Overlap of self-force, post-Newtonian and effective-one-body approaches

Chris Kavanagh
Institut des Hautes Études Scientifique
Bures S/Yvette, Paris.

Overview

Effective-one-body

…an attempted overview.

SF<—>EOB

(Gauge invariants)

EMRI evolution with EOB

Conservative PN

Conservative PN 2-body dynamics can be described using a Hamiltonian

$$\mathcal{H} = \mathcal{H}_N + \frac{1}{c^2}\mathcal{H}_{1\mathrm{PN}} + \frac{1}{c^4}\mathcal{H}_{2\mathrm{PN}} + \ldots$$

EOB: Mapping the two systems

2-body Taylor-expanded N + 1PN + 2PN Hamiltonian

$$H_{\rm N}(\mathbf{x}_a, \mathbf{p}_a) = \frac{\mathbf{p}_1^2}{2m_1} - \frac{1}{2}\frac{Gm_1m_2}{r_{12}} + (1 \leftrightarrow 2)$$

$$c^2 H_{\rm 1PN}(\mathbf{x}_a, \mathbf{p}_a) = -\frac{1}{8}\frac{(\mathbf{p}_1^2)^2}{m_1^3} + \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(-12\frac{\mathbf{p}_1^2}{m_1^2} + 14\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1m_2} + 2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right) + \frac{1}{4}\frac{Gm_1m_2}{r_{12}}\frac{G(m_1+m_2)}{r_{12}} + (1 \leftrightarrow 2),$$

$$\begin{aligned} c^4 H_{\rm 2PN}(\mathbf{x}_a, \mathbf{p}_a) = {} & \frac{1}{16}\frac{(\mathbf{p}_1^2)^3}{m_1^5} + \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\left(5\frac{(\mathbf{p}_1^2)^2}{m_1^4} - \frac{11}{2}\frac{\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2m_2^2} - \frac{(\mathbf{p}_1\cdot\mathbf{p}_2)^2}{m_1^2m_2^2} + 5\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right. \\ & \left. - 6\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2m_2^2} - \frac{3}{2}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2m_2^2}\right) \\ & + \frac{1}{4}\frac{G^2m_1m_2}{r_{12}^2}\left(m_2\left(10\frac{\mathbf{p}_1^2}{m_1^2} + 19\frac{\mathbf{p}_2^2}{m_2^2}\right) - \frac{1}{2}(m_1+m_2)\frac{27(\mathbf{p}_1\cdot\mathbf{p}_2) + 6(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\right) \\ & - \frac{1}{8}\frac{Gm_1m_2}{r_{12}}\frac{G^2(m_1^2 + 5m_1m_2 + m_2^2)}{r_{12}^2} + (1 \leftrightarrow 2), \end{aligned}$$

EOB: Mapping the two systems

2-body Taylor-expanded 3PN Hamiltonian [JS 98, DJS 01]

$$
\begin{aligned}
c^6 H_{3\mathrm{PN}}(\mathbf{x}_a, \mathbf{p}_a) = & -\frac{5}{128}\frac{(\mathbf{p}_1^2)^4}{m_1^7} + \frac{1}{32}\frac{Gm_1m_2}{r_{12}}\Bigg(-14\frac{(\mathbf{p}_1^2)^3}{m_1^6} + 4\frac{((\mathbf{p}_1\cdot\mathbf{p}_2)^2 + 4\mathbf{p}_1^2\mathbf{p}_2^2)\mathbf{p}_1^2}{m_1^4 m_2^2} + 6\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^4 m_2^2} \\
& - 10\frac{(\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2 + \mathbf{p}_2^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2)\mathbf{p}_1^2}{m_1^4 m_2^2} + 24\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^4 m_2^2} \\
& + 2\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3 m_2^3} + \frac{(7\mathbf{p}_1^2\mathbf{p}_2^2 - 10(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2^3} \\
& + \frac{(\mathbf{p}_1^2\mathbf{p}_2^2 - 2(\mathbf{p}_1\cdot\mathbf{p}_2)^2)(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2^3} + 15\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^3 m_2^3} \\
& - 18\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3 m_2^3} + 5\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)^3}{m_1^3 m_2^3}\Bigg) + \frac{G^2 m_1 m_2}{r_{12}^2}\Bigg(\frac{1}{16}(m_1 - 27m_2)\frac{(\mathbf{p}_1^2)^2}{m_1^4} \\
& - \frac{115}{16}m_1\frac{\mathbf{p}_1^2(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1^3 m_2} + \frac{1}{48}m_2\frac{25(\mathbf{p}_1\cdot\mathbf{p}_2)^2 + 371\mathbf{p}_1^2\mathbf{p}_2^2}{m_1^2 m_2^2} + \frac{17}{16}\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^3} + \frac{5}{12}\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^4}{m_1^3} \\
& - \frac{1}{8}m_1\frac{(15\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2) + 11(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1))(\mathbf{n}_{12}\cdot\mathbf{p}_1)}{m_1^3 m_2} - \frac{3}{2}m_1\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^3(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^3 m_2} \\
& + \frac{125}{12}m_2\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1^2 m_2^2} + \frac{10}{3}m_2\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2} \\
& - \frac{1}{48}(220m_1 + 193m_2)\frac{\mathbf{p}_1^2(\mathbf{n}_{12}\cdot\mathbf{p}_2)^2}{m_1^2 m_2^2}\Bigg) + \frac{G^3 m_1 m_2}{r_{12}^3}\Bigg(-\frac{1}{48}\left(425m_1^2 + \left(473 - \frac{3}{4}\pi^2\right)m_1m_2 + 150m_2^2\right)\frac{\mathbf{p}_1^2}{m_1^2} \\
& + \frac{1}{16}\left(77(m_1^2 + m_2^2) + \left(143 - \frac{1}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{p}_1\cdot\mathbf{p}_2)}{m_1m_2} + \frac{1}{16}\left(20m_1^2 - \left(43 + \frac{3}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)^2}{m_1^2} \\
& + \frac{1}{16}\left(21(m_1^2 + m_2^2) + \left(119 + \frac{3}{4}\pi^2\right)m_1m_2\right)\frac{(\mathbf{n}_{12}\cdot\mathbf{p}_1)(\mathbf{n}_{12}\cdot\mathbf{p}_2)}{m_1m_2}\Bigg) \\
& + \frac{1}{8}\frac{G^4 m_1 m_2^3}{r_{12}^4}\left(\left(\frac{227}{3} - \frac{21}{4}\pi^2\right)m_1 + m_2\right) + (1 \leftrightarrow 2).
\end{aligned}
$$

