno-Damour (1998)]. At 3PN order, non-geodesic terms must be inserted in a quartic-momenta term Q. [Damour-Jaranowski-Schaefer (2000)]
- Q depends in principle on p_r and p_ϕ. In DJS2000, Q only depends on p_r. This is the DJS gauge.

THE LIGHT RING DIVERGENCE

- [Le Tiec et al.(2011), Barausse et al. (2011)] used the first law of binary black hole mechanics to calculate the linear in ν correction to the potential A(u, ν):

$$A(u,\nu) = A_{Schw} + \nu a_{SMR}[u, z_{SMR}(u)] = 1 - 2u + \nu\left[z_{SMR}(u)\sqrt{1-3u} - u\left(1 + \frac{1-4u}{\sqrt{1-3u}}\right)\right]$$

- The Detweiler redshift $z_{SMR}(u)$ incorporates the SMR data in the EOB theory.
- Possible presence of a divergence in A(u, ν) at the Schwarzschild LR ($u_{LR} = 1/3$). [Barausse et al. (2011)]

Why do we expect the procedure to lead to a divergence?

- In the circular orbit limit, p_ϕ is: $\left.\frac{\partial H_{EOB}}{\partial u}\right|_{p_r=0} = 0 \rightarrow \left.p_\phi^2\right|_{circ} \sim (1-3u)^{-1}$
- At the LR, the circular orbit binding energy $\left.E_B\right|_{circ} = \frac{\left.H_{EOB}\right|_{circ} - 1}{\nu}$ is dominated by:

$$\left.E_B\right|_{circ} \sim \left.H_{Eff}^2\right|_{p_r=0} \xrightarrow{u \to u_{LR}} a_{SMR}[u, z_{SMR}(u)] \times \left.p_\phi^2\right|_{circ} \sim (1-3u)^{-3/2} \rightarrow a_{SMR}[u, z_{SMR}(u)] \sim (1-3u)^{-1/2}$$

THE LIGHT RING DIVERGENCE

- [Le Tiec et al.(2011), Barausse et al. (2011)] used the first law of binary black hole mechanics to calculate the linear in ν correction to the potential A(u, ν):

$$A(u,\nu) = A_{Schw} + \nu a_{SMR}[u, z_{SMR}(u)] = 1 - 2u + \nu\left[z_{SMR}(u)\sqrt{1-3u} - u\left(1 + \frac{1-4u}{\sqrt{1-3u}}\right)\right]$$

