First order Self-force on Kerr generic orbits

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Outline

1. Generic orbit self-force in 4 easy steps

2. Overview of Results

Generic orbit self-force in 4 “easy” steps

Step 1; Solve Teukolsky equation

Separation of variables; frequency domain

$$\psi_4 = \rho^{-4} \sum_{lm\omega} Z_{lm\omega\, s} R_{lm\omega}(r)_s S_{lm\omega}(\theta) e^{i(m\phi - \omega t)}$$

MST solutions

$$_s R_{lm\omega}(r) = \mathcal{C} \sum_{n=-\infty}^{\infty} a_n^\nu \, F_n^\nu(r)$$

$F_n^\nu(r)$: Hypergeometric function, a_n^ν coefficients, n "renormalized angular momentum"

Variation of Parameters + extended homogeneous solutions

$$R_{lm\omega} = \begin{cases} Z^+_{lm\omega} R^+_{lm\omega} & r > r_0 \\ Z^-_{lm\omega} R^-_{lm\omega} & r < r_0 \end{cases},$$

$$Z^\pm_{lm\omega} = \int_{r_{\min}}^{r_{\max}} \frac{R^\mp_{lm\omega}(r) T_{lm\omega}(r)}{W[R^+, R^-]}$$

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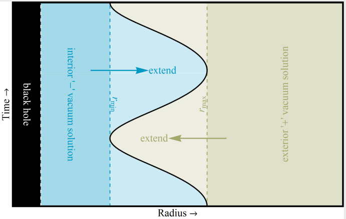

Step 2: Metric Reconstruction

A tale of two maps [Chrzanowksi, Cohen, Kegeles, Wald, 1970s]

$$\hat{\psi}_4 : h_{\mu\nu} \mapsto \psi_4$$

$\mathcal{S}^\dagger_{\mathrm{CCK}}$: (vacuum solutions of spin-2 teuk. eq.) $\to$ (vacuum metric perturbations)

"Radial" inversion [Ori, 2001][MvdM, 2014]

$\hat{\psi}_4 \circ \mathcal{S}^\dagger_{\mathrm{CCK}}$ is not the identity map. However, it is diagonal on a basis of frequency domain in/out (up/down) going radial modes. Ergo, easy to invert mode-by-mode.

Metric reconstruction

$\mathcal{S}^\dagger_{\mathrm{CCK}} \circ (\hat{\psi}_4 \circ \mathcal{S}^\dagger_{\mathrm{CCK}})^{-1}$ is a right inverse for $\hat{\psi}_4$ restricted to vacuum solutions.

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Step 3: Completion

[Wald 1968]

$$\ker \hat{\psi}_4 = (\delta M \text{ and } \delta J \text{ perturbations} + \text{gauge modes})$$

δM and δJ perturbations [Merlin et al., 2016][MvdM, 2017]

- Abbott-Deser integrals measure mass/angular momentum inside closed surfaces.
- $\mathrm{im}(\mathcal{S}^{\dagger}_{\mathrm{CCK}})$ does not contribute to Abbott-Deser integrals [MvdM, 2017].

Gauge modes

- Care only about 2d family of linearly growing gauge vectors that produce bounded metric perturbations. (but not today)
- Can be fixed by a generalization of the methods of [Merlin et al., 2016].
- Not done (yet) for generic orbits.

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Mode sum [Pound,Merlin&Barack, 2013]

$$F^{\mu} = \left(\sum_{\ell=0}^{\infty} \frac{F^{\mu,+}_{\ell,\mathrm{Rad}} + F^{\mu,-}_{\ell,\mathrm{Rad}}}{2} - B^{\mu}_{\mathrm{Lor}} - \frac{C^{\mu}_{\mathrm{Lor}}}{L} \right) - D^{\mu}_{\mathrm{Lor}}.$$