EOB in newtonian gravity..
very quick motivation

In Newtonian mechanics the total energy of a two body system is given by

$$E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{Gm_1 m_2}{|r_1 - r_2|}$$

$$L = m_1 r_1 \times v_1 + m_2 r_2 \times v_2$$

$$r = r_1 - r_2, \quad v = v_1 - v_2, \quad M = m_1 + m_2, \quad \mu = m_1 m_2 / M$$

$$E = \frac{1}{2}\mu v^2 - \frac{GM\mu}{r}$$

$$L = \mu r \times v$$

EOB in GR and the PN approximation

[Buonanno, Damour 1998,2000]

In GR, the natural generalisation of this would be motion in a central Schwarzschild Spacetime of mass M

$$g^{\text{eff}}_{\mu\nu} dx^\mu dx^\nu = -A(R)c^2 dT^2 + B(R)dR^2 + R^2(d\theta^2 + \sin^2(\theta)d\varphi^2)$$

- This can be used to define an effective action

$$g^{\mu\nu}_{\text{eff}} P_\mu P_\nu + \mu^2 c^2 = 0$$

- Hamilton-Jacobi equation-Geodesics

$$P_\mu = \frac{\partial S^{\text{eff}}}{\partial x^\mu}$$

$$S^{\text{eff}} = -Et + J\varphi + S_R(R, E, J)$$

$$\mathcal{H}^{\text{eff}} = \mathcal{H}^{\text{eff}}(R, P_R, J)$$

EOB: Mapping the two systems

Conservative PN 2-body dynamics can be described using a Hamiltonian

$$\mathcal{H} = \mathcal{H}_N + \frac{1}{c^2}\mathcal{H}_{1\mathrm{PN}} + \frac{1}{c^4}\mathcal{H}_{2\mathrm{PN}} + \dots$$

Encode this information somehow into our test-body motion in an effective spacetime:

$$M = m_1 + m_2, \quad \mu = m_1 m_2 / M$$

$$g^{\mathrm{eff}}_{\mu\nu} dx^\mu dx^\nu = -A(R)c^2 dT^2 + B(R) dR^2 + R^2 (d\theta^2 + \sin^2(\theta) d\varphi^2)$$

$$A(R) = 1 - 2\frac{GM}{c^2 r} + a_1(\nu)\left(\frac{GM}{c^2 r}\right)^2 + \dots$$

$$g^{\mu\nu}_{\mathrm{eff}} P_\mu P_\nu + \mu^2 c^2 = Q(R, P_\mu)$$

3PN onwards

[Damour Jaranowski Schaefer 2000]

EOB: Mapping the two systems

$$\mathcal{H}_{\rm eff} = f(\mathcal{H}_{\rm real}) = \mathcal{H}_{\rm real}(1 + \frac{1}{c^2}\alpha_1 \mathcal{H}_{\rm real} + \ldots)$$

effective coordinates PN coordinates

$$(R, P_R, J) \Leftrightarrow (r, p_r, p_\varphi)$$

—> Demand canonical transformation between coordinates

$$p_i = P_i + \frac{1}{c^2}\frac{\partial G(p,q)}{\partial q^i}$$

$$q_i = Q_i - \frac{1}{c^2}\frac{\partial G(p,q)}{\partial p_i}$$

Determines everything up to 4PN so far (modulo some freedom used to simplify!)

$$\alpha_{i\geq 2} = 0$$

By inverting the energy map, somehow..