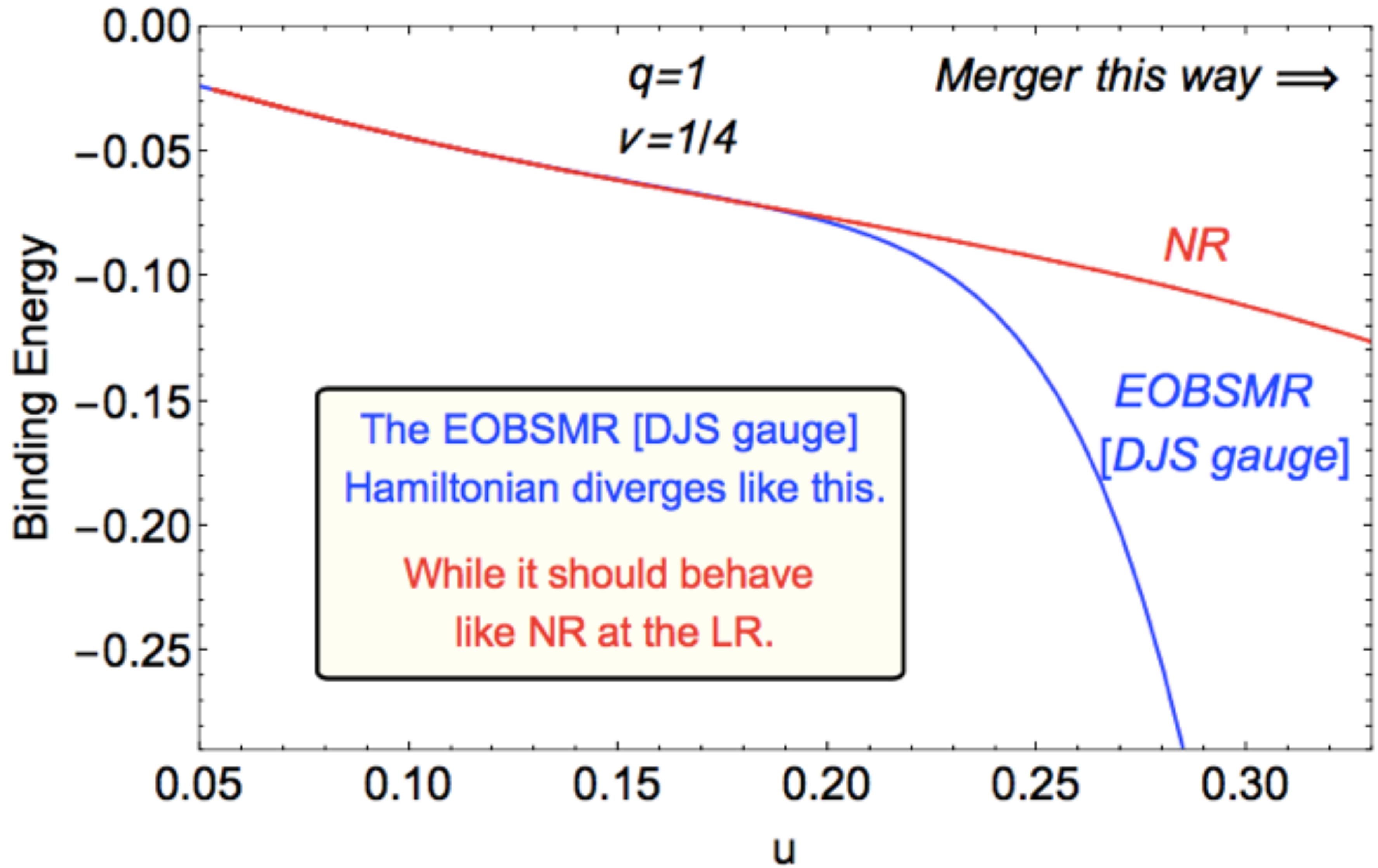
- The Detweiler redshift $z_{SMR}(u)$ incorporates the SMR data in the EOB theory.
- Possible presence of a divergence in A(u, ν) at the Schwarzschild LR ($u_{LR} = 1/3$). [Barausse et al. (2011)]

- [Akcay et al. (2012)] confirmed this divergence when data for $z_{SMR}(u)$ were made available up to the LR.

THE LIGHT RING DIVERGENCE

Why is the divergence a problem?

- The EOBSMR [DJS gauge] contains a divergence at the LR.
- Comparison with a Numerical Relativity (NR) simulation from [Ossokine et al. (2017)]



A NEW GAUGE

- [Damour (2017)] introduced the Energy gauge in the context of PM calculations.

$$H_{Eff} = \sqrt{H_{Schw}^2 + \delta H_{Eff}^2[u, H_{Schw}(u, p_r, p_\phi)]}$$

- The gauge depends on a new variable, the Schwarzschild Hamiltonian H_{Schw}.

In the circular orbit limit, H_{Schw} diverges at the LR, but it is regular at the LR for generic orbits.

$$H_{Schw} = \sqrt{(1-2u)[1+p_\phi^2 u^2 + (1-2u)p_r^2]} \xrightarrow[p_r=0]{p_{\phi,circ}=[u(1-3u)]^{-1/2}} H_{Schw}\big|_{circ} = \frac{1-2u}{\sqrt{1-3u}}$$

- The key idea is to push the divergence onto H_{Schw}, so to recover it only in the circular orbit limit, where we physically expect it.