Re-expansion to spherical harmonics

$$\mathcal{F}^{\mu,\pm}_{\mathrm{Rad}} = \sum_{\substack{m\omega\mathfrak{l}\\ ij}} \mathcal{C}^{\mu,\pm}_{\mathfrak{l}m\omega ij}(r, z)\, {}_2R^{\pm,(\mathrm{i})}_{\mathfrak{l}m\omega}(r)\, {}_2S^{(\mathrm{j})}_{\mathfrak{l}m\omega}(z) e^{im\phi - i\omega t} + c.c.$$

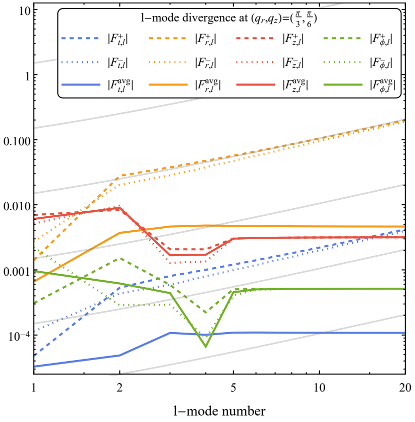
becomes

$$= \sum_{\substack{m\omega ijk\\ l_1 l_2 l_3 \mathfrak{l}}} \left(\tilde{\mathcal{C}}^{\mu,\pm}_{\mathfrak{l}m\omega ijk}\, {}_2R^{\pm,(\mathrm{i})}_{\mathfrak{l}m\omega} ({}_2b_{m\omega})^{\mathfrak{l}}_{l_1}\, {}^{m}_{j}\mathcal{A}^{l_1}_{l_2}\, {}^{k}_{m}\mathcal{B}^{l_2}_{\ell}\, e^{im\phi - i\omega t} + c.c. \right) Y_{\ell m}(z)$$

Overview of Results

l-mode convergence test

$a = 0.9M$, $p = 10$, $e = 0.1$, and $z_{\max} = \cos\theta_{\min} = 0.1$



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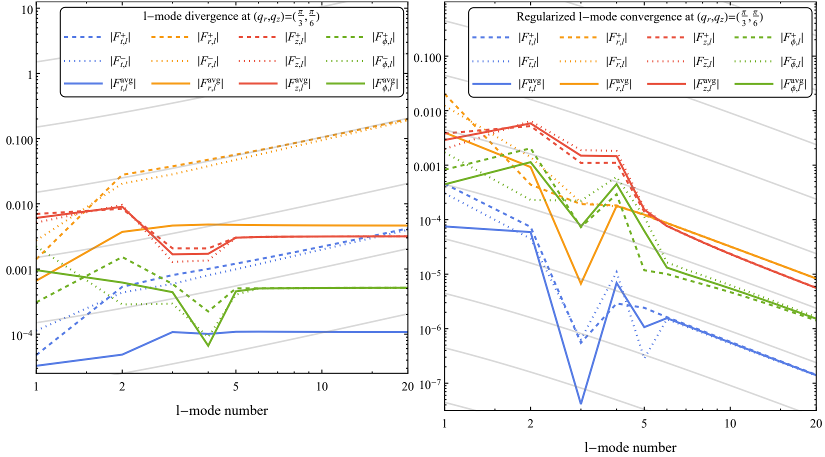
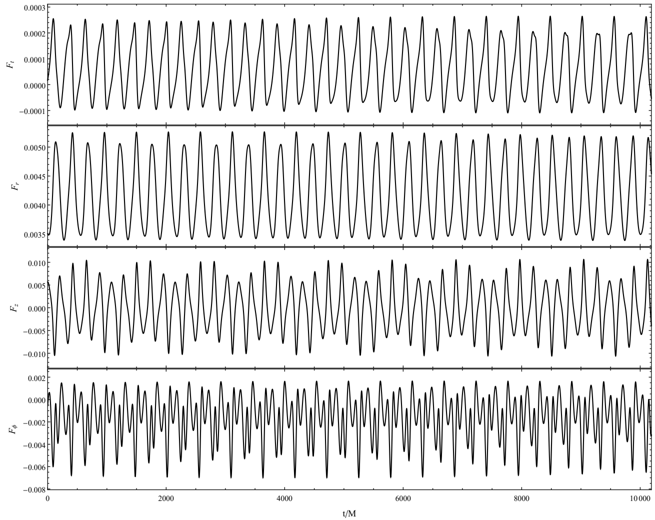
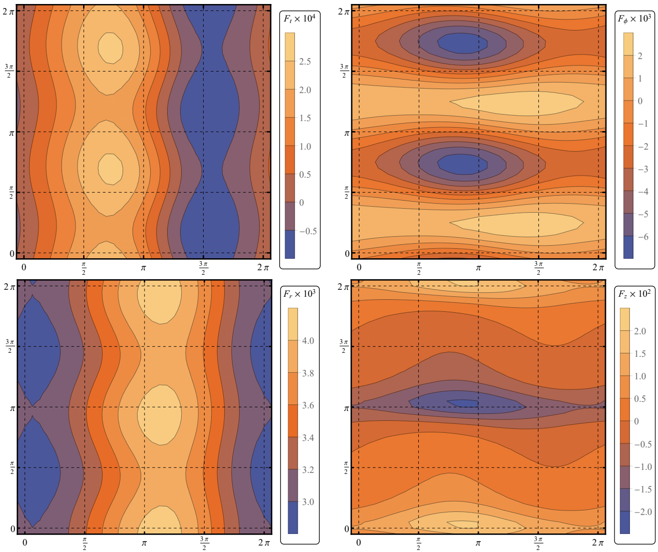




