Signatures of Extra Dimensions in Gravitational Waves from Black Hole Quasi-Normal Modes

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Outline

1. Introduction to Background Spacetime.
2. Perturbation Equation in presence of Extra Dimensions.
3. Possible signatures of extra dimensions in the Quasi-Normal Modes.

References

- **SC**, K. Chakravarti, S. Bose and S. SenGupta, PRD 97, 104053 (2018) [arXiv:1710.05188].
- S.S. Seahra, C. Clarkson and R. Maartens, PRL 94, 121302 (2005).

Why Extra Dimensions?

- The basic motivation for existence of extra dimensions is the renormalization of Higgs mass.
- The counter-term needed for mass renormalization corresponds to,

Mass Renormalization

$$\delta m_{\mathrm{H}}^2 = \frac{\Lambda^2}{8\pi^2}\left(\lambda_{\mathrm{H}} - \lambda_{\mathrm{F}}^2\right) + \mathrm{log.\ div.} + \mathrm{finite\ terms}$$

- Since the cutoff scale Λ is in the Planck regime, we must have a fine tuning of the couplings to get renormalized Higgs mass at the Electro-weak scale.
- Extra dimension is one particular method, which was invoked to solve the above issue.

The background spacetime

- The five dimensional gravitational field equations read

Field Equations

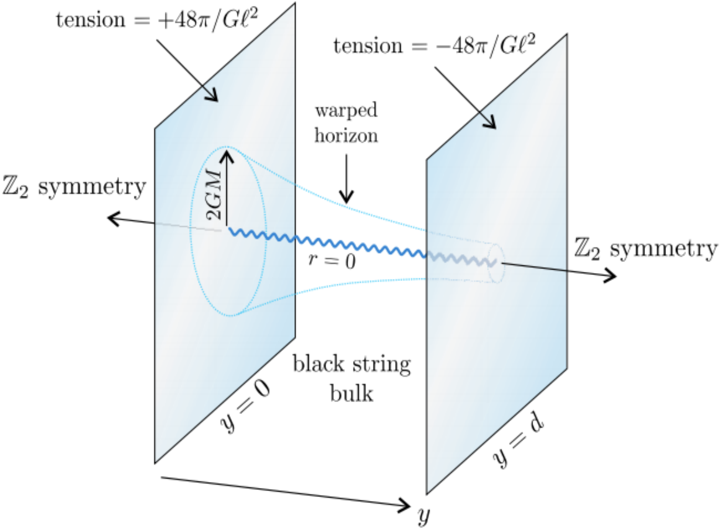
$$G_{AB} = 8\pi G_{(5)} T_{AB}$$

- When the bulk energy momentum tensor is originating from a negative cosmological constant Λ, one arrives at the following static and spherically symmetric solution on the brane,

Background Metric

$$ds^2_{\mathrm{unperturbed}} = e^{-2ky}\left(-f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2\right) + dy^2$$

Pictorial Visualization



Effective Field Equations

T. Shiromizu, K. Maeda and M. Sasaki, PRD 62, 024012 (2000).

R. Maartens and K. Koyama, Liv. Rev. Rel. 13, 5 (2010)

- The normal $n_A = \nabla_A y$, yields the induced metric on the brane hypersurface to be $h_{AB} = g_{AB} - n_A n_B$, such that $n_A h^A_B = 0$.

Effective Field Equations

$$^{(4)}G_{\mu\nu} + E_{\mu\nu} = 0$$

- Here $E_{\mu\nu}$ stands for a particular projection of the bulk Weyl tensor C_{ABCD} on the brane hypersurface

Weyl Stress

$$E_{\mu\nu} = C_{ABCD} e^A_\mu n^B e^C_\nu n^D$$

Perturbation to first order

- Perturbation of the effective field equations around the bulk metric g_{AB}, such that $g_{AB} \rightarrow g_{AB} + h_{AB}$.
- There are redundant gauge degrees of freedom. The following gauge conditions (known as the Randall-Sundrum gauge)

Gauge Condition

$$\nabla_A h^A_B = 0; \qquad h^A_A = 0; \qquad h_{AB} = h_{\alpha\beta} e^\alpha_A e^\beta_B$$