ABSORBING THE LIGHT RING DIVERGENCE

- The fit for the Detweiler redshift from [Akcay et al. (2012)] has the form:

$$z_{SMR} = \frac{1}{(1-3u)^{3/2}}\left[z_0(u) + z_1(u)\sqrt{1-3u} + z_2(u)\ln\left(\frac{(1-2u)^2}{1-3u}\right)\right]$$

- Since $H_{Schw}\big|_{circ} = \frac{1-2u}{\sqrt{1-3u}}$, we propose the following Hamiltonian:

$$H^2_{Eff} = H^2_{Schw} + (1-2u)\nu\left[X_0 H^3_{Schw} + X_1 H^2_{Schw} + X_2 H^3_{Schw} \ln(H^2_{Schw})\right]$$

1) Calculate linear in ν, circular orbit binding energy as a function of frequency.
2) Equate to binding energy from [Le Tiec et al. (2011)] at fixed frequency.
3) Impose that the X_i coefficients are smooth at the LR, in order to get:

$$X_0 = \frac{z_0(u) - (1-4u)u}{(1-2u)^3}$$

$$X_1 = \frac{z_1(u) - u}{(1-2u)^2}$$

$$X_2 = \frac{z_2(u)}{(1-2u)^3}$$

The X_i coefficients are regular at the LR.

$\Longrightarrow$ The divergence has been absorbed by the Schwarzschild Hamiltonians.

EVOLUTION OF THE MODEL

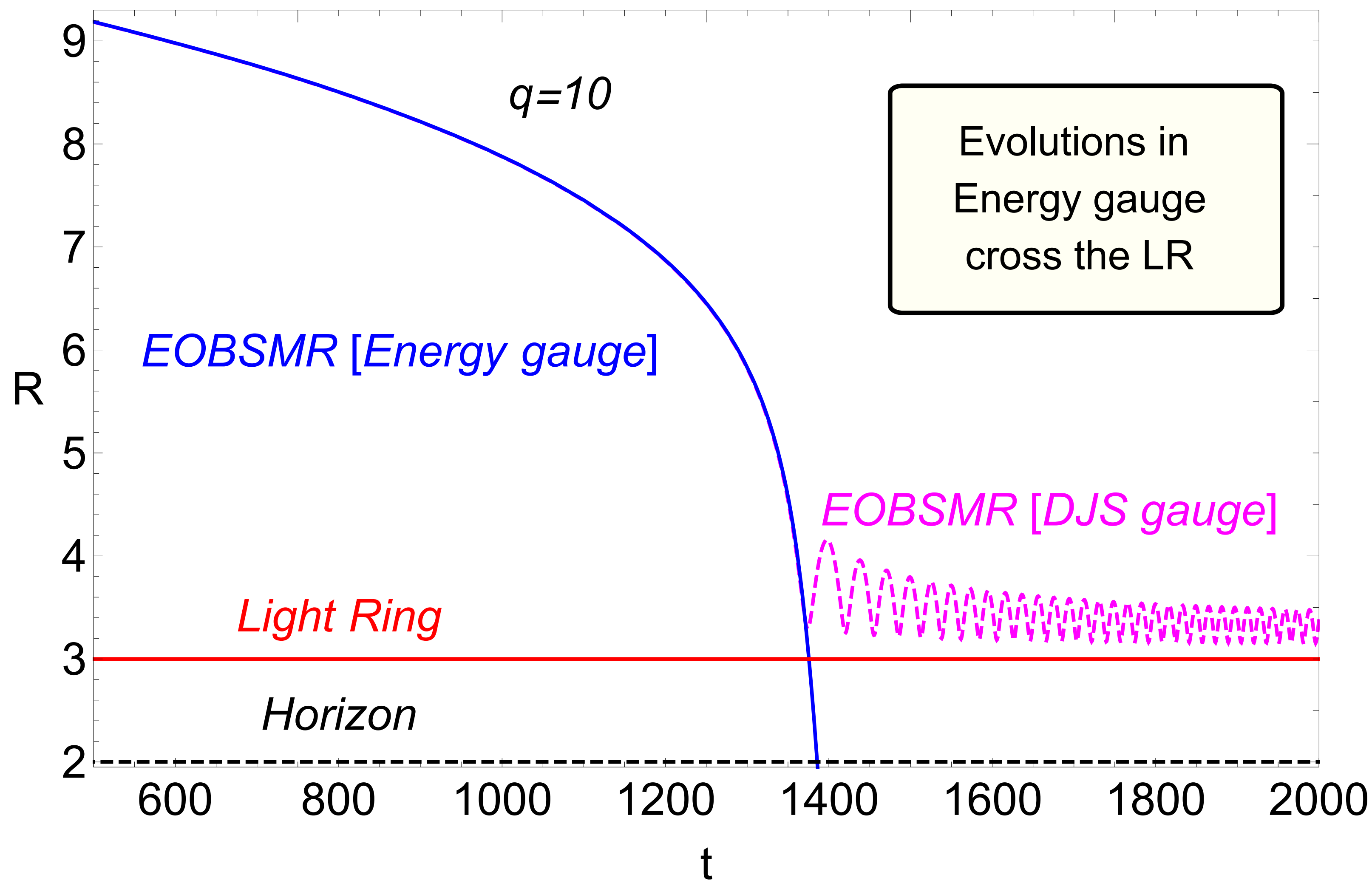
- We evolve EOB Hamiltonians via the Hamilton equations (with EOB flux F_ϕ):