$$\mathcal{H}_{\rm eob} = \mathcal{H}_{\rm real}(\mathcal{H}_{\rm eff}(R, P_R, J))$$

will be much simpler than

$$\mathcal{H}_{\rm real}(r, p_r, p_\varphi)$$

EOB: Mapping the two systems

$$\mathcal{H}_{\rm eob} = \mathcal{H}_{\rm real}(\mathcal{H}_{\rm eff}(R, P_R, J))$$

$$\mathcal{H}_{\rm eob} = Mc^2\sqrt{1 + 2\nu\left(\frac{\mathcal{H}_{\rm eff}}{\mu c^2} - 1\right)}$$

$$\frac{\mathcal{H}_{\rm eff}}{\mu c^2} = \sqrt{A(R)\left(1 + A(R)D(R)\frac{P_R^2}{\mu^2 c^2} + \frac{P_\varphi^2}{\mu^2 c^2 R^2} + \frac{Q(R)}{\mu}\right)}$$

$$D = (AB)^{-1}$$

$$A(u) = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41\pi^2}{32}\right)\nu u^4 + O(u^5)$$

$$D(u) = 1 + 6u^2\nu + (52\nu - 6\nu^2)u^3 + O(u^4)$$

$$Q(u) = q_0(u)P_R^4 + q_1(u)P_R^6 + \dots$$

$$q_0(u) = 2(4 - 3\nu)\nu u^2 + O(u^3)$$

$$u = \frac{GM}{c^2 R}$$

EOB:
Equations of motion

In standard coordinates the dynamics are determined by Hamiltons equations

$$\frac{dr}{dt} = \frac{\partial \mathcal{H}_{\rm real}}{\partial p_r}, \qquad \frac{dp_r}{dt} = -\frac{\partial \mathcal{H}_{\rm real}}{\partial r}$$

$$\frac{d\varphi}{dt} = \frac{\partial \mathcal{H}_{\rm real}}{\partial p_\varphi}, \qquad \frac{dp_\varphi}{dt} = 0$$

canonical transformation →

$$\frac{dR}{dt} = \frac{\partial \mathcal{H}_{\rm eob}}{\partial P_R}, \qquad \frac{dP_R}{dt} = -\frac{\partial \mathcal{H}_{\rm eob}}{\partial R}$$

$$\frac{d\Phi}{dt} = \frac{\partial \mathcal{H}_{\rm eob}}{\partial J}, \qquad \frac{dJ}{dt} = 0$$

Dissipation is included using PN

$$\frac{dR}{dt} = \frac{\partial \mathcal{H}_{\rm eob}}{\partial P_R}, \qquad \frac{dP_R}{dt} = -\frac{\partial \mathcal{H}_{\rm eob}}{\partial R} + \mathcal{F}_r$$

$$\frac{d\Phi}{dt} = \frac{\partial \mathcal{H}_{\rm eob}}{\partial J}, \qquad \frac{dJ}{dt} = \mathcal{F}_\varphi$$

circular orbits

$$\mathcal{F}_\varphi = \frac{1}{\dot{\Phi}}\frac{dE}{dt}$$

$$\mathcal{F}_\varphi = 0$$

EOB and gravitational self force

what can SF offer?

…this is capra

- small mass ratio, high accuracy, strong field

'Easy' to extract conservative information

$$f_\mu^{\mathrm{Cons}} = \frac{1}{2}(f_\mu^{\mathrm{Ret}} + f_\mu^{\mathrm{Adv}})$$

- frequency shifts, change in ISCO locations
- periastron advances
- redshift, spin precessions, tidal effects

EOB and GSF:
What do we hope to learn

$$A(R) = 1 - 2u + a_1(\nu)u^2 + \ldots$$
$$D(R) = 1 + d_1(\nu)u^2 + \ldots$$

as PN

$$A(R) = 1 - 2u + a_1^{\mathrm{GSF}}(u)\nu + a_2^{\mathrm{GSF}}(u)\nu^2 + \ldots$$
$$D(R) = 1 + d_1^{\mathrm{GSF}}(u)\nu^2 + \ldots$$

as GSF

e.g. from the PN series we know

$$a_1^{\mathrm{GSF}}(u) = 2u^3 + \left(\frac{94}{3} - \frac{41\pi^2}{32}\right)u^4 + \ldots$$

EOB and GSF:
How do you import the information

Idea: compare 'observables', use gauge invariance

[Damour 09, Barack, Damour, Sago 10]

just like comparing SF codes..

Lorenz		Regge-Wheeler	
$f_\mu(x)$	$\neq$	$f_\mu(x)$	
$z(x)$	$\neq$	$z(x)$	
$z(y)$	$=$	$z(y)$	$y = (m_1\Omega)^{2/3}$

EOB and GSF:
Initial constraints: ISCO shift & periastron advance

Damour initially suggested comparing the ISCO and the periastron precession

$$(M\Omega_{\rm ISCO})^{3/2} = \frac{1}{6}\left(1+\nu\left(a_1(1/6)+\frac{1}{6}a_1'(1/6)+\frac{1}{18}a_1''(1/6)\right)\nu+\ldots\right)$$
$$= \frac{1}{6}(1+.8342\nu+\ldots) \quad \text{[Barack, Sago 2009]}$$