- The perturbed bulk metric takes the following form,

Perturbed Bulk Metric

$$ds^2_{\rm perturbed} = \Big[q_{\alpha\beta}(y, x^\mu) + h_{\alpha\beta}(y, x^\mu)\Big] dx^\alpha dx^\beta + dy^2$$

The Imprints of Extra Dimensions

- The imprints of the presence of extra dimensions are through two quantities — (a) Size of the extra dimension d and (b) the bulk curvature scale $\ell = 1/k$.
- The dimensionless ratio d/ℓ is an important one and if one wishes to solve the hierarchy problem we must have $d/\ell \geq 12$.
- The above model can also be written as a Brans-Dicke theory, with the Brans-Dicke parameter $\omega_{\rm bd}(d/\ell)$.
- Thus to be consistent with local physics we must have $d/\ell \geq 5$.
- Finally, the black hole mass and the bulk curvature scale has to satisfy some constraint to avoid the Gregory-Laflamme instability.

The Evolution of Perturbations

- Assuming a separable perturbation $h_{\alpha\beta}(y, x^\mu) = h_{\alpha\beta}(x^\mu)\chi(y)$, the perturbed effective equations can be decomposed into two parts:

Separability

$$e^{-2ky}\left\{-k^2\chi + 3k\partial_y\chi + \partial_y^2\chi\right\} = -\mathcal{M}^2\chi(y)$$
$$^{(4)}\Box h_{\mu\nu} + 2h_{\alpha\beta}\ ^{(4)}R^{\alpha\ \beta}_{\ \mu\ \nu} - \mathcal{M}^2 h_{\mu\nu} = 0$$

- With $\mathcal{M} = 0$, one immediately recovers the dynamical equation governing gravitational perturbation in general relativity.

The Kaluza-Klein Mass Modes

- The equation for $\chi(y)$ is essentially Bessel's differential equation and hence it's two independent solutions are

Solutions

$$\chi(y) = e^{-\frac{3}{2}ky}\left[C_1 J_\nu\left(\frac{me^{ky}}{k}\right) + C_2 Y_\nu\left(\frac{me^{ky}}{k}\right)\right]$$

- The boundary conditions imposed are derivatives of $\chi = 0$ at $y = 0$ and also on $y = d$. This leads to the following algebraic equation

KK Modes

$$Y_{\nu-1}(m_n/k)J_{\nu-1}(z_n) - J_{\nu-1}(m_n/k)Y_{\nu-1}(z_n) = 0$$

- Here $m_n = \{z_n k\}e^{-kd}$ are Kaluza-Klein mode masses.

The axial Perturbation equations on the brane

- In this case there are two master variables, $u_{n,l}$ and $v_{n,l}$ respectively and their evolution equations read

axial perturbation

$$\mathcal{D}u_{n,l} + f(r)\left\{m_n^2 + \frac{l(l+1)}{r^2} - \frac{6}{r^3}\right\}u_{n,l} + f(r)\frac{m_n^2}{r^3}v_{n,l} = 0$$

$$\mathcal{D}v_{n,l} + f(r)\left\{m_n^2 + \frac{l(l+1)}{r^2}\right\}v_{n,l} + 4f(r)u_{n,l} = 0$$

- Here, $\mathcal{D}$ is the differential operator $\partial_t^2 - \partial_{r_*}^2$, where r_* is the tortoise coordinate defined using $f(r)$ as $dr_* = dr/f(r)$.

Quasi-Normal Modes

Table: Imaginary parts of the quasi-normal mode frequencies have been presented for $d/\ell = 20; 1/\ell = 6 \times 10^7$.

$m = 0.44, l = 2$	$m = 0.83, l = 2$
Imaginary	Imaginary
-0.051	-0.038
-0.071	-0.104
-0.197	-0.168
-0.239	-0.369

Quasi-Normal Modes for General Relativity

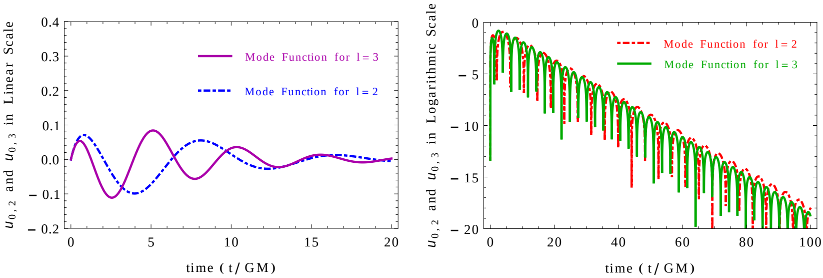


Figure: Time evolution of the master mode function $u_{n,l}(t)$ associated with axial gravitational perturbation for two different values of angular momentum l in the context of general relativity have been depicted.