1) $\frac{dR}{dt} = \frac{A(R)}{\sqrt{D(R)}} \frac{\partial H_{EOB}}{\partial p_{R*}}$ 2) $\Omega = \frac{d\phi}{dt} = \frac{\partial H_{EOB}}{\partial p_\phi}$ 3) $\frac{dp_{R*}}{dt} = -\frac{A(R)}{\sqrt{D(R)}} \frac{\partial H_{EOB}}{\partial R} + F_\phi \frac{p_{R*}}{p_\phi}$ 4) $\frac{dp_\phi}{dt} = F_\phi$

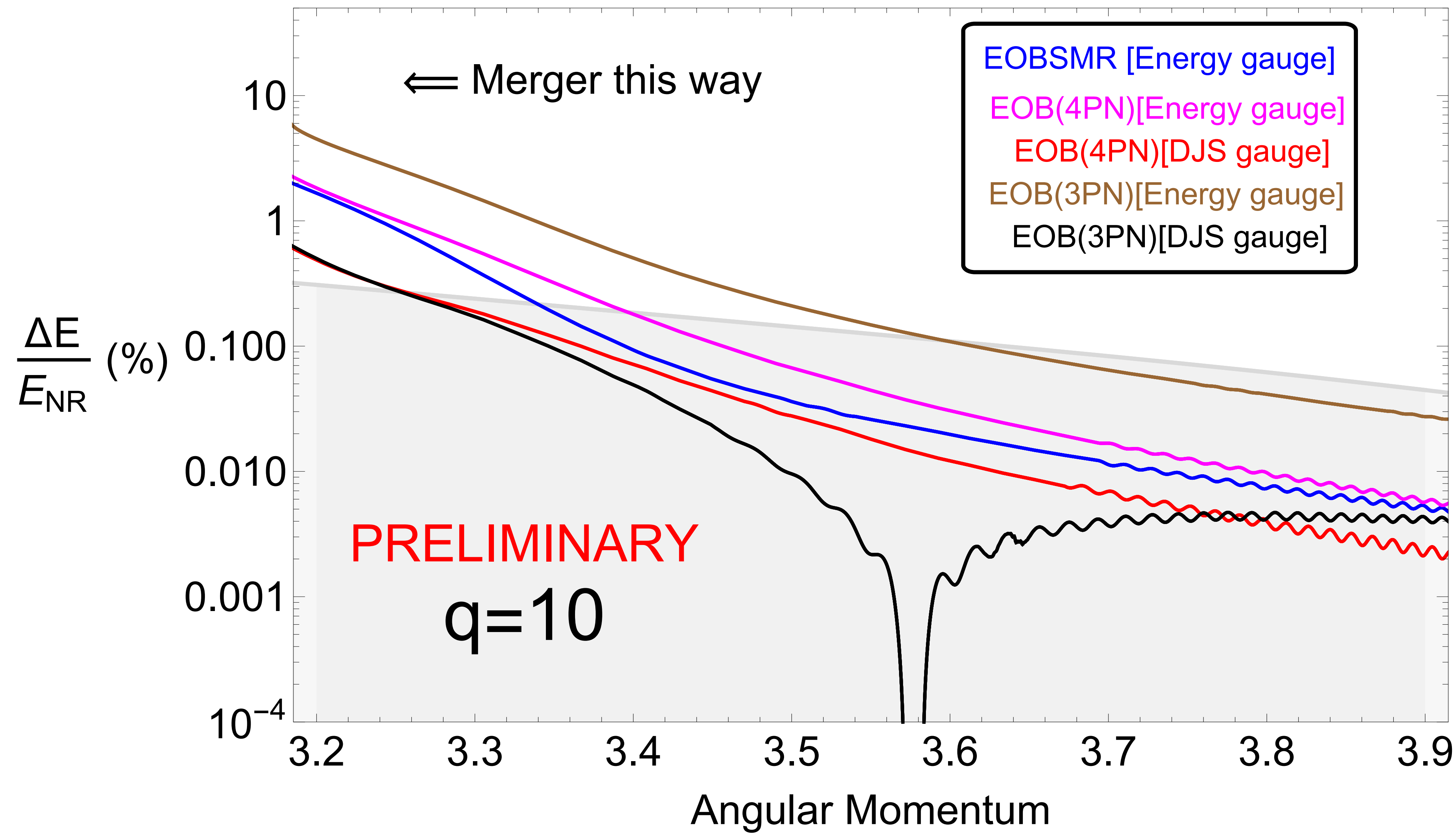
- Here the radius $R = 1/u$ is used. The radial momentum is calculated in tortoise coordinates.

- We use R(t) as a proxy for the behaviour of the EOBSMR dynamics at the LR.

- The EOBSMR [DJS gauge] Hamitonian is the one analytically calculated in [Barausse et al. (2012)].