Likewise, the periastron advance:

$$\frac{\Omega_r}{\Omega_\varphi} = 1-6y+\nu\rho(y)+\ldots$$

$$\rho(y) \sim a(y), a'(y), a''(y), d(y)$$

constrains a linear combination of the potentials

EOB and GSF:
First law and binding energy

LeTiec et al [Le Tiec et al 2012] derived '1st law for binary BH', relating Mass, AM and redshifts:

$$\delta M - \Omega \delta J = z_1 \delta m_1 + z_2 \delta m_2$$

SF:

In the extreme mass ratio limit, the 1st law relates the binding energy to the redshift invariant

$$E_{\rm SF}(y) = \frac{1}{2} z_{\rm SF}(y) - \frac{y}{3} z'_{\rm SF}(y) - 1 + \sqrt{1-3y} + \frac{y}{6} \frac{7-24y}{(1-3y)^{3/2}}$$

EOB: The Hamiltonian is the energy of the system..

$$E(x) \rightarrow A(x)$$

Expanding in the mass-ratio, equating the two:

$$a_{1\rm sf} = \sqrt{1-3x}\, z_{1\rm sf} - x \left(1 + \frac{1-4x}{\sqrt{1-3x}} \right)$$

[Barausse et al 2012]

EOB and GSF: First law and binding energy

Akcay et al 2012

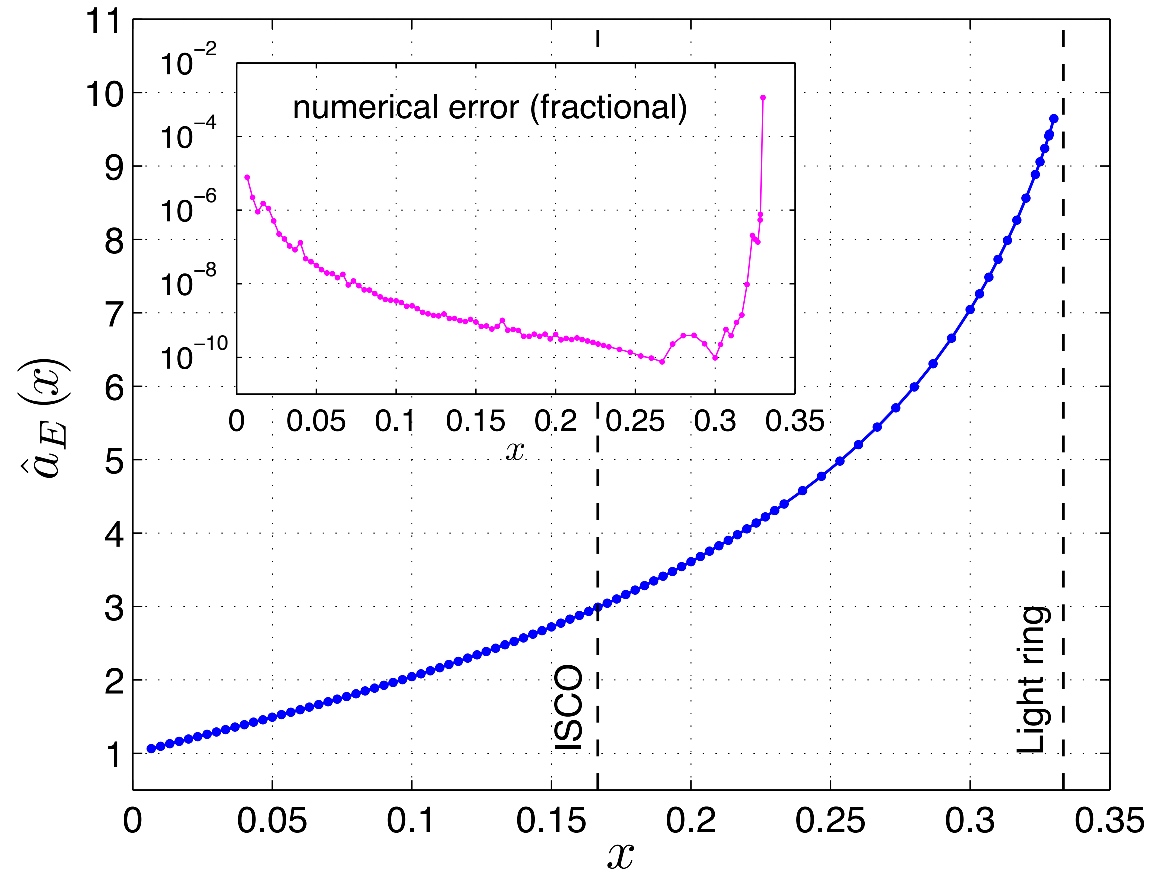


FIG. 3: Numerical data for the doubly-rescaled function $\hat{a}_E(x)$ [see Eq. (50)]. The solid line is a cubic interpolation of the numerical data points (beads). The inset shows, on a semi-logarithmic scale, the relative numerical error in the $\hat{a}_E$ data, computed based on the estimated errors tabulated in Appendix A. Note that the relative error is between 10^{-8} and 10^{-10} over most of the domain, and it never exceeds 10^{-5} (except at a single point, closest to the LR, where it is $\sim 0.1\%$).

1st law + periastron advance—-> d(y)!