Quasi-Normal Modes — I

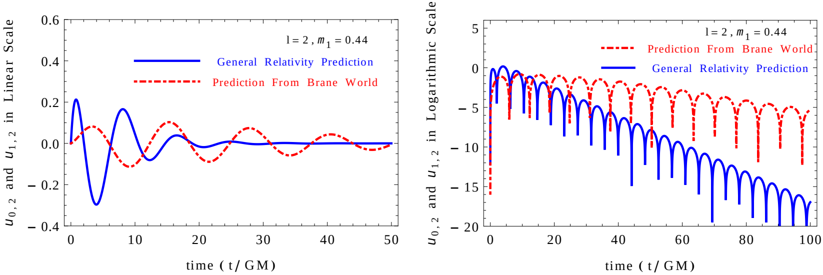


Figure: Time evolution of the master mode function $u_{n,l}(t)$ for general relativity ($n = 0$) as well as with the lowest lying Kaluza-Klein mode mass $m_1 = 0.44$ and $l = 2$.

Quasi-Normal Modes — II

V. Cardoso, E. Franzin and P. Pani, PRL 116, 171101 (2016)

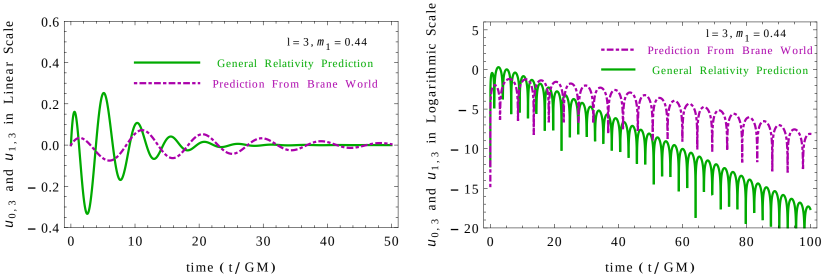
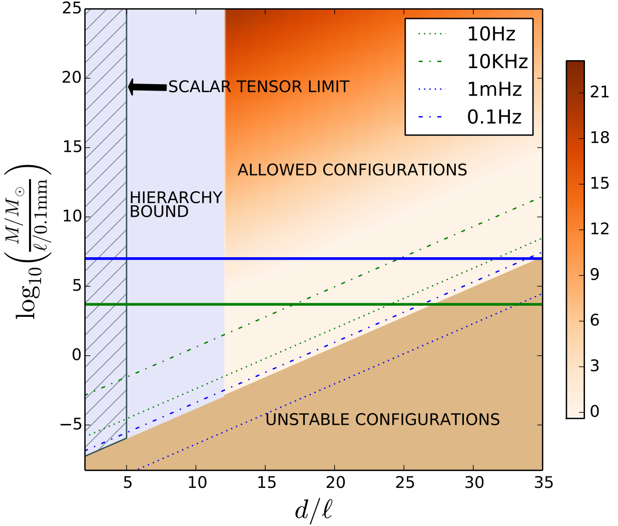


Figure: Time evolution of the master mode function $u_{n,l}(t)$ for general relativity ($n = 0$) as well as with the lowest lying Kaluza-Klein mode mass $m_1 = 0.44$ and l=3.

The Late-Time Behaviour



Summary

- We have discussed how the presence of extra dimensions will modify the black hole perturbation equations.
- Possible modifications of the black hole quasi-normal modes and distinct features.
- Late time behaviour of the black hole perturbations.

Thank You

The Late-Time Behaviour

- At late times the frequencies can be written in an analytical manner, such that,

Late Time Behaviour

$$f_n = z_n e^{27-(d/\ell)}(0.1\mathrm{mm}/\ell)\mathrm{Hz}$$