$a = 0.9M$,
$p = 10$,
$e = 0.1$, and
$z_{\max} = \cos\theta_{\min} = 0.5$

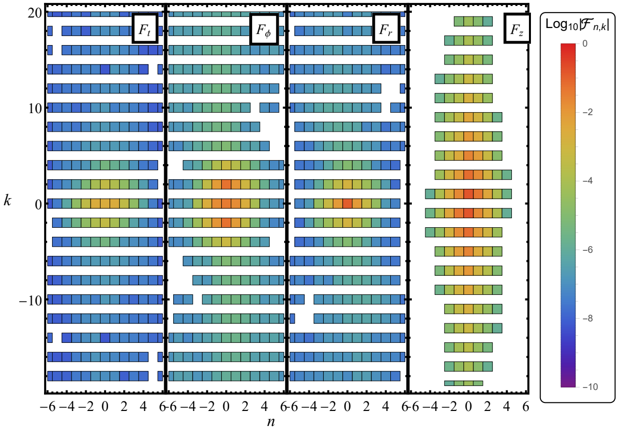
- F_μ is biperiodic
- not very informative



$a = 0.9M$,
$p = 10$,
$e = 0.1$, and
$z_{\max} = 0.9$

- q_r vs q_z
- angles conjugate to radial and polar action
- F_t, F_r, and F_ϕ are π periodic in q_z

$$F(q_r, q_z) = \sum_{nk} \mathcal{F}_{nk} e^{inq_r + ikq_z}$$



$a = 0.9M$, $p = 10$, $e = 0.1$, and $z_{\max} = \cos\theta_{\min} = 0.9$

- Half of the modes vanish due to up/down symmetry.

The good, the bad, and the ugly

The good ...

- We can calculate the self-force on generic orbits in Kerr spacetime.
- High inclinations pose little problems.

The bad ...

- Slow ($\sim 10^4$ CPU hours/geodesic)
- Memory intensive.

The ugly ...

- Re-expansion to spherical harmonic modes is unpleasant.

Summary and Outlook

Summary

Calculation of the GSF corrections to generic orbits is a reality.

To do...

- spherical orbits
- explore resonances
- invariants (redshift, spin precession, ISSO shift)
- fill parameter space

Thank you for listening!

Acknowledgments

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