EOB and GSF:
First law generalisations

Le Tiec 2015: First law for eccentric orbits

[Barack Sago-11']

Binding energy now in terms of the orbit averaged redshift $\langle z \rangle$

$$\frac{\mathcal{H}_{\text{eff}}}{\mu c^2} = \sqrt{A(R)\left(1 + A(R)D(R)\frac{P_R^2}{\mu^2 c^2} + \frac{P_\varphi^2}{\mu^2 c^2 R^2} + \frac{Q(R)}{\mu}\right)}$$

$$Q(u) = q_0(u)P_R^4 + q_1(u)P_R^6 + \dots$$

Doing a low eccentricity expansion..

$$\langle z \rangle = \langle z \rangle_0 + \langle z \rangle_1 e^2 + \langle z \rangle_2 e^4 \dots$$

$$\langle z \rangle_0 \to A$$

$$\langle z \rangle_0, \langle z \rangle_1 \to D$$

$$\langle z \rangle_0, \langle z \rangle_1, \langle z \rangle_2 \to q_0$$

etc.

—-Entire non-spinning Hamiltonian just from the redshift invariant

EOB and GSF:
Overview of results for non-spinning EOB-GSF

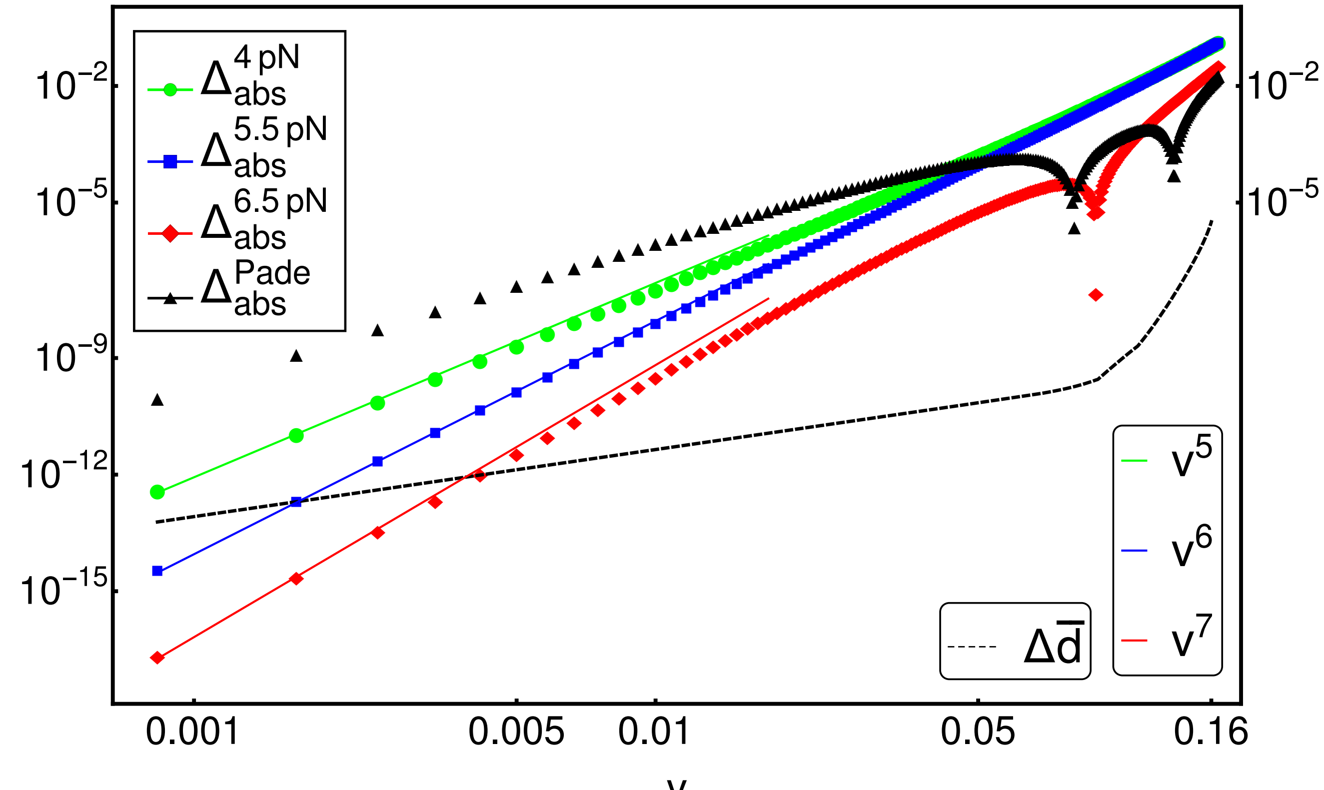
	$a^{1\mathrm{SF}}(u)$	$d^{1\mathrm{SF}}(u)$	e^2 $q_0^{1\mathrm{SF}}(u)$	e^4 $q_1^{1\mathrm{SF}}(u)$
PN	22.5	9.5	9.5	4
Numerics	[1]	[1,2]	[2]	–

PN-GSF work

Bini, Damour 2013, 2014, 2014, 2015, 2016
Kavanagh, Ottewill, Wardell 2015
Shah, Whiting, Johnson McDaniel 2015
Hopper, Kavanagh, Ottewill 2016