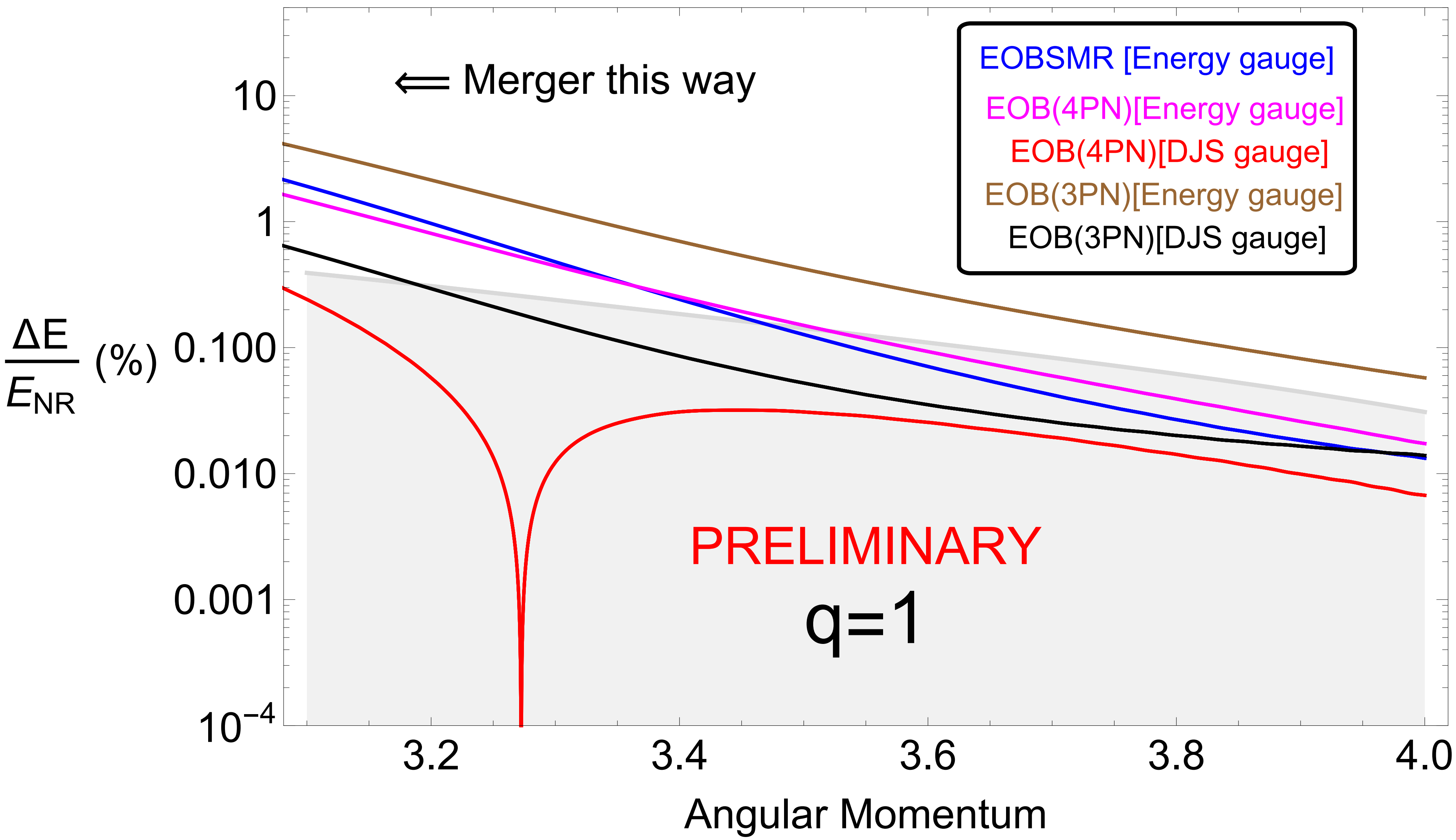


BINDING ENERGY VS NR



- We compare the **fractional difference of energy** $\Delta E_{bind} / E_{NR}(\%)$ until merger between the EOB and NR as a function of the angular momentum.
- We stop the evolution at the Schwarzschild LR.
- NR data for the binding energy from [Ossokine et al. (2017)].

BINDING ENERGY VS NR



- We compare the **fractional difference of energy** $\Delta E_{bind} / E_{NR} (\%)$ until merger between the EOB and NR as a function of the angular momentum.
- We stop the evolution at the Schwarzschild LR.
- NR data for the binding energy from [Ossokine et al. (2017)].

CONCLUSIONS

WHAT WAS DONE:

- We built a first example of EOB Hamiltonian informed by the SMR approximation that contains terms linear in ν. The Hamiltonian can be evolved smoothly through the LR.
- We found that the EOBSMR differs by around 2% from NR at merger.
- We found that EOB binding energy performs slightly better against NR when the DJS gauge, instead of the Energy gauge, is used.

TO DO:

- Better fit for the redshift, with new SMR data from M. van de Meent.
- Include higher orders in ν from PN expansion in the Energy gauge.
- Compare the Hamiltonian to a larger set of NR data to assess its accuracy.
- Build an EOBNR-SMR waveform model based on the new gauge.