Numerical work

[1] Akcay, Barack, Damour, Sago 2012
[2] Akcay, van de Meent 2016



EOB and GSF: Spinning EOB & spin precessions

$$\mathcal{H}(R,P,S_1,S_2) = Mc^2\sqrt{1+2\nu\left(\frac{\mathcal{H}_{\rm eff}}{\mu c^2}-1\right)}$$

$$\mathcal{H}_{\rm eff} = \mathcal{H}_{\rm eff}^{\rm O} + \mathcal{H}_{\rm eff}^{\rm SO}$$

$$\mathcal{H}_{\rm eff}^{\rm SO} = \frac{G}{c^2 R^3}\left(g_{\rm S}\mathbf{L}\cdot\mathbf{S} + g_{\rm S^*}\mathbf{L}\cdot\mathbf{S}^*\right)$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

$$\mathbf{S}^* = \frac{m_2}{m_1}\mathbf{S}_1 + \frac{m_1}{m_2}\mathbf{S}_2$$

g_S : effects from the big BH

Bini, Damour 2016: use first law w/spinning binaries [Blanchet et al 2013]

Spin corrections to A, g_S

Both as high PN, and using strong field data of Shah et al (all circular)

g_{S*} : effects from the small BH

→ Use the GSF spin precession invariant : Dolan et al 2014

EOB and GSF: Spinning EOB & spin precessions

$$\frac{d\mathbf{S}_a}{dt} = \{\mathcal{H}, \mathbf{S}_a\}$$
$$= \Omega_{\mathbf{S}_a} \times \mathbf{S}_a$$

$$\Omega_{\mathbf{S}_a} = \frac{\partial \mathcal{H}}{\partial \mathbf{S}_a}$$

Set up same situation as SF spin precession calculation:

$$S_1 \ll 1, \quad S_2 = 0, \quad L \cdot S = P_\varphi s$$

[Bini, Damour]: $$\psi = \frac{\Omega_{S_1}}{\Omega_\varphi}$$

Extract SF info by equating

$\mathcal{O}(\nu)$ piece of ψ^{EOB} $\longleftrightarrow$ $\Delta\psi$

(via gauge inv. parameterisation.)

EOB and GSF: Spinning EOB & spin precessions, everything else

[Akcay, Dolan, Dempsey 2016]- Eccentric generalization of GSF spin precession (schw)

$$\langle \Delta\psi \rangle = \langle \Delta\psi \rangle^0 + \langle \Delta\psi \rangle^1 e^2 + \dots$$

$$g_{S*} = g_{S*}^0 + g_{S*}^1 P_R^2 + \dots$$

[Akcay 2017]- Eccentric spin prec in Kerr (formulation)

GSF tidal invariants → Tidal EOB

[Dolan et al 2015]

[Bini, Damour 2015]

EOB and GSF:
Lightring behaviour

See Andrea Antonelli next!

EOB and EMRI evolution: why?

- → meeting point of NR, PN and SF
- → conservative information is neatly packaged in gauge invariant manner
- → it's a different method

EOB and EMRI evolution:
Yunes et al 2010: Quasi-circular equatorial inspiral, Kerr

Include conservative PN information in the EOB hamiltonian

$$\frac{dR}{dt} = \frac{\partial \mathcal{H}_{\rm eob}}{\partial P_R}, \qquad \frac{dP_R}{dt} = -\frac{\partial \mathcal{H}_{\rm eob}}{\partial R}$$

$$\frac{d\Phi}{dt} = \frac{\partial \mathcal{H}_{\rm eob}}{\partial J}, \qquad \frac{dJ}{dt} = \mathcal{F}_\varphi$$

$\mathcal{H}_{\rm eob}$ —3PN conservative info via A,D

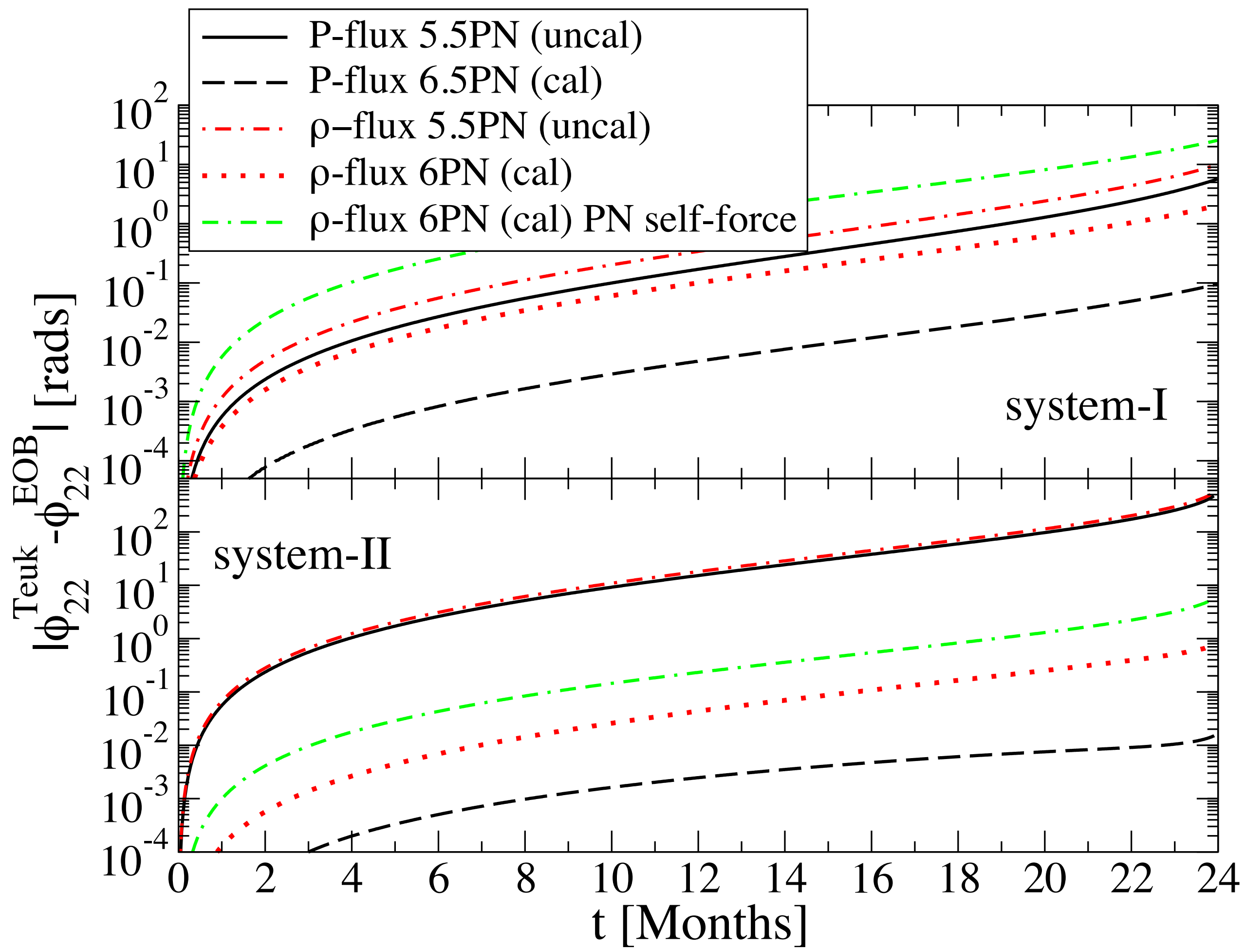
$\mathcal{F}_\varphi$ —Semi-analytic Teukolsky fluxes (PN/calibrated PN)

$$\mathcal{F}_\varphi = \frac{1}{\dot{\Phi}} \frac{dE}{dt}$$

Fit to high accuracy numerics

$$\frac{dE}{dt} = \left(\frac{dE}{dt}\right)^{n\rm PN} + (a_1 + a_2 log(u))u^{(n+1)\rm PN}$$

EOB and EMRI evolution:
Yunes et al 2010: Quasi-circular equatorial inspiral, Kerr



With conservative SF turned on in the EOB potentials, found ~6-27 rad phase difference/two year inspiral

EOB and EMRI evolution:
Up to date information

Schwarzschild

→ High eccentricities > .5 ?

	$a^{1SF}(u)$	$d^{1SF}(u)$	$q_0^{1SF}(u)$ (e^2)	$q_1^{1SF}(u)$ (e^4)
PN	22.5	9.5	9.5	4
Numerics	[1]	[1,2]	[2]	–

$$\frac{dR}{dt} = \frac{\partial \mathcal{H}_{\rm eob}}{\partial P_R}, \qquad \frac{dP_R}{dt} = -\frac{\partial \mathcal{H}_{\rm eob}}{\partial R}$$

$$\frac{d\Phi}{dt} = \frac{\partial \mathcal{H}_{\rm eob}}{\partial J}, \qquad \frac{dJ}{dt} = \mathcal{F}_\varphi$$

EOB and EMRI evolution:
Preliminary-eccentric inspiral

Schwarzschild → High eccentricities > .5 ?

	$a^{1\mathrm{SF}}(u)$	$d^{1\mathrm{SF}}(u)$	$q_0^{1\mathrm{SF}}(u)$ (e^2)	$q_1^{1\mathrm{SF}}(u)$ (e^4)
PN	22.5	9.5	9.5	4
Numerics	[1]	[1,2]	[2]	–

$$\frac{dR}{dt} = \frac{\partial \mathcal{H}_{\rm eob}}{\partial P_R}, \qquad \frac{dP_R}{dt} = -\frac{\partial \mathcal{H}_{\rm eob}}{\partial R} + \mathcal{F}_r$$

$$\frac{d\Phi}{dt} = \frac{\partial \mathcal{H}_{\rm eob}}{\partial J}, \qquad \frac{dJ}{dt} = \mathcal{F}_\varphi$$

These are realllly slow to evolve.. SEE N Warburton tomorrow!

EOB and EMRI evolution:
Preliminary-eccentric inspiral

Using action angle-type variables (similar to Hinderer & Barak 2017)

$$\frac{d\theta_r}{dt} = \omega_r \quad \frac{dp}{dt} = \nu \mathcal{F}_p$$
$$\frac{d\theta_\varphi}{dt} = \omega_\varphi \quad \frac{de}{dt} = \nu \mathcal{F}_e$$

$$r_{\text{min}} = \frac{p}{1+e}, r_{\text{max}} = \frac{p}{1-e}$$

To adiabatic order, i.e. ignoring oscillatory dissipative pieces, and second order SF

$$\mathcal{F}_p, \mathcal{F}_e \sim \langle \frac{dE}{dt} \rangle, \langle \frac{dJ}{dt} \rangle$$

using numerical SF data

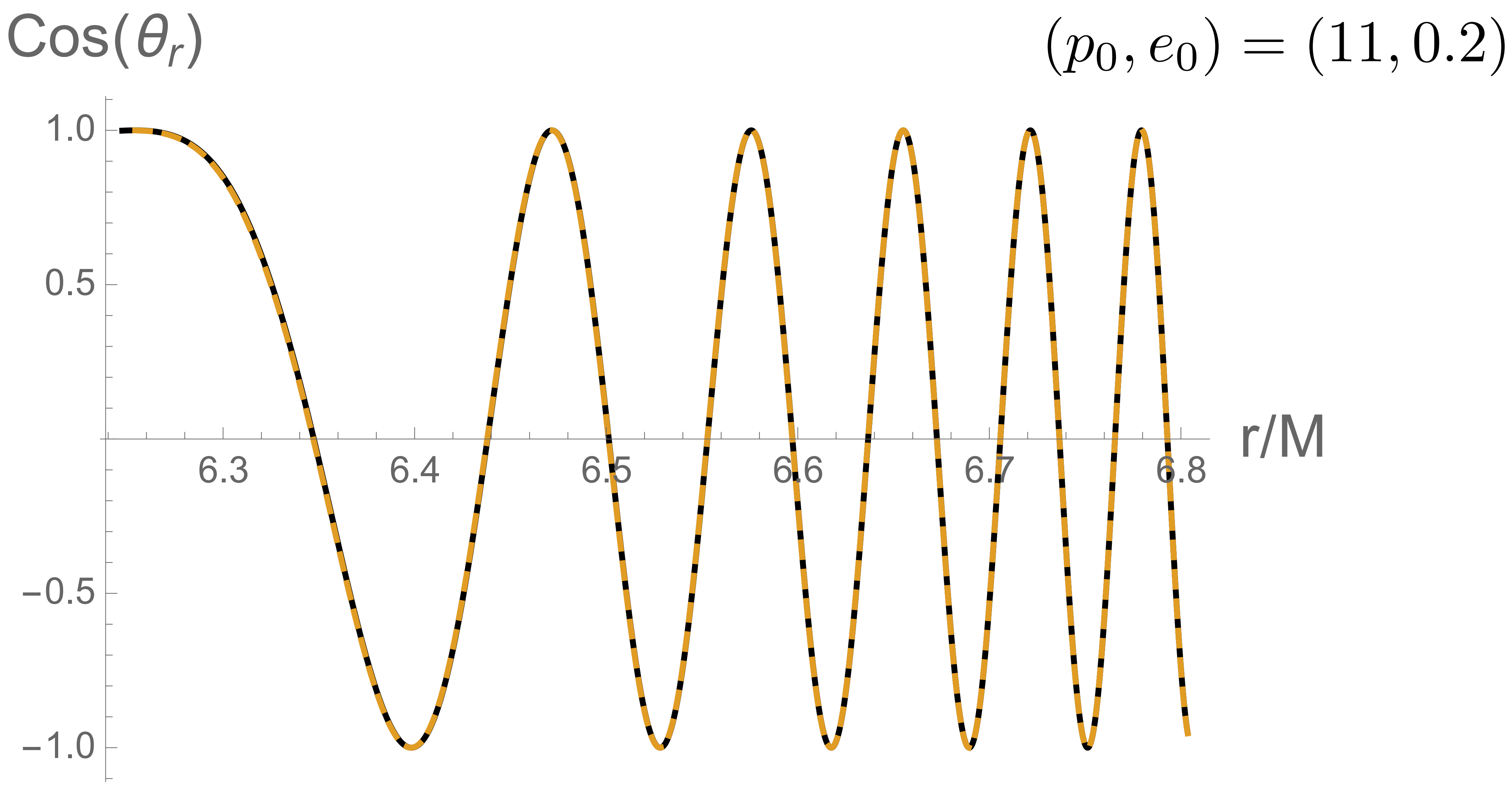
$$\omega_i = \omega_i^0 + \nu \omega_i^1 + O(\nu^2)$$

ω_i^0 → Schwarzschild orbital frequencies

EOB and EMRI evolution:
Preliminary-eccentric inspiral

Two plots— Using PN+strong field data for a,d, q potentials

Using purely high order PN



Conclusions/what's next

- ★ No eccentric effects yet transcribed in Kerr
- ★ To date, all work assumed aligned spins → Equatorial orbits

  Generic orbit informations is becoming available [See van de Meent]
- ★ Current formulations of EOB are divergent at the light-ring (see next talk?)
- ★ Deeper understanding needed of inclusion of radiation reaction

obvious questions:

How does conservative PN SF fare in a 'proper' self-force inspiral?

Can the SF equations of motion be formulated with conservative information only entering in a gauge invariant